**Vassilis ZAMARIAS**

**NTUA**

# Regularized Codimension-2 Brane Cosmology

***In collaboration with***

***E. PAPANTONOPOULOS***

***and A. PAPAZOGLOU***

**hep-th/0611311**

**and 0707.1396 [hep-th]**

**1st ANNUAL SCHOOL OF THE EU NETWORK "UNIVERSENET"**

**MYTILINI**

28 Sept. 2007

# OUTLINE

I. Setup and Static Solution of the 6D warped flux compactified model.
II. Cosmology of the 4-Brane.
III. Conclusions.

# I. SETUP AND STATIC SOLUTION

M. Peloso, L. Sorbo, G. Tasinato (2006)
E. Papantonopoulos, A. Papazoglou, VZ (2006)

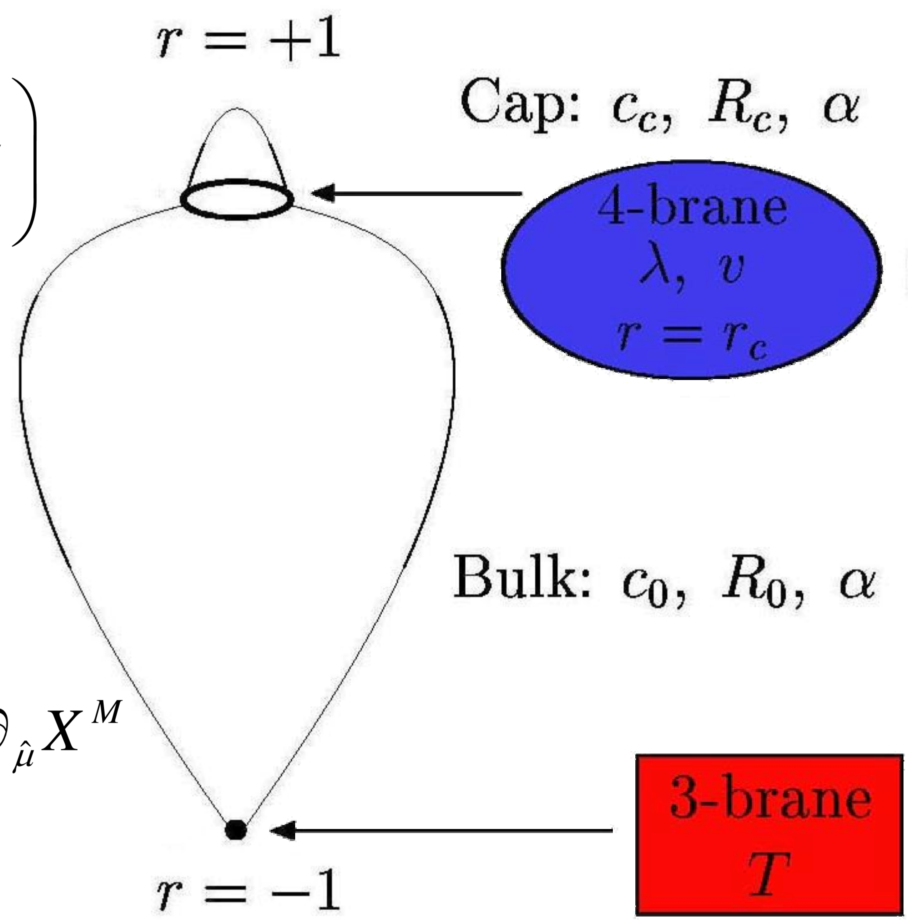
- Action:

$$S=\int d^6x\sqrt{-g}\left(\frac{M^4}{2}R-\Lambda_i-\frac{1}{4}F^2\right)$$

$$-\int d^5x\sqrt{-\gamma_+}\left(\lambda+\frac{v^2}{2}\left(\tilde{D}_{\hat{\mu}}\sigma\right)^2\right)$$

$$-\int d^4x\sqrt{-\gamma_-}T$$

with $\tilde{D}_{\hat{\mu}}\sigma=\partial_{\hat{\mu}}\sigma-ea_{\hat{\mu}},\quad a_{\hat{\mu}}=A_M\partial_{\hat{\mu}}X^M$



