

# Black holes in String Theory

Lesvos, 4<sup>th</sup> Aegean Summer School: Black Holes

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# Introduction

Black holes in String Theory very **broad subject**

⇒ string theory gives rise to (more general) black hole solutions:  
charged **black branes** + extremal (supersymmetric) versions of these

two main themes:

- use black branes in connection with gauge/gravity correspondence to learn about **thermal phases** of dual non-gravitational theories (e.g. super Yang-Mills theory) living on brane (holography)
- microscopic description of **black hole entropy** via string theory

◀ Focus in these lectures:

- **black holes, strings and rings** in higher dimensions  
(intimately connected to black objects in string theory)
- **black branes in string theory** (U-duality, thermodynamics, stability, .. )

Many other interesting recent developments in string theory:

- black hole entropy and attractors, relation to topological strings
- fuzzball conjecture (see lecture Mathur) .

# Outline

- Introduction to black branes and their thermodynamics + relevance for the gauge/gravity correspondence

## Part 1:

- Black objects in **higher dimensional spaces**
  - **Kaluza-Klein black holes** (spaces with extra compact directions)
  - **rotating** (stationary) black objects (in asymptotically flat space)
  - newly found **connection between the phase structure** of these two

## Part 2:

- **non-and near- extremal branes in string theory** via boost/U-duality map
- applications of KK black holes to **string theory**:
  - **correlated stability conjecture**  
(relation between thermodynamic and classical stability)

See e.g. Review articles by: **Kol (PhysRept)/Harmark,Niarchos,NO (CQG)**  
also Tasi lecture of **Peet** (+ various other reviews)

# Motivation (higher D gravity + String Theory)

## ■ Study black objects in higher dimensional gravity

- richer phase structure
- (non)-uniqueness theorems in higher dimensional gravity
- new topologies of event horizons possible
- gravitational phase transitions between different solutions with event horizons (topology change)
- Gregory-Laflamme instability (new phases)
- possible objects in universe/accelerators
- + many ST applications (black branes, BH entropy, AdS/CFT)

► Two cases studied      most progress in recent years      -      less explored

- asymptotically flat spaces:      five dimensions      six and beyond  
(stationary solutions)      - MP black holes, black rings,  
black Saturns, black di-rings,

- Kaluza-Klein spaces:       $d$ -dim Minkowski x circle (tori)      other Ricci flat..  
(static solutions)      -non-uniform strings, localized black holes      e.g. CY  
bubble-black hole sequences, merger point  
evolution of GL instability

# String/Gauge Theory Motivations

- phase structure of Kaluza-Klein black holes related to objects and phenomena in string theory/gauge theory

Bostock, Ross/Aharony, Marsano, Minwalla, Wiseman/  
Harmark, NO

→ phase structure of non- + near-extremal branes (with circle in transverse space)

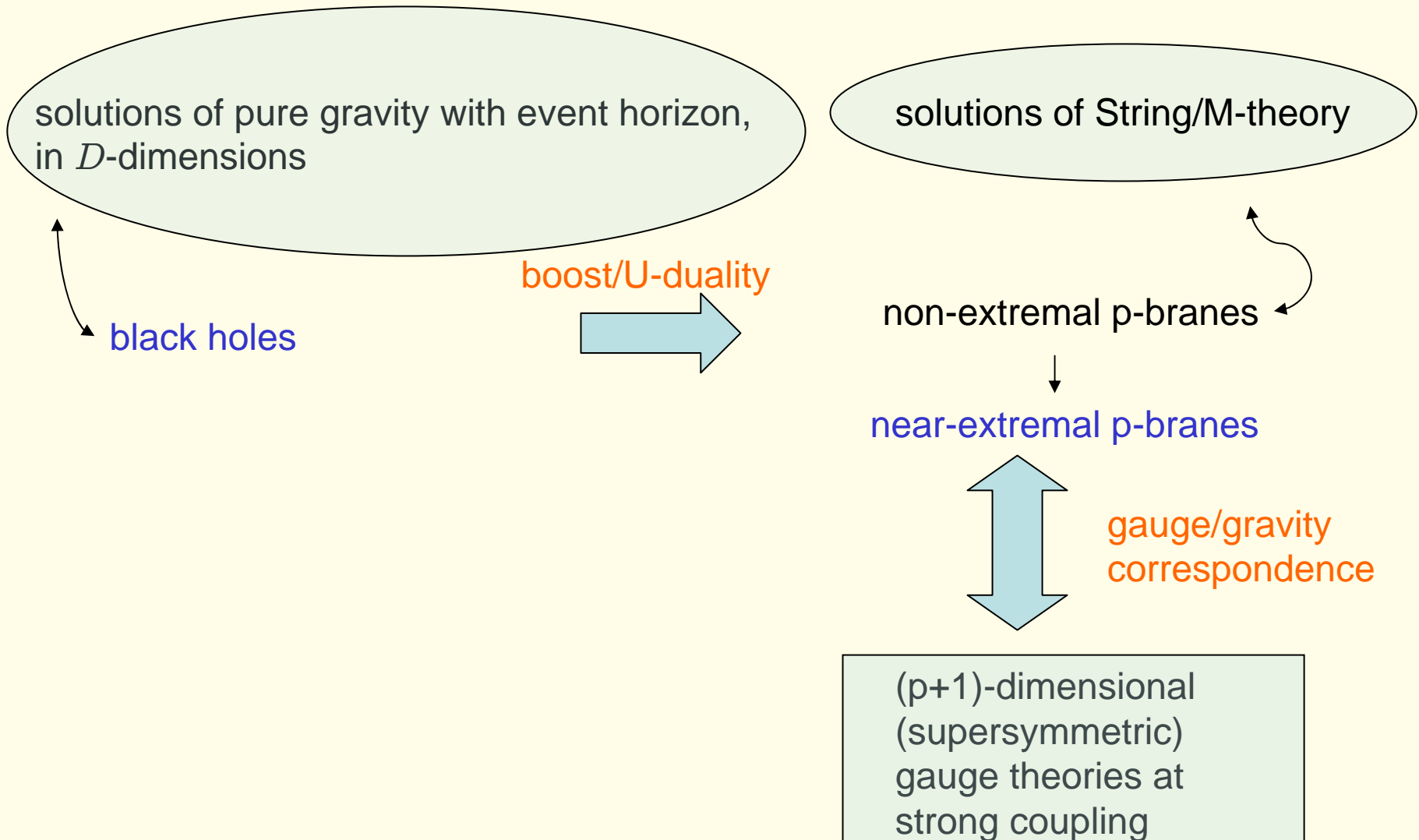
- new insights into phase structure of **strongly coupled large  $N$  theories**
  - qualitative/quantitative tests of gauge/gravity correspondence
- correlated stability conjecture
- new stable phase of LST
- entropy of 3-charge BHs on circle

- **Black rings + supersymmetric cousins play important role in string theory**

- supersymmetric black rings, supertubes
- microscopic counting of entropy, 4D-5D connection
- foaming black rings and fuzzball proposal
- plasma rings.....

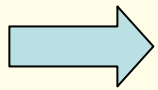
# From black hole to gauge theory thermodynamics

Black hole thermodynamics is intimately related to gauge theory dynamics in very precise way, via brane solutions in string theory



# black holes in higher dimensions

What do we know about black objects (i.e. with event horizon)  
in **higher dimensional gravity**

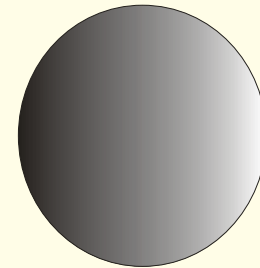
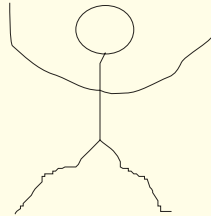


some, but still lot to discover



stationary black holes

$\mathcal{M}^D$



$\mathcal{M}^{D-1} \times S^1$



KK black holes

# Black hole non-uniqueness

in 4 dimensions: given mass, angular momentum and charge:  
unique black hole solution

for  $D$ -dimensional asymptotically flat space times:  
only static and neutral black hole in pure gravity is Schwarzschild-  
Tangherlini black hole

$$ds^2 = - \left( 1 - \frac{r_0^{d-3}}{r^{d-3}} \right) dt^2 + \left( 1 - \frac{r_0^{d-3}}{r^{d-3}} \right)^{-1} dr^2 + r^2 d\Omega_{d-2}^2$$

event horizon at  $r_0$

mass  $M = \frac{\Omega_{d-2}(d-2)}{16\pi G} r_0^{d-3}$

Q: Does this uniqueness extend to higher dimensional GR ?

