



# The Need for Dark Matter in MOND on Galactic Scales

Muhammad Furqaan Yusaf

Kings College London,  
Strand Campus,  
Department of Physics.

`muhammad.yusaf@kcl.ac.uk`

September 28, 2007

As presented by I. Ferreras, M. Sakelleriadou, M. F. Yusaf, [arXiv:0709.3189]

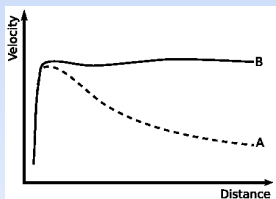
- 1 Introduction: Rotation Curves and the Call for Dark Matter
  - The Observations
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  - The Density Parameters
  - $\Lambda$ CDM
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  - The Deflection Angle
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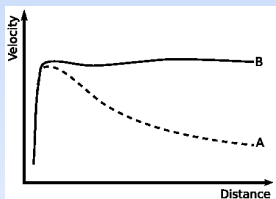
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- Line A - The Keplerian Prediction

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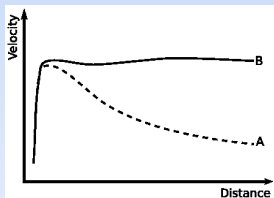
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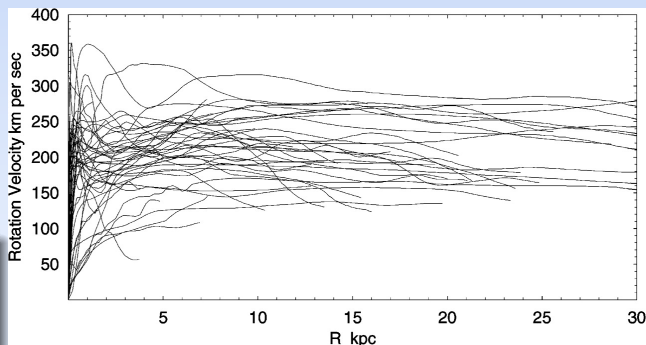
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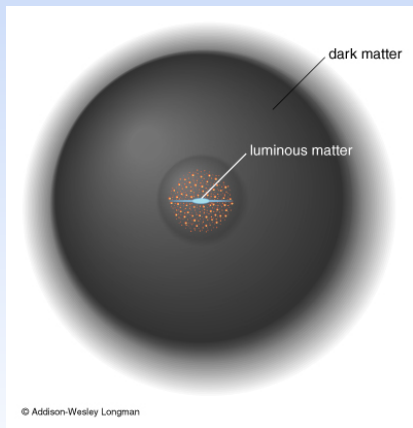
The observed results from many galaxies.

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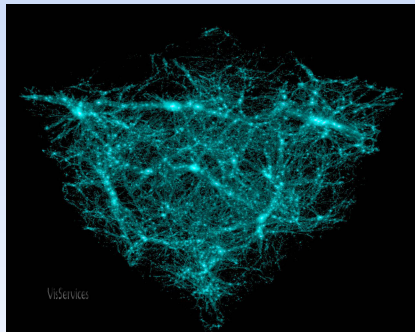
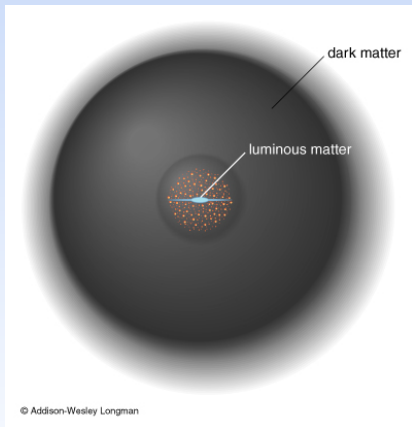


Introduce large quantities of invisible massive matter in a halo around galaxies.



# Introduction: Rotation Curves and the Call for Dark Matter

The Solution:



A numerical simulation  
showing clumping  
strands of dark matter.

Introduce large quantities of invisible  
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# Cosmological Structure and Dark Matter

## The Density Parameters:

Three types of energy density observed in the Universe.