- Metric Solution:

$$ds_6^2 = z(r,\alpha)^2 \eta_{\mu\nu} dx^\mu dx^\nu + R_i^2 \left[ \frac{dr^2}{f(r,\alpha)} + c_i^2 f(r,\alpha) d\varphi^2 \right],$$

$$F_{r\varphi} = -c_i R_i M^2 S(\alpha) \frac{1}{z(r,\alpha)^4}$$

with $z(r,\alpha) = \frac{1}{2}\left[(1-\alpha)r + (1+\alpha)\right],$

$$f(r,\alpha) = \frac{1}{5(1-\alpha)^2}\left[ -z(r)^2 + \frac{1-\alpha^8}{1-\alpha^3}\frac{1}{z(r)^3} - \alpha^3 \frac{1-\alpha^5}{1-\alpha^3}\frac{1}{z(r)^6} \right],$$

$$S(\alpha) = \sqrt{\frac{3}{5}\alpha^3 \frac{1-\alpha^5}{1-\alpha^3}}.$$

- ✓ The cap is smooth at $r = +1$ as long as

$$c_c = \frac{1}{X_+} \quad \text{with} \quad X_+ = \frac{20(1-\alpha)(1-\alpha^3)}{5+3\alpha^8-8\alpha^3}$$

- ✓ The conical singularity at $r = -1$ is supported by a codim-2 brane with tension

$$T = 2\pi M^4 (1 - c_o X_-) \quad \text{with} \quad X_- = \frac{3+5\alpha^8-8\alpha^5}{20\alpha^4(1-\alpha)(1-\alpha^3)}$$

- ✓ Restriction condition from quantum numbers

$$n = \frac{N}{2}\frac{2}{(1-\alpha^3)}\left[\frac{5(1-\alpha^3)}{8(1-\alpha^5)} - \alpha^3\right] \qquad \Rightarrow \alpha \text{ takes discrete values}$$

# II. COSMOLOGY OF THE 4-BRANE

**E. Papantonopoulos, A. Papazoglou, VZ (2007)**

- In general: inclusion of matter contribution $\Rightarrow$ evolution of ***both*** brane and bulk. But as a first step we want to avoid time dependent bulk solution,

$\Rightarrow$ Approach: Bulk remains static while the brane matter merely makes the brane to move between the static bulk and the static cup with position $R(t)$. Here the brane is not merely a probe brane and we will use Junction Conditions (we include back-reaction of the brane energy density).

- Brane coordinates: $\sigma^{\hat{\mu}} = (t, x^i, \varphi).$
- Brane embedding $X^M$: $X^i = x^i,\ X^r = R(t)$ and $X^\varphi = \varphi,$

Outer bulk section: $X^0_{(out)} = t,$

Inner cap section: $X^0_{(in)} = T(t).$

- ✓ We put *matter* on the brane (perfect fluid), 6 parameters: $\rho, P, \hat{P}, l, L$ and $\hat{L}$.

⇒ The brane *MOVES* in the *static* bulk.

⇒ This movement *induces cosmology* on the brane with the need of *warping* in the bulk.

- ✓ Induced metric on the brane

$$ds^2_{(5)} = -d\tau^2 + a^2(\tau)d\vec{x}^2 + b^2(\tau)\,d\varphi^2.$$

- ✓ Relations derived from the continuity of the induced metric.

$$c_0 R_0 = c_c R_c, \qquad \dot{T}^2\left(1-\beta_+^2\frac{\dot{R}^2}{\dot{T}^2}\frac{R_0^2}{fz^2}\right)=\left(1-\dot{R}^2\frac{R_0^2}{fz^2}\right)$$

➢ Hubble parameters for the two scale factors:

$$H_a \equiv \frac{1}{a}\frac{da}{d\tau} = \frac{z'}{z}\frac{\dot{R}}{\sqrt{1-\dot{R}^2\frac{R_0^2}{fz^2}}}, \qquad H_b \equiv \frac{1}{b}\frac{db}{d\tau} = \frac{f'}{2fz}\frac{\dot{R}}{\sqrt{1-\dot{R}^2\frac{R_0^2}{fz^2}}}$$

⇒ The two Hubble rates are related: $\boxed{H_b = \frac{zf'}{2fz'}H_a}$

Close to the would-be conical singularity we have: $f' < 0$

$$\text{If} \quad H_a > 0 \Rightarrow H_b < 0$$

(If 4D space expands then internal space shrinks)

- Junction conditions:

$$\left\{\hat{K}_{\hat{\mu}\hat{\nu}}\right\} = -\frac{1}{M^4} t^{(br)}_{\hat{\mu}\hat{\nu}}, \qquad \text{where } \left\{H\right\} = H^{in} + H^{out}$$

$$\left\{n_M F^M_N \partial_{\hat{\kappa}} X^N\right\} = -\frac{\delta S_{br}}{\delta a^{\hat{\kappa}}}.$$

- RESULTS:

- ✓ 2 Junction conditions for the gauge field (the ($\tau$) and ($i$) components) give for the coupling: $l = L = 0$,
- ✓ The (φ) component of the gauge field junction and the (φφ) component of the metric junction determine $\hat{P}, \hat{L}$,
- ✓ The 2 remaining metric junction conditions give the Friedmann equation and the acceleration equation.