→ Recent years of research gives answer: No !

two examples of such non-uniqueness known:



# Rotating ring in five dimensions

5D asymptotically flat rotating BH solutions

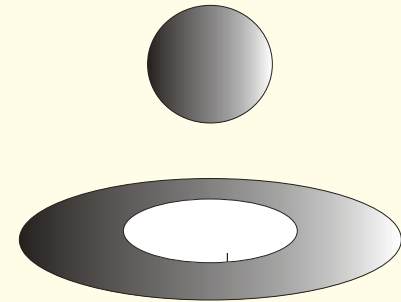
horizon topology

- **Myers-Perry black hole**  
(generalizes 4D Kerr solution)

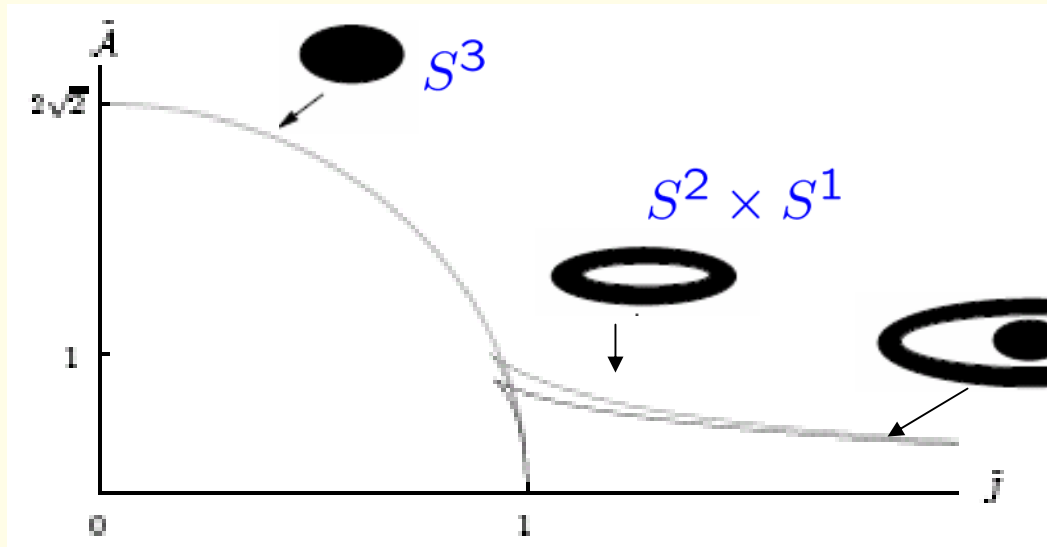
$$S^3$$

- **rotating black ring**  
(Empanan-Reall)

$$S^2 \times S^1$$



+ black saturn, black di-rings etc. (exact solutions)



EmpananReall/Elvang, Figueras  
/Elvang, Empanan, Figueras

infinite non-uniqueness  
for configurations not  
in thermal equilibrium

Elvang, Empanan, Mateos, Reall

non-uniqueness even persists for supersymmetric generalization in ST

# Kaluza-Klein black holes

- ▶ black holes asymptoting to d-dimensional Minkowski space times a circle (Kaluza-Klein spaces) =  $\mathcal{M}^d \times S^1$   
= time x d-dimensional cylinder  $\mathbb{R}^{d-1} \times S^1$

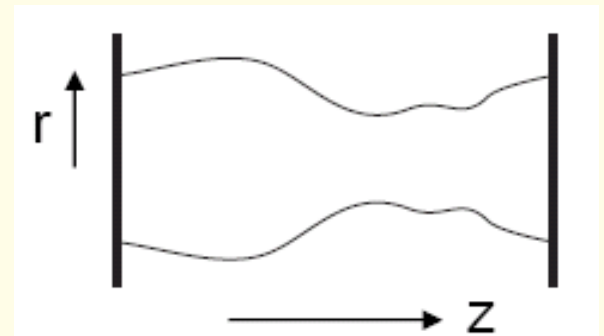
circle direction breaks symmetry  $\longrightarrow$  gives rise to new possibilities of BH solutions

- what are the static & neutral BH solutions on the cylinder ?
  - why richer phase structure ?
  - how can we parameterize extra freedom ?
- ▶ consider case with spherical symmetry for  $\mathbb{R}^{d-1}$  part of cylinder:

at  $\infty$  can think of any BH solution as coming from Newtonian source located at origin of  $\mathbb{R}^{d-1}$  but with mass distribution in circle direction: source  $\rho(z)$

measure at horizon proper radius of  $S^{d-2}$  around cylinder  $\longrightarrow$  profile  $r(z)$

- heuristically we can connect  $r(z)$  to mass distribution  $\rho(z)$  by imagining d-dim static Sch BH for each value of z

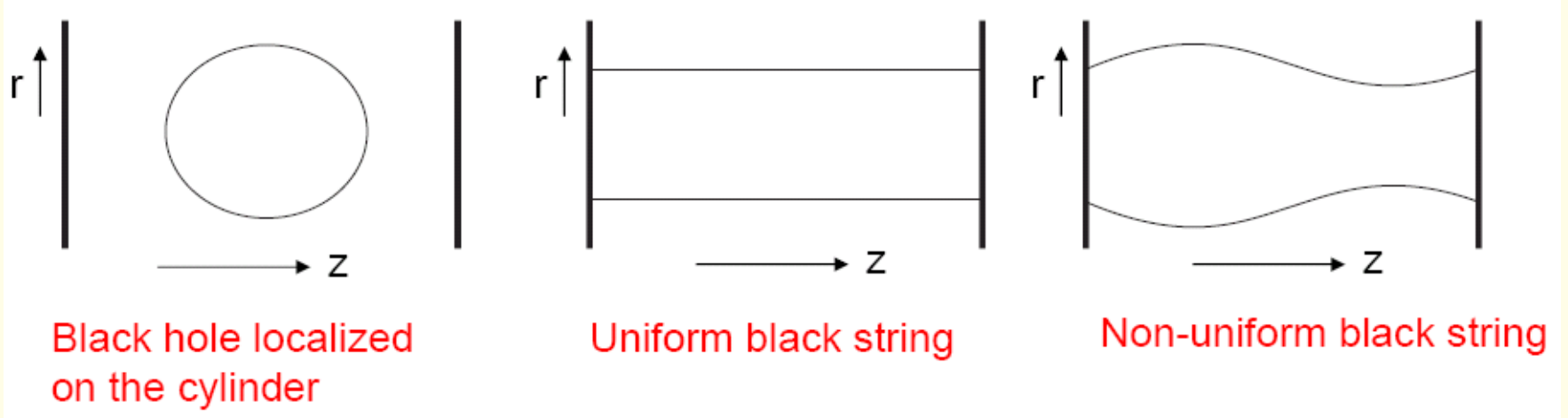


# Possible BH solutions

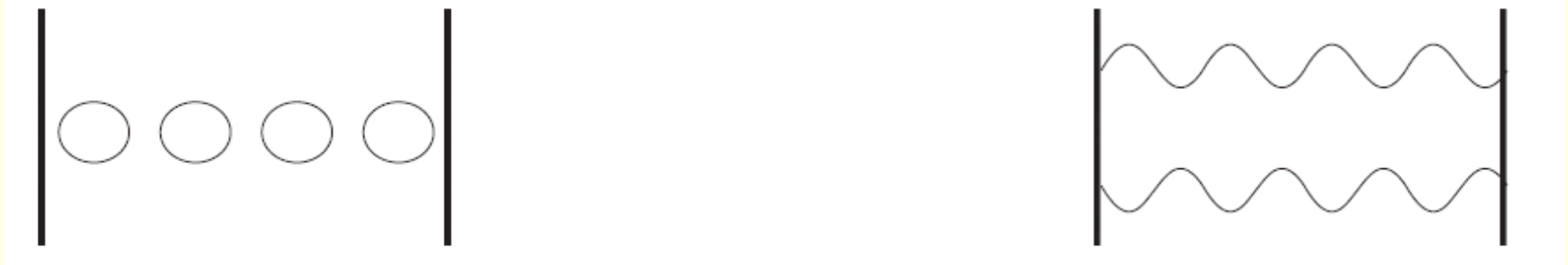
► do all profile/mass distributions correspond to BH solutions ?

