Non-canonical scalar fields, superluminal propagation and black holes

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based on JHEP 0609:061,2006 [hep-th/0604075]
arXiv:0708.0561

picture from www.duke.edu

Motivation

- Non-canonical (non-quadratic) kinetic terms are rather common for effective fields theories.
- Interesting applications to cosmology:
  - Inflation without $V(\phi)$.
    [Armendariz-Picon, Damour, Mukhanov'99]
  - Kinetically driven quintessence (k-essence).
    [Armendariz-Picon, Mukhanov, Steinhardt'00]
  - Inflation with $c_s > 1$.
    [Mukhanov, Vikman'05]
- Interesting behavior of exotic matter in the neighborhood of a black hole:
  - Decreasing of a black hole mass due to the accretion of phantom.
    [E.B, Dokuchaev, Eroshenko'04]
  - Accretion of ghost condensate.
    [Frolov'04; Mukohyama'05; Dubovsky,Sibiryakov'06]
- What happens with k-essence in the neighborhood of a black hole?

- scalar field ϕ with action:

$$S_\phi = \int d^4x \sqrt{-g}\, p(X), \qquad X \equiv \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi.$$

- $p(X)$ is a non-linear function, therefore the small perturbations propagate in a new metric (different from $g^{\mu\nu}$)

$$G^{\mu\nu} \equiv \frac{c_s}{p_{,X}}\left[g^{\mu\nu} + \frac{p_{,XX}}{p_{,X}}\nabla^\mu\phi\nabla^\nu\phi\right].$$

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! if $p_{,XX}/p_{,X} < 0$ then superluminal propagation of perturbation.

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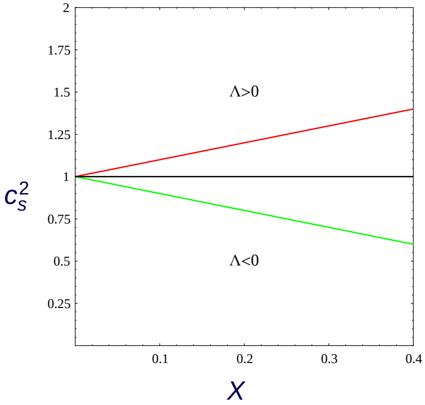

Accretion onto a Black Hole

- take Schwarzschild BH $\longrightarrow$
- find spherically symmetric, stationary accretion of a scalar field without backreaction (test fluid)
- consider small perturbations in the background solution, $G_{\mu\nu}$

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- take Schwarzschild BH $\longrightarrow$
- find spherically symmetric, stationary accretion of a scalar field without backreaction (test fluid)
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- guess: one is able to look inside BH.

- ingoing Eddington-Finkelstein coordinates
- looking for solution to:

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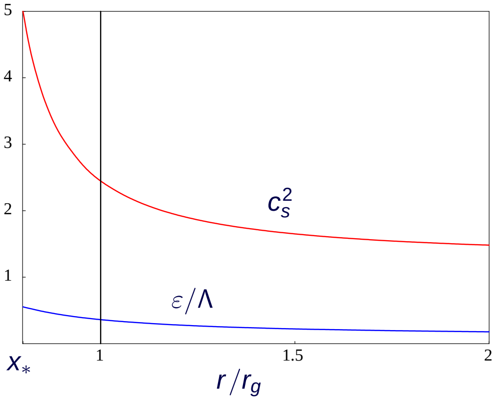
$$\phi(V,x) = \alpha\sqrt{c_{\infty}^2 - 1}\left(V + r_g \int F(x)dx\right), \quad x = r/r_g$$

$$F(x) = \frac{1}{f}\left(\sqrt{\frac{c_{\infty}^2 + f - 1}{fx^4 c_{\infty}^8 (c_{\infty}^2 - 1)}} - 1\right), \quad f \equiv 1 - r_g/r$$

- provides spherical symmetry and the stationarity condition
- recovers the cosmological solution at $r \to \infty$
- non-singular at the horizon
- depends on conditions at infinity: c_{∞}

Background solution for $c_s > 1$



Propagation of perturbations

$\phi(V, r)$ (depending on c_∞) $\rightarrow$

$$G^{-1}_{\mu\nu}\eta^\mu\eta^\nu = 0$$

Sound horizon (SH)

- No sound signals can escape from inside SH;
- Being emitted from the outside of the SH they can reach a distant observer.

