STANDARD MODEL and BEYOND: SUCCESSES and FAILURES of QFT

(Two lectures)

Lecture 1: Mass scales in particle physics - naturalness in QFT

Lecture 2: Renormalisable or non-renormalisable effective electroweak theory?

Lecture 3 (Graham Ross' suggestion): Failures

Hierachical mass scales in particle physics and naturalness

Proton mass and Planck scale

In QCD, the proton mass is determined by the confinement scale = the scale of quark-antiquark condensate = the scale of spontaneous chiral symmetry breaking

QCD with massless quarks is classically a scale invariant theory!

$$\mathcal{L} = -\frac{1}{4} G^{\alpha}_{\mu\nu} G^{\alpha\,\mu\nu} + \sum_f \bar{\psi}_f \, i \not{D} \, \psi_f$$

invariant under

$$x \to x' = e^{\varepsilon} x \qquad (\text{or} \quad p \to p' = e^{-\varepsilon} p \;)$$

But scale invariance is broken by the renormalisation procedure i.e. by the renormalisation scale

Take a dimensionless quantity, e.g. the strong coupling constant, which could depend on only one dimensionful kinematical variable Q

$$\alpha(Q^2)$$

Dimensional analysis tells us that in a scale invariant theory (no dimensionful parameters) such a quantity must be a constant

$$\alpha(Q^2) = \mathrm{const}$$

Pure dimensional analysis breaks down in QFT because our quantity also depends on the dimensionless free parameters of the theory, which must be taken from experiment – measured at some scale

$$\alpha(Q^2) = \alpha(Q^2, \alpha_0)$$

$$\alpha_0 = \alpha(Q_0^2)$$

One parameter ambiguity for $\alpha(Q^2)$ is reflected in the fact that the theory specifies the first derivative of $\alpha(Q^2)$ rather than $\alpha(Q^2)$ itself

$$\frac{d\alpha}{dt} = \beta(\alpha)$$

where $\quad Q = e^t Q_0$

$$\beta(\alpha) = (b/\pi)\,\alpha^2 + O(\alpha^3)$$

Integrating

$$\ln(Q/Q_0) = F(\alpha(Q)) - F(\alpha(Q_0))$$

$$F(x) = \int dx/\beta(x)$$

Choose Q_0 so that $F(\alpha(Q_0)) = 0$ and call it Λ :

$$\alpha(Q) = -\pi/b\ln Q/\Lambda$$

Hence $\alpha(\Lambda) = \infty$

$$\Lambda/Q = \exp\{-\pi/b\,\alpha(Q)\}$$

Given (e.g. measured) the strong coupling constant at some Q,
the theory predicts the confinement scale (proton mass!)

The mechanism is know as DIMENSIONAL TRANSMUTATION

IN ASYMPTOTICALLY FREE THEORY, IT EXPLAINS THE
HIERARCHY OF SCALES: CONFINEMENT SCALE (proton
mass) VERSUS UV CUT-OFF TO QCD

$K^0_L - K^0_S$ mass difference

$$\Delta M_K = 3 \times 10^{-12}\ \mathrm{MeV}$$

$$K^0_L = \frac{1}{\sqrt{2}}\left(K^0 - \bar{K}^0\right)$$

$$K^0_S = \frac{1}{\sqrt{2}}\left(K^0 + \bar{K}^0\right)$$

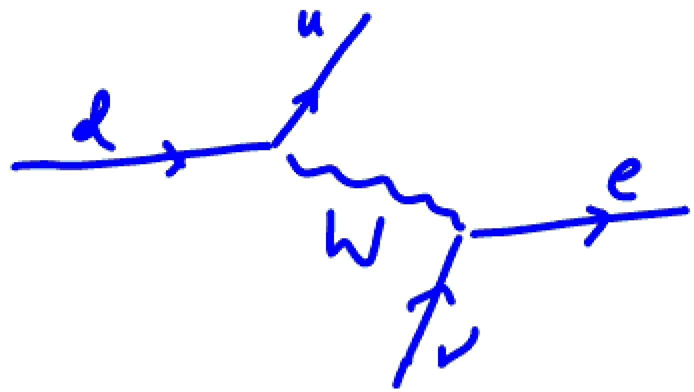
$$K^0 = K^0(d\bar{s})$$

$$\bar{K}^0 = \bar{K}^0(\bar{d}s)$$

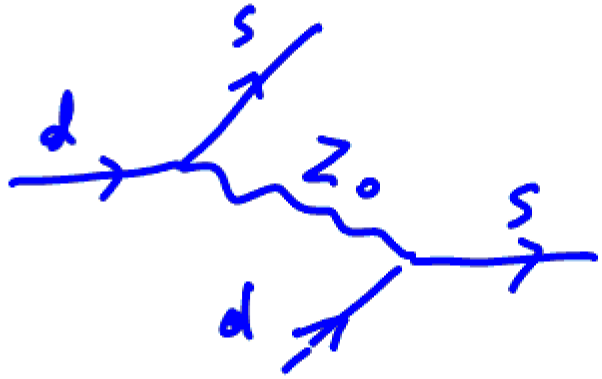
The mass difference between the kaon mass eigenstates is a good measure of the mixing between the charge conjugation eigenstates, caused by weak interaction

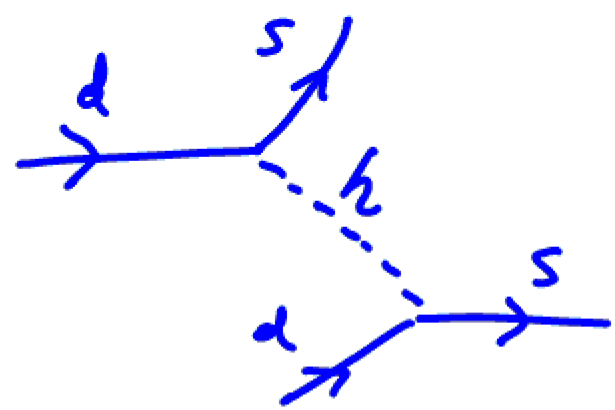
$$\langle K^0 | H_{weak} | \bar{K}^0 \rangle$$

The magnitude of the mass difference tells us that the weak mixing is strongly suppressed in comparison with the amplitudes for the charged current transitions like neutron decay into proton, electron and antineutrino neutron decay



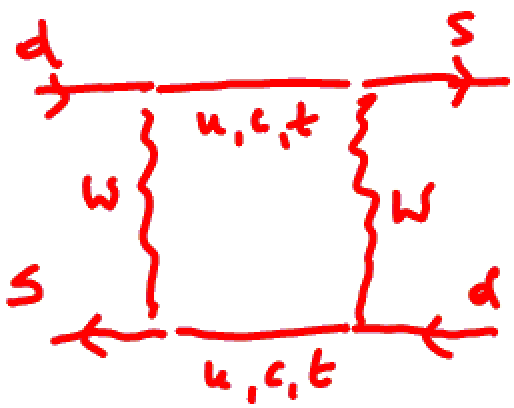
$\bar{K}^0 - K^0$ mixing ?