- clearly No – BH solution in GR automatically takes into account self-gravitation of the mass distribution, so not even for Newtonian matter would we expect that

what are possible BH solutions ?



plus **copies**: repeat same profile number of times (e.g. 4 times)



# Mass and tension for KK BHs

consider static solutions of vacuum Einstein equations

- coordinates for  $\mathcal{M}^d \times S^1$  ( $d \geq 4$ )

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-2}^2 + dz^2 \quad z \sim z + L$$

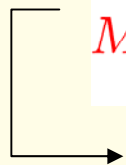
asymptotics  $g_{tt} \simeq -1 + \frac{c_t}{r^{d-3}}, \quad g_{zz} \simeq 1 + \frac{c_z}{r^{d-3}}$



2 (gauge-invariant) asymptotic quantities

Harmark,NO/Kol,Sorkin,Piran/  
Traschen,Fox/Towsend,Zamaklar

$$M = \frac{\Omega_{d-2} L}{16\pi G_N} [(d-2)c_t - c_z], \quad \mathcal{T} = \frac{\Omega_{d-2}}{16\pi G_N} [c_t - (d-2)c_z]$$



mass



tension

1<sup>st</sup> law of thermo  $\delta M = T\delta S + \mathcal{T}\delta L$

Smarr formula

$$TS = \frac{d-2}{d-1} M - \frac{\mathcal{T}L}{d-1}$$

using Komar integral  
(time-translation sym.)

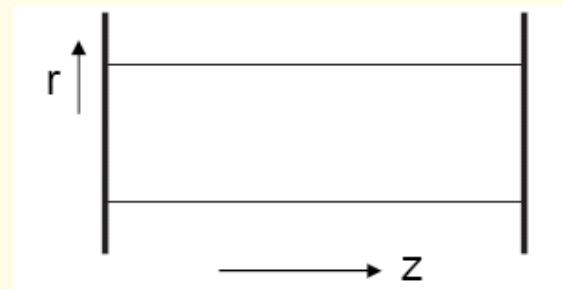
dimensionless  
quantities:

$$\mu = \frac{16\pi G_N}{L^{d-2}} M, \quad n = \frac{\mathcal{T}L}{M} \quad \mu \geq 0, \quad 0 \leq n \leq d-2$$

# Uniform black string (UBS) + Gregory-Laflamme instability

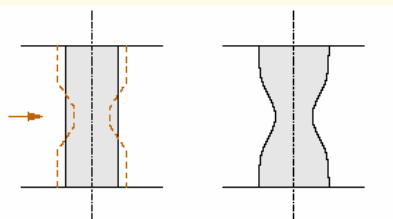
$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_{d-2}^2 + dz^2$$

$$f = 1 - \frac{r_0^{d-3}}{r^{d-3}}$$



d-dim Schw-Tang. BH x flat compactified direction

GL found:



classically  
unstable

$$\mu < \mu_{GL}$$

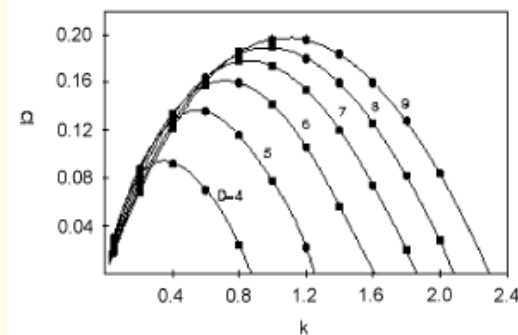
classically  
stable

Gregory, Laflamme

$$\delta g_{\mu\nu} \sim e^{ikz} e^{\Omega t}$$

threshold mode  $\Omega = 0$

non-uniform static  
solution emerges



long wave-length instability

– present when critical GL wavelength  
can fit in the compact direction

global thermodynamic argument:

$$\mu \ll \mu_{GL}, \quad S_{BH}(M) > S_{BS}(M)$$

suggests that UBS decays to LBH

(subject of discussion)

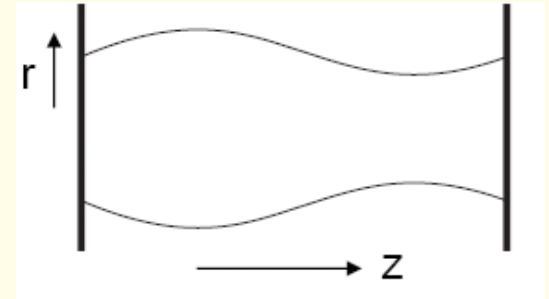
Horowitz, Maeda

# Non-Uniform black string (NUBS)

► UBS has marginal mode at  $\mu = \mu_{GL}$

→ new branch of solutions emerges from this point

- not translational invariant along circle direction



Gubser,/Wiseman,/Sorkin

Kleihaus,Kunz,Radu

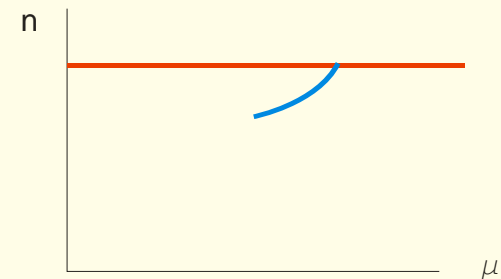
$d < 13$  : branch numerically known

$$n(\mu) = \frac{1}{2} - 0.14(\mu - \mu_{GL}) + \mathcal{O}((\mu - \mu_{GL})^2), \quad 0 \leq \mu - \mu_{GL} \ll 1$$

• **critical dimension  $D=14$ :**  
slope of non-uniform branch reverses

Sorkin

(2<sup>nd</sup> order transition instead of 1<sup>st</sup> order)



• analytical result: ansatz for metric of NUBS

Harmark,NO/Wiseman

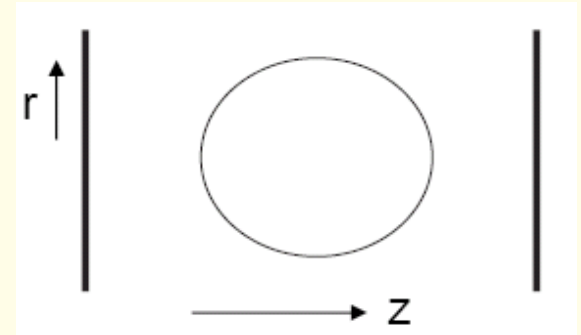


# Localized black hole on cylinder (LBH)

- ▶ for small mass (large circle radius) LBH becomes more and more like  $d+1$  dimensional Schwarzschild-Tangherlini BH

tension vanishes in limit of vanishing mass

$$n \rightarrow 0 \quad \text{as} \quad \mu \rightarrow 0$$



- analytical results

- ansatz for metric

- first-order metric for LBH for  $\mu \ll 1$

(found using method of **matched asymptotic expansion**)