Escaping!

- Sound horizon is inside the Schwarzschild horizon:

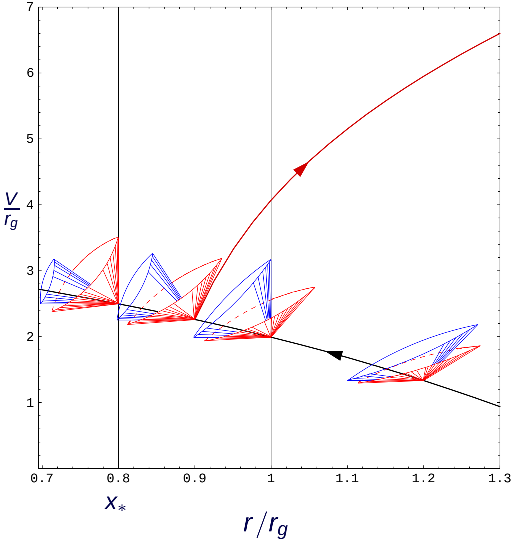
$$\frac{r_{SH}}{r_g} = \frac{1}{c_\infty^2}$$

- As long as the signals are emitted at $x > x_*$, they reach the spatial infinity propagating along η_+.
- At the Schwarzschild horizon:

$$\eta_{\pm H} = \frac{1}{2}\frac{\left(c_\infty^4 \pm 1\right)^2}{c_\infty^2 - 1}.$$

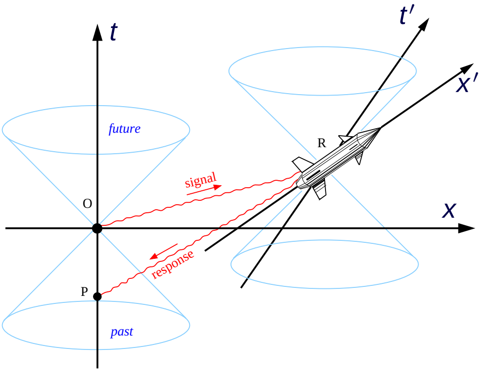
$\eta_{+H} \neq \infty$! Signals can freely penetrate the Schwarzschild horizon and move *towards infinity*.

Falling satellite

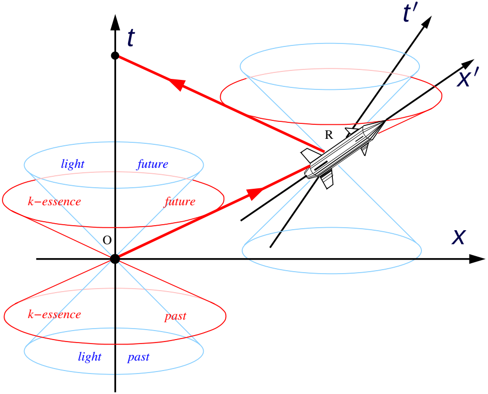


ARE THERE ANY TROUBLES WITH
WELL-POSEDNESS OF CAUCHY PROBLEM,
STABILITY OR THERMODYNAMICS?

Violation of causality?



In this simplest case NO



For the case of BH the answer is again NO

- The theorem on stable causality: *A spacetime $(M, g_{\mu\nu})$ is stably causal if and only if there exists a differentiable function f on M such that $\nabla^\mu f$ is a future directed timelike vector field.*
- The scalar field ϕ itself serves as such a global time function.