Such flavour conserving tree level diagrams are absent because of the structure of the theory (SU(2) quark and lepton doublets, only one Higgs doublet; remember: quark flavour (mass eigenstates) is defined by strong interactions and SU(2) doublets are in the electroweak basis; the two basis are related by a unitary transformation)

But what about loops?



$$\mathcal{L}_{eff} = C(\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma_\mu d_L)$$

Generically

$$C \sim \frac{\alpha^2}{M_W^2} \sim \alpha G_F$$

Factor 10^4 too much

Let's calculate the above diagram (plus similar but crossed and also with unphysical Goldstone boson exchange, if in covariant gauges).
The amplitude originating from them is finite and has dimension (mass)-2.

In the limit of vanishing external momenta, the contribution from the W- boxes can be expressed as a tree – level contribution from the following effective local interaction term:

$$\mathcal{L}_{eff} = \frac{1}{2}\left(\frac{e}{\sqrt{2}\sin\theta_W}\right)^4 \sum_{i,j=u,c,t} V_{is}^* V_{id} V_{js}^* V_{jd} \int \frac{d^4 q}{(2\pi)^4} \, *$$

$$* \frac{[\bar{s}_L \gamma_\mu (\not{q} + m_{q_i}) \gamma_\nu d_L][\bar{s}_L \gamma^\nu (\not{q} + m_{q_j}) \gamma^\mu d_L]}{(q^2 - m_{q_i}^2)(q^2 - m_{q_j}^2)(q^2 - M_W^2)^2}$$

The top quark contribution is suppressed by the smallness of the CKM matrix elements $V_{ts}^* V_{td}$. For the rest we get

$$A_{u,c} \approx \left(\frac{e}{\sqrt{2}\sin\theta_W}\right)^4 \frac{1}{M_W^2} \sum_{i,j=u,c} V_{is}^* V_{id} V_{js}^* V_{jd}^*$$

$$* \left[1 + O\left(\frac{m_{q_i}^2}{M_W^2}, \frac{m_{q_j}^2}{M_W^2}\right]\right.$$

$$\sim \alpha G_F \left[(V_{ts}^* V_{td})^2 + O\left(\sum_{u,c} V_{is}^* V_{id} V_{js}^* V_{jd} \frac{m_q^2}{M_W^2}\right)\right]$$

In the last line, unitarity of CKM matrix has been used

$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} = -V_{ts}^* V_{td}$$

We see that the generic result is suppressed

$$O(\alpha G_F) \rightarrow \begin{array}{l} O\left(\alpha G_F V_{ts}^* V_{td}\right) \\ O\left(\alpha G_F \frac{m_c^2}{M_W^2}\right) \end{array}$$

We get the necessary 10^{-4} suppression!

CONCLUSION: kaon mass difference is naturally small (no large cancellations) thanks to the symmetries of the SM and the pattern of quark masses and mixing.

It is one of the main challanges for physics beyond SM to preserve natural explanation of the suppresion of FCNC

NO LIGHT SCALARS IN NATURE EXCEPT FOR NAMBU - GOLDSTONE BOSONS (PIONS)

Indeed, light scalars are unnatural in QFT

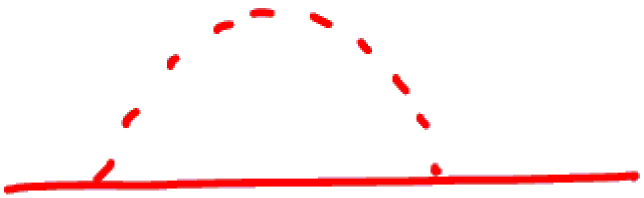
Quantum corrections to fermion and scalar masses

Simple example

$\mathcal{L} = \text{kin terms} - m^2 \varphi^2 - \tilde{m} \bar{\psi} \psi - \varphi \bar{\psi} \psi - \varphi^4$

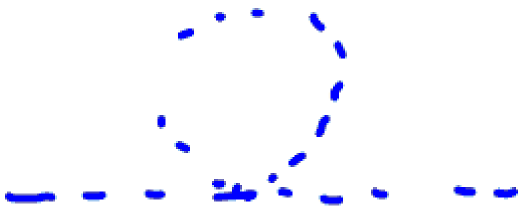
$$G_f = \frac{i}{\not{p} - \tilde{m} - \Sigma_f} \qquad G_s = \frac{i}{p^2 - m^2 - \Sigma_s}$$

Σ_f : 1-loop contribution in two different renormalisation schemes

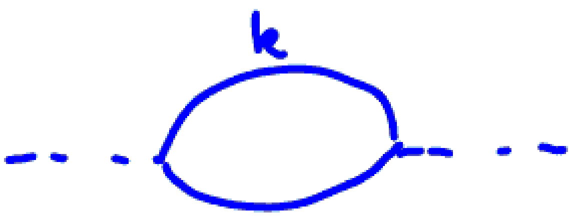


Cut-off Λ	$\overline{MS}$
$\tilde{m} \ln \Lambda$	$\tilde{m} \ln \frac{\tilde{m}^2}{\mu^2}$