$$n(\mu) = \frac{(d-2)\zeta(d-2)}{2(d-1)\Omega_{d-1}} \mu + \mathcal{O}(\mu^2)$$

Harmark,NO/Wiseman

Harmark/Kol,Gorbonos

Karasik et al/Chu,Goldberger,Rothstein

- higher order corrections to metric and thermodynamics

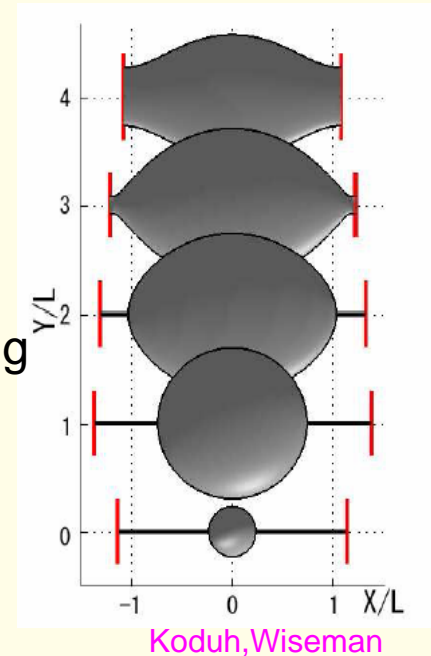
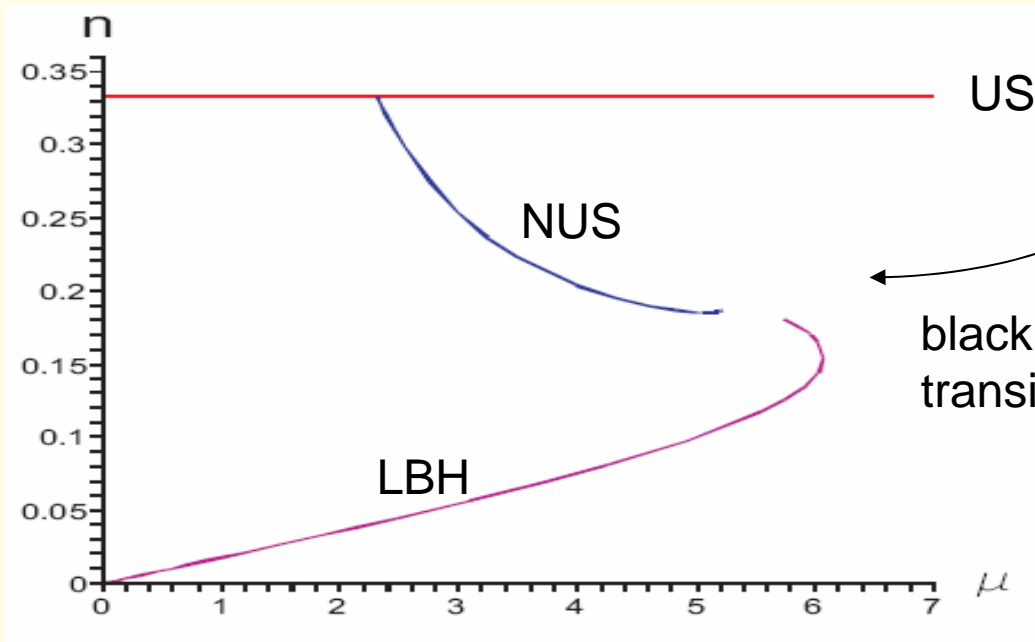
- numerical results

Sorkin,Kol,Piran/Kudoh,Wiseman

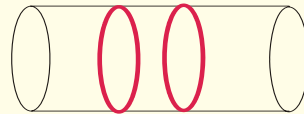
$d=4,5$ : LBH solution found with impressive accuracy



# Phase Diagram in six dimensions

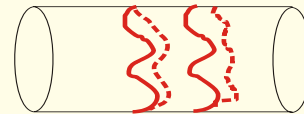


• uniform black string  
(US)  $S^{d-2} \times S^1$



Schwarzschild  $(d) \times S^1$

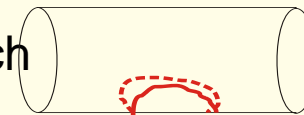
• non-uniform black string  
(NUS)  $S^{d-2} \times S^1$



emanates from uniform at  
Gregory-Laflamme point

Gregory,Laflamme/Gubser/Wiseman/Sorkin  
motivated in part by: Horowitz, Maeda

• localized black hole branch  
(LBH)  $S^{d-1}$



Schwarzschild  $(d + 1) + \mathcal{O}(\mu)$

Harmark,NO/Harmar/Kol,Gorbonos/  
Sorkin,Kol,Piran/Koduh,Wiseman

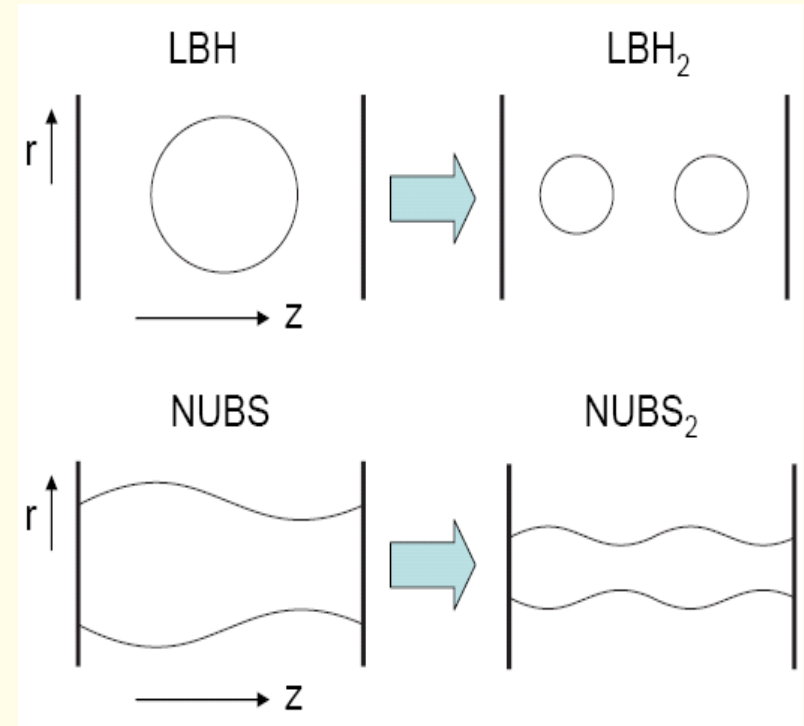


# Copies of LBH and NUBS phases

- for solutions that vary in circle direction

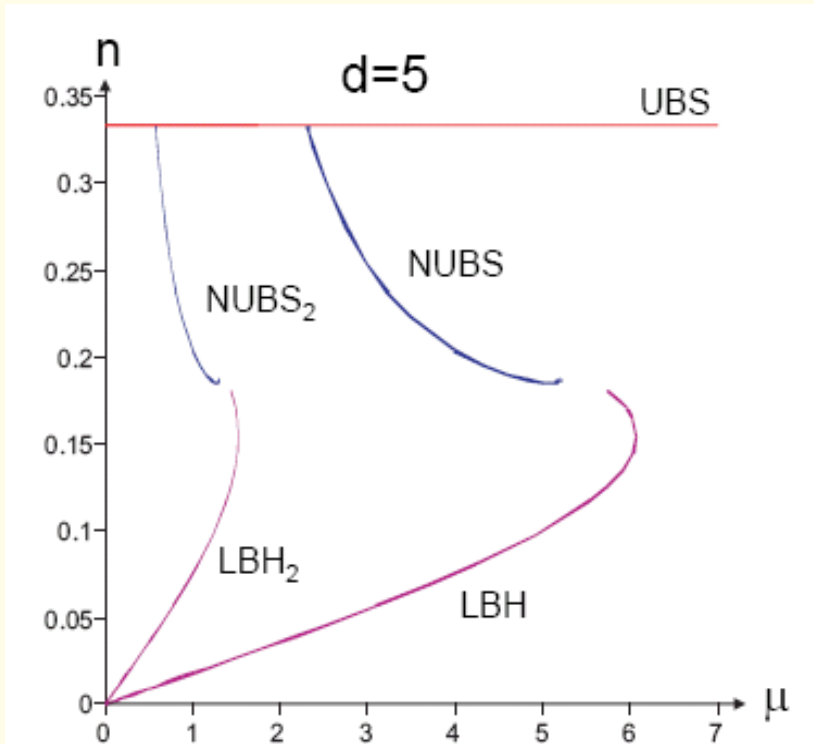
Horowitz/Harmark,NO

→ can generate **copies** by copying  $k$  times on circle and scaling appropriately in  $\mathbb{R}^{d-1}$  part



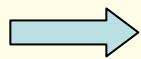
◀ from this we can argue existence of localized multi-black configurations of different mass/size