- The Friedmann Equation:

$$H_a^2 = C_1(a)\rho^2 + \frac{C_2(a)}{\rho^2} + C_3(a),$$

where $C_i\,(a)$ are expressed as known functions of f and z.

- Expansion around the static case $\rho_m^{(4)} << \rho_0$ :

where the effective 4D $\rho_m^{(4)} = \int d\varphi \sqrt{g_{\varphi\varphi}}\,\rho_m = \frac{2\pi\beta_+}{X_+} R_0 \sqrt{f}\,\rho_m$

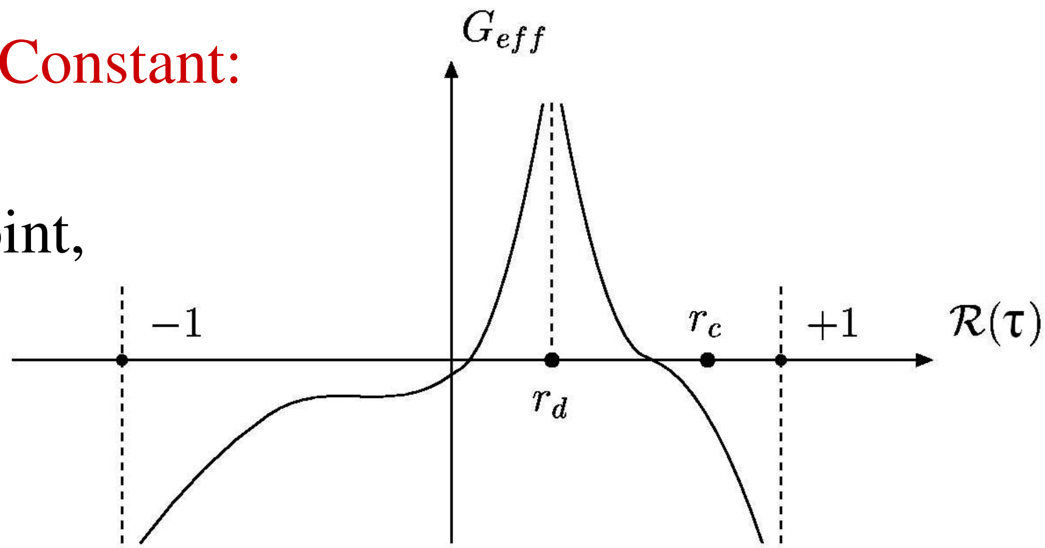
$$H_a^2 = \frac{8\pi}{3} G_{eff}(a)\rho_m^{(4)} + \Lambda_{eff}(a) + O(\rho_m^{(4)2})$$

$\Lambda_{eff}$ : Mirage matter contribution

- The Effective Newton's Constant:

$r_c$ : Static Equilibrium Point,

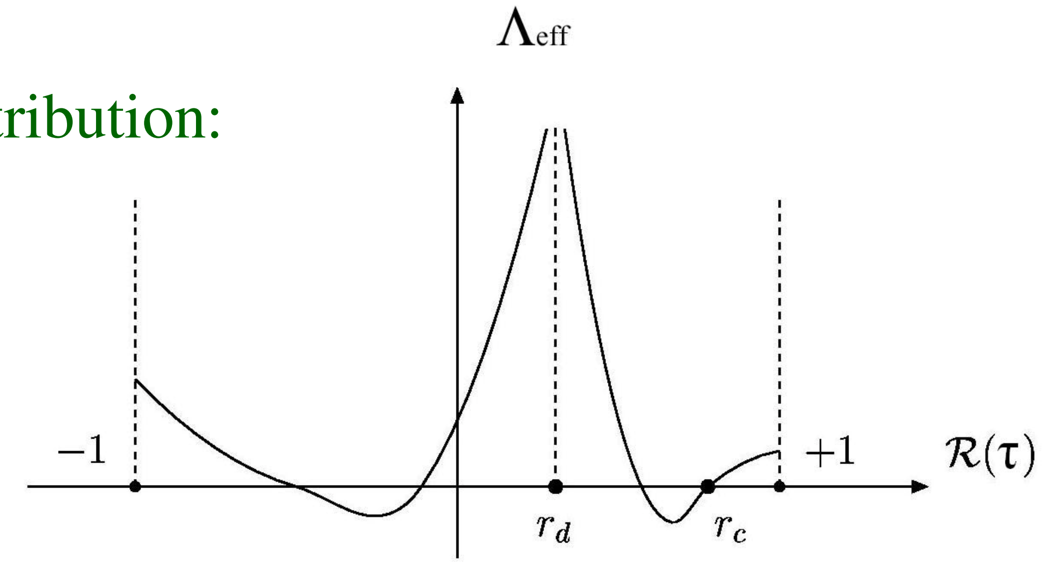
$$r_d : \quad z_d = \left( \frac{3(1-\alpha^8)}{8(1-\alpha^3)} \right)^{1/5} .$$




