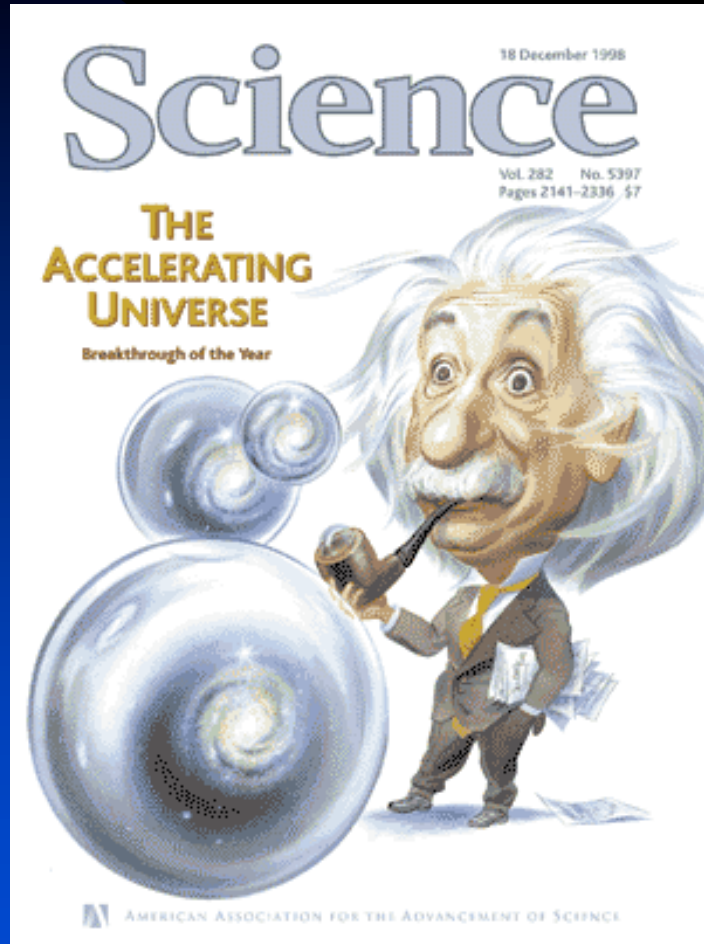


Dark Energy: the evidence and possible physical implications

Ed Copeland -- Nottingham University

1. Evidence for Dark Energy
2. Models of Lambda
3. Scalar field models
4. Coupled dark energy models
5. Dark energy and varying constants
6. Modified Gravity Models
7. Observational features

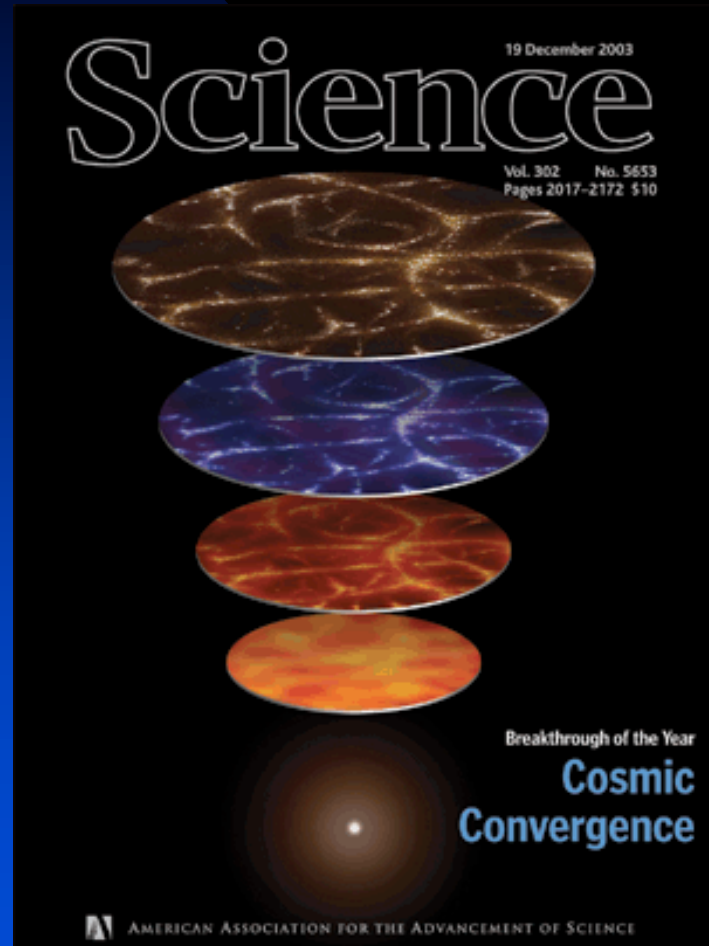
Science Magazine -- Breakthrough of the year -Dec 1998



“Einstein watches in surprise as a universe expands exponentially, its galaxies rushing apart ever faster. Evidence for an accelerating universe, the Breakthrough of the Year for 1998, resurrects Einstein's discarded idea of an energy called lambda, or λ , which counteracts gravity and pushes space apart.”

So good -- they named it twice

Science Magazine -- Breakthrough of the year -Dec 2003

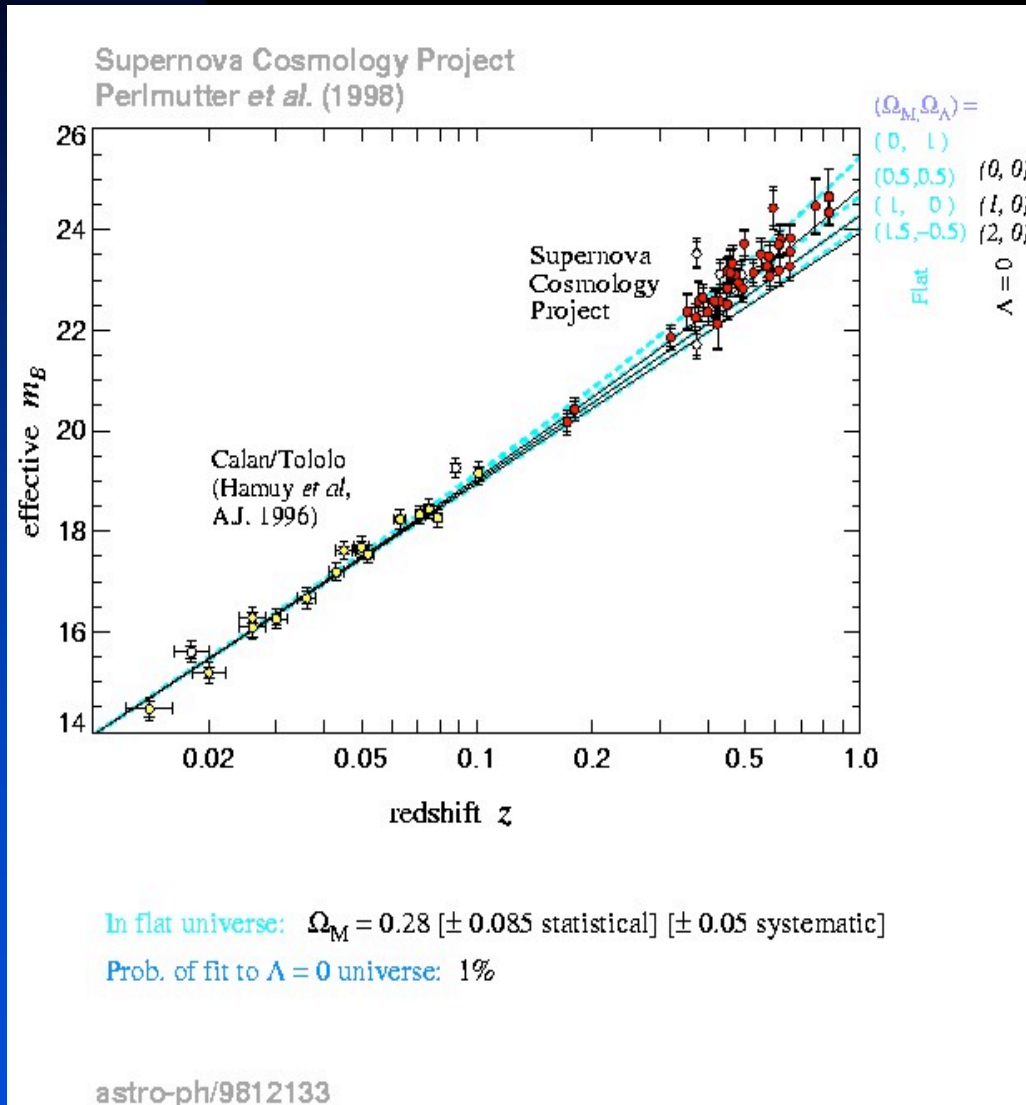


“Disks represent an aging and expanding universe.

Work this year confirmed a bizarre story of how the cosmos was born and what it is made of.

Dark energy is the primary ingredient in a universe whose expansion rate and age are now known with unprecedented precision.”

1. The Big Bang – (1sec → today)



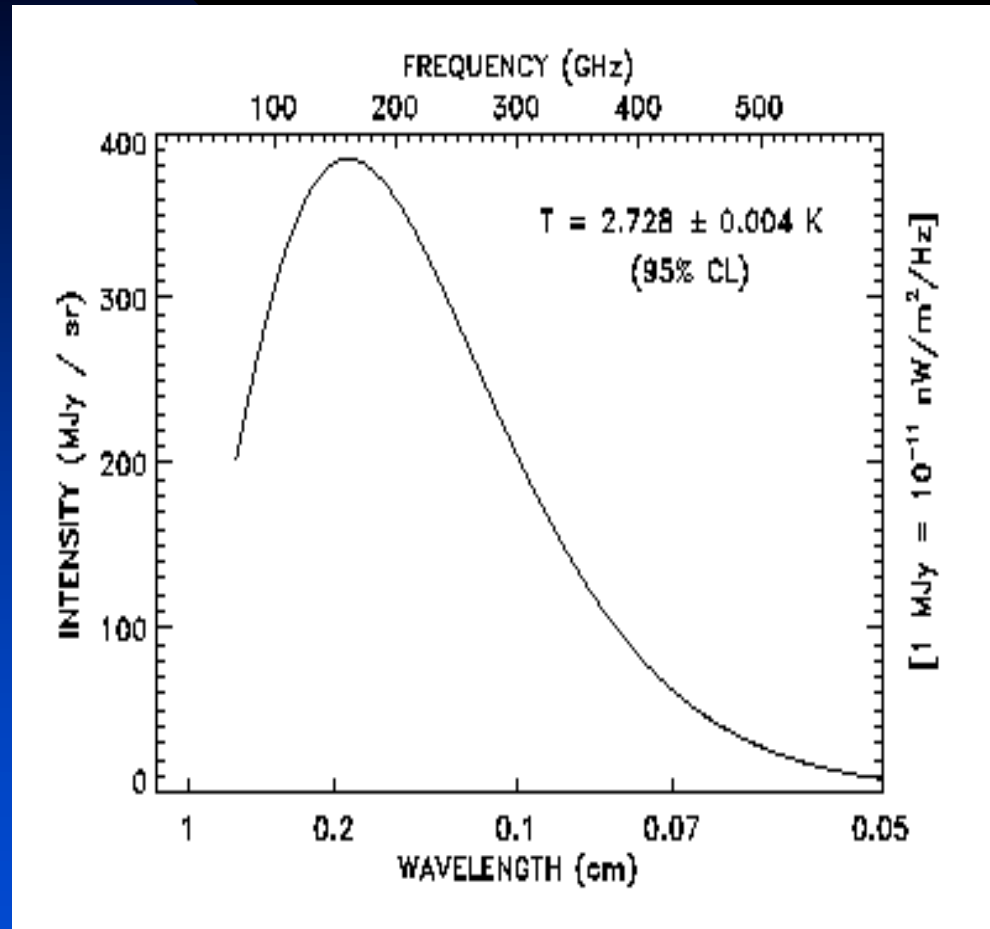
Test 1

• The expansion of the Universe

$$H_0 = 72^{+8}_{-8} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

(Freedman *et al.*, 2001)

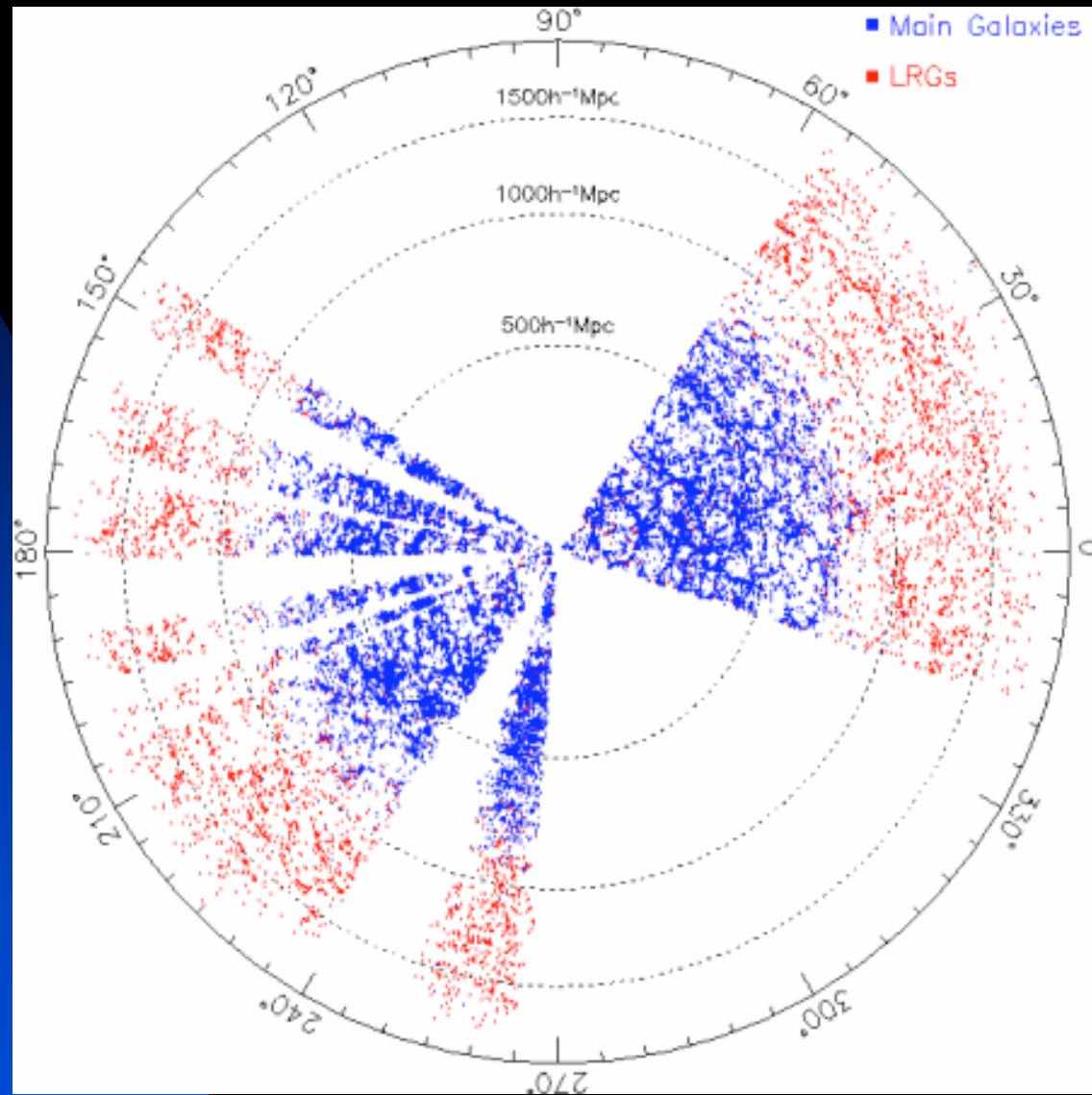
The Big Bang – (1sec → today)



Test 2

- The existence and spectrum of the CMBR
- $T_0 = 2.728 \pm 0.004 \text{ K}$

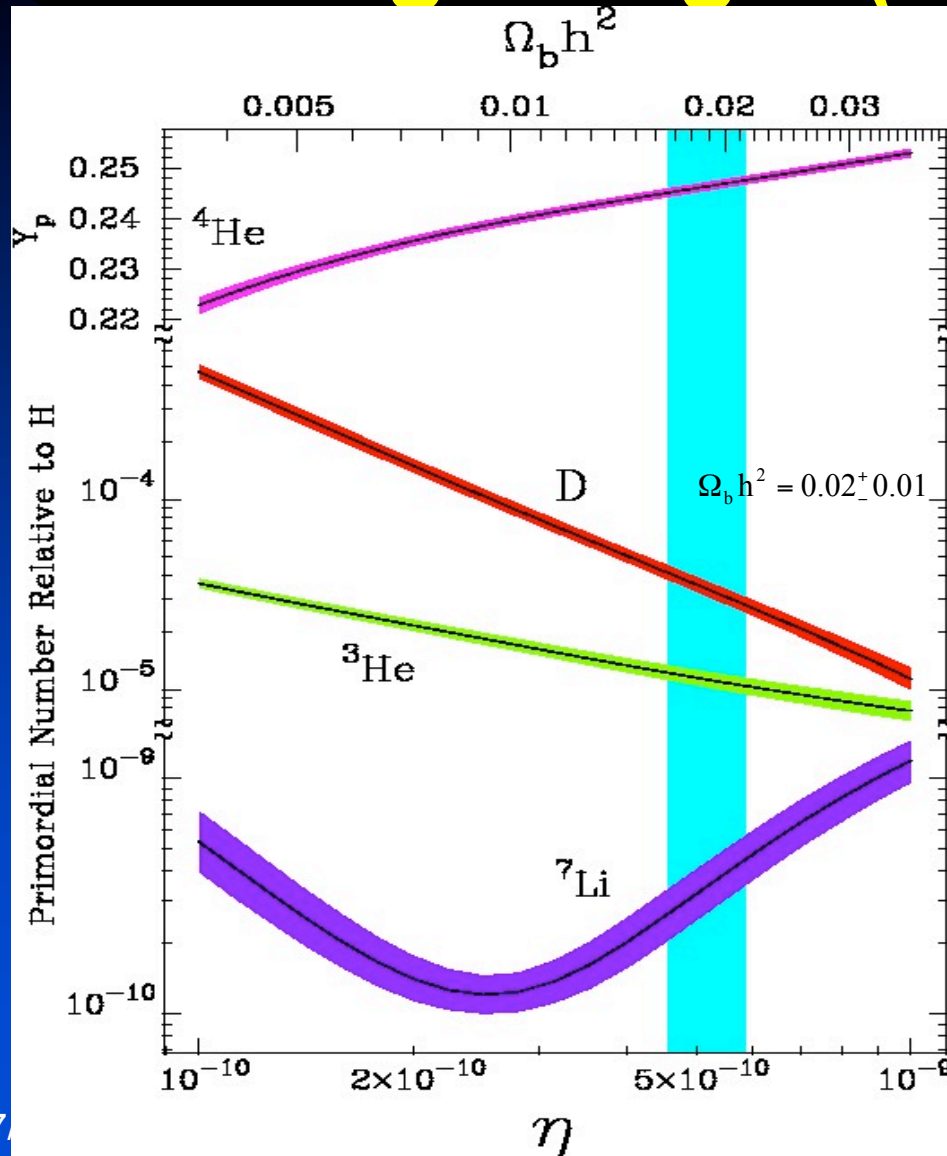
Sloan Digital Sky Survey



9/27/07

Homogeneous on large scales?