Dias.Harmark,MyersNO



# Multi-black hole configurations in phase diagram

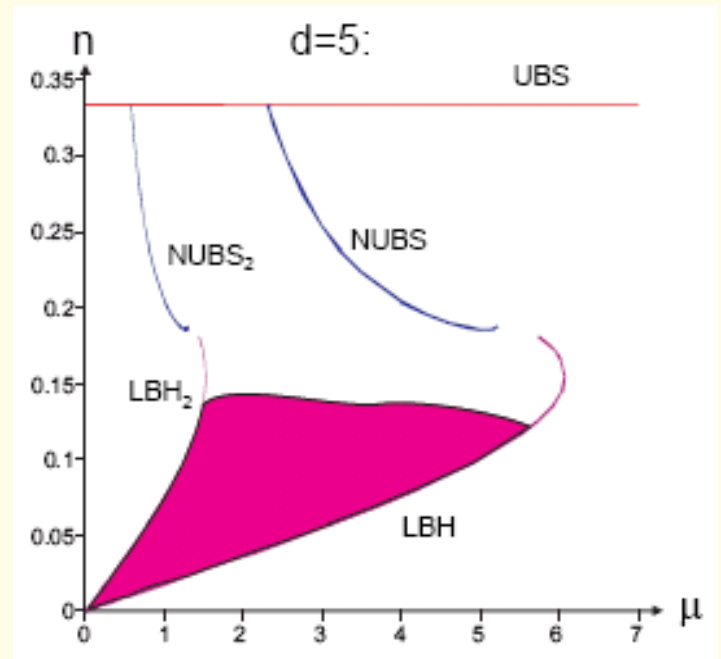
- ▶ range of  $n$  is such that it can take all values between small LBH solution and the small  $\text{LBH}_k$  solution



continuous non-uniqueness

for give mass  $\mu$

- takes infinite amount of continuous parameters to point to specific solution !  
(non thermal equilibrium solutions)

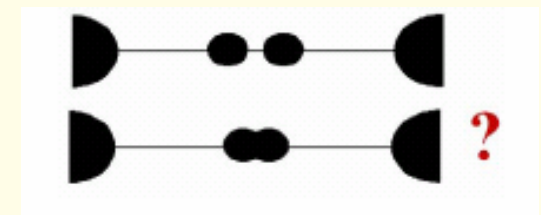


- ▶ multi BH solutions are in **unstable mechanical equilibrium**

in accordance with this: entropy of single BH is greater than entropy of mulit BH with  $k$  black holes (at same total mass)

$$S_k(\mu) < S(\mu)$$

- ▶ speculation: existence of (static) lumpy black holes  
(one big BH + two small BHs: small ones can merge into lumpy object before all horizons merge)

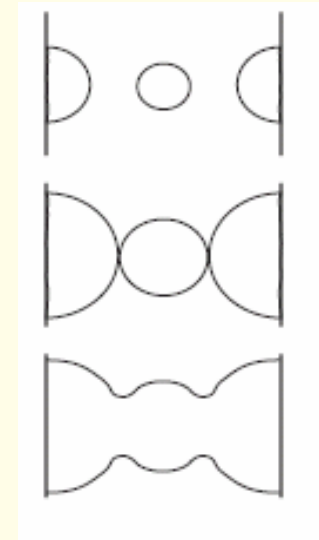


# Consequences for non-uniform black strings

► what happens when we crank up mass for multi BH configuration

→ could point to existence of **new non-uniform black strings** (bumpy black strings)

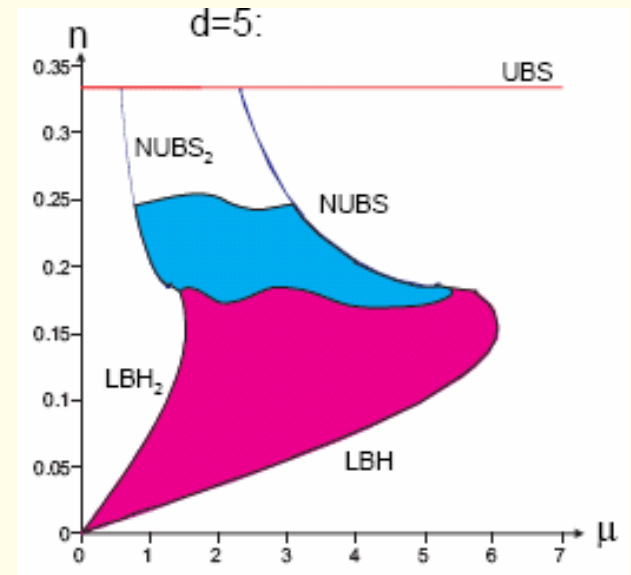
Open Q: if exist, how connected to GL critical masses ?



possibility:

GL instability first decays to non-uniform black string, then bifurcation into new non-uniform black strings

→ could change our understanding of GL instability as happening between uniform black string and single black hole without any intermediate steps



# Black rings and holes in higher dimensions

The non-uniqueness + phase structure of rotating rings in higher dimensions turns out to be intimately related to that of black hole solutions of vacuum Einstein equations in **six or more dimensions**  
Empanan, Harmark, Niarchos, NO, Rodriguez

► construct **thin rotating black rings in higher dimensions**

( $D \geq 6$ ) with horizon topology  $S^{D-3} \times S^1$

- matched asymptotic expansion Harmark/Gorbonos, Kol
  - metric of thin black ring in linearized gravity
  - near-horizon metric (dipole perturbations corresponding to bending a boosted black string)
  - match in overlap zone + require regular event horizon
    - zero pressure condition is required (balancing of ring)

► first steps to qualitatively complete **phase diagram** of asymptotically flat, neutral and rotating blackfolds

- exploit connection between black holes/strings/branes in KK spacetimes and higher-dim rotating black holes Empanan, Myers
  - use the analogy and known phase structure of KK BHs to uncover the phase diagram of stationary BHs
  - conjecture existence of infinite number of pinched black holes connection to black Saturns configs thru merger transitions

# Thin black rings from circular boosted black strings

- ▶ in limit of very large radius  $R$  of the  $S^1$  of 5D black ring metric becomes that of a (critically) boosted 5D black string

Emparan/Hovdebo, Myers/  
Emparan, Elvang, Virmani

aim: use **perturbative approach** to construct **thin black rings** in  $\mathcal{M}^{n+4}$  starting from **boosted black string** in  $\mathcal{M}^{n+4}$

horizon topology  $S^1 \times S^{n+1}$

thin means:  $R \gg r_0$

**zeroth order** solution in  $1/R$ : **straight** boosted black string

$$ds^2 = - \left( 1 - \cosh^2 \alpha \frac{r_0^n}{r^n} \right) dt^2 - 2 \frac{r_0^n}{r^n} \cosh \alpha \sinh \alpha dt dz + \left( 1 + \sinh^2 \alpha \frac{r_0^n}{r^n} \right) dz^2 \\ + \left( 1 - \frac{r_0^n}{r^n} \right)^{-1} dr^2 + r^2 d\Omega_{n+1}^2$$

- distributional source of energy/momentum for thin black ring

$$T_{tt} = \frac{r_0^n}{16\pi G} (n \cosh^2 \alpha + 1) \delta^{(n+2)}(r)$$

$$T_{tz} = \frac{r_0^n}{16\pi G} n \cosh \alpha \sinh \alpha \delta^{(n+2)}(r)$$

$$T_{zz} = \frac{r_0^n}{16\pi G} (n \sinh^2 \alpha - 1) \delta^{(n+2)}(r)$$

# Equilibrium condition

- ▶ boosted black string limit of black ring is described by **three parameters**

$$r_0, R, \alpha$$

expect physically: **two parameters** (e.g. given radius and mass, spin is fixed)

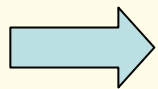
**dynamical balance condition** relates the three parameters

$$K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0 \longrightarrow \frac{T_{zz}}{R} = 0 \longrightarrow \sinh^2 \alpha = \frac{1}{n}$$

EOM for probe  
brane-like objects

Carter

critical boost:



enables computation of **all leading order thermodynamic quantities** !

$$M = \frac{\Omega_{n+1}}{8G} R r_0^n (n+2)$$

$$J = \frac{\Omega_{n+1}}{8G} R^2 r_0^n \sqrt{n+1}$$

$$A = \Omega_{n+1} 2\pi R r_0^{n+1} \sqrt{\frac{n+1}{n}}$$

$$R = \frac{n+2}{\sqrt{n+1}} \frac{J}{M}$$

valid in large  $J$  limit of black ring

crucial assumption: **horizon remains regular** when boosted black string is curved

- important check: rederive equilibrium condition from regularity condition

→ shows how GR encodes EOM of BHs as regularity conditions on geometry

# Matched asymptotic expansion

- ▶ MAE = systematic approach to **iteratively construct solution** given known solution in some limit + then correcting it in perturbative expansion