Σ_s | cut-off Λ | $\overline{MS}$



$\sim \Lambda^2$ | $m^2 \ln \frac{m^2}{\mu^2}$



$\sim \Lambda^2$ | $\tilde{m}^2 \ln \frac{\tilde{m}^2}{\mu^2}$

$$\int \frac{d^4 k}{k^2 - m^2}$$

The result obtained with cut-off means that there is a hierachy problem: in the presence of a large physical scale, light scalars are highly unnatural

Let's see it using MS as the renormalisation scheme:

$$\mathcal{L} = \frac{1}{2}\left[(\partial \varphi)^2 + (\partial \phi)^2 - m^2 \varphi^2 - M^2 \phi^2\right]$$

$$- \frac{\lambda_1}{4!}\varphi^4 - \frac{\lambda_2}{4!}\varphi^2 \phi^2 - \frac{\lambda_3}{4!}\phi^4$$

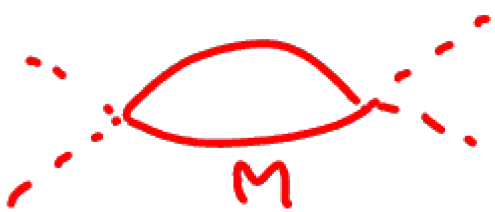
$$m << M$$

Quantum correction to the effective low energy theory ($E << M \Rightarrow$ only φ) in the $\overline{MS}$ scheme :

$$\mathcal{L}_{eff} = \frac{1}{2}\left((\partial\varphi)^2 - (m^2 + \delta m^2)\varphi^2\right) - \frac{\lambda_1 + \delta\lambda_1}{4!}\varphi^4 + \ldots$$

$$\delta m^2 = \frac{\lambda_2}{(4\pi)^2} M^2 \left[-1 + \ln\frac{M^2}{\mu^2}\right]$$

hierarchy problem!



$$\delta\lambda_1 = \frac{\lambda_2}{(4\pi)^2} \ln\frac{M^2}{\mu^2}$$

Large cancellations between parameters m and M are necessary to keep the physical scalar mass small

Exception: Goldstone boson

Toy model - $U(1)$ global symmetry

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2$$
$$+ i \psi_1 \bar{\sigma}_\mu \partial^\mu \psi_1 + i \psi_2 \bar{\sigma}_\mu \partial^\mu \psi_2$$
$$- g(\phi \psi_1 \psi_2 + \phi^* \bar{\psi}_1 \bar{\psi}_2)$$

ψ_1, ψ_2 - Weyl fermions; $U(1)$ charges

$\phi : +1$

$\psi_1 : -1$, $\psi_2 : 0$

Spontaneous symmetry breaking

$$\phi = \frac{1}{\sqrt{2}}\left(\varphi + v + i\chi\right)$$

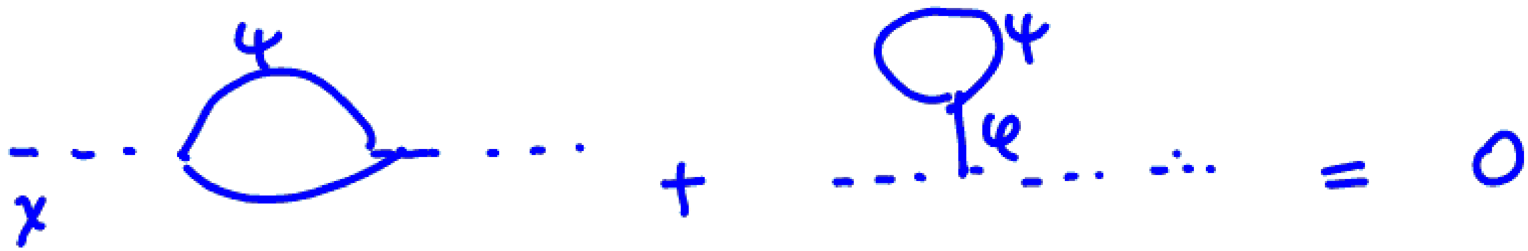
Dirac fermion $\quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}(\partial\chi)^2 + \bar{\psi}\not{\partial}\psi - \frac{1}{4}\underbrace{\lambda v^2}_{m_\varphi^2}\varphi^2$$

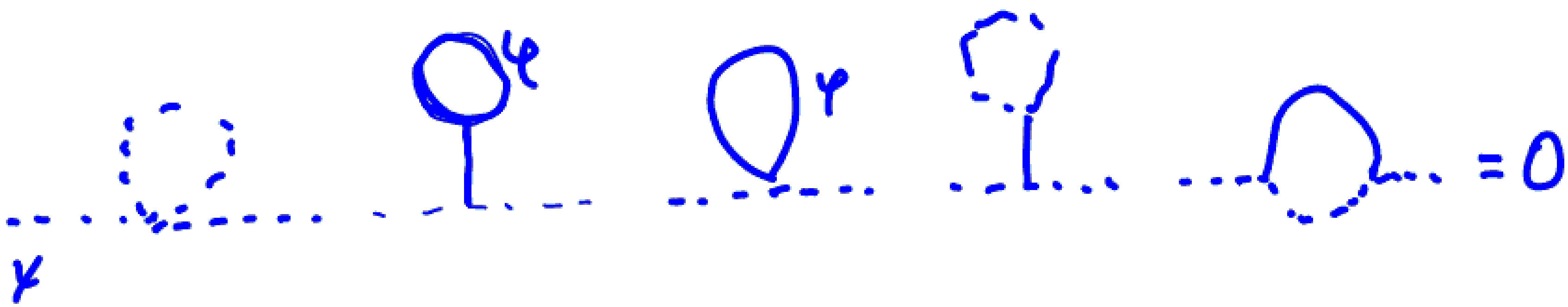
$$- \frac{\lambda v}{4}\varphi\left(\varphi^2 + \chi^2\right) - \frac{\lambda}{16}\left(\varphi^2+\chi^2\right)^2$$

$$- \underbrace{\frac{g v}{\sqrt{2}}}_{m_\psi}\bar{\psi}\psi - \frac{1}{\sqrt{2}} g \varphi \bar{\psi}\psi - \frac{i}{\sqrt{2}} g \chi \bar{\psi}\gamma_5\psi$$

Corrections to the χ mass (Goldstone)



(no dependence on cut-off Λ !)