The Big Bang – (1sec → today)



Test 3

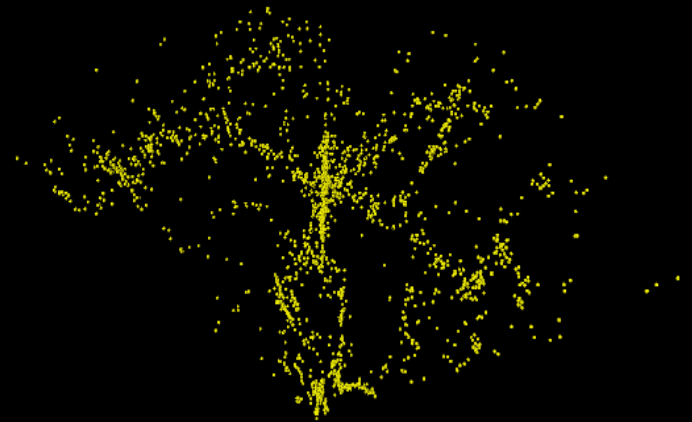
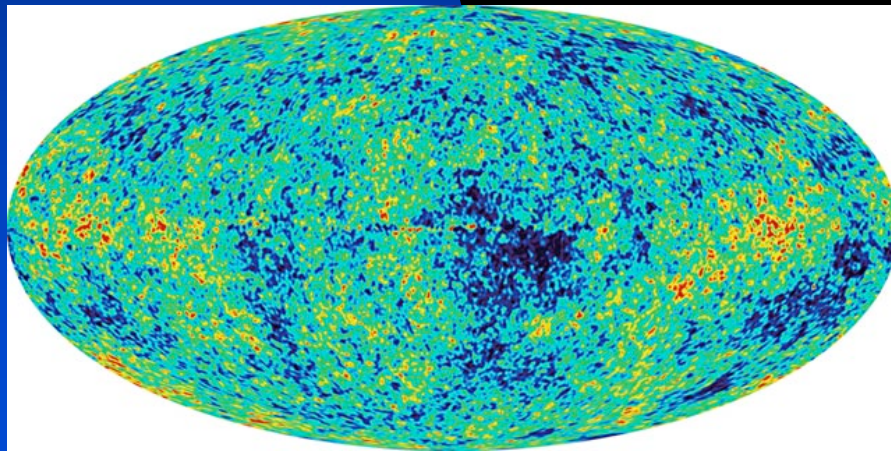
- The abundance of light elements in the Universe.
- Most of the visible matter just hydrogen and helium.

$$\Omega_b h^2 = 0.02^{+0.01}_{-0.01}$$

The Big Bang – (1sec → today)

Test 4

- **Given** the irregularities seen in the CMBR, the development of structure can be explained through **gravitational collapse.**



Some basic equations

Friedmann:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$a(t)$ depends on matter.

Energy density $\rho(t)$: Pressure $p(t)$

Related through : $p = w\rho$

$w=1/3$ – Rad dom: $w=0$ – Mat dom: $w=-1$ – Vac dom

Eqns ($\Lambda=0$):

**Friedmann +
Fluid
conservation**

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$
$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

Combine

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G (\rho + 3p) \text{ --- Accn}$$

$$\text{If } \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G \rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p) \frac{\dot{a}}{a} = 0$$

$$\rho(t) = \rho_0 \left(\frac{a}{a_0} \right)^{-3(1+w)} ; \quad a(t) = a_0 \left(\frac{t}{t_0} \right)^{2/3(1+w)}$$

$$\text{RD: } w = \frac{1}{3} : \rho(t) = \rho_0 \left(\frac{a}{a_0} \right)^{-4} ; \quad a(t) = a_0 \left(\frac{t}{t_0} \right)^{1/2}$$

$$\text{MD: } w = 0 : \rho(t) = \rho_0 \left(\frac{a}{a_0} \right)^{-3} ; \quad a(t) = a_0 \left(\frac{t}{t_0} \right)^{2/3}$$

$$\text{VD: } w = -1 : \rho(t) = \rho_0 ; \quad a(t) \propto e^{Ht}$$

A neat equation

$$\rho_c(t) \equiv \frac{3H^2}{8\pi G} \quad ; \quad \Omega(t) \equiv \frac{\rho}{\rho_c}$$

Friedmann eqn

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

$$\rho_c(t_0) \equiv 1.88h^2 * 10^{-29} \text{ g cm}^{-3}$$

Critical density

Weighing the Universe

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

1. Ω_m

- a. Cluster baryon abundance using X-ray measurements of intracluster gas, or SZ measurements.
- b. Weak grav lensing and large scale peculiar velocities.
- c. Large scale structure distribution.
- d. Lyman alpha forest
- e. Numerical simulations of cluster formation.

$$\Omega_m = 0.266 \pm 0.02$$

$$\Omega_m \ll 1$$

Growth of structure by gravity -- sensitive to dark matter and dark energy

◆ Perturbations can be measured at different epochs hence probes different physics contributions:

1. CMB $z=1000$
2. 21cm $z=10-20$ (?)
3. Ly-alpha forest $z=2-4$
4. Weak lensing $z=0.3-2$
5. Galaxy clustering $z=0-2$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}\delta \rightarrow \delta(t)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8}{3}\pi G\bar{\rho} - Ka^{-2}$$

$$\bar{\rho} = \rho_m a^{-3} + \rho_{de} a^{-3(1+w)} + \rho_\gamma a^{-4} + \rho_\nu F(a)$$

Evidence for Dark Energy?

Enter CMBR:

$$3. \Omega_0 = \Omega_m + \Omega_\Lambda$$

Provides clue. 1st angular peak in power spectrum.

$$l_{\text{peak}} \approx \frac{220}{\sqrt{\Omega_0}}$$

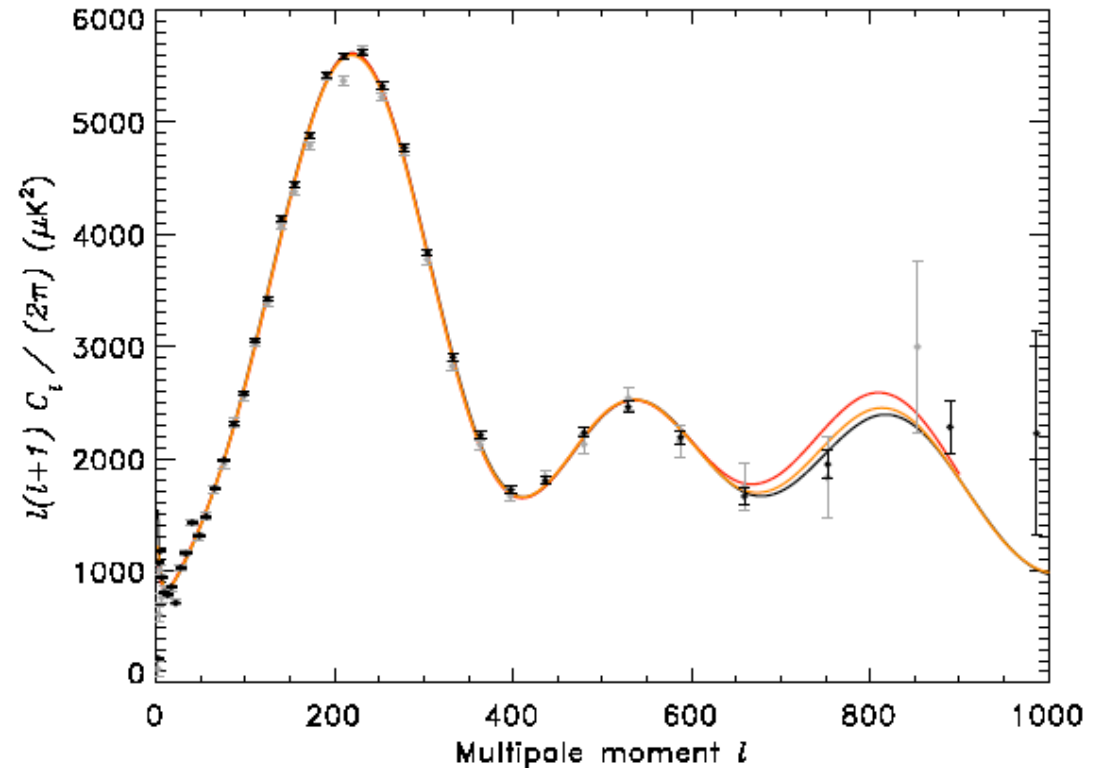


$$|1 - \Omega_0| = 0.03^{+0.026}_{-0.025}$$

WMAP3-Depends on assumed priors

Spergel et al 2006

9/27/07



WMAP3 and dark energy

Assume flat univ
+ SNLS:

$$w = -0.97^{+0.07}_{-0.09}$$

Rules out
frustrated
networks of walls:

If assume $w = -1$,
then with SNLS:

$$\Omega_k = -0.015^{+0.020}_{-0.016}$$

WMAP + HST:

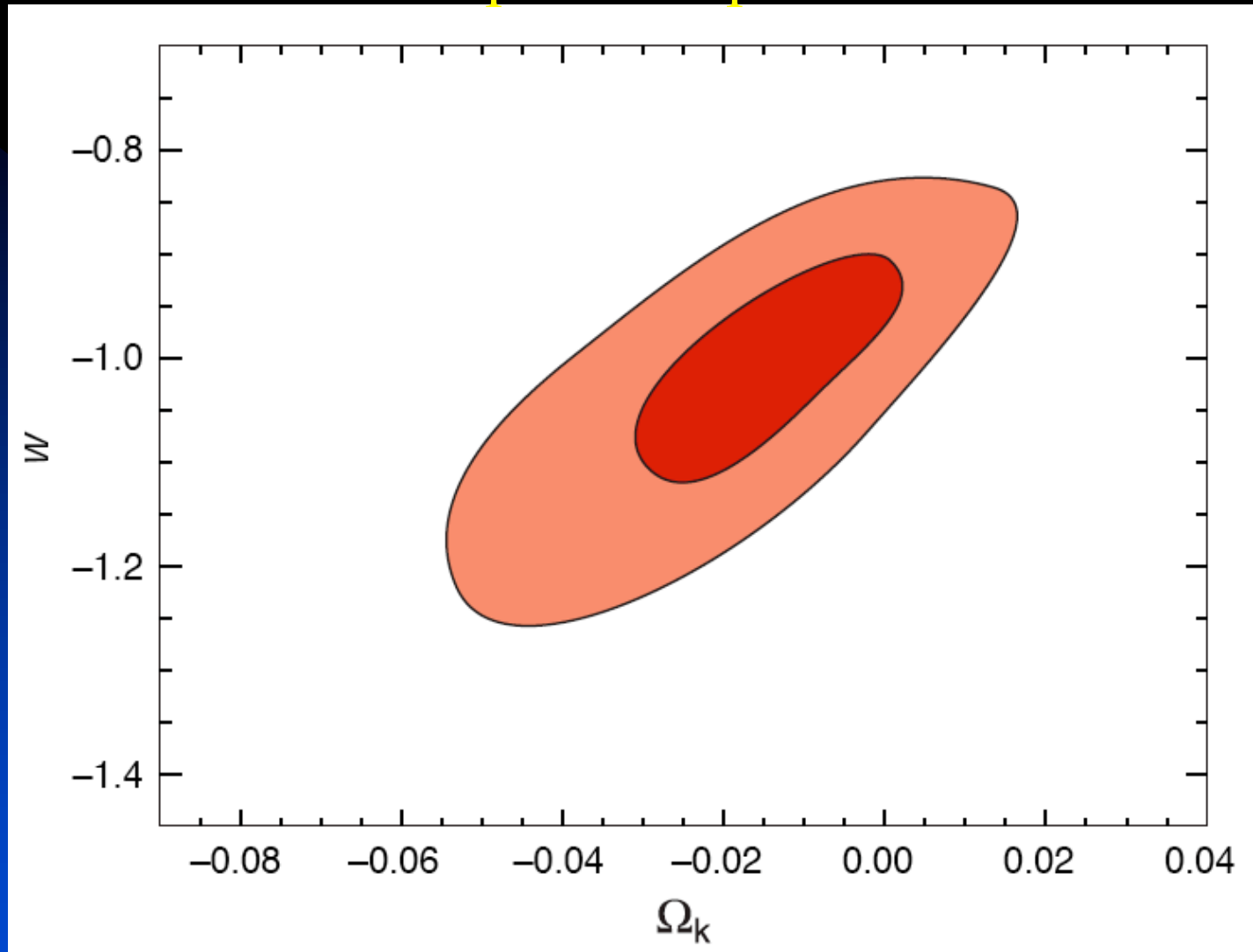
$$\Omega_K = -0.010^{+0.016}_{-0.009} \text{ and } \Omega_\Lambda = 0.72 \pm 0.04.$$

Drop prior of flat
univ: WMAP +
LSS+ SNLS:

$$w = -1.06^{+0.13}_{-0.08}$$

Spergel et al 2006

Relax the prior of spatial flatness.