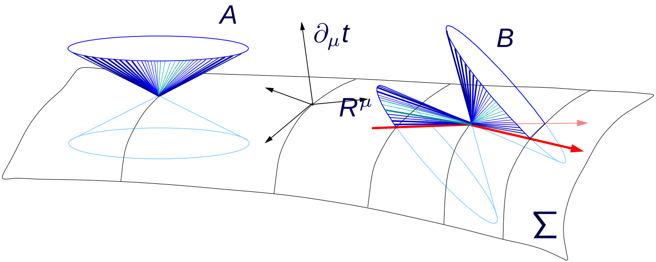
Cauchy problem

- EOM are hyperbolic provided

$$1 + 2X\frac{p_{,XX}}{p_{,X}} > 0$$

- “Good” initial hypersurface and initial data:

$$1 + c_s^2\left(\vec{\nabla}\phi(\mathbf{x})\right)^2\frac{p_{,XX}}{p_{,X}} > 0$$



Troubles with well-posedness of Cauchy problem?

Is well-posedness lost?

$$G^{\mu\nu}\nabla_\mu\nabla_\nu\phi = 0, \quad \phi = \phi_0 + \delta\phi$$

background → ϕ_0 perturbations → $\delta\phi$

$$G^{\mu\nu}(\phi_0, \nabla\phi_0)\nabla_\mu\nabla_\nu\delta\phi = 0$$

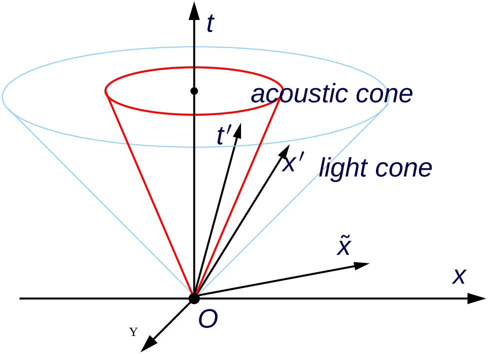
In Minkowski with $\phi_0 = \dot{\phi}_0 t \quad \rightarrow \quad \delta\ddot{\phi} - c_s^2\triangle\delta\phi = 0$
in a rocket with velocity β along $\vec{x}$:

$$(1 - c_s^2\beta^2)\partial_t^2\delta\phi + 2\beta(1 - c_s^2)\,\partial_t\partial_x\delta\phi - (c_s^2 - \beta^2)\,\partial_x^2\delta\phi - (1 - \beta^2)\,c_s^2\,\partial_\perp^2\delta\phi = 0$$

If $\beta > 1/c_s$ then Cauchy problem for $\delta\phi$ is ill posed

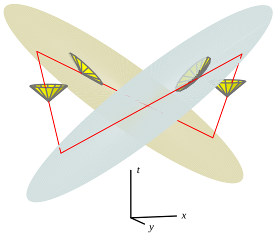
[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi'06]

Again no!



- “Bad” hypersurface to set up initial conditions.
- Physically: any device poses (initial) conditions correctly, on “good” hypersurface. One has to use another way of clock synchronization — by fastest signals.

Time machines — closed causal curves for inhomogeneous backgrounds?



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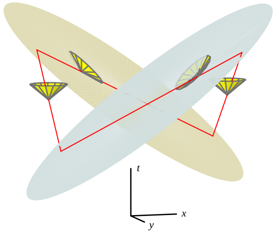

[*Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi'06*]

Chronology protection conjecture

Similar to GR: wormholes, Gödel's cosmological model, Stockum's rotating dust cylinder, Gott's solution for two infinitely long strings.

Ori's time machine

[*Ori'07*]

Closed causal curves for two pairs of Casimir plates.

[*Liberati, Sonego, Visser'01*]

Problems with Thermodynamics?

- Hawking radiation from two horizons. *[Dubovsky,Sibiryakov'06]*
- Classical energy-extraction method? *[Eling, Foster, Jacobson, Wall'07]*

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In our case there is an external flux of energy (accretion)

Conclusion

- one can send signal from the interior of BH
  - no quantum phenomena involved
  - The null energy condition is not violated as well.
- the universal meaning of the Schwarzschild horizon as the event horizon changes.
- Consequences for the thermodynamics of black holes.
- Non-causal behavior is excluded by Chronology protection (as in the case of two pairs of Casimir plates).
- No troubles with initial data problem, Green functions etc.
- Thermodynamics? Instability between two horizons? Cherenkov radiations? Need to understand...