- 
- 
- 

These densities are related by  $\Omega_m + \Omega_\Lambda + \Omega_k = 1$ . Big Bang Nucleosynthesis constrains the Baryonic component of  $\Omega_m$  to be approximately 0.03. Other constraints on these parameters come from observations of supernovas and the cosmic microwave background.

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$$(\Omega_m, \Omega_\Lambda, \Omega_k) = (0.25, 0.75, 0)$$

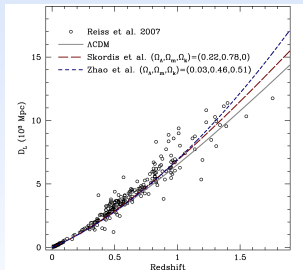
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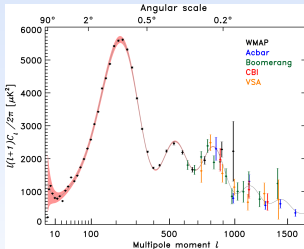
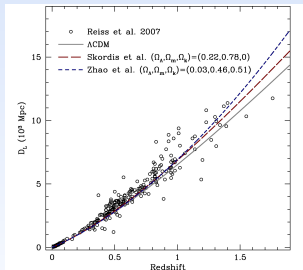


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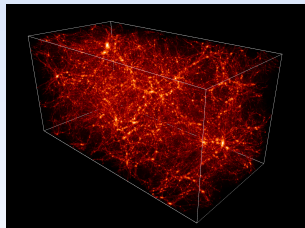
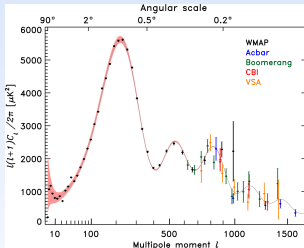
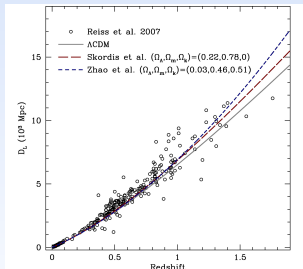


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# Modified Newtonian Dynamics

## Objections and Alternatives:

### The Objections

- Dark Matter has yet to be directly observed (LHC?)
- Tully-Fisher relation
- Fine tuning

### An Alternative

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- MOND insensitive to exact form of  $f(x)$  function.

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[N. E. Mavromatos and M. Sakellariadou, PLB 652 (2007) 97]
- In the Newtonian limit, TeVeS reproduces MOND. TeVeS very complex, and lensing results from simpler MOND can be extended to TeVeS.

# Modified Newtonian Dynamics

Testing MOND:

MOND shows very good fit to the rotation curves of spiral galaxies. There is some debate over dwarf galaxies and galactic clusters, but there is some evidence that MOND can even work in some of these cases.

Results taken from R. Bottema, J. L. G. Peñarrubia, B. Rothberg, R. H. Sanders

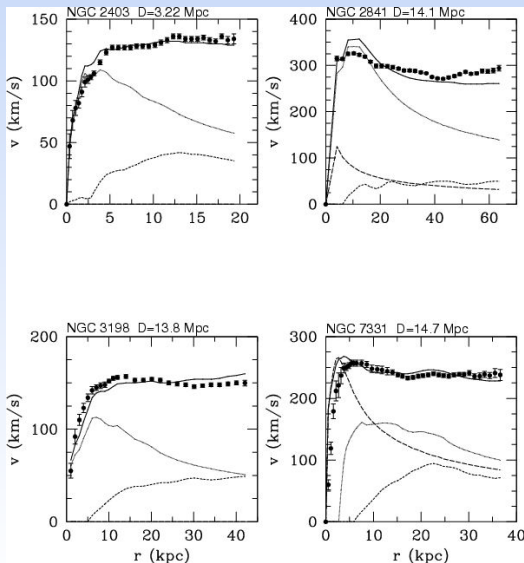
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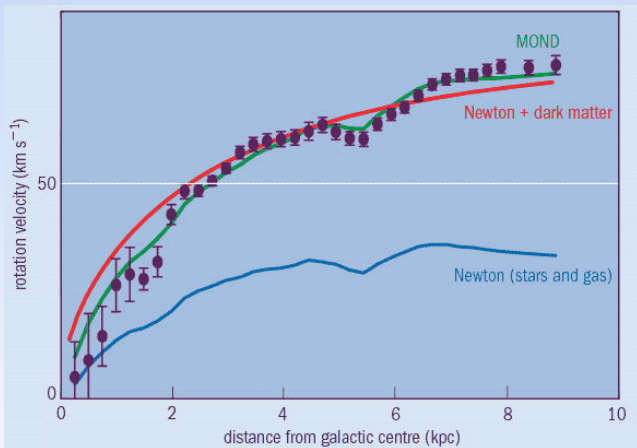