Spergel et al 2006

WMAP+LSS+SN

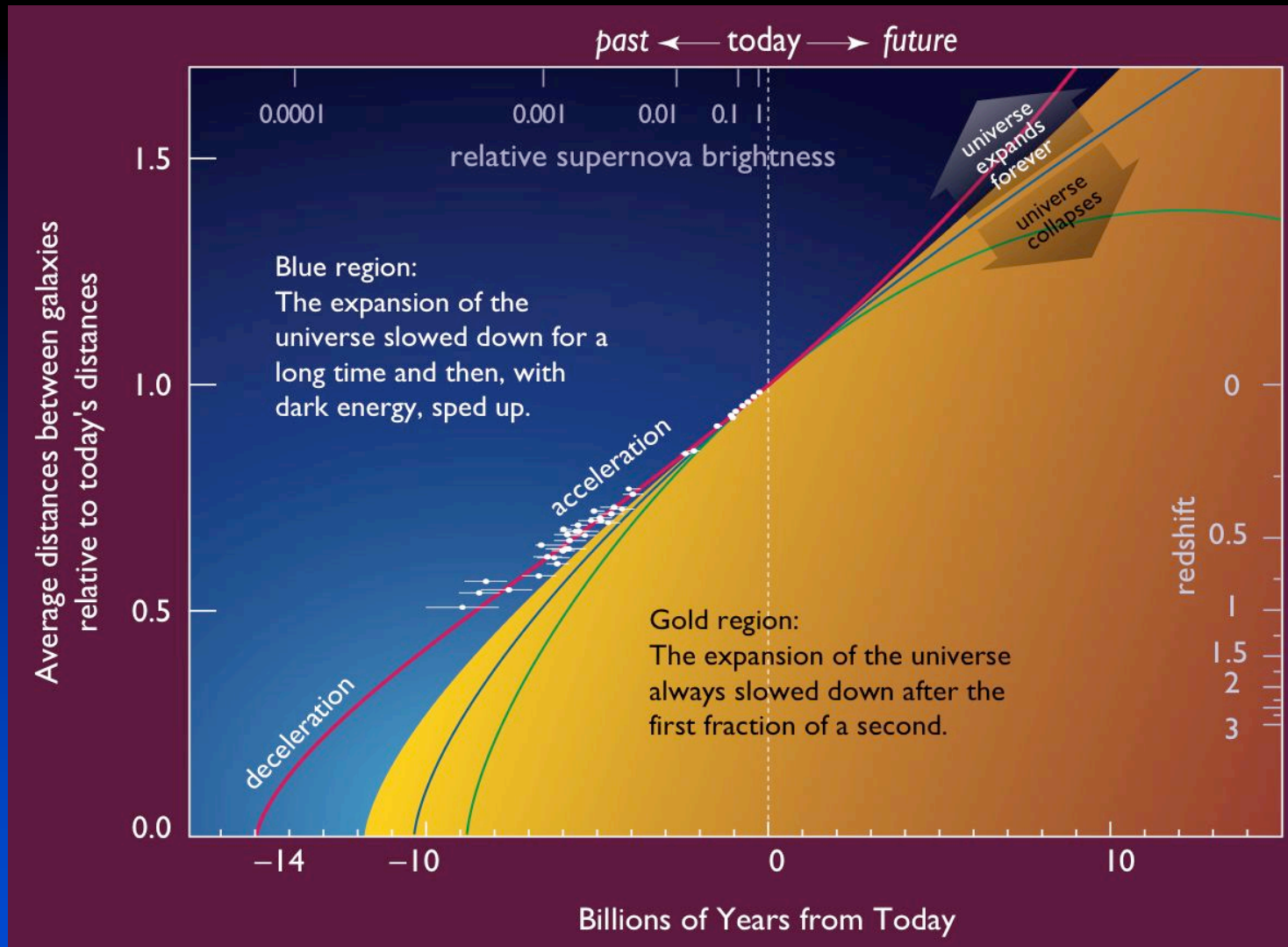
9/27/07

Best fit values:

$$w = -1.062^{+0.128}_{-0.079} \text{ and } \Omega_k = -0.024^{+0.016}_{-0.013}$$

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Evidence for Acceleration

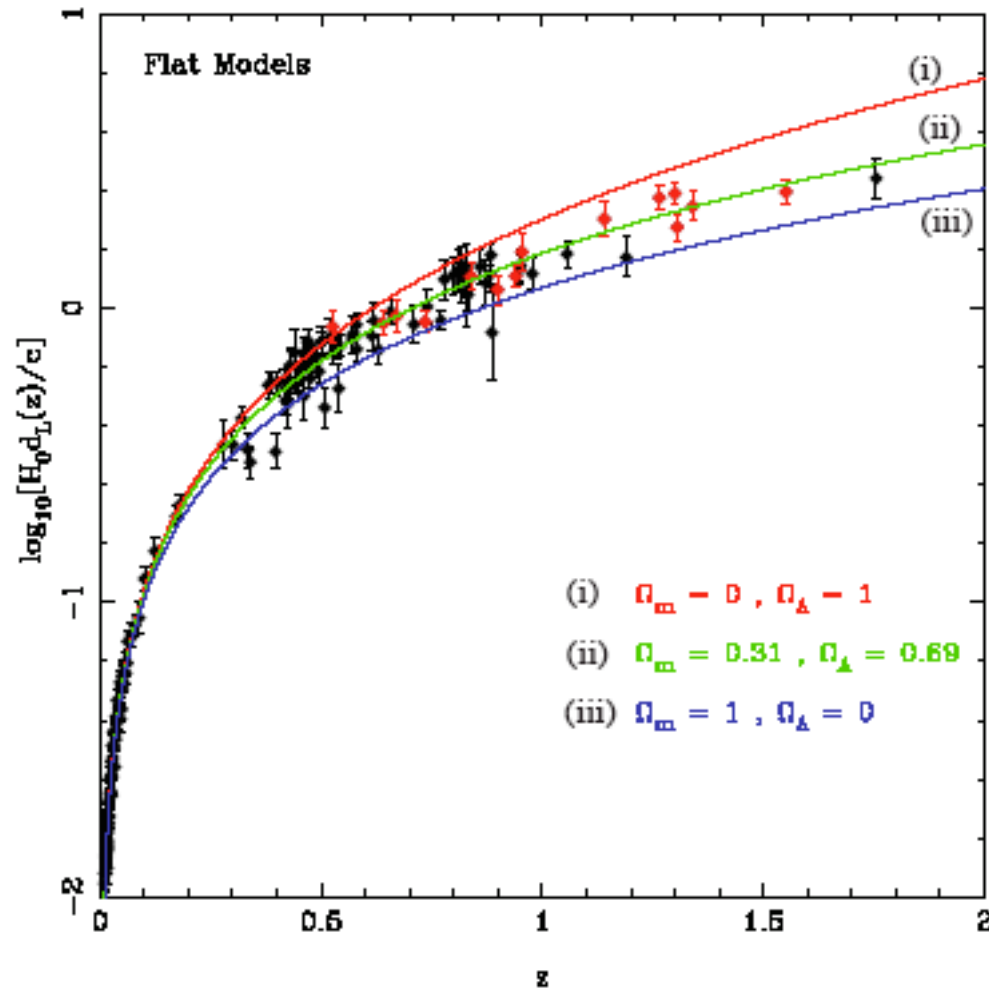


data from Supernova
Cosmology Project
(LBL)

graphic by Barnett,
Linder, Perlmutter &
Smoot (for OSTP)

Exploding stars – supernovae – bright beacons that allow us to measure the expansion over the last 10 billion years.

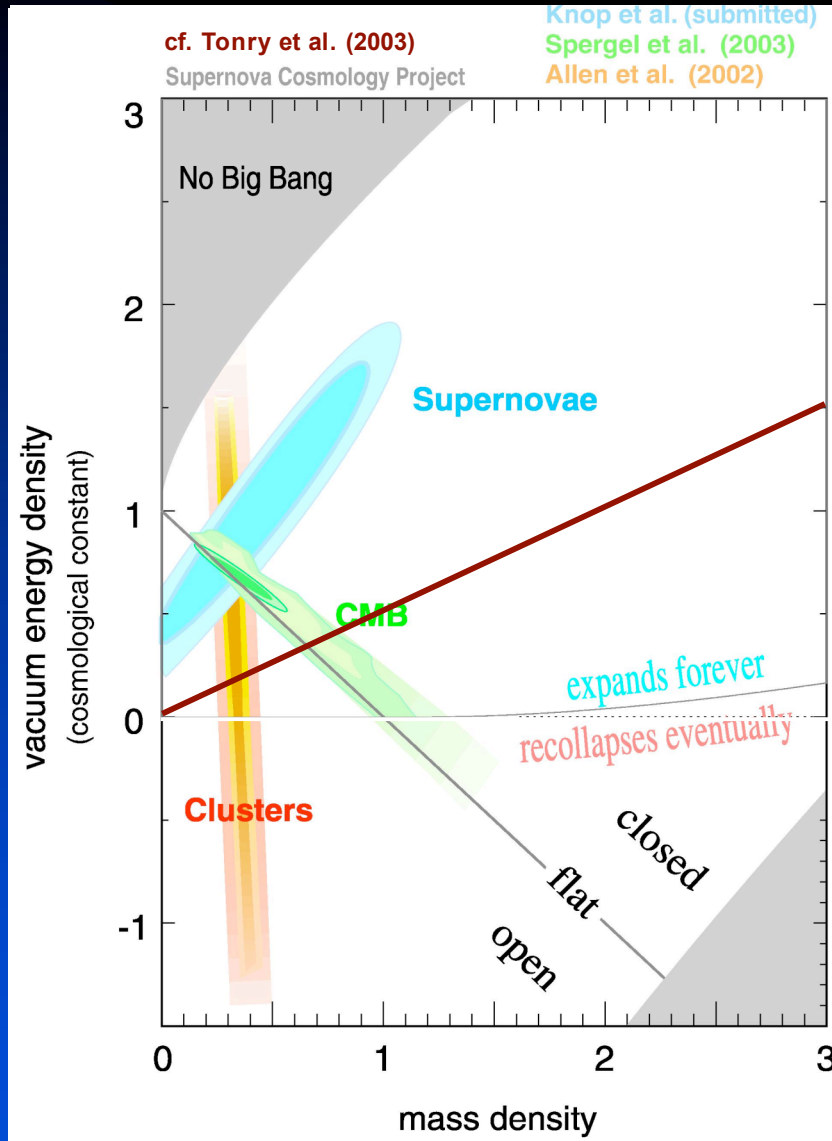
Type Ia Luminosity distance v Z [Reiss et al 2004]



Flat model
Black dots --
Gold data set
Red dots -- HST

(i) $\Omega_m = 0, \Omega_\Lambda = 1$ (ii) $\Omega_m = 0.31, \Omega_\Lambda = 0.69$ (iii) $\Omega_m = 1, \Omega_\Lambda = 0$

Cosmic Concordance



- **Supernovae alone**
⇒ **Accelerating expansion**

- ⇒ $\Lambda > 0$

- **CMB (plus LSS)**

- ⇒ **Flat universe**

- ⇒ $\Lambda > 0$

- **Any two of SN, CMB, LSS**

- ⇒ **Dark energy ~70%**

Different approaches to Dark Energy include amongst many:

- A true cosmological constant -- if so, why this value?
- Solid –dark energy such as arising from frustrated network of domain walls.
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.
- Anthropic arguments

Over 1200 papers on archives with dark energy in title -- we will go through each one.

The problem with the cosmological constant

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Einstein (1917) -- static universe
with dust

Not easy to get rid of it, once universe found to be
expanding.

Anything that contributes to energy density of vacuum
acts like a cosmological constant

$$\langle T_{\mu\nu} \rangle = \langle \rho \rangle g_{\mu\nu}$$

Lorentz inv

$$\lambda_{eff} = \lambda + 8\pi G \langle \rho \rangle$$

or

$$\rho_V = \lambda_{eff} / 8\pi G$$

Effective cosm const

Effective vac energy

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} \text{yr}^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - \langle \rho \rangle| \leq \frac{3H_0^2}{8\pi G}$$

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} \text{yr}^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - \langle \rho \rangle| \leq \frac{3H_0^2}{8\pi G}$$

Hence: $\lambda_{eff} \leq H_0^2$ or $|\rho_V| \leq 10^{-29} \text{gcm}^{-3} \simeq 10^{-47} \text{GeV}^4$

Problem: expect $\langle \rho \rangle$ of empty space to be much larger.

Consider summing zero-point energies of all normal modes of some field of mass m up to wave number cut off

$\Lambda \gg m$:

$$\langle \rho \rangle = \int_0^\Lambda \frac{4\pi k^2 dk}{2(2\pi)^3} \sqrt{k^2 + m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

Planck scale: $\Lambda \simeq (8\pi G)^{-1/2} \rightarrow \langle \rho \rangle \simeq 2 \times 10^{71} \text{GeV}^4$

But: $|\rho_V| = |\langle \rho \rangle + \lambda/8\pi G| \leq 2 \times 10^{-47} \text{GeV}^4$

Must cancel to better than 118 decimal places.

Even at QCD scale require 41 decimal places!

Coincidence problem – why now?

Recall: $\frac{\ddot{a}}{a} \geq 0 < - > = (\rho + 3p) \leq 0$

If: $\rho_x = \rho_x^0 a^{-3(1+w_x)}$

Universe dom by
Quintessence at:

$$z_x = \left(\frac{\Omega_x}{\Omega_m} \right)^{\frac{1}{3w_x}} - 1$$

$$\left(\frac{\Omega_x}{\Omega_m} \right) = \frac{7}{3} \rightarrow z_x = 0.5, 0.3 \text{ for } w_x = -\frac{2}{3}, -1$$

Univ
accelerates at:

$$z_a = \left(- (1 + 3w_x) \frac{\Omega_x}{\Omega_m} \right)^{\frac{-1}{3w_x}} - 1$$

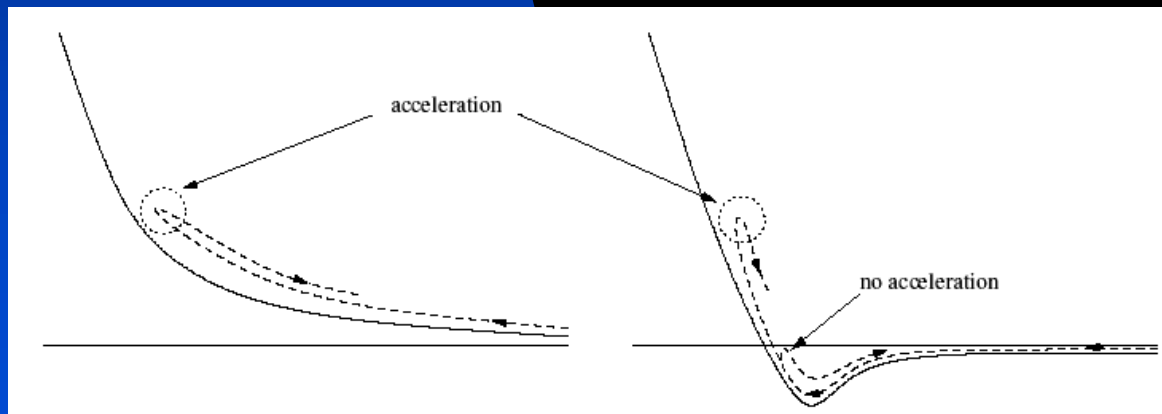
$$z_a = 0.7, 0.5 \text{ for } w_x = -\frac{2}{3}, -1$$

Quintessence and M-theory -- where are the realistic models?

'No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGRA models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent, analogue of time-dependent scalar fields (radion)



But no sustained inflation.

Current realistic potentials are too steep

Models kinetic, not matter domination before entering accelerated phase.

Four form Flux and the cosm const: [Bousso and Polchinski]

Effective 4D theory from $M^4 \times S^7$ compactification

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R + \Lambda_b - \frac{1}{2 \cdot 4!} F_4^2 \right)$$

Negative bare cosm const: $-\Lambda_b$

EOM: $\nabla_\mu (\sqrt{-g} F^{\mu\nu\rho\sigma}) = 0 \implies F^{\mu\nu\rho\sigma} = c \epsilon^{\mu\nu\rho\sigma}$

Eff cosm const:

$$\Lambda = -\Lambda_b - \frac{1}{48} F_4^2 = -\Lambda_b + \frac{c^2}{2}$$

Quantising c and
considering J fluxes

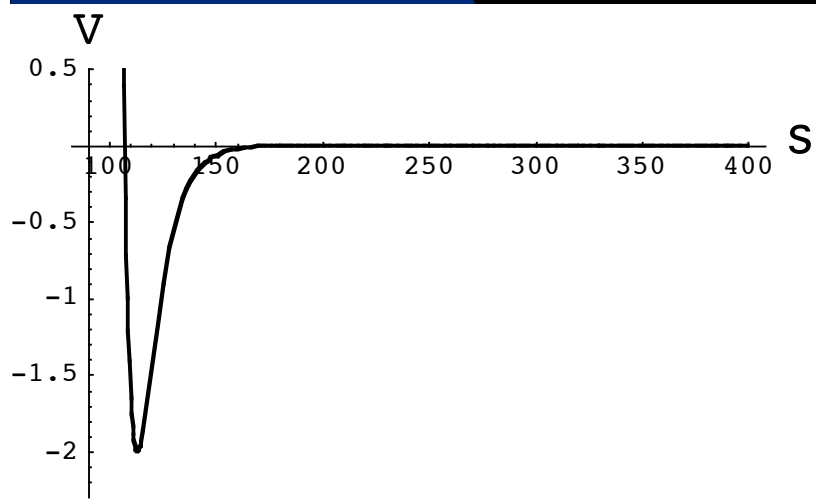
$$\Lambda = -\Lambda_b + \frac{1}{2} \sum_{i=1}^J n_i^2 q_i^2$$

Observed cosm const with $J \sim 100$

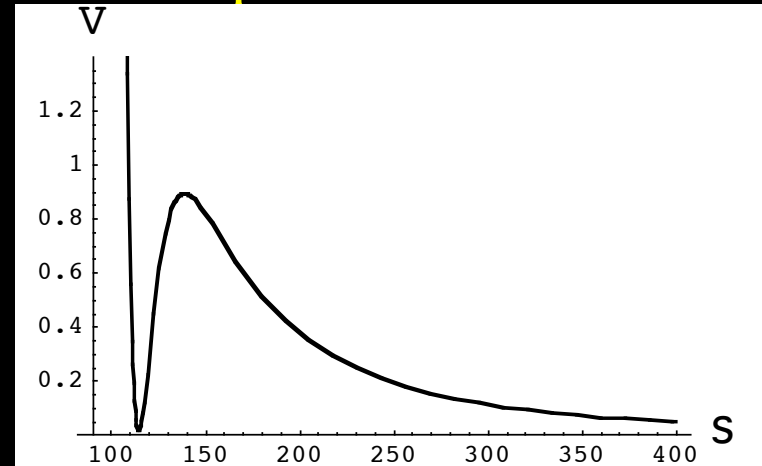
Still needed to stabilise moduli but opened up way of obtaining
many de Sitter vacua using fluxes -- String Landscape

Example of stabilised scenario: Metastable de Sitter string vacua in Type IIB string theory, based on stable highly warped IIB compactifications with NS and RR three-form fluxes. [Kachru, Kallosh, Linde and Trivedi 2003]

Metastable minima arises from adding positive energy of anti-D3 brane in warped Calabi-Yau space.