(applied e.g. to construct metric of small black holes on circle)

thin black rings:

two scales  $r_0$ ,  $R$



$r_0$ :  $S^{n+1}$  radius

asymptotic zone:  $r \gg r_0$

near-horizon zone:  $r \ll R$

$r$  is distance from ring

- **step 0**: solution in near-horizon zone to 0<sup>th</sup> order in  $1/R$   
= boosted black string of infinite length
- **step 1**: solve Einstein eqs. in **linearized approximation** around flat space for appropriate source (circular distribution of given mass/angular momentum density)  
- valid to first order in  $r_0^n \propto GM/R$ ,  $r_0 \ll r$
- **step 2**: find linear corrections to boosted black string for perturbation that is small in  $1/R$ , i.e. analyze effect of slightly bending the string  
- BCs fixed by matching to step 2 in **overlap region**:  $r_0 \ll r \ll R$   
+ require regularity at horizon
- **step 3**: solve next-to-linearized solution in asymptotic zone + use BCs from step 2  
- not necessary for phys. quantities, since can use corrections near horizon + Smarr

# Solution method

**Step 1a:** find metric in the **linearized approximation** around flat space sourced by a thin black ring localized on circle of radius  $R$

$$\square \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu} \quad T_{tt} \neq 0, \quad T_{t\psi} \neq 0 \quad \psi \simeq \frac{z}{R}$$

$$r_0 \gg r \quad T_{\mu\nu} \sim \delta(r)$$

**Step 1b:** Consider **overlap region**  $r_0 \ll r \ll R$

- effect of curving thin black string into locally arc of constant curvature radius  $R$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{r_0^n}{r^n} \left( h_{\mu\nu}^{(0)} + \frac{r \cos \theta}{R} h_{\mu\nu}^{(1)} \right) \quad \eta_{\mu\nu} = \text{flat space metric in ring-adapted coords}$$

Note: can explicitly check that regular solution requires  $T_{\psi\psi} = 0$   
 expected: additional stresses lead to singularities

**Step 2:** Find general solution **near black string** (hardest part)  $r_0 \leq r \ll R$   
 + match to metric in overlap region

$$g_{\mu\nu} = g_{\mu\nu}^{(bs)} + \frac{\cos \theta}{R} h_{\mu\nu}(r)$$

- can be solved exactly in terms of hypergeometric functions
- involves only **dipole perturbations**: thermodynamics not corrected thru this order (exception: five dimensions)



# Dimensionless quantities

- ▶ meaningful comparison in terms of **dimensionless quantities**  
classical GR does not possess intrinsic scale: use **mass**

$$j^{n+1} \propto \frac{J^{n+1}}{GM^{n+2}}, \quad a_H^{n+1} \propto \frac{\mathcal{A}^{n+1}}{(GM)^{n+2}}, \quad \omega_H \propto (GM)^{\frac{1}{n+1}} \Omega_H, \quad t_H \propto (GM)^{\frac{1}{n+1}} T_H$$

- thin black ring  $j^{n+1} \sim \left(\frac{R}{r_0}\right)^n$   $j$  very large
- ultraspinning MP BH  $j^{n+1} \sim \left(\frac{a}{r_0}\right)^{n-1}$   
( $a$  is rotation parameter)

- ▶ compare the “phase structure” for large  $j$

$$a_H(j), \quad \omega_H(j), \quad t_H(j)$$

↙ same as studying  $\mathcal{A}(M, J)$  at fixed  $M$

# Higher-dimensional black rings vs. MP black holes

black ring

$$a_H \sim \frac{1}{j^{1/n}}$$

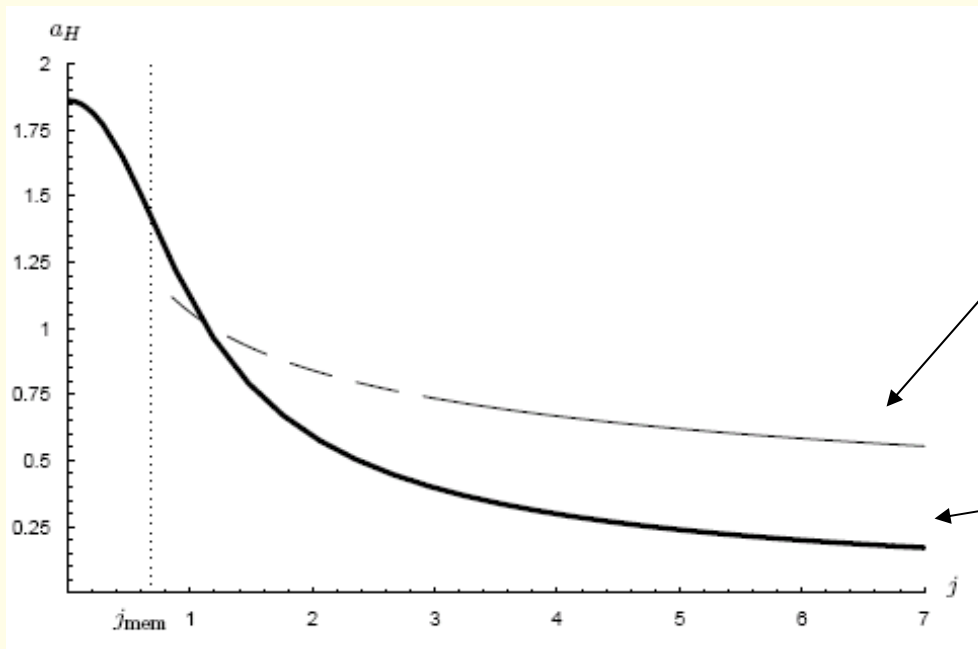
black hole

$$a_H \sim \frac{1}{j^{2/(n-1)}}$$

area

⇒ black rings dominate entropically in ultraspinning regime ( $n > 1$ )

e.g. in 7D



$$a_H^{\text{ring}} \sim \frac{1}{j^{1/3}}$$

$$a_H^{\text{hole}} \sim \frac{1}{j}$$

onset of membrane-like behavior of MP BH

# Higher-dimensional black rings vs. MP black holes (cont'd)

	black ring	black hole
angular velocity	$\omega_H \sim \frac{1}{2j}$	$\omega_H \sim \frac{1}{j}$

- ▶ black ring has hole in middle (wheel), more efficient to carry spin than packed MP BH (disk) → **ring rotates more slowly**

temperature	$t_H \sim j^{1/n}$	$t_H \sim j^{2/(n-1)}$
-------------	--------------------	------------------------

- ▶ i.e. **black ring cooler** than MP BH at fixed mass:  
same mass in a wheel makes it thicker than in a pancake  
+ temperature inversely proportional to thickness

# GL Instability of ultraspinning MP BH

MP BH approaches **black membrane** geometry  $\mathbb{R}^2 \times S^n$  for large  $j$