Figure: Taken from [Astrophys. J. 634 70]

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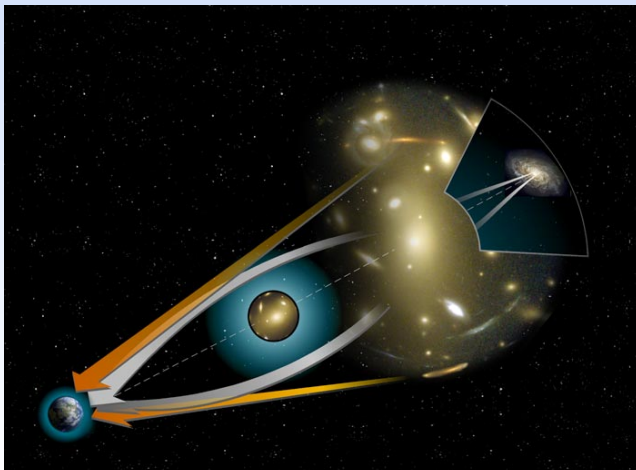
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Gravitational lensing provides a way to probe the matter distribution of galaxies.

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# Lensing in MOND

The Deflection Angle:

## The Deflection Equation

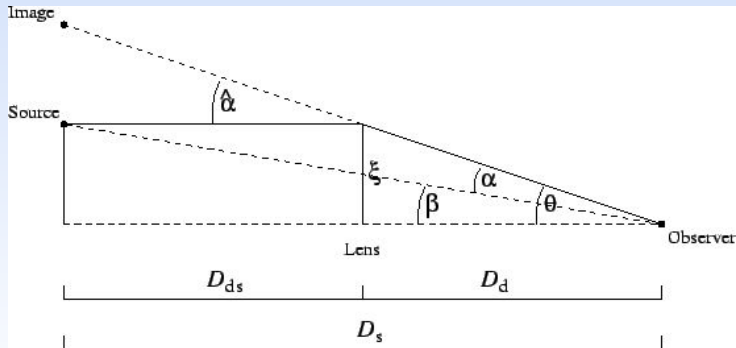
$$\alpha(b) = -\frac{4Gb}{c^2} \int_0^\infty f^{-1/2} \left[ \frac{GM(<\sqrt{b^2+z^2})}{(b^2+z^2)a_0} \right] \frac{M(<\sqrt{b^2+z^2})}{(b^2+z^2)^{3/2}} dz .$$

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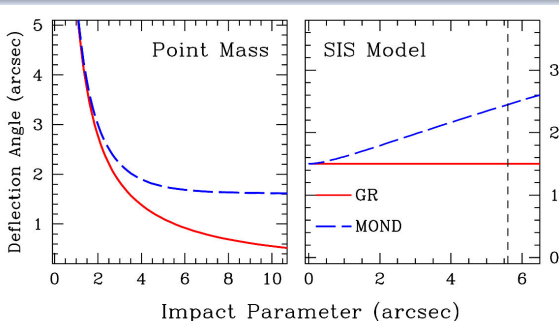
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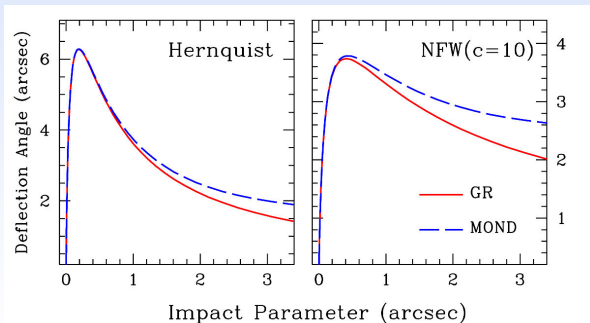
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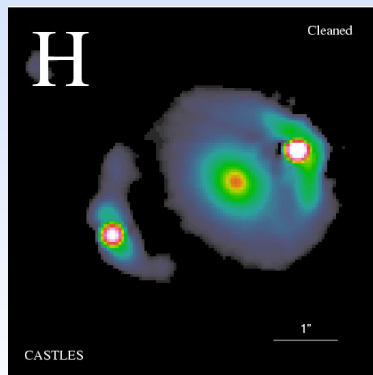