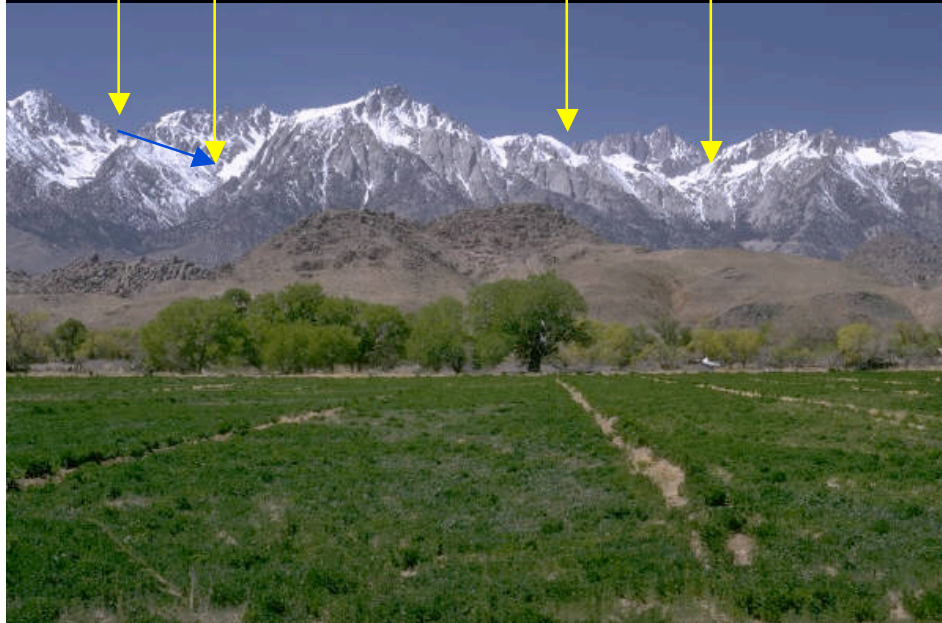
AdS minimum



Metastable dS minimum

$$V_{\text{KKLT}} = V_{\text{AdS}} + \frac{D}{\sigma^2}$$

1. The String Landscape approach



Type IIB String theory
compactified from 10
dimensions to 4.

Internal dimensions
stabilised by fluxes.

Many many vacua $\sim 10^{500}$!

Typical separation $\sim 10^{-500} \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between
vacua --> separate universes .

Anthropic : Galaxies require vacua $< 10^{-118} \Lambda_{pl}$ [Weinberg]

9/27/07

Most likely to find values not equal to zero!

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2. Λ from a self-tuning universe [Feng et al 2001].

Λ relaxes through nucleation of branes coupled to gauge potential, the particular branes depending on the compactification assumed. Need rapid relaxation from high energy scales but remains stable over age of universe today.

Leads to constraint $M_{\text{SUSY}}^2 \leq (10^{-3} \text{ eV})(M_{\text{Planck}})$

3. Relaxation of Λ [Kachru et al 2000, Arkani Hamad et al 2000].

Relies on presence of extra dimension to remove the gravitational effect of the vacuum energy.

3 brane solns in 5D eff theories leads to standard model vacuum energy warping the higher dimensional spacetime while preserving 4D flatness with no cosm constant. Quantum treatment of standard model implies result stable against quantum loops and changes to standard model couplings. Problems with singularities [Nilles et al²⁸]

5. Supersymmetric Large Extra Dims and Λ [Burgess et al, 2003-2006].

Solutions to 6D Supergravity

In more than 4D, the 4D vacuum energy can curve the extra dimensions instead of the observed 4 dimensions [Carroll and Guica; Aghababaie et al]

Proposal: Physics is 6D above 10^{-2} eV scale with supersymmetric bulk. We live in 4D brane with 2 extra dim.

Integrate out brane physics leads to large 4D vacuum energy, but it is localised in extra dimensions.

Integrate out classical contributions in bulk and find tensions cancel between bulk and brane.

Static and time dependent solutions exist, most of them runaway with rapid growing or shrinking dimensions.

Albrecht-Skiordis type quintessence evolution leads to late time acceleration and testable predictions.

6. Anthropic selection of Λ [Weinberg, Linde, Vilenkin, Efstathiou ...].

Weinberg pointed out that once Λ dominates energy density, structure formation stops because density perturbations cease to grow. Need structure formation to complete before this otherwise no observers today. Leads to

$$\rho_{\Lambda} < 500\rho_m^{(0)}$$

Two orders of magnitude out.

What if Λ differs in different parts of universe? [Efstathiou et al (1990) , Garriga and Vilenkin (2000)].

Intro conditional prob density

$$d\mathcal{P}(\rho_{\Lambda}) = \mathcal{P}_{*}(\rho_{\Lambda})n_G(\rho_{\Lambda})d\rho_{\Lambda}$$

$$n_G(\rho_{\Lambda})$$

Ave number of galaxies that can form per unit vol

$$\mathcal{P}_{*}(\rho_{\Lambda})$$

A Priori probability density distribution

For a flat a priori probability density distribution it has been shown that $\mathcal{P}(\rho_\Lambda)$ peaks around

$$\rho_{\text{vac}} \sim \delta\rho_m^{(0)} \quad [\text{Martel et al (1998)}]$$

Two important aspects to Anthropic argument:

1. Prediction of a priori probability
2. Assuming Λ takes on diff values in diff parts of universe.

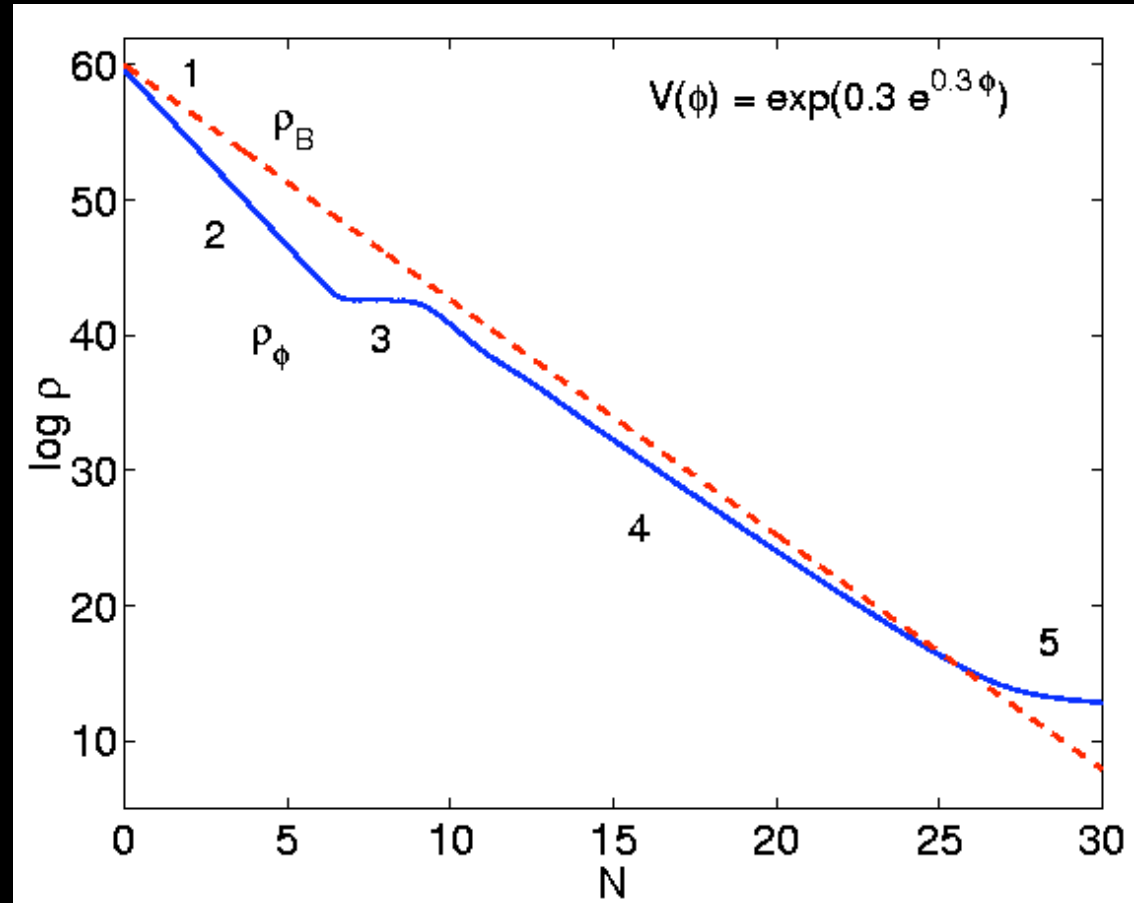
How are we going to determine the a priori probability?

See also [Garriga and Vilenkin (2000), Linde (2007), Bousso et al (2007)...]

Slowly rolling scalar fields

Quintessence - Generic behaviour

1. PE \rightarrow KE
2. KE dom scalar field energy den.
3. Const field.
4. Attractor solution: almost const ratio KE/PE.
5. PE dom.



Tracker solutions

Wetterich,

Peebles and Ratra,

Zlatev, Wang and Steinhardt

Scalar field:

$$\phi: \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi); \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

EoM:

$$\dot{H} = -\frac{\kappa^2}{2} (\dot{\phi}^2 + \gamma \rho_B)$$

$$\dot{\rho}_B = -3\gamma H \rho_B$$

$$\ddot{\phi} = -3H \dot{\phi} - \frac{dV}{d\phi}$$

+ constraint:

$$H^2 = \frac{\kappa^2}{3} (\rho_\phi + \rho_B)$$

Intro:

$$x = \frac{\kappa \dot{\phi}}{\sqrt{6}H}$$

$$y = \frac{\kappa \sqrt{V}}{\sqrt{3}H}$$

$$\lambda \equiv \frac{-1}{\kappa V} \frac{dV}{d\phi}$$

$$\Gamma - 1 \equiv \frac{d}{d\phi} \left(\frac{1}{\kappa \lambda} \right)$$

Eff eqn of state:

$$\gamma_\phi = \frac{\dot{\phi}^2}{v + \frac{\dot{\phi}^2}{2}} = \frac{2x^2}{x^2 + y^2}$$

$$\Omega_\phi = \frac{\kappa^2 \rho_\phi}{3H^2} = x^2 + y^2$$

Friedmann eqns and fluid eqns become:

$$x' = -3x + \lambda \sqrt{\frac{3}{2}} y^2 + \frac{3}{2} x [2x^2 + \gamma (1 - x^2 - y^2)]$$

$$y' = -\lambda \sqrt{\frac{3}{2}} xy + \frac{3}{2} y [2x^2 + \gamma (1 - x^2 - y^2)]$$

$$\lambda' = -\sqrt{6} \lambda^2 (\Gamma - 1)$$

$$\frac{\kappa^2 \rho_\gamma}{3H^2} + x^2 + y^2 = 1$$

where

$$' \equiv d / d(\ln a)$$

Note: $0 \leq \gamma_\phi \leq 2 : 0 \leq \Omega_\phi \leq 1$

Scaling solutions: ($\dot{x} = \dot{y} = 0$)

$$V = V_0 e^{-\lambda \kappa \phi}$$

No:	x_c	y_c	Existance	Stability	Ω_ϕ	γ_ϕ
1	0	0	$\forall \lambda, \gamma$	SP: $0 < \gamma$ SN: $\gamma = 0$	0	Undefined
2a	1	0	$\forall \lambda, \gamma$	UN: $\lambda < \sqrt{6}$ SP: $\lambda > \sqrt{6}$	1	2
2b	-1	0	$\forall \lambda, \gamma$	UN: $\lambda > -\sqrt{6}$ SP: $\lambda < -\sqrt{6}$	1	2
3	$\frac{\lambda}{\sqrt{6}}$	$\left(1 - \frac{\lambda^2}{6}\right)^{1/2}$	$\lambda^2 \leq 6$	SP: $3\gamma < \lambda^2 < 6$ SN: $\lambda^2 < 3\gamma$	1	$\frac{\lambda^2}{3}$
4	$\left(\frac{3}{2}\right)^{1/2} \frac{\gamma}{\lambda}$	$\left[\frac{3(2-\gamma)\gamma}{2\lambda^2}\right]^{1/2}$	$\lambda^2 \geq 3\gamma$	SN: $3\gamma < \lambda^2 < \frac{24\gamma^2}{9\gamma-2}$ SS: $\lambda^2 > \frac{24\gamma^2}{9\gamma-2}$	$\frac{3\gamma}{\lambda^2}$	γ

Late time attractor is scalar field dominated

$$\lambda^2 \leq 6$$

Field mimics background fluid. $\lambda^2 \geq 3\gamma$

Nucleosynthesis bound $\rightarrow \lambda^2 > 20$

Original Quintessence model

Peebles and Ratra;

Zlatev, Wang and Steinhardt

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha}$$

$$\lambda = \frac{-\alpha}{\kappa\phi}$$

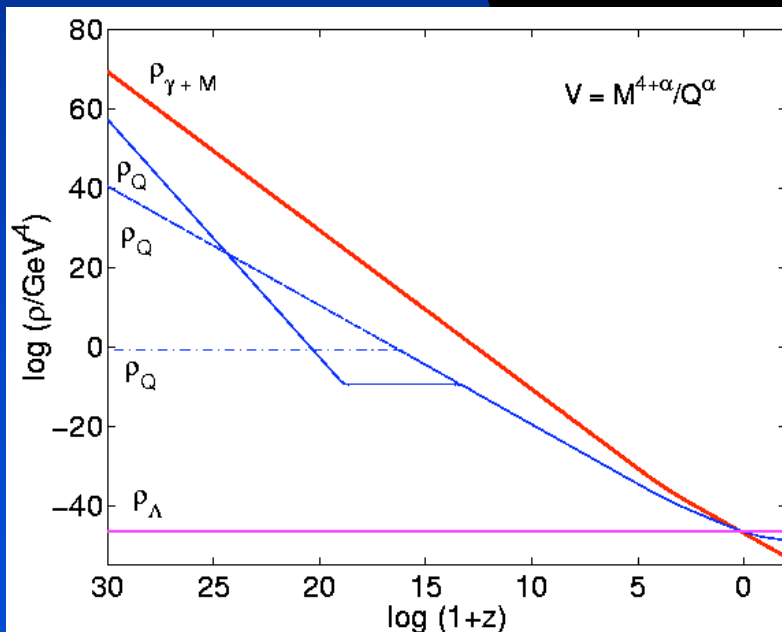
$$\Gamma - 1 \equiv \frac{1}{\alpha}$$

Find:

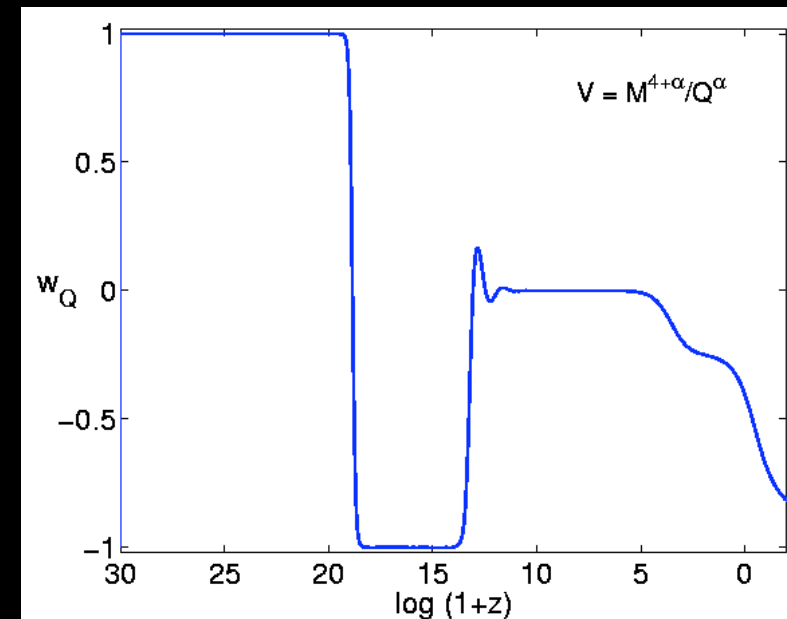
$$\phi = \phi_i \left(\frac{a}{a_i} \right)^{\frac{3(1+w_B)}{(2+\alpha)}}$$

and

$$w_\phi = \frac{\alpha w_B - 2}{2 + \alpha}$$



$$\alpha = 6$$



Fine Tuning in Quintessence

Need to match energy density in Quintessence field to current critical energy density.