fermion - fermion
boson - boson

cancellations
(conspiracy of couplings)

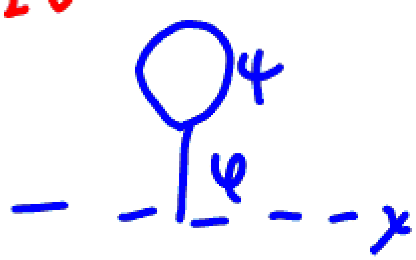
With non-linear parametrization

$$\phi = \frac{1}{\sqrt{2}}(\varphi + v)e^{i\chi/v}$$

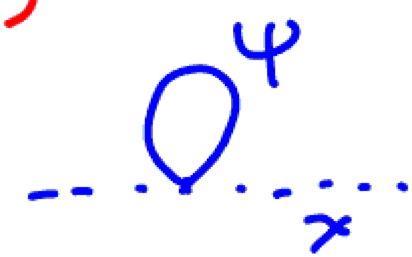
$$-g(\phi\psi_1\psi_2 + \phi^*\bar{\psi}_1\bar{\psi}_2) = -\frac{gv}{\sqrt{2}}\Big(e^{i\frac{\chi}{v}}\psi_1\psi_2 +$$

$$+ e^{-i\frac{\chi}{v}}\bar{\psi}_1\bar{\psi}_2\Big) = \frac{-ig}{\sqrt{2}}(\chi\psi_1\psi_2 - \chi\bar{\psi}_1\bar{\psi}_2)$$

$$+\frac{g}{\sqrt{2}v}\chi^2(\psi_1\psi_2 + \bar{\psi}_1\bar{\psi}_2) + \ldots\ldots$$



$\longrightarrow$



Corrections to a Goldstone boson mass vanish independently of the scale of new physics.

In reality, no Goldstone bosons but pseudo-Goldstone bosons: some potential generated by explicit (weak) symmetry breaking;

some cut-off dependence is in general reintroduced (see electromagnetic pion mass difference) and the question about naturalness becomes relevant for such corrections

Let us now return to our original calculation in MS scheme in the absence of any new large mass scale.

E.G. suppose the SM is the Theory of Everything and that gravity is irrelevant for particle physics.

Then a light scalar does not create any hierachy problem! In fact, one can invoke scale symmetry to justify MS scheme (Coleman-Weiberg, Bardeen,…)
and break electroweak symmetry by quantum corrections.

BUT THE ABOVE ASSUMPTIONS ARE UNLIKELY TO BE CORRECT…..

EXERCISE

CHECK ALL THAT USING EFFECTIVE POTENTIAL FORMALISM

Electromagnetic $\pi^+ - \pi^0$ mass difference

$$(m_{\pi^+} - m_{\pi^0}) \sim 5\ \text{MeV}$$

Pions — Goldstone bosons of broken chiral symmetry $SU(2)_L \times SU(2)_R \to SU(2)_V$

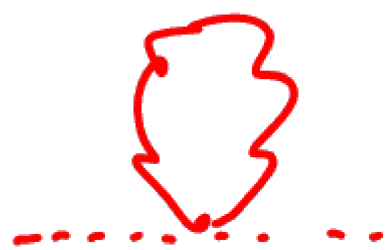
$$\phi(2, \bar{2}) \to U = f_\pi \, e^{i \pi^a \tau^a / f_\pi}$$

non-linear σ model as a low energy approximation to non-perturbative QCD

Electromagnetic interactions break explicitly chiral invariance

$$\mathcal{L} = \mathrm{Tr}\, D_\mu U \, D^\mu U$$

$\left(D_\mu = \partial_\mu + ieA_\mu [\tau_3, U] \right)$ and generate corrections to π^+ mass

 $+$  $\simeq \dfrac{3\alpha}{4\pi} \Lambda^2$

Λ is the cut-off to non-linear σ model (the next mass scale in non-perturbative QCD)

$$\Lambda \sim m_\rho \;!$$

$\pi^+ - \pi^0$ mass difference is correctly explained

CONLUSION

ALL MASS SCALES IN THE SM , EXCEPT THE ELECTROWEAK SCALE, HAVE NATURAL EXPLANATION (NO LARGE CANCELLATIONS NEEDED!)

WHAT ABOUT THE ELECTROWEAK SCALE THEN? CAN WE UNDERSTAND ITS STABILITY WITH RESPECT TO QUANTUM CORRECTIONS?

CAN WE UNDERSTAND ITS VALUE?

TACIT ASSUMPTION: THERE ARE NEW MASS SCALES IN PHYSICS

END OF LECTURE 1

LECTURE 2: RENORMALISABLE OR NON-RENORMALISABLE EFFECTIVE ELECTROWEAK THEORY?

A question relevant for its extensions

What could it be – a nonrenormalisable effective electroweak theory?

Spontaneously broken gauge SU(2)xU(1), structure of currents, etc- well tested;

But the mechanism of the spontaneous breaking of the gauge symmetry still unknown and the effect of nonrenormalisability could in principle appear in that mechanism

CONLUSIONS of the 1st Lecture

ALL MASS SCALES IN THE SM , EXCEPT THE ELECTROWEAK SCALE, HAVE NATURAL EXPLANATION (NO LARGE CANCELLATIONS NEEDED!)

WHAT ABOUT THE ELECTROWEAK SCALE THEN? CAN WE UNDERSTAND ITS STABILITY WITH RESPECT TO QUANTUM CORRECTIONS?

CAN WE UNDERSTAND ITS VALUE?

TACIT ASSUMPTION: THERE ARE NEW MASS SCALES IN PHYSICS

Saying it diiferently,

WHAT BREAKS

ELECTROWEAK SYMMETRY ?

SM answer: the Higgs potential of a single Higgs doublet (renormalisable theory)

$$\partial_\mu H \gamma^\mu H^+ - m_H^2 \; H H^+ + \frac{1}{2}\lambda (H H^+)^2$$

$$v^2 = \frac{-2 m_H^2}{\lambda} \; , \quad m_h^2 = \lambda v^2$$

$$\left(H = \frac{1}{\sqrt{2}} v \right) \qquad v \to M_Z$$

Renormalisable QFT : UV sensitivity hidden in a finite number of free parameters, to be taken from experiment

Why useful if one expects UV completion, with new physical mass scales?