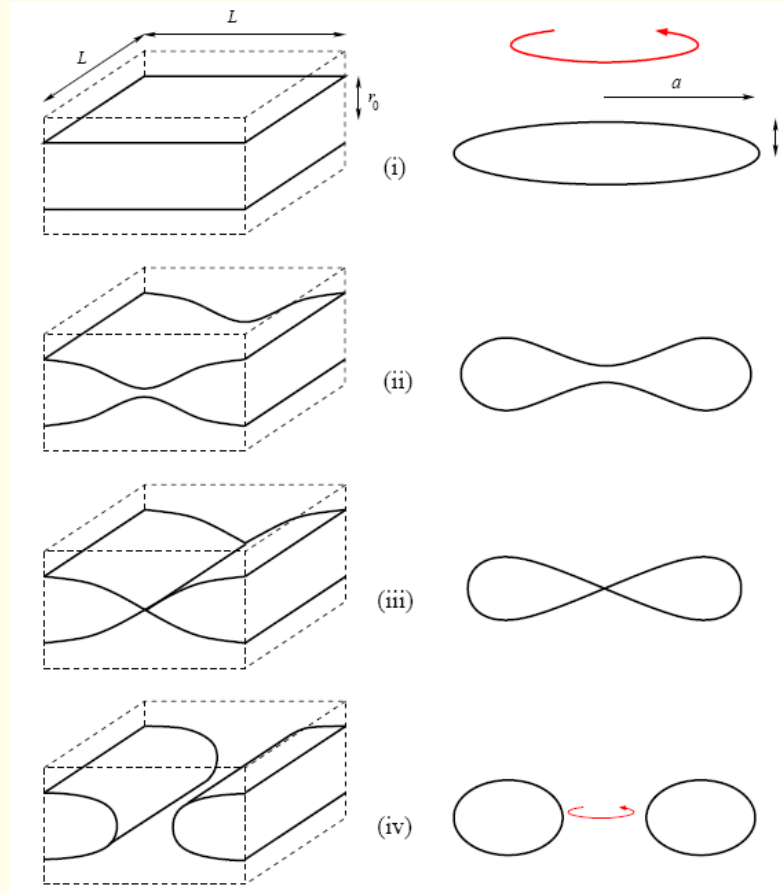
Empanan, Myers

$$a \gtrsim r_0$$

$$a \sim J/M$$

rotational axial symmetry so  
one translational symmetry along  $\mathbb{T}^2$

**uniform** black  
membrane



**ultraspinning** MP  
black hole

**pinched** membrane  
(GL instability)  
non-uniform phase

**lumpy** (pinched)  
black holes

**pinched-off**  
membrane

**pinched-off**  
black hole

**localized** black  
string

**black ring**

# Phase Diagram for Kaluza-Klein BHs on two-torus

- import knowledge of **KK black holes on circle** + add one uniform direction

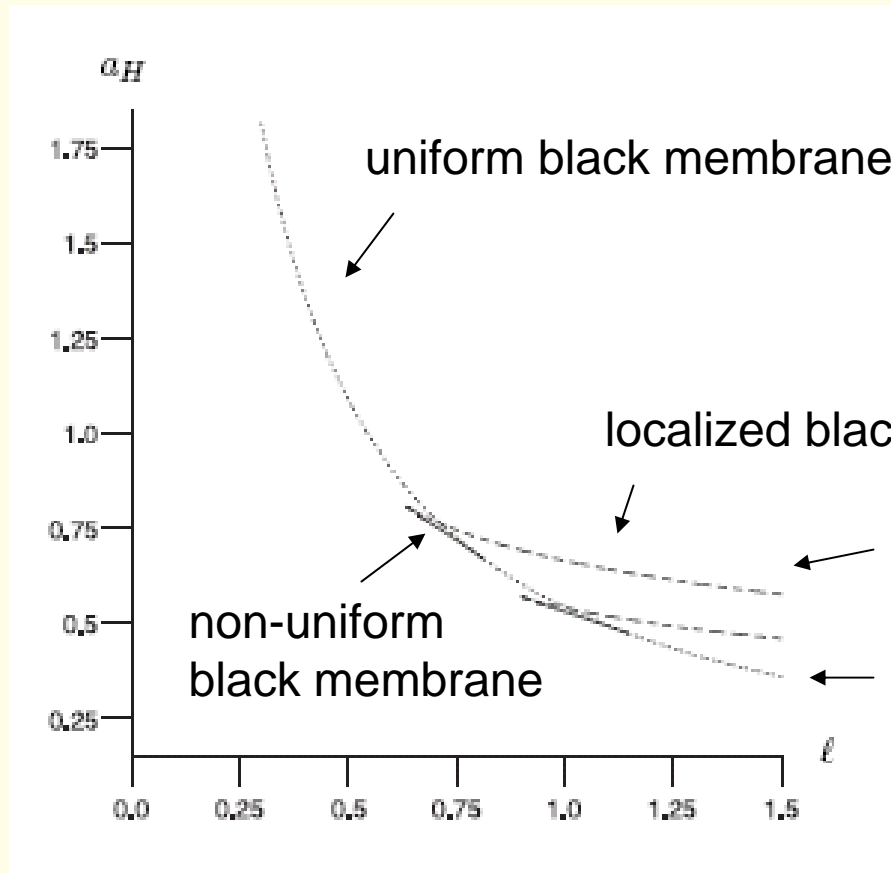
see e.g. review: [Harmark, Niarchos, NO](#)

- square two-torus sides  $L$

identify:  $j \leftrightarrow \ell$

$$\ell^{n+1} \propto \frac{L^{n+1}}{GM}$$

(measure linear size of horizon along rotation plane/torus for fixed mass)



example:  
 KK BHs in 7 dimensions  
 on  $\mathcal{M}^5 \times \mathbb{T}^2$

$$a_H^{\text{lbs}}(\ell) \sim \ell^{-\frac{1}{n}} \quad (\text{cf. thin BR})$$

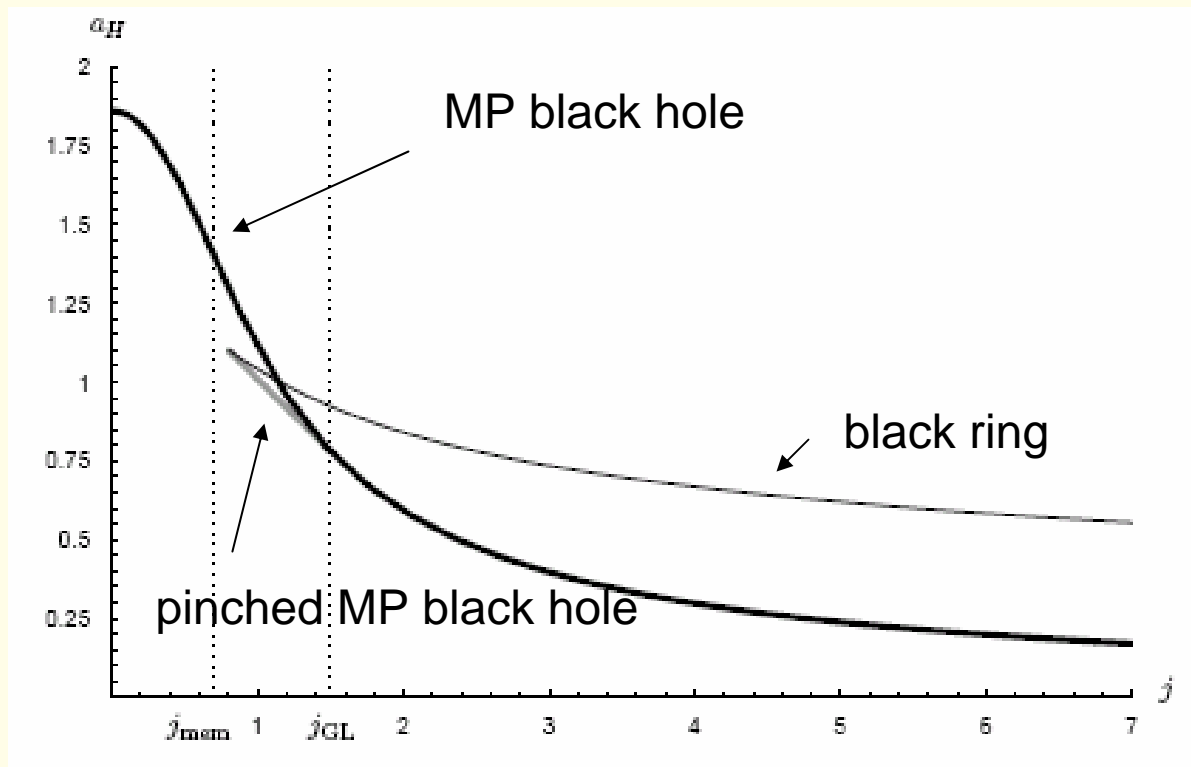
$$a_H^{\text{ubm}}(\ell) \sim \ell^{-\frac{2}{n-1}} \quad (\text{cf. ultraspinning MP BH})$$

+ **copies** by copying the solutions  $k$  times on the circle

# Towards completing the phase diagram

◀ based on analogy with phase diagram for KK BHs on torus:  
extrapolate to  $j = \mathcal{O}(1)$  regime

- proposal for **phase diagram of stationary BHs** (one angular momentum)  
in asymptotically flat space: **main sequence** = MP BH, pinched MP BH, black ring  
(uniform, non-uniform, localized)