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha}$$

$$\rho_\Lambda \leq \frac{H_0^2}{\kappa^2} \approx 10^{-47} \text{ GeV}^4$$

Find: $y_c^2 = \frac{\kappa^2 V}{3H^2} \propto \kappa^2 \phi^2$

so: $H^2 = \frac{V}{\phi^2} \propto \kappa^2 \rho_\phi \Rightarrow \phi_0 \approx M_{pl}$

Hence: $M = \left[\rho_\phi^0 M_{pl}^\alpha \right]^{1/4+\alpha} \Rightarrow \alpha = 2; M = 1 \text{ GeV}$

A few models

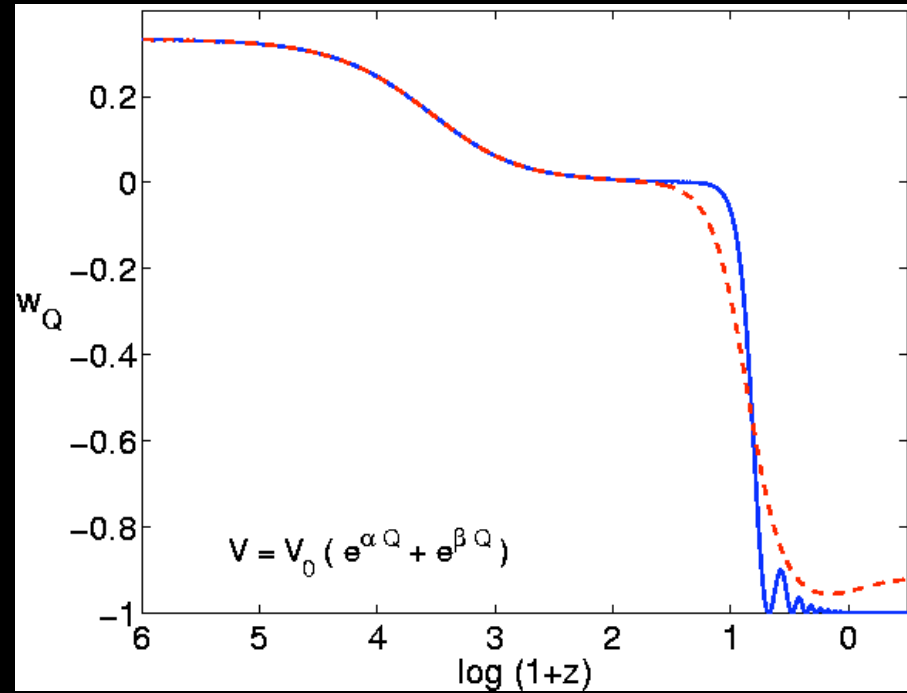
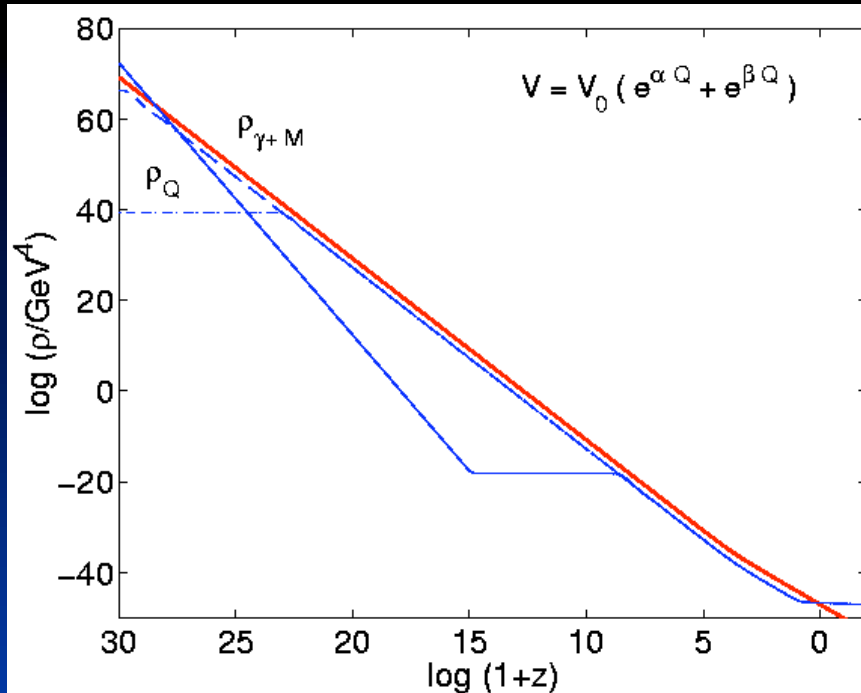
1. Inverse polynomial – found in SUSY QCD - Binetruy
2. Multiple exponential potentials – SUGRA and String compactification.

$$\begin{aligned} V(\phi) &= V_1 + V_2 \\ &= V_{01} e^{-\kappa\lambda_1\phi} + V_{02} e^{-\kappa\lambda_2\phi} \end{aligned}$$

Barreiro, EC,
Nunes

Enters two scaling regimes depends on lambda, one tracking radiation and matter, second one dominating at end. Must ensure do not violate nucleosynthesis constraints.

$$\alpha = 20; \beta = 0.5$$



Scaling for wide range of i.c.

Fine tuning: $V_0 \approx \rho_\phi \approx 10^{-47} \text{ GeV}^4 \approx (10^{-3} \text{ eV})^4$

Mass:

$$m \approx \sqrt{\frac{V_0}{M_{\text{pl}}^2}} \approx 10^{-33} \text{ eV}$$

Fifth
force !

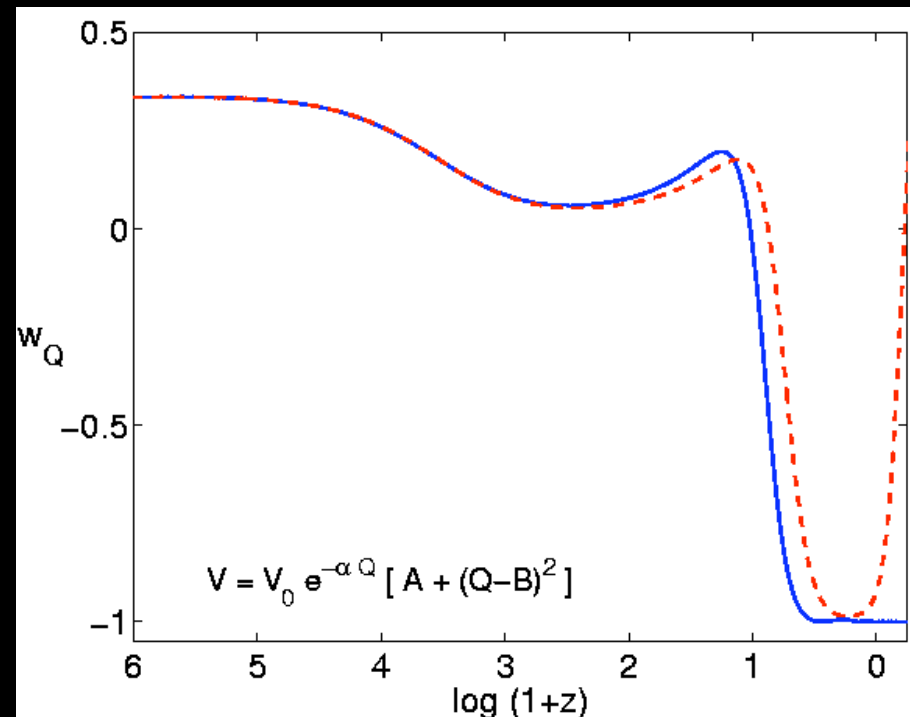
3. Albrecht-Skordis model — Albrecht and Skordis

$$V(\phi) = V_0 e^{-\alpha\kappa\phi} \left[A + (\kappa\phi - B)^2 \right]$$

-- Brane models

Early times: exp
dominates and scales as
rad or matter.

Field gets trapped in
local minima and univ
accelerates



Fine tuned as in previous cases.

4. Quintessential Inflation – Peebles and Vilenkin

Same field provides both initial inflaton
and today's Quintessence – not tracker.

$$V(\phi) = \lambda(\phi^4 + M^4) \quad \text{for } \phi < 0$$
$$= \frac{\lambda M^4}{1 + (\phi/M)^\alpha} \quad \text{for } \phi \geq 0$$

Reheating at end of inflation from grav particle production

Ford

Avoids need for minima in inflaton potential

$$\lambda = 10^{-14} : \Omega_{\phi_0} = 0.7 \Rightarrow \alpha = 4 ; M = 10^5 \text{ GeV},$$

9/27/07 Need to be careful do not overproduce grav waves.

5. Supergravity inspired models — Brax and Martin; Choi; EC, Nunes, Rosati; ...

$$W = \Lambda^{3+\alpha} \phi^{-\alpha}$$

$$K = \phi\phi^*$$

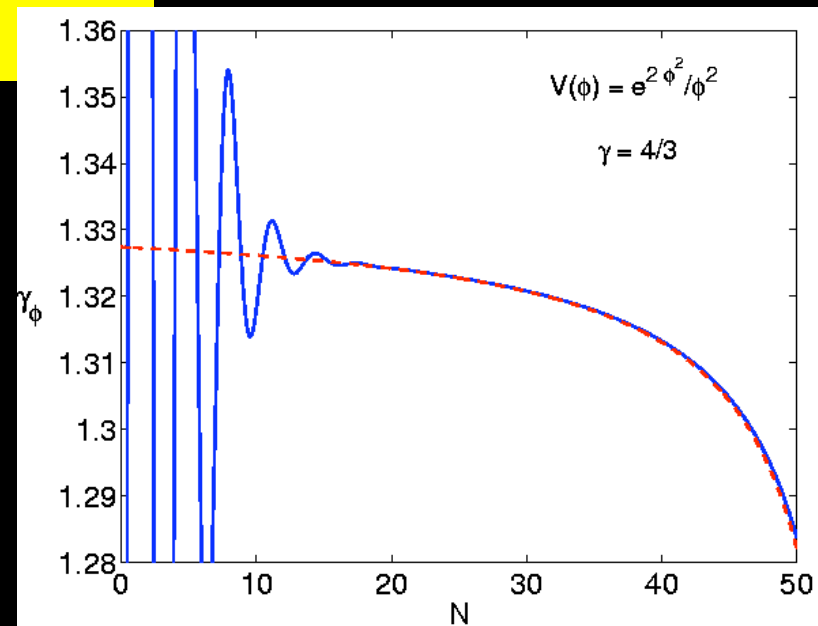
$$\text{If } \langle W \rangle = 0 \Rightarrow V(\phi) = \frac{\Lambda^{6+2\alpha}}{\phi^{2\alpha+2}} e^{\frac{\kappa^2}{2}\phi^2}$$

$$\alpha = 11 \Rightarrow w_{\phi_0} = -0.8$$

Issues over flatness of
potential — Lyth and Kolda

9/27/07

Many more models!!



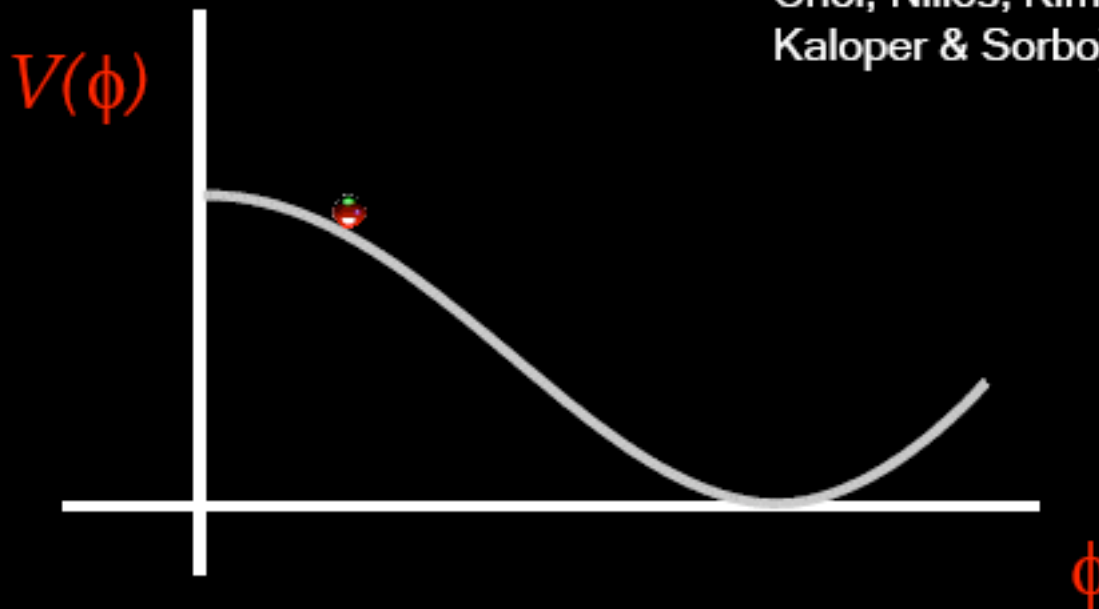
Particle physics inspired models?

Pseudo-Goldstone Bosons -- approx sym $\phi \rightarrow \phi + \text{const.}$

Leads to naturally small masses, naturally small couplings

[Hill, Freiman, et al;
Choi; Nilles; Kim;
Kaloper & Sorbo]

Barbieri et al



$$V(\phi) = \mu^4 [1 + \cos(\phi)]$$

Quintessential Axion -- Kim and Nilles

Linear combination of two axions together through hidden sector supergravity breaking.

Light CDM axion (solve strong CP problem) with decay const through hidden sector squark condensation:

$$F_a \sim 10^{12} \text{GeV}$$

Quintaxion (dark energy) with decay const as expected for model independent axion of string theory:

$$F_q \sim 10^{18} \text{GeV}$$

Model works because of similarities in mass scales:

Scale of susy breaking and scale of QCD axion.

Scale of vacuum energy and mass of QCD axion.

$$\lambda^4 \sim (0.003 \text{eV})^4$$

Potential for quintaxion remains very flat, because of smallness of hidden sector quark masses, ideal for Quintessence. Quintessence mass protected through existence of global symmetry associated with pseudo Nambu-Goldstone boson.

K-essence v Quintessence

K-essence -- scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model? -- Armendariz-Picon, Mukhanov, Steinhardt

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form.

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{16\pi G} R + K(\phi) \tilde{p}(X) \right]$$

$$K(\phi) > 0, \quad X = \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$$

Eqn of state

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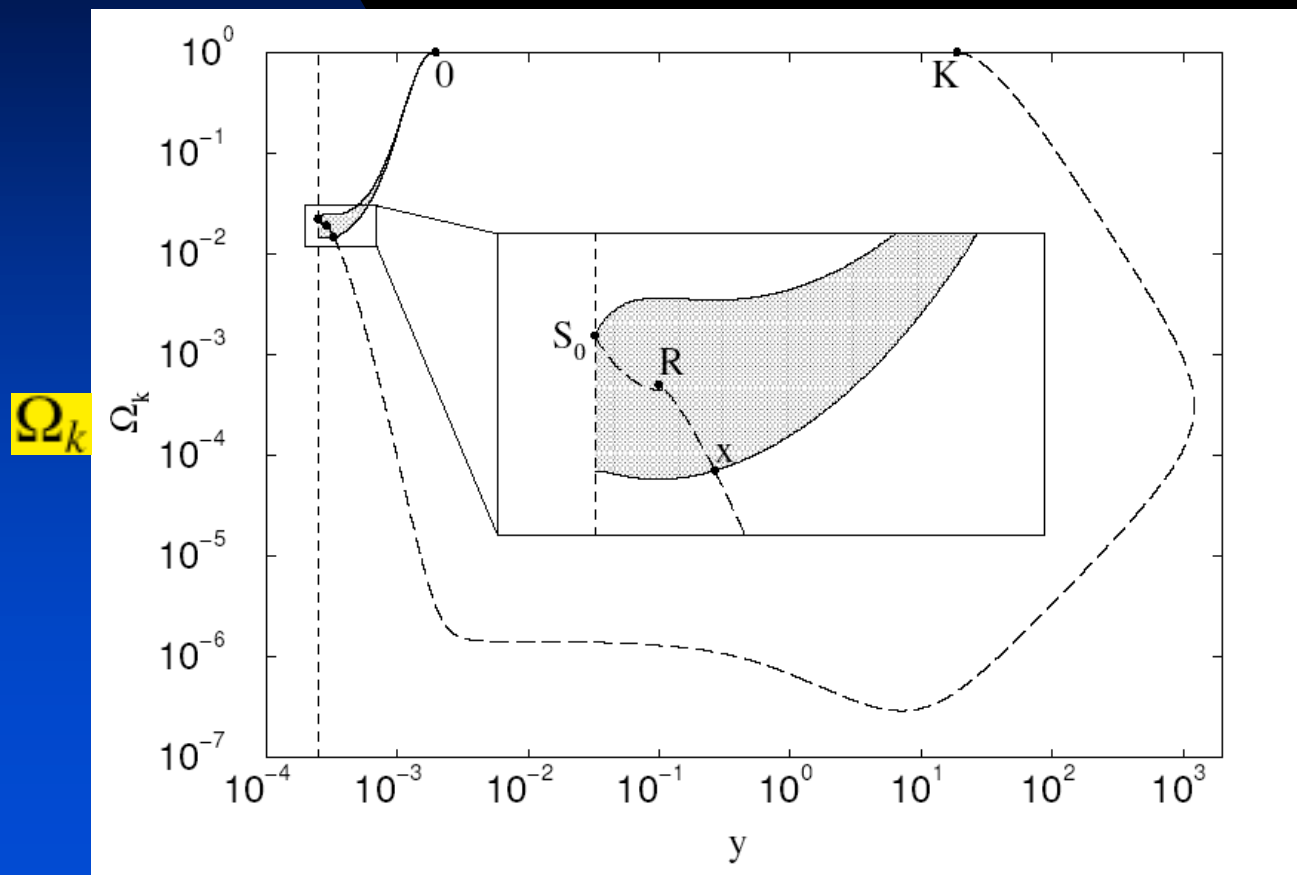
$$w_k = \frac{\tilde{p}(X)}{\tilde{\epsilon}(X)} = \frac{\tilde{p}(X)}{2X\tilde{p}'(X) - \tilde{p}(X)}$$

can be < -1

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Fine tuning in K-essence as well: -- Malquarti, EJC, Liddle

Not so clear that K-essence solves the coincidence problem. The basin of attraction into the regime of tracker solutions is small compared to those where it immediately goes into K-essence domination.