Because calculations with arbitrary precision are possible, with no information about higher mass scales

What do we learn if a description of a set of physical phenomena in terms of a renormalisable QFT is correct?

Technically, we learn that a lagrangian

with up to dim 4 operators is consistent

with the symmetries of the problem

(e.g. QED with its gauge symmetry,
contrary to the non-linear sigma model
– effective theory of pions- with its
chiral symmetry)

Physically, we learn then that the next mass

scale is well separated and can be decoupled

in the Appelquist-Carazzone sense;   e.g. QED

as a renormalisable low energy approximation

to SM after decoupling of W and Z bosons,

contrary to the non-decoupling of sigma meson

in linear sigma model of pions and sigma
meson

For a renormalisable but effective theory we expect

$$\mathcal{L} = \mathcal{L}_{ren} + \sum_n \frac{c_n}{\Lambda^n} \hat{O}_{4+n}$$

$$c_n \sim \begin{cases} 1/16\pi^2 & \text{if perturbative} \\ O(1) & \text{if non-perturbative} \end{cases}$$

with the new scale high enough for the higher dim operators to be neglected in the „first” approximation

QED as an effective renormalisable theory

Renormalisability means calculability with arbitrary precision with a finite number of input parameters

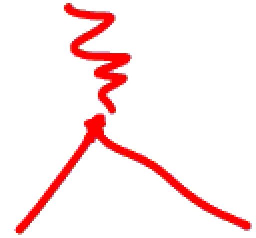
But it is only an effective theory –low energy approximation to SM, so its predictions disagree with experiment at the level E/Mw

Example: lepton magnetic moment

Dirac equation

$$(\not{p} - e\not{A} - m)\psi = 0$$

$\mathcal{L}: j_\mu A_\mu$

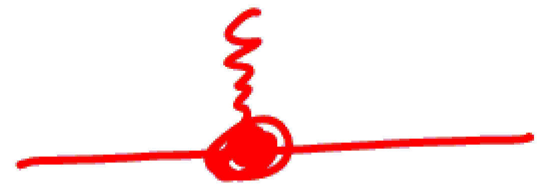


Non-relativistic limit (φ - two-component spinor with $s = \pm 1/2$):

$$i\frac{\partial \varphi}{\partial t} = \left[\frac{p^2}{2m} - \frac{e}{2m}\left(\vec{L} + \underbrace{2}_{g=2}\vec{S}\right)\vec{B}\right]\varphi$$

Anomalous magnetic moment =

= effective vertex



described by the operator

$$C \, \frac{e m_\ell}{\Lambda^2} \, \bar{\psi}_\ell \sigma^{\mu\nu} \psi_\ell F^{\mu\nu} \longrightarrow 2C \, \frac{e m_\ell}{\Lambda^2} \, \varphi_\ell^{+} \vec{S} \vec{B} \varphi_\ell$$

Anomalous magnetic moment of the lepton ℓ

$$a = (g-2)/2 = C \, \frac{e m_\ell}{\Lambda^2} \Big/ \frac{e}{2 m_\ell} = C \, \frac{2 m_\ell^2}{\Lambda^2}$$

why m_ℓ appears in the coefficient of the effective operator ?

$$\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu , \gamma_\nu\right]$$

$$\bar{\psi}\,\sigma_{\mu\nu}\,\psi = \left.\bar{\psi}_R\,\sigma_{\mu\nu}\,\psi_L\right\}\ \text{chirality flip}$$

$$\psi_R = \frac{1}{2}(1+\gamma_5)\psi \qquad \bar{\psi}_R \equiv \bar{\psi}\,\frac{(1-\gamma_5)}{2}$$

$$\psi_L = 1/2\,(1-\gamma_5)\psi$$

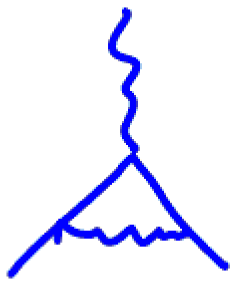
$$\psi = \psi_R + \psi_L$$

To generate effective vertex $\bar{\psi}_R \sigma^{\mu\nu} \psi_L$, we need interactions that generate chirality flip.

In QED and in the SM the only source of chirality flip is the mass term

$$m_e \bar{\psi}_R \psi_L \ !$$

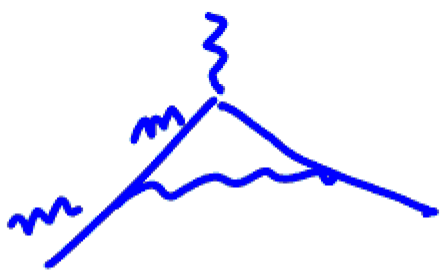
Schwinger (QED)



$$\frac{e}{2m}\,\frac{\alpha}{2\pi}\,\bar{\psi}\,\sigma^{\mu\nu}\psi F^{\mu\nu}$$