- The Mirage Matter Contribution:

At $r = r_c$ : $\Lambda_{eff} = 0$

but $G_{eff} < 0$ !!!



- Early Times Expansion of the Friedmann Equation: $\rho_m^{(4)} >> \rho_0$ :

$$H_a^2 = C_4(a)\rho_m^2, \qquad \text{5D Freidmann Law}$$

- We can also derive the energy continuity equation:

$$\frac{d\rho_{tot}}{d\tau} + 3(\rho + P)H_a + (\rho + \hat{P})\frac{zf\,'}{2fz\,'}H_a = -W(a, H_a)$$

Significant Energy Flow
from brane to Bulk !!!

# III. CONCLUSIONS

- First step for getting cosmology in a regularized codimension-2 brane.

- We use the simplest scenario of a static bulk and a moving brane $\Rightarrow$ unsatisfactory cosmology!

- D. Langlois & M. Minamitsuji [0707.1426 hep-th]: Time-dependent Scalar Field.

- Need of ***time-dependent bulk solution***. (work in progress…)

  K. i. Maeda & H. Nishino, Phys. Lett. B 154 (1985) 358

  C. P. Burgess & al. [hep-th/0608083]

  T. Kobayashi & M. Minamitsuji [0705.3500 hep-th]

  E. J. Copeland & O. Seto [0705.4169 hep-th]

# IV. EXTRA SLIDES

- Brane matter (Energy Momentum Tensor):

$$t_{\hat{\mu}}^{\hat{\nu}(br)} = -\frac{2}{\sqrt{-\gamma_+}} \frac{\delta S_{br}}{\delta \gamma_{+\nu}^{\mu}} = \text{diag}\left(-\rho, P, P, P, \hat{P}\right)$$

with

$$\rho = \rho_0 + \rho_m = \lambda + \frac{v^2\left(n - eA_\varphi^+\right)^2}{2c_0^2 R_0^2 f(r_c)} + \rho_m$$

$$P = P_0 + P_m = -\rho_0 + P_m$$

$$\hat{P} = \hat{P}_0 + \hat{P}_m = -\lambda + \frac{v^2\left(n - eA_\varphi^+\right)^2}{2c_0^2 R_0^2 f(r_c)} + \hat{P}_m$$

- Coupling of the bulk gauge field to the brane matter :

$$\frac{\delta S_{br}}{\delta a^{\hat{\kappa}}} = \left(l, L, L, L, \hat{L}\right)$$

with

$$l = l_m$$

$$L = L_m$$

$$\hat{L} = \hat{L}_0 + \hat{L}_m = ev^2\left(n - eA_\varphi^+\right) + \hat{L}_m$$

- Induced metric on the brane:

$$ds^2_{(5)} = -z^2\left(1-\dot{R}^2\frac{R_0^2}{fz^2}\right)dt^2 + z^2 d\vec{x}^2 + c_0^2 R_0^2 f\, d\varphi^2 .$$

Brane Proper Time

$$\dot{\tau}^2 = z^2(1-\dot{R}^2\frac{R_0^2}{fz^2})$$

$$a = z(R(\tau))$$

$$b = c_0 R_0\sqrt{f(R(\tau))}$$

$$\Downarrow$$

$$ds^2_{(5)} = -d\tau^2 + a^2(\tau)d\vec{x}^2 + b^2(\tau)\,d\varphi^2 .$$

***Need of warping***