Shaded region is basin of attraction for stable tracker solution at point R. All other trajectories go to K-essence dom at point K.

Based on K-essence model astro-ph/0004134, Armendariz-Picon et al.

Dark energy from Tachyon fields [Sen (2002), Garousi (2002), Gibbons (2002) ...]

Introduced by Sen as a way of understanding the decay of D-branes, it has been noted that a rolling tachyon has an equation of state which varies between -1 and 0. Difficult to use it to have early Inflation but possible to have late time acceleration.

Tachyon on non BPS D3 brane:

$$S = - \int d^4x V(\phi) \sqrt{-\det(g_{ab} + \partial_a \phi \partial_b \phi)}$$

$$V(\phi) = \frac{V_0}{\cosh(\phi/\phi_0)}$$

Density and pressure and EOM:

$$\rho = -T_0^0 = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}},$$

$$p = T_i^i = -V(\phi) \sqrt{1 - \dot{\phi}^2}$$

$$H^2 = \frac{8\pi G V(\phi)}{3\sqrt{1 - \dot{\phi}^2}},$$

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + \frac{1}{V} \frac{dV}{d\phi} = 0.$$

Accn:
$$\frac{\ddot{a}}{a} = \frac{8\pi G V(\phi)}{3\sqrt{1 - \dot{\phi}^2}} \left(1 - \frac{3}{2}\dot{\phi}^2\right)$$

Accn for:
$$\dot{\phi}^2 < 2/3.$$

Eqn of state:

$$w_\phi = \frac{p}{\rho} = \dot{\phi}^2 - 1$$

Note, indep of steepness of potential, eos varies between 0 and -1

Phantom fields [Caldwell (2002) ...]

The data does not rule out $w < -1$. Can not accommodate in standard quintessence models but can by allowing negative kinetic energy for scalar field (amongst other approaches). Can arise from two time models in Type IIA strings, or low energy limit of F-theory in 12D Type IIB action.

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} (\nabla\phi)^2 - V(\phi) \right]$$

leads to

$$w_\phi = \frac{p}{\rho} = \frac{\dot{\phi}^2 + 2V(\phi)}{\dot{\phi}^2 - 2V(\phi)}$$

$$w_\phi < -1 \text{ for } \dot{\phi}^2/2 < V(\phi).$$

Super
inflationary soln

$$a(t) = (t_s - t)^{\frac{2}{3(1+w)}}$$

$$H = \frac{n}{t_s - t}, \quad n = -\frac{2}{3(1+w)} > 0,$$

$$R = 6 \left(2H^2 + \dot{H} \right) = \frac{6n(2n+1)}{(t_s - t)^2}.$$

Big Rip Singularity as $t \rightarrow t_s$

Depending on potential can avoid Big Rip but concerns over UV quantum instabilities. Vacuum unstable against production of ghosts and normal (+ve energy fields) [Carroll et al(2002), Cline et al (2004)]

Chameleon fields [Khoury and Weltman (2003) ...]

Key idea: in order to avoid fifth force type constraints on Quintessence models, why not have a situation where the mass of the field depends on the local matter density, so it is massive in high density regions and light ($m \sim H$) in low density regions (cosmological scales).

In that way can explain dark energy without violating solar system bounds.

Mass Varying Neutrino Models (MaVaNs). [Hung;Li et al; Fardon et al]

Coincidence ? $\rho_\Lambda \sim \Delta m_\nu^2(\text{solar}) \sim (10^{-3})^4 \text{eV}^4$

Perhaps neutrinos coupled to dark energy with a mass depending on a scalar field -- acceleron

Field has instantaneous min which varies slowly as function of neutrino density. It can be heavy relative to Hubble rate (unlike standard Quintessence).

Eff pot for MaVaNs: $V = n_\nu m_\nu(\mathcal{A}) + V_0(\mathcal{A})$ with: $n_\nu = -\frac{\partial V_0}{\partial m_\nu}$

EOS for system (ignoring KE of acceleron):

$$w = \frac{p}{\rho} = -1 + \frac{n_\nu m_\nu}{V}$$

$$w \sim -1 \text{ for } n_\nu m_\nu \ll V_0$$

Many authors studied cosmology -- interesting model, example of coupled dark energy scenarios [Amendola]

Chaplygin gases -- acceleration by changing the equation of state of exotic background fluid rather than using a scalar field potential. [Kamenshchik, Moshella, Pasquier 2001]

$$p = -\frac{A}{\rho}$$

Sub in energy-momentum conservation

$$\rho = \sqrt{A + \frac{B}{a^6}}$$

Interpolates: dust dom --> De Sitter phase via stiff fluid

$$\rho = \sqrt{B}a^{-3}$$

$$p = -\rho$$

$$p = \rho$$

Representation in terms of generalised d-branes evolving in (d+1,1) dimensional spacetime [Bento et al, 2002]

Nice feature -- does not introduce new scalar field. Provides way of unifying dark matter and dark energy under one umbrella. (Note can write it as a potential if you want)

Need to understand ways of testing it observationally. Must link LSS and current acceleration.

Acceleration from new Gravitational Physics? Starobinski 1980, Carroll et al 2003

$$S = \frac{M_{\text{P}}^2}{2} \int d^4x \sqrt{-g} \left(R - \frac{\mu^4}{R} \right) + \int d^4x \sqrt{-g} \mathcal{L}_M$$

Modify Einstein

Const curv vac
solutions:

$$\nabla_{\mu} R = 0, \rightarrow R = \pm \sqrt{3} \mu^2$$

de Sitter or Anti
de Sitter

Transform to EH
action:

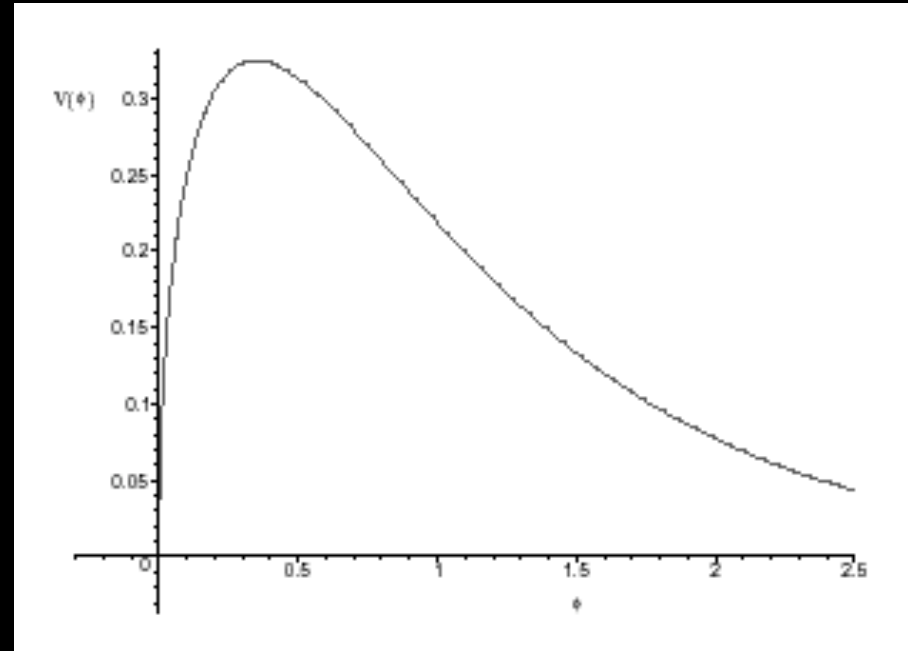
$$\tilde{g}_{\mu\nu} = p(\phi) g_{\mu\nu}, \quad p \equiv \exp \left(\sqrt{\frac{2}{3}} \frac{\phi}{M_{\text{P}}} \right) \equiv 1 + \frac{\mu^4}{R^2}$$

Scalar field min coupled to gravity and non minimally
coupled to matter fields with potential:

$$V(\phi) = \mu^2 M_{\text{P}}^2 \frac{\sqrt{p-1}}{p^2}$$

Cosmological solutions:

1. **Eternal de Sitter** - ϕ just reaches V_{\max} and stays there. Fine tuned and unstable.
2. **Power law inflation** -- ϕ overshoots V_{\max} , universe asymptotes with $w_{\text{DE}} = -2/3$.
3. **Future singularity**-- ϕ doesn't reach V_{\max} , and evolves back towards $\phi=0$.



Fine tuning needed so acceleration only recently: $\mu \sim 10^{-33} \text{eV}$

Also, any modification of Einstein-Hilbert action needs to be consistent with classic solar system tests of gravity. These models are not.

More general $f(R)$ models [Loads of people]

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{2\kappa^2} + \mathcal{L}_m \right] \quad \text{No } \Lambda$$

Usually $f(R)$ struggles to satisfy both solar system bounds on deviations from GR and late time acceleration. It brings in extra light degree of freedom --> fifth force constraints.

Get out clause: Make scalar dof massive in high density solar vicinity and hidden from solar system tests by chameleon mechanism.

Requires form for $f(R)$ where mass squared of scalar is large and positive at high curvature.

In fact has to look like a standard cosmological constant

Designer $f(R)$ models [Hu and Sawicki (2007)]

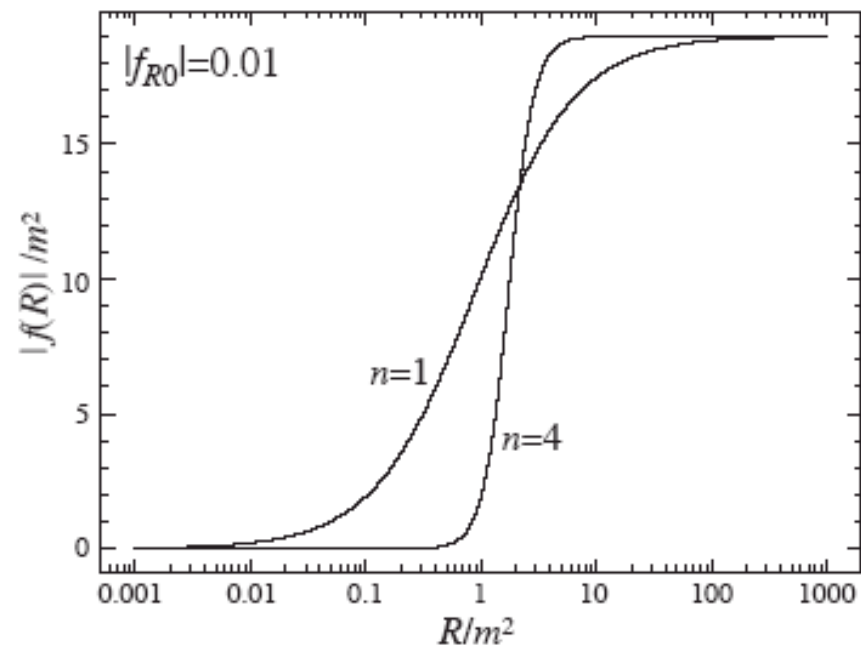
Construct a model to satisfy observational requirements:

1. Mimic LCDM at high z as required by CMB
2. Accelerate univ at low z
3. Include enough dof to allow for variety of low z phenomena
4. Include phenom of LCDM as limiting case.
5. Quantum corrections?

$$\lim_{R \rightarrow \infty} f(R) = \text{const.},$$
$$\lim_{R \rightarrow 0} f(R) = 0,$$

$$f(R) = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1},$$

$$f_{RR} \equiv \frac{d^2 f(R)}{dR^2} > 0$$



Modifications of Friedmann equation in 4D:

Write:

$$H^2 = \frac{8\pi}{3m_4^2} \rho L^2(\rho)$$

$$L(\rho) = 1$$

Standard Friedmann

$$L(\rho) = \sqrt{1 + \frac{\rho}{2\sigma}}; \quad \sigma^{1/4} > 2.0 \text{ MeV}$$

Randall-Sundrum II: co-dimension one brane, embedded in 5D AdS space.

$$L(\rho) = \sqrt{1 - \frac{\rho}{2|\sigma|}}; \quad \sigma < 0$$

Shtanov-Sahni: co-dimension one brane, negative tension embedded in 5D conformally flat Einstein space where signature of 5th dim is timelike

$$L(\rho) = \sqrt{1 + A\rho^n}; \quad n < -1/3$$

Cardassian: only matter present --> late time acceleration. Freese & Lewis

$$L = \frac{1}{\sqrt{B\rho}} \left[\mp 1 + \sqrt{1 + B\rho} \right]; \quad B \equiv \frac{8\pi m_4^2}{3m_5^6}$$

Dvali-Gabadadze-Porrati: 3-brane embedded in flat 5D Minkowski with Ricci scalar term included in brane action. Bulk empty.

DGP model:
$$H^2 \pm \frac{H}{r_0} = \frac{8\pi}{3m_4^2} \rho; \quad r_0 \equiv m_4^2 / (2m_5^3)$$

Gravity like 4D gravity on short scales, but propagates into bulk on large scales. Induces corrections to Friedmann eqn, characterised by length r_0 .

Two ways of embedding brane in bulk given by \pm

-sign --> self accelerating phase (deS) for any decreasing energy density -- ($w \rightarrow -1$)

+sign --> Minkowski phase. Brane extrinsically curved so that for $H \sim r_0^{-1}$ gravity screens the effects of the brane energy momentum

Consider our univ (brane) with homogeneous dust and lambda:

$$H^2 + \frac{H}{r_0} = \frac{8\pi}{3m_4^2} \rho_M(t) + \lambda$$

Infer effective dark energy :

$$\frac{8\pi}{3m_4^2} \rho_{DE}^{eff}(t) = \lambda - \frac{H}{r_0}$$

Lue & Starkman

H decreases with time, effective dark energy increases! For DE domination $w_{eff} < -1$ (mimics effect of phantom energy).

As universe evolves, screening term becomes weaker and eff dark energy density appears to increase

Degree of growth modulated by r_0 . As $r_0 \rightarrow \infty$ recover standard Λ CDM.

For any cut off r_0 , $w_{eff} \rightarrow -1$ with time and pure Λ cosmology recovered in future.

Possible concern over entering strong coupling regime for large distances.

Self acceleration branch contains ghost in spectrum for any value of
brane tension -- instability Charmousis et al 2006

How about no dark energy and no modification of gravity?

Kolb, Matarrese and Riotto

Idea: take perturbed Einstein equation

$$G_{00} + \delta G_{00} = 8\pi G(T_{00} + \delta T_{00})$$

Treat the averaged 00 component of the Einstein tensor as an effective energy density

$$H^2 = \frac{8\pi G}{3}(\rho + \delta\rho) + \frac{1}{3} \langle \delta G_{00} \rangle$$

Calculate to second order and check to see whether it acts as dark energy in magnitude and evolution.

In many ways the ideal solution, it is all down to perturbations on large scales due to backreaction effects - generated plenty of reaction saying it can't work.

Evidence for dynamical dark energy ?