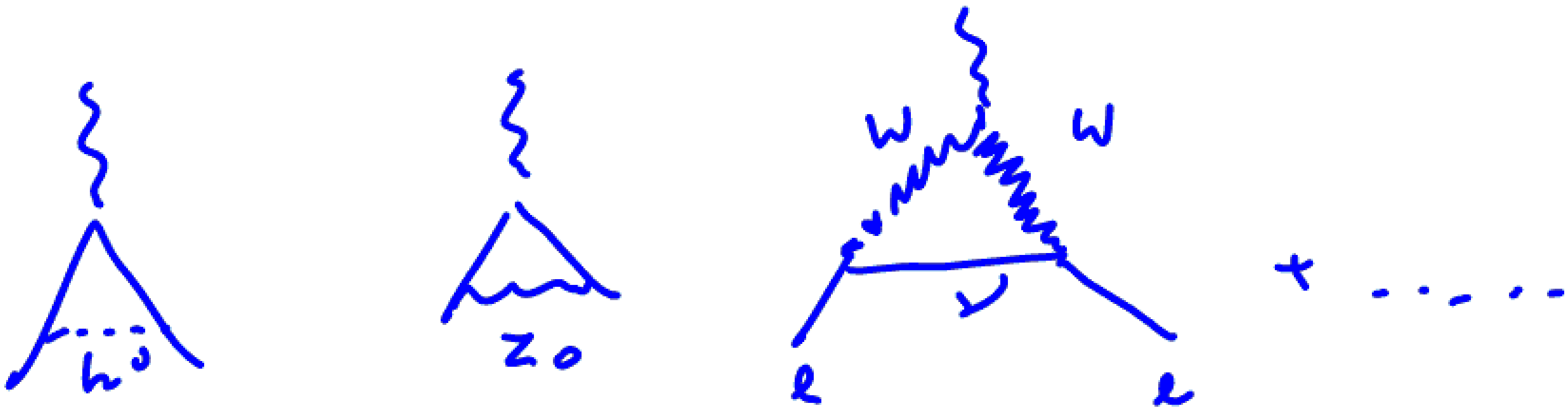
$$a = \frac{\alpha}{2\pi}$$

How to compare with $C\,\frac{e\,m}{\Lambda^2}\,\bar{\psi}\,\sigma_{\mu\nu}\psi F^{\mu\nu}$



$$\Lambda \to m \;\Rightarrow\; \frac{m}{\Lambda^2} \to \frac{1}{m}$$

SM additional contributions



$$a \sim \frac{5}{3} \frac{G_F}{8\sqrt{2}\pi^2} m_e^2$$

G_F - Fermi constant $\qquad G_F = \sqrt{2} \dfrac{e^2}{8 s_w^2 M_W^2}$

we identify $\quad \Lambda \equiv M_W \quad , \quad C = \dfrac{5}{3} \dfrac{1}{16\pi^2} \alpha$

Effective low energy theory ($E \ll M_W$)

$$\mathcal{L} = \mathcal{L}_{QED} + C \frac{m_\ell}{M_W^2} \bar{\psi}_\ell \sigma^{\mu\nu} \psi_\ell F^{\mu\nu}$$

$$+ \ldots\ldots$$

Exp is now sensitive to higher dim operator corrections

3σ discrepancy with SM?

New physics?

$$\frac{e}{2m}\vec{S}\cdot\vec{B}\left[2+\underbrace{\frac{\alpha}{\pi}+}_{g_\mu}\dots\dots+\dots\dots\right]$$

$$a_\mu^{Exp}=\frac{g_\mu-2}{2}=(116\,592\,023\pm 145)\times 10^{-11}$$

$$a_\mu^{QED}=(116\,584\,705\pm 3)\times 10^{-11}$$

$$a_\mu^{h}=(6739\pm 67)\times 10^{-11}$$

$$a_\mu^{W}=(152\pm 4)\times 10^{-11}$$

$$a_\mu^{SM}=(116\,591\,597\pm 67)\times 10^{-11}$$

$$a_\mu^{Exp}-a_\mu^{SM}=(426\pm 165)\times 10^{-11}$$

SM is a renormalisable theory. Is it the correct effective electroweak theory?

History repeats itself?

QED → SM → Beyond?

Mixed signals:

Stunning theoretical consistency of the SM, e.g. fermion mass generation, chiral anomaly cancellation;

SM is anomaly free- an accident or a hint?

But, on the other hand,
hierachy problem $\rightarrow$
new nearby mass scale very
welcome;

No real benefit from a renormalisable
effective electroweak theory?

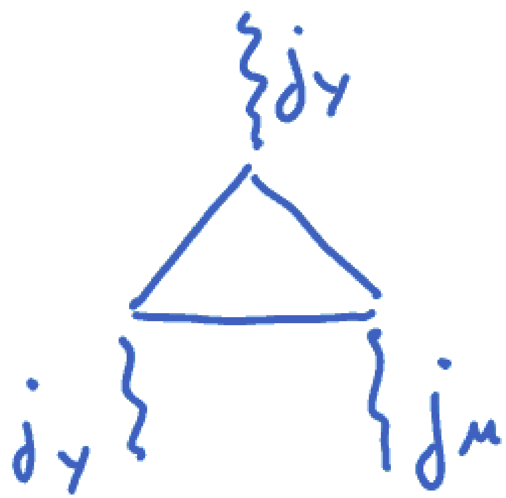
It depends on its extension…e.g. for minimal supersymmetric extension anomaly cancellation and renormalisability of the SM are relevant.

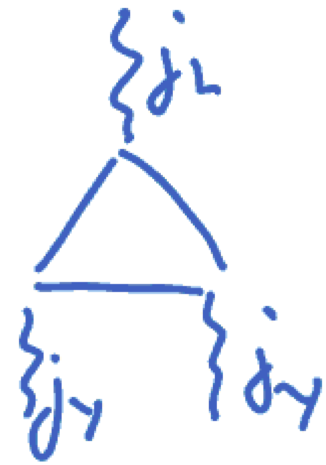
And finally, is the (renormalisable) SM really consistent with exp. data?

That fuzzy picture is behind various different attempts to go beyond the electroweak scale

Renormalisability and chiral anomaly cancellation

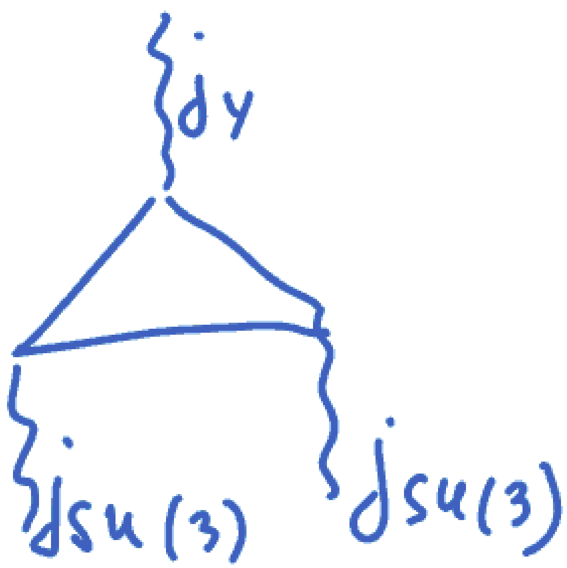
Potential source of chiral anomalies in the SM:





Anomaly cancellation condition

$$\sum_{\text{fermions}} Q_i = 0$$



Anomalies and renormalisablity – a toy model- instead of full SM let's take a U(1) invariant model with only a left-handed fermion coupled to gauge boson

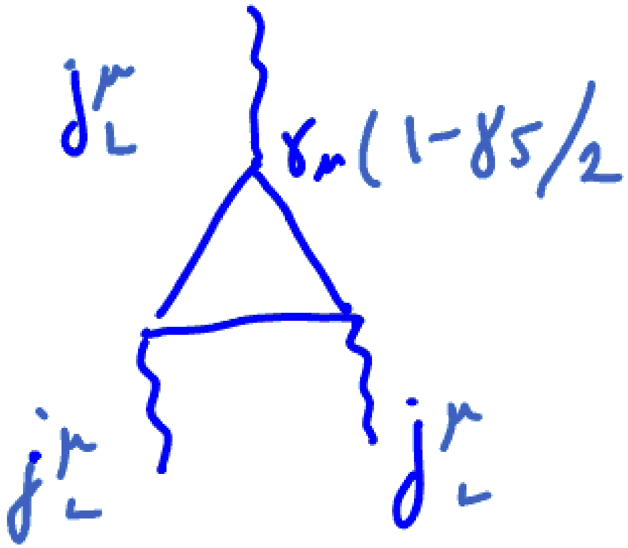
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D_\mu \phi + \mu^2 \phi^* \phi$$

$$- \lambda (\phi^* \phi)^2 + \bar{\psi}_R i \not{\partial} \psi_R$$

$$+ \bar{\psi}_L [i \not{\partial} + \not{A}] \psi_L$$

$$- y \phi (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

$D_\mu = \partial_\mu + i A_\mu$



$$\Rightarrow \quad \partial_\mu j_L^\mu = -\frac{g^2}{48\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

What about gauge invariance?

$$A'_\mu = A^\mu + \partial_\mu \theta(x)$$

$$\phi' = e^{i\theta(x)}\phi \approx \phi + i\theta\phi$$

$$\psi'_L = e^{i\theta(x)}\psi_L \approx \psi_L + i\theta\psi_L \,; \quad \psi'_R = \psi_R$$

We can study the gauge dependence of the theory, after inclusion of quantum effects from fermions by checking the response to the gauge transformation of the generating functional

$$W[A] = \int \mathcal{D}\phi \, \mathcal{D}\psi \, e^{i \int d^4x \, \mathcal{L}[\phi, \psi, A]}$$

$$\mathcal{L} = \cdots + j_\mu A_\mu$$

Under the gauge transformation

$$\delta W = \frac{\delta W}{\delta A_\mu}\bigg|_{\theta=0} \delta A_\mu = \int d^4x \int \mathcal{D}\phi\, \mathcal{D}\psi\, e^{i\int d^4x\, \mathcal{L}} \, *$$

$$* \, j^\mu \partial_\mu \theta = \int d^4x \, \underbrace{\partial_\mu j^\mu}_{F\tilde{F}} \, \theta \int \mathcal{D}\phi\, \mathcal{D}\psi \ldots$$

Hence

$$\delta W \sim \int d^4x \, F_{\mu\nu} \tilde{F}^{\mu\nu} \theta \, * \ldots$$

Gauge invariance can be restored by introducing a new field χ, transforming like

$$\chi' = \chi - v\theta(x)$$

with

$$\mathcal{L}_\chi = \frac{1}{v} F \tilde{F} \chi \quad \Big\} \text{ dim } 5$$

also known as Green-Schwarz mechan

where v is some scale generic for the anomalous theory

All is more transparent in a theory with the Higgs mechanism for U(1)

$$\phi = \frac{1}{\sqrt{2}}(v + \varphi + i\chi) = (\tilde{\rho} + v)e^{i\tilde{\chi}/v}$$

$$m_A = 2v \quad , \quad m_\psi = (y/\sqrt{2})\, v \quad , \quad m_{\tilde{\rho}} = \lambda v$$

when the anomaly is present, unitarity is violated: there remain poles in $k^2 = 0$ in the spectrum

We can restore gauge invariance by adding

$$\mathcal{L}_{\tilde{\chi}} \sim \frac{1}{v} F \tilde{F} \tilde{\chi}$$

(indeed, $\tilde{\chi} \to \tilde{\chi} + v\theta$)

but then the model becomes non-renormalisable

The scale v is the scale where new physics should show up, to cancel the anomalies

Is the anomaly cancellation – with the spectrum of known fermions – a hint for the correctness of SM as the effective electroweak theory?

But . . hierarchy problem

For $m_h \sim O(100)\,\text{GeV} - O(1)\,\text{TeV}$

unitarity bound

$$\lambda \lesssim O(1)$$

Higgs mass parameter $|m_H^2| \sim 10^{-2}\,\text{TeV} - 3\times 10^{-2}\,\text{TeV}$

$$m_H^2 = m_H^2\Big|_{\text{tree}} + \delta m_H^2 \sim M_Z^2$$

loop corrections

Options for (still unknown) physics beyond the electroweak scale (not just beyond the SM!) that could make the electroweak scale natural:

- supersymmetric extensions of (renormalisable) SM
- extensions of non-renormalisable effective electroweak theory

Two candidate class of models

- Higgless models à la chiral symmetry breaking in QCD ($SU(2) \times SU(2) \to SU_V(2)$)

$$U = v\, e^{i G^a \tau^a / v}$$

- Models with the Higgs <u>doublet</u> as a Goldstone boson (e.g. $SO(5) \to SU(2) \times SU(2)$)

$$U = f\, e^{i H^a T^a / f}$$

{ can be perturbative
{ or non-perturbative

Is the SM confirmed by the data?