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The Lensing Equation:

## The Geometric Lens Equation

$$\beta = \theta - \alpha(\beta, \theta, M) \frac{D_{LS}}{D_S}$$



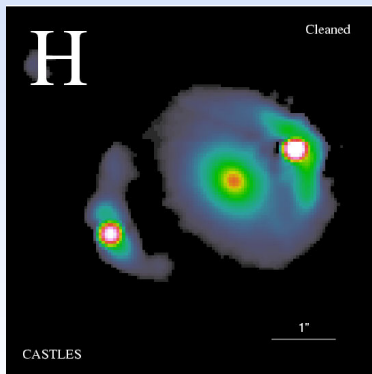
[cfa-www.harvard.edu/castles/](http://cfa-www.harvard.edu/castles/)

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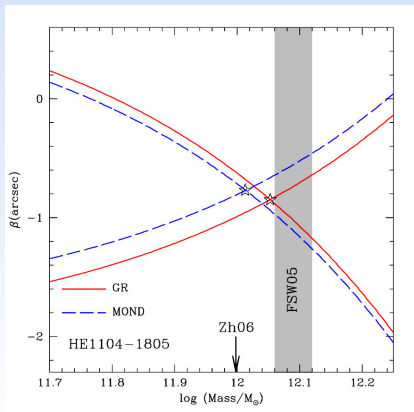
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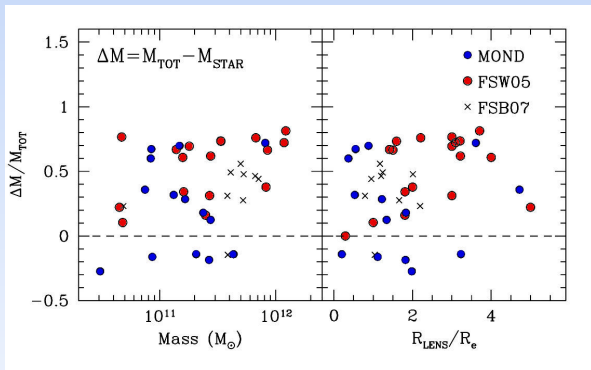
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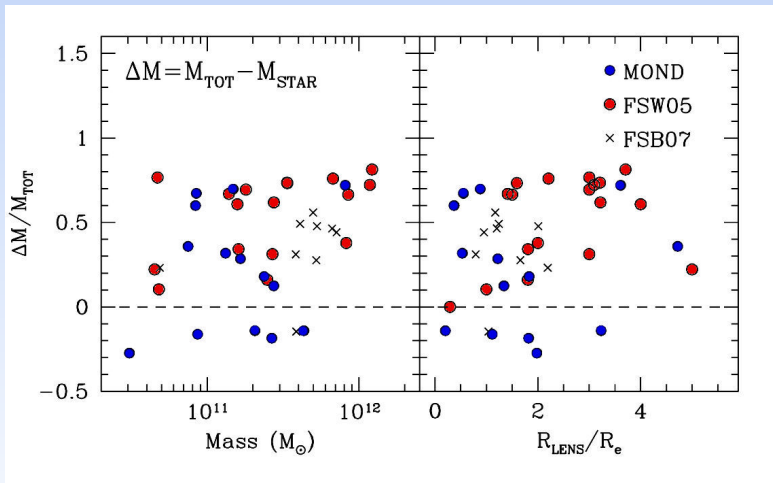
Solving The Lens Equation:



FSB07: I. Ferreras, P. Saha., L. L. R. Williams and S. Burles [astro-ph/0708.2151]

FSW05: I. Ferreras, P. Saha and L. L. R. Williams, [Astrophys. J. **623** (2005) L5.]

# Conclusion



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