1. Precision CMB anisotropies – lots of models currently compatible.
2. Combined LSS , SN1a and CMB data – tend to give $w_Q < -0.85 \rightarrow$ best fit remains cosmological constant.
3. Look for more SN1a – SNAP will find over 2000 at large redshift – can then start to constrain eqn of state.
4. Constraining eqn of state with SZ cluster surveys – compute number of clusters for given set of cosm parameters.
5. Baryon Acoustic Oscillations in the LSS as a probe of dark energy.
6. Reconstruct eqn of state from observation – offers hope of method indep of potentials.
7. Look for evidence in variation of fine structure constant.
8. Using Gravitational lensing to constrain w --Dark Energy Survey
9. Sandage Loeb test -- measuring quasar spectra at different redshift between $2 < z < 5$. [Corasaniti et al 2007]

Evolution of Fine Structure Constant

Olive and Pospelov

Non-trivial coupling to emg:

$$L_m = -\frac{1}{4} B_F(\phi) F_{\mu\nu} F^{\mu\nu}$$

Expand about current value of field:

$$B_F(\phi) = 1 + \zeta_F \phi + \frac{1}{2} \xi_F \phi^2$$

Bekenstein

Eff fine structure const depends on value of field

$$\alpha(\phi) = \frac{e_0^2}{4\pi B_F(\phi)}$$
$$\frac{\Delta\alpha}{\alpha} = \zeta_F \phi + \frac{1}{2} (\xi_F - 2\zeta_F^2) \phi^2$$

Claim from analysing quasar absorption spectra:

$$\frac{\Delta\alpha}{\alpha} (z = 0.5 - 3.5) \approx 10^{-5}$$

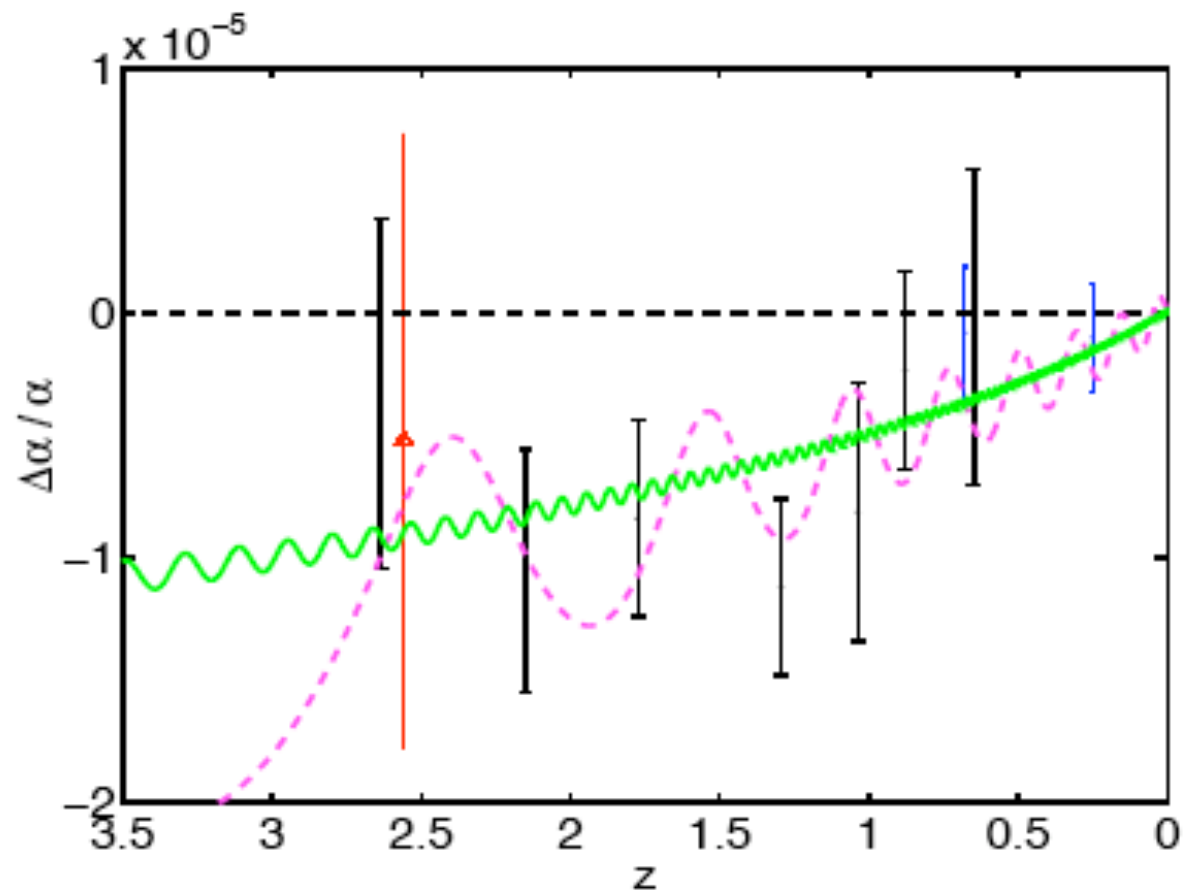
Webb et al

$$V = V_0 e^{-\lambda \kappa \phi}$$

Nunes

$\lambda = 100$ – solid

$\lambda = 10$ – dashed



A way of constraining the eqn of state?

Dynamical evolution of w ?

Weller and Albrecht; Kujat et al; Maor et al;
Gerke and Efstathiou, Kratochvil et al

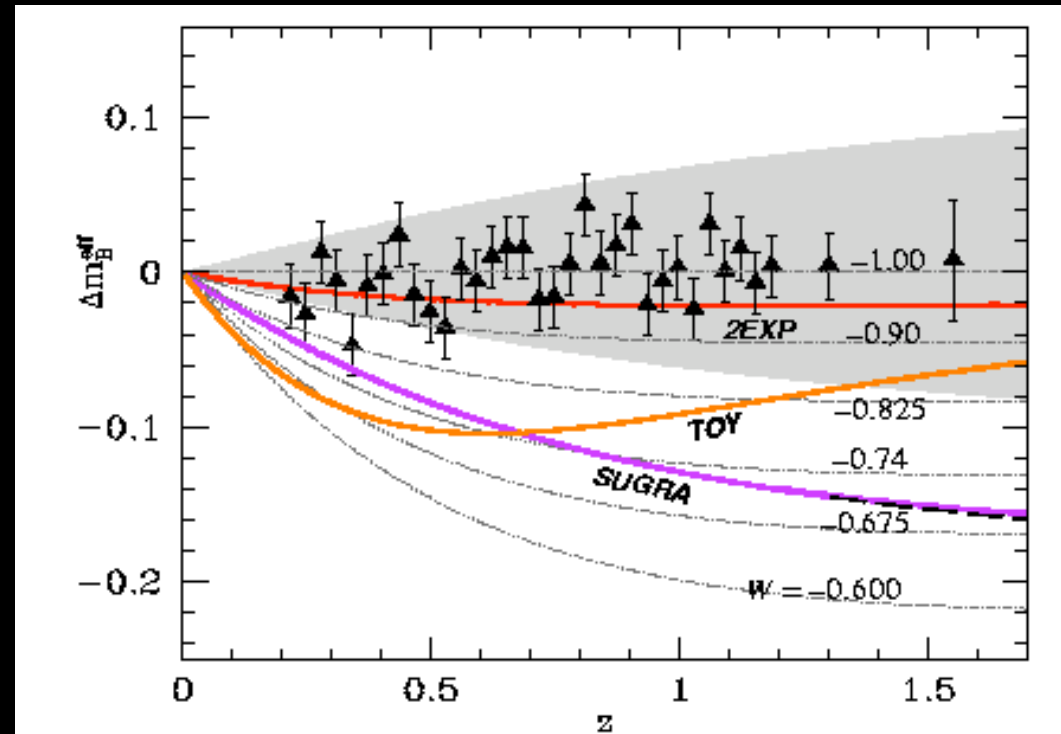
SNAP as a
discriminator

Write:

$$w(z) = \sum_{i=0}^N w_i z^i$$

or:

$$w(z) = \sum_{i=0}^N w_i \ln(1+z)^i$$



Evaluate magnitude difference for each
model and compare with Monte Carlo
simulated data sets.

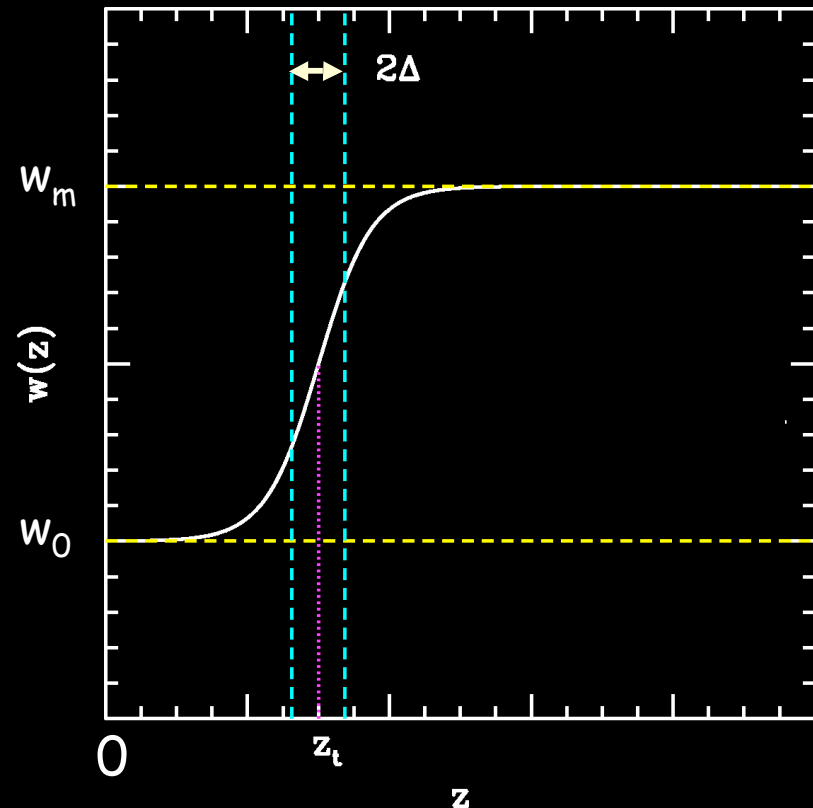
Modelling quintessence

Impose an equation of state $w(z)$ which captures the essential features of quintessence.

typical expectations:

- recent acceleration
→ $w_0 < -1/3$
- avoid fine tuning the initial energy density
→ $w_m > -1/3$
- there is a **transition** at a given redshift z_+ with a given width Δ .
- Λ corresponds to $w_0 = -1$ and either $w_m = -1$ or $z_+ \gg 1$.

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Strategy:

- compute predictions for many models with different parameters (ie H_0 , w_0 , w_m , n_s , t and the normalisation)
- compare with data sets (we use WMAP + SN-Ia)
- derive constraints on parameters (Markov-Chain Monte Carlo code with modified cmbfast)
- draw conclusions about the physical nature of the models.

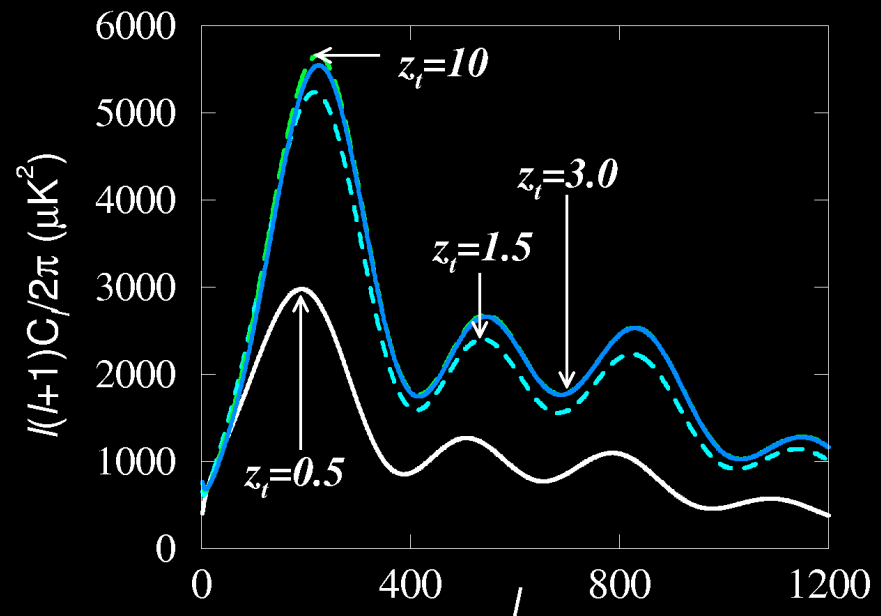
Kunz et al astro-ph/0307346; Corasaniti et al astro-ph/0406608

w(z) impact on the CMB through ISW

$$\left[\frac{\delta T(e)}{T} \right]_{SW} = \frac{1}{3} \Phi(ex_{ls}) + 2 \int_{\tau_{ls}}^{\tau_0} \frac{\partial \Phi(ex, \tau)}{\partial \tau} d\tau$$

rapid transition :

- late onset of expansion changes ISW effect which acts at large l
- peak lower after COBE normalisation



- Cosmic variance makes the effect hard to observe, especially for models with slowly varying equation of state.
- A data set which connects large and small angular scales is crucial for a correct normalisation → WMAP.

cosmological parameters --WMAP1

- limits slightly wider, but no clear difference
- **NO** new degeneracies!

$$\Omega_m = 0.29 \pm 0.04$$

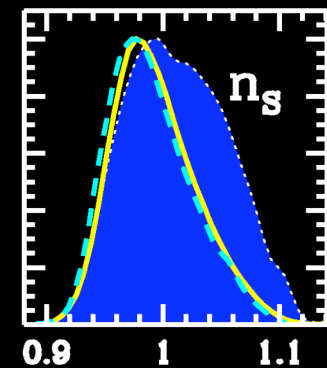
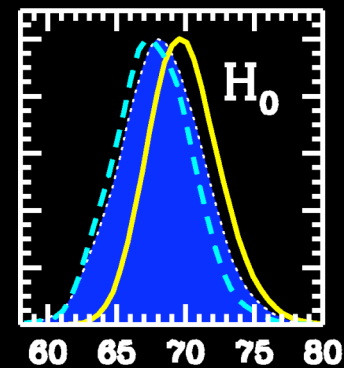
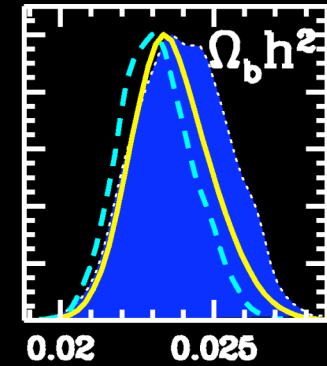
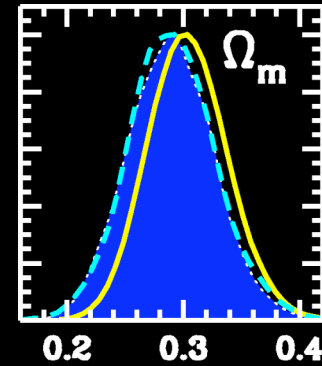
$$\Omega_b h^2 = 0.0240 \pm 0.0015$$

$$H_0 = 68 \pm 3$$

$$n_s = 1.01 \pm 0.04$$

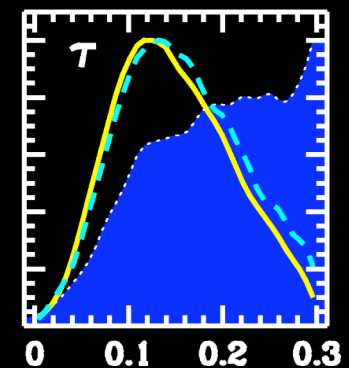
$$\tau = 0.19 \pm 0.07$$

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quintessence
with Ω_b prior

pure Λ CDM



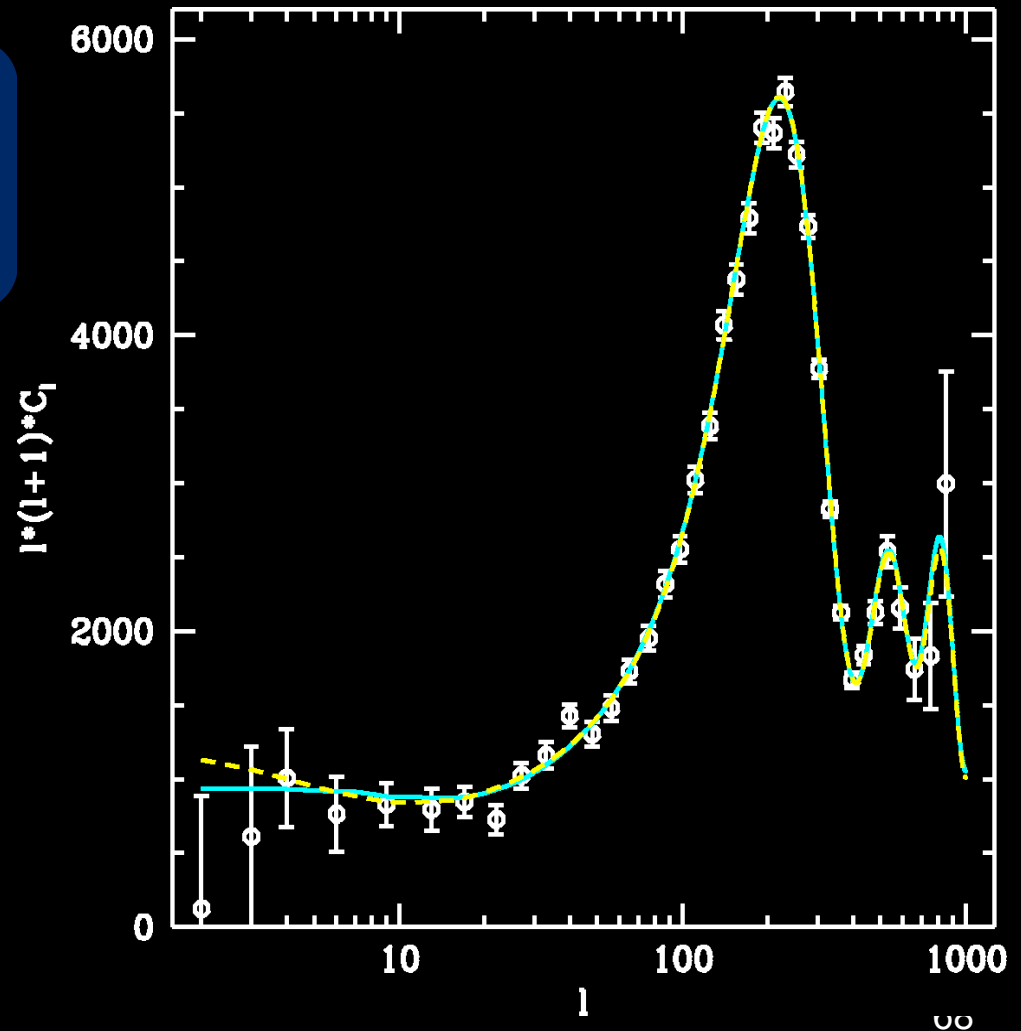
dark energy parameters

$w_0 < -0.80$ at 95% CL
 $z_t > 0.6$ (fast transitions)