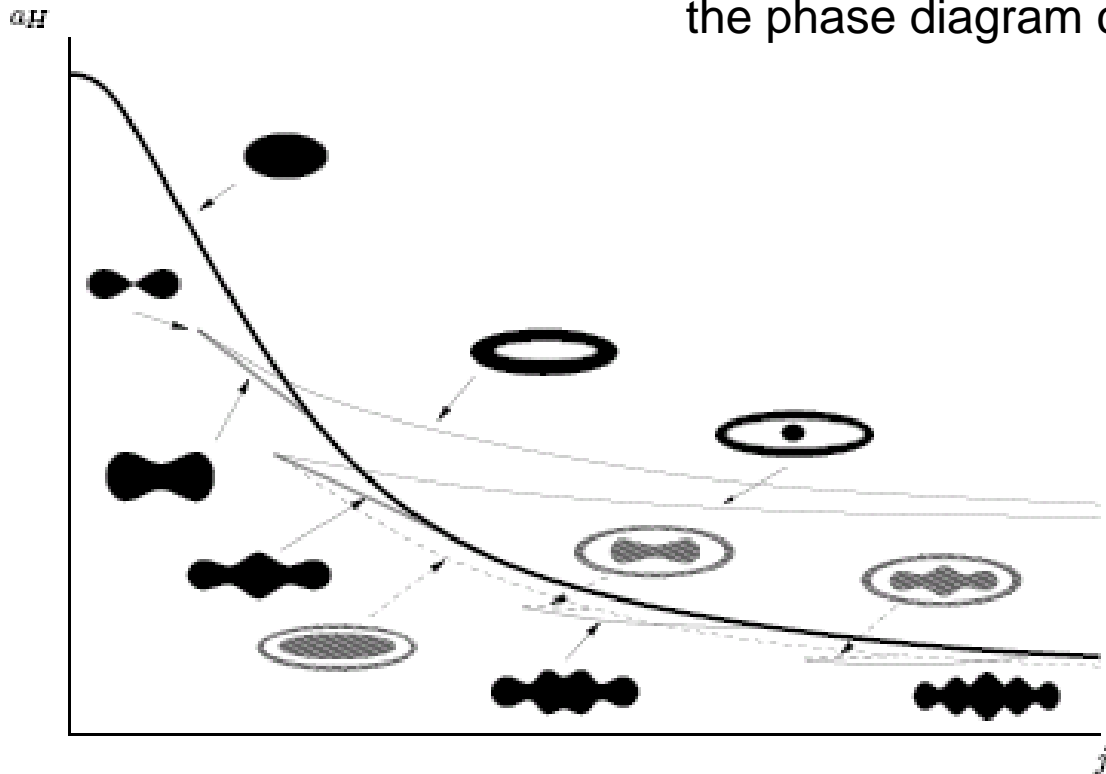
# Black saturns and multi-pinches

most likely features

- **main sequence**: BH with pinch at rotation axis meets black ring phase
- **infinite sequence of pinched BHs** emanating from BH curve (from copies of the GL zero mode)
- upper **black Saturn** curve + merger to circular pinch

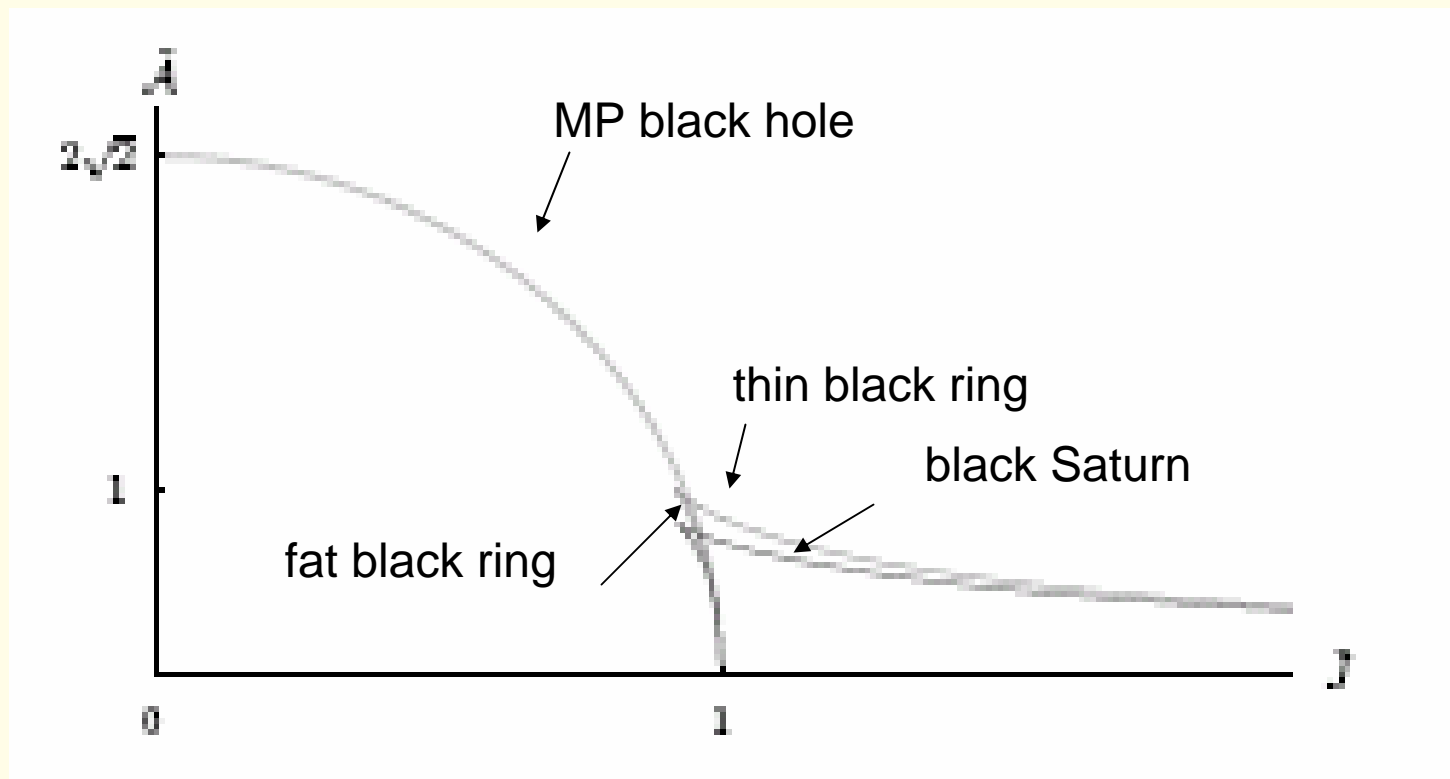
less compelling arguments for: **pancaked + pinched black Saturns**

(but admit a simple and natural way for completing the phase diagram consistent with available info)



# Comparison to five dimensions

- 5D : MP BHs have **upper bound** on  $j$  (black membranes do not exist in 5D)
- **fat black ring** instead of pinched MP (behave like drilled-through MP BH)
  - only one type of black Saturn at large  $j$





## Further properties

- ◀ so far: **thermal equilibrium phases** (equal temperatures/angular velocities when more than one black object present)

**non-thermal equilibrium** phases perfectly valid as stationary multi-BH configurations of GR → continuous families of solutions, e.g

·  
more general black Saturns, di-rings etc.  
(analogue of localized multi-BH solutions in KK space with unequal mass BHs)

Dias, Harmark, Myers, NO

- ◀ **stability properties**

(like in 5D) black rings at large  $j$ , **GL-type instability** creating ripples on  $S^1$   
fragments the ring into black holes flying apart

- MP BH + pinched BH could suffer from same type of instability
- black ring solutions below the cusp could be unstable towards **radial perturbations** causing collaps into MP BH

# Future directions + outlook

## ► KK black holes

- further examine possibility of **new non-uniform strings and lumpy black holes**
- analytical second order corrections, higher D compact spaces + adding J
  - apply effective FT technique of **Chu, Goldberger, Rothstein**
- numerical studies
  - extend into non-perturbative domain, temp. converge for multi BH ?
- applications to ST + dual gauge theories (see next time)
  - multiply gapped eigenvalue distributions as saddle points in finite T SYM ?
  - 3-charge multi BHs on circle + microscopic entropy **Harmark, Kristjansen, NO, Roenne/ Chowdhury, Giusto, Mathur**
- examine fluid analogy

## ► higher D black rings

- black saturns, rings in (A)dS spaces, dipoles, etc.
- other **topologies**  $S^2 \times S^2$ ,  $S^2 \times T^2$ ,  $S^2 \times \Sigma_{g>1}$  (in six dimensions)
  - boosted black p-branes:  $S^{D-2-p} \times T^p$  (in progress)
- balance conditions, global properties (embedding of the different topologies)
- exact solutions ? - numerics, effective FT techniques for thermodynamics
- SUSY black rings + ST
- duality to **plasma balls + rings** in AdS (cf. **Lahiri, Minwalla**) – many similar features

- further explore relation between **phase structure of KK BHs and rotating black objects**