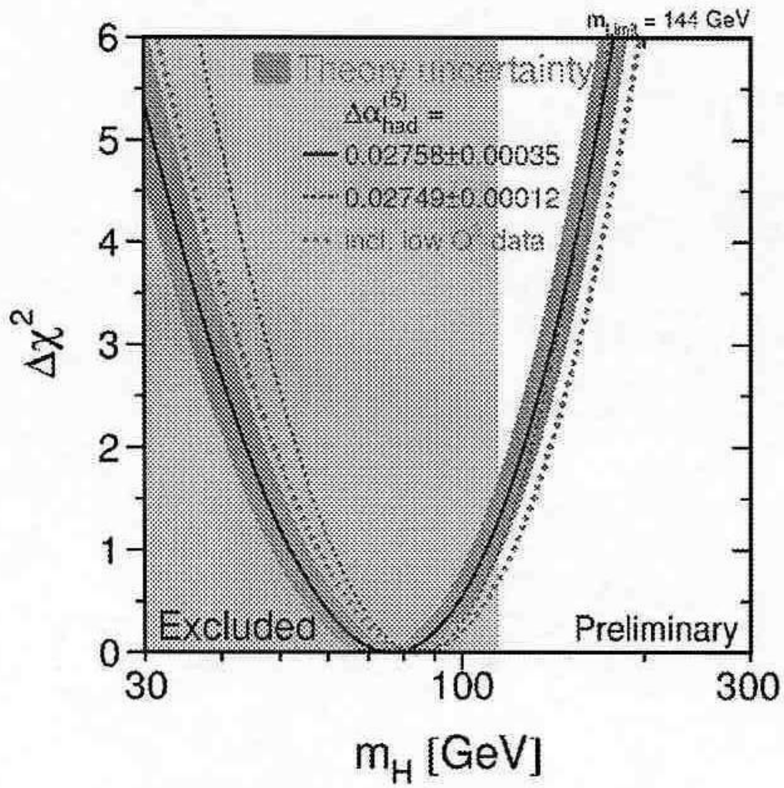
The prediction from the fits: the Higgs boson mass- to be yet confirmed!

If not confirmed, the data are not consistent with the SM:
it would need additions that do not decouple at the electroweak scale.

If confirmed- one can ask a reversed question: what are then the limits on the new scale

1) Is there a Higgs?

- Most economical solution for EW breaking
- LEP gives indications for a light Higgs



Preferred value $m_H = 76^{+33}_{-24}\,\mathrm{GeV}$

Upper limit $m_H < 144\,\mathrm{GeV}$ (95% CL)

including direct limit of $114\,\mathrm{GeV}$:

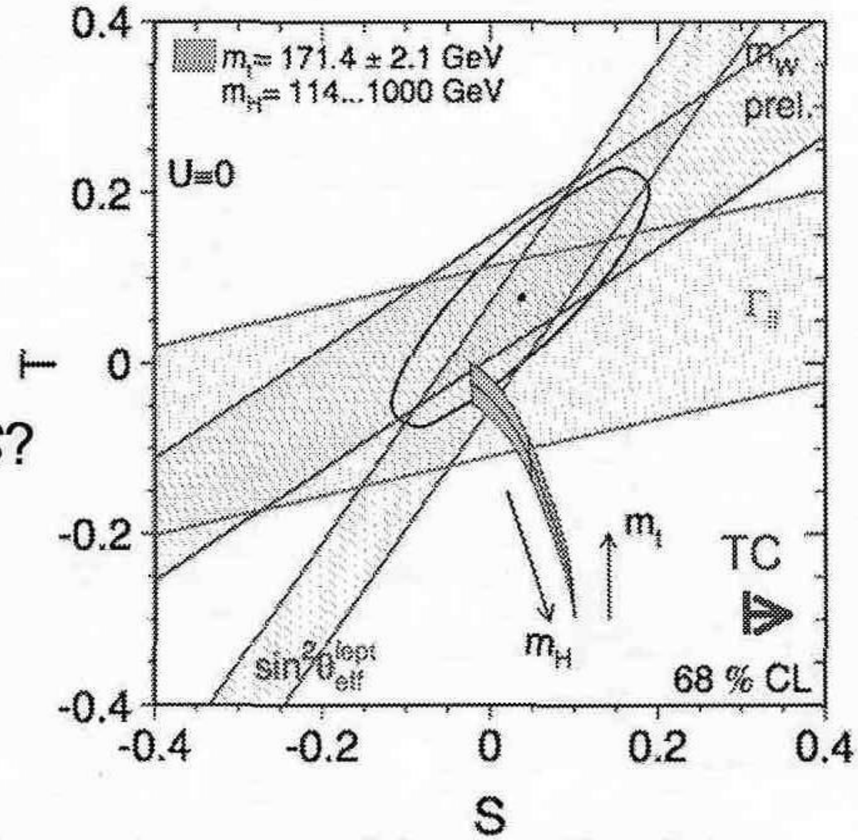
$m_H < 182\,\mathrm{GeV}$ (95% CL)

LEPEWWG 07

Still open is the possibility of no Higgs, and new strong dynamics

- flavour?
- EW precision data?

$$\Delta S \approx \frac{1}{6\pi}\left(n_{TF} N_{TC} + \ln \frac{\Lambda_{TC}}{m_Z}\right)$$



Negative contributions to *S*?

Hirn-Sanz; Delgado-Falkowski; Agashe-Csaki-Grojean-Reece

Walking at small N_{TC}?

Foadi-Frandsen-Ryttov-Sannino; Piai

Higgsless? EW broken by boundary conditions with KK gauge bosons curing unitarity up to about 10 TeV (or less)

Csaki-Grojean-Pilo-Terning

Suppose there exist a light Higgs boson, as predicted by the SM fits; supersymmetry is consistent with it (generically susy models give a light Higgs boson).

But Msusy > 500 Gev (from the electroweak fits in particular) and there remains a little hierarchy problem.

No full satisfaction → models with a Higgs doublet as a (pseudo)-Goldstone boson; The lightest Higgs boson is generically light because its mass is generated at 1 loop by explicit symmetry breaking potential; but no full satisfaction either- as „unnatural" as MSSM

If the Higgs boson is heavy (or no Higgs boson)
- there must be new contributions to the electroweak observables to improve the precision data fits; unfortunately, at present no models consistent with precison data; usually, corrections from new physics go wrong way

Double protection –by supersymmetry and by pseudo-Goldstone nature of the Higgs bosons- the only models with no hierarchy problem – but very complicated

The third lecture – on a failure- after the LHC data!

Shall we or we shall not understand the Fermi scale in the QFT framework?

THE END