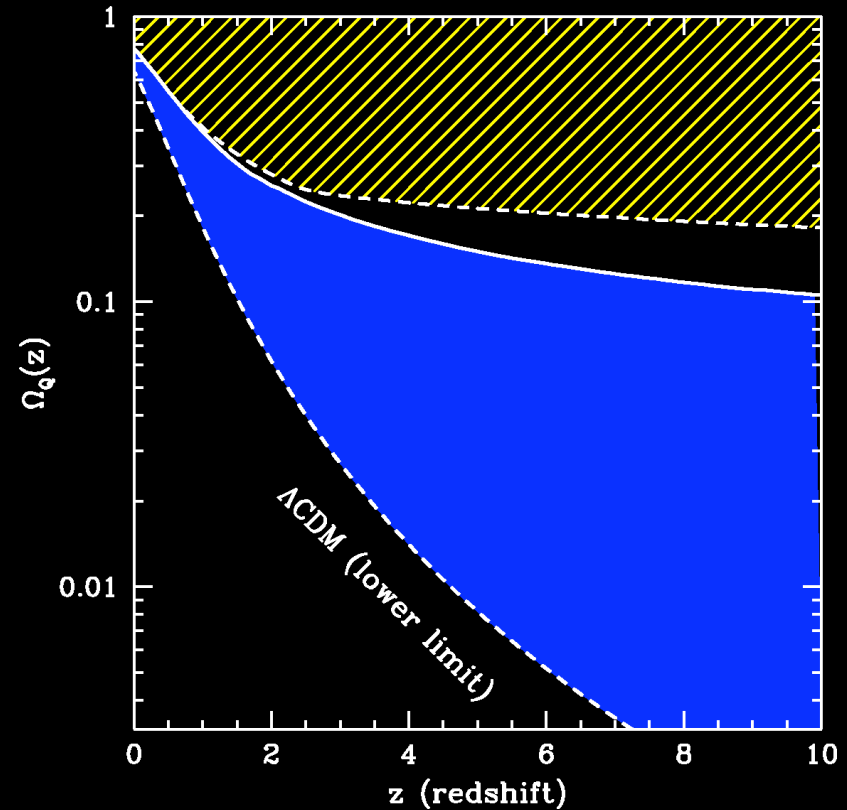
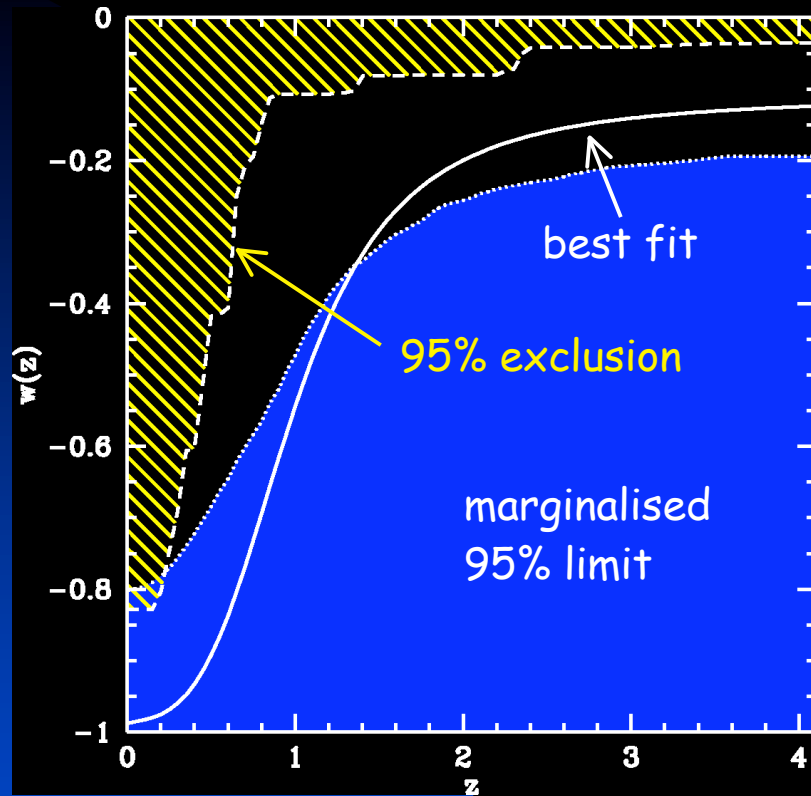
best-fit quintessence model:

- $w_0 = -1$
- $w_m = -0.13$
- $a_t = 0.5$ ($z_t = 1$)
- effective $\chi^2 = 1603$

best Λ CDM : $\chi^2 = 1606$



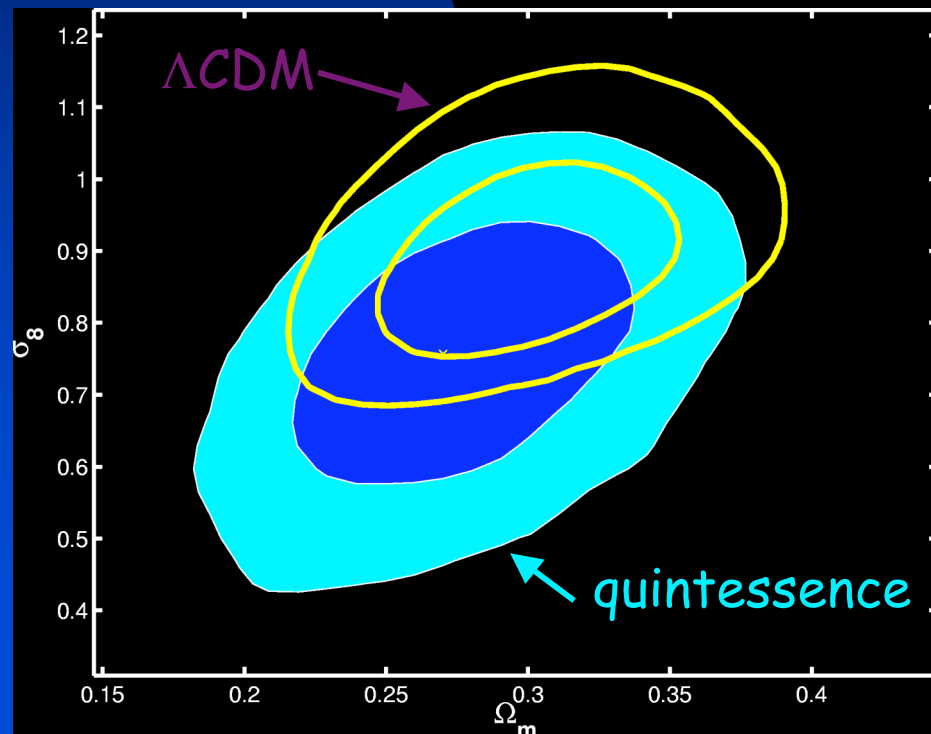
time behaviour of the DE



- really strong constraints on w only for $z < 0.2$

the effect on clustering

- the ISW changes the overall normalisation
 - this in turn changes the normalisation of the matter $P(k)$
 - we can detect this if we know the amplitude of $P(k)$ or σ_8
 - **BUT:** we can only observe **galaxies**
- we don't know σ_8 very well!



Do we need Dark Energy?

Hunt and
Sarkar (2007)

Attempts to describe universe without recourse to the fine tuned cosmological constant we appear to need.

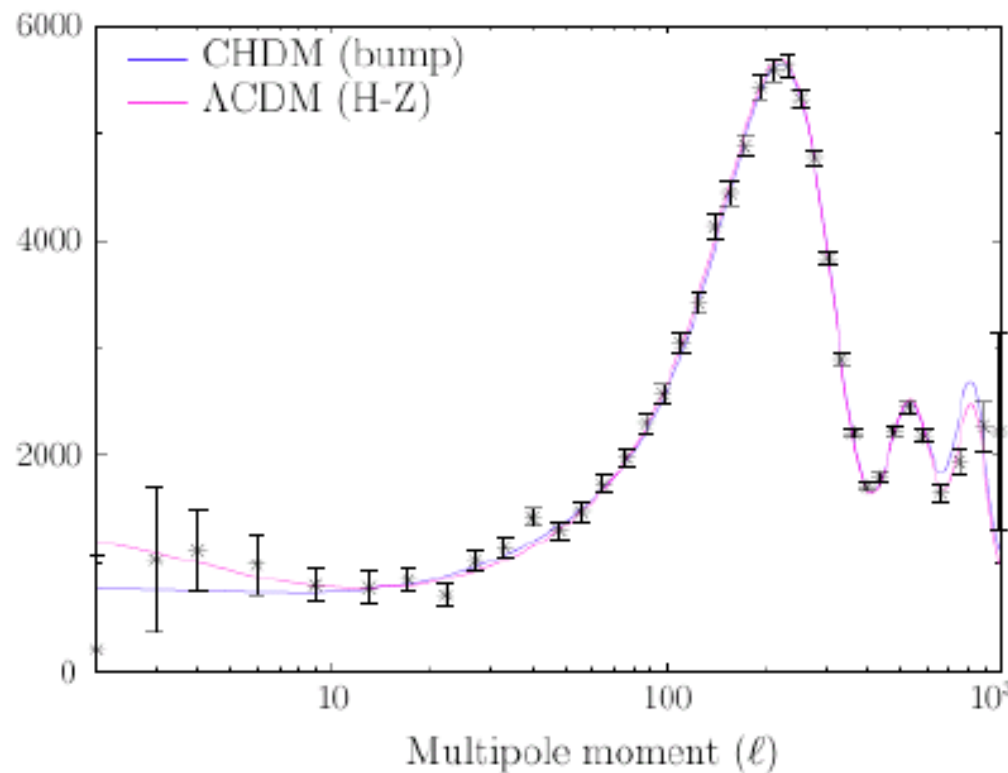
Allow for possibility we live in Inhomogeneous universe, inflation proceeds leading to features (bumps) in the primordial spectrum so that it is not scale free.

We could be living in a local void where Hubble flow is 30% faster than global rate.

Possible problem with obtaining observed baryon oscillations in power spectra.

Hunt and
Sarkar (2007)

The *WMAP* data can then be fitted
just as well with *no dark energy*
($\Omega_m = 1$, $\Omega_\Lambda = 0$, $h = 0.46$)



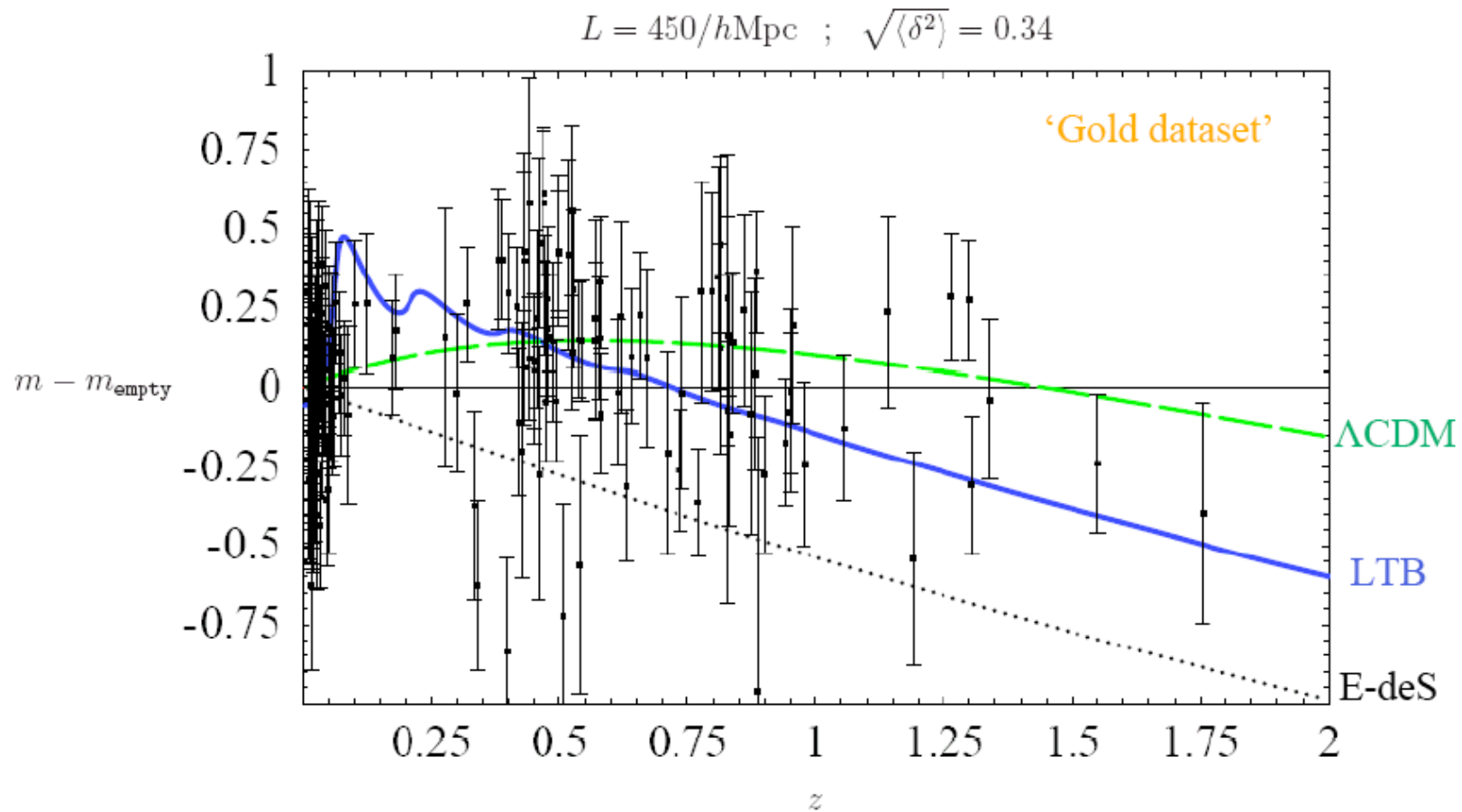
$h = 0.46$ is inconsistent with Hubble Key Project value ($h = 0.72 \pm 0.08$)
but is in fact *indicated* by direct (and much deeper) determinations
e.g. gravitational lens time delays ($h = 0.48 \pm 0.03$)

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May be we are in a void expanding faster than global rate

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Such a Lemaitr -Tolman-Bondi model may even explain the SNIa Hubble diagram *without* acceleration!



Biswas, Mansouri & Notari (2006)

The small-scale power would be excessive unless damped by free-streaming

But adding 3 ν s of mass 0.8 eV ($\Rightarrow \Omega_{\nu} \approx 0.14$) gives *good* match to large-scale structure

(note that $\Sigma m_{\nu} \approx 2.4$ eV – well above ‘*WMAP* bound’)

Summary

- Observations transforming field, especially CMBR and LSS. -- everything consistent with a pure cosmological constant.
- Why is the universe inflating today?
- Is $w = -1$, the cosmological constant? If not, then what value has it?
- Is $w(z)$ -- dynamical?
- New Gravitational Physics -- perhaps modifying Friedmann equation on large scales?
- Lots of models of dark energy but may yet prove too difficult to separate one from another such as cosmological const – need to try though!
- Perhaps we will only be able to determine it from anthropic arguments and not from fundamental theory.
- or -- could we all be wrong and we do not need a lambda term?