Stability of the Hořava-Witten Model

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Abstract

We consider scalar perturbations in the time-dependent Hořava-Witten Model in order to study its stability. We show that during the pre-big bang epoque the model evolves without instabilities until it encounters the curvature singularity where the big bang is supposed to happen. We compute the frequencies of the scalar field oscillation during the stable period and show how the oscillations encounter the singularity.

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P.Hořava, E.Witten, *NPB* **460**, 506 (1996)

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- By dimensional reduction the 5D supergravity solution is

$$ds_5^2 = \tilde{H}(-dt^2 + d\vec{x}^2) + \tilde{H}^4 d\tilde{y}^2\,, \qquad (1)$$

where $\tilde{H} = 1 + \tilde{k}|\tilde{y}|\,, \quad \phi = -3\log\tilde{H}\,.$

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- The equations of motion are obtained from

$$\mathcal{L}_5 = \sqrt{-g}\,(R - \frac{1}{2}(\partial\phi)^2 - m^2 e^{2\phi})\,. \qquad (2)$$

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♣ The model is not realistic.

♣ Attempts to incorporate HW Model into Braneworld cosmology: Ekpyrotic Universe, Cyclic Universe.

Time-Dependent Hořava-Witten Model

W. Chen, Z.-W. Chong, G.W. Gibbons, H. Lu, C.N. Pope, *NPB* **732, 118 (2006)**

The metric is given by

$$ds_5^2 = H^{1/2}(-dt^2 + d\vec{x}^2) + H dy^2 \,, \tag{3}$$

with

$$H = ht + k|y| \,, \quad \phi = -\frac{3}{2}\log H \,. \tag{4}$$

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♣ $t < 0 \longrightarrow$ Two 3-branes approaching.

♣ $t = 0 \longrightarrow$ Curvature singularity on negative tension brane $\longrightarrow$ reaches positive tension brane at $t = kL/(-h)$.

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or using the metric (3)

$$\Big\{-H^{-1/2}\partial_t^2 - hH^{-3/2}\partial_t + H^{-1/2}\partial_r^2 + \frac{2}{r}H^{-1/2}\partial_r + \frac{H^{-1/2}}{r^2}\times$$
$$\times\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta) + \frac{1}{\sin^2\theta}\partial_\phi^2\right] + H^{-1}\partial_y^2 + \frac{k}{2}H^{-2}\mathsf{sgn}(y)\partial_y - m^2\Big\}\,\Phi = 0\,. \tag{6}$$

We decompose the scalar field as

$$\Phi(t,r,\theta,\phi,y) = Z(t,r,y)Y_{\ell m}(\theta,\phi)\,, \tag{7}$$

where the spherical harmonic part obeys

$$\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta Y_{\ell m}) + \frac{1}{\sin^2\theta}\partial_\phi^2 Y_{\ell m} = -\ell(\ell+1)Y_{\ell m}\,. \tag{8}$$

A further variable separation $Z(t,r,y) = \Psi(t,y)R(r)$ produces

$$\partial_r^2 R + \frac{2}{r}\partial_r R + \left(\alpha^2 - \frac{\ell(\ell+1)}{r^2}\right) R = 0\,, \tag{9}$$

which solution is

$$R(r) = \frac{A}{\sqrt{r}}\, J\left(\frac{1}{2}+\ell, \alpha r\right) + \frac{B}{\sqrt{r}}\, Y\left(\frac{1}{2}+\ell, \alpha r\right)\,. \tag{10}$$

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And

$$\partial_t^2 \Psi + \frac{h}{H}\partial_t \Psi - \frac{1}{\sqrt{H}}\partial_y^2 \Psi - \frac{k\,\mathrm{sgn}(y)}{2H^{3/2}}\partial_y \Psi + (\alpha^2 + m^2 H^{1/2})\Psi = 0\,. \qquad (11)$$

Figure 1: Potential $\alpha^2 + m^2\, H^{1/2}$ for $\alpha^2 = 1$, $m = 0.1$.

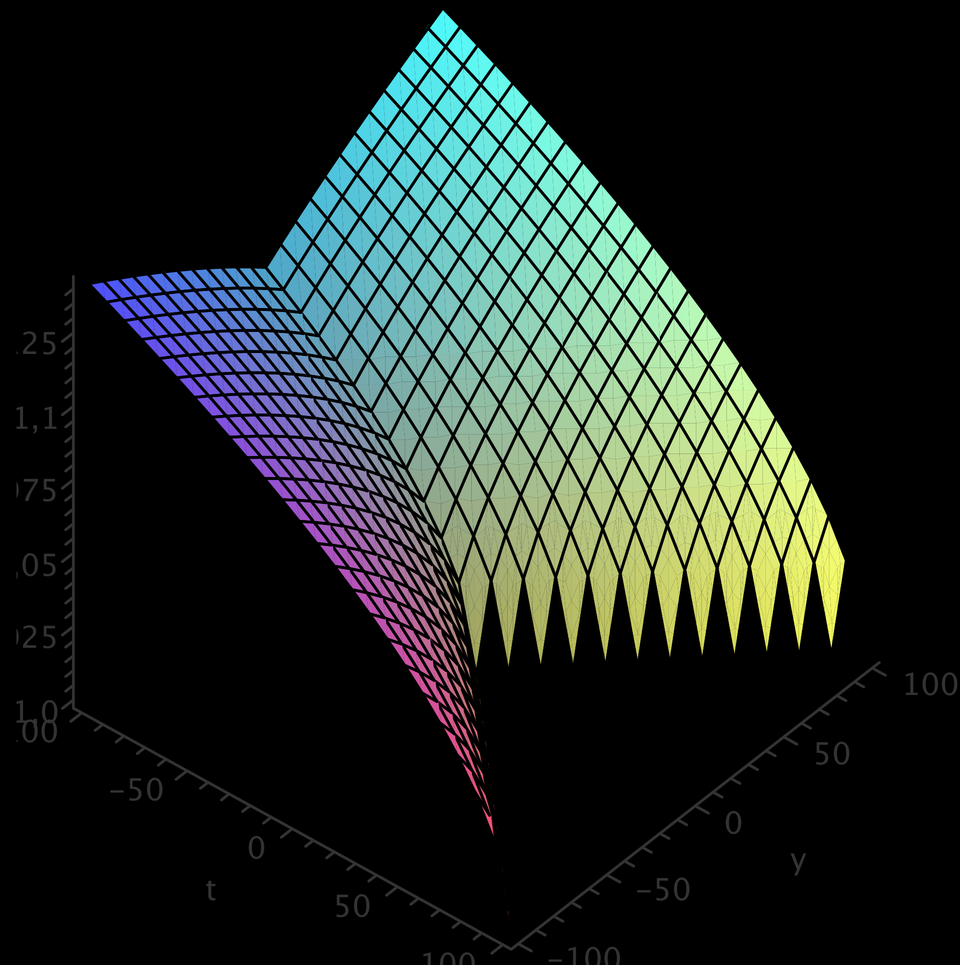


Figure 2: Quasinormal modes at $y = 0$ for $m = 0$ and different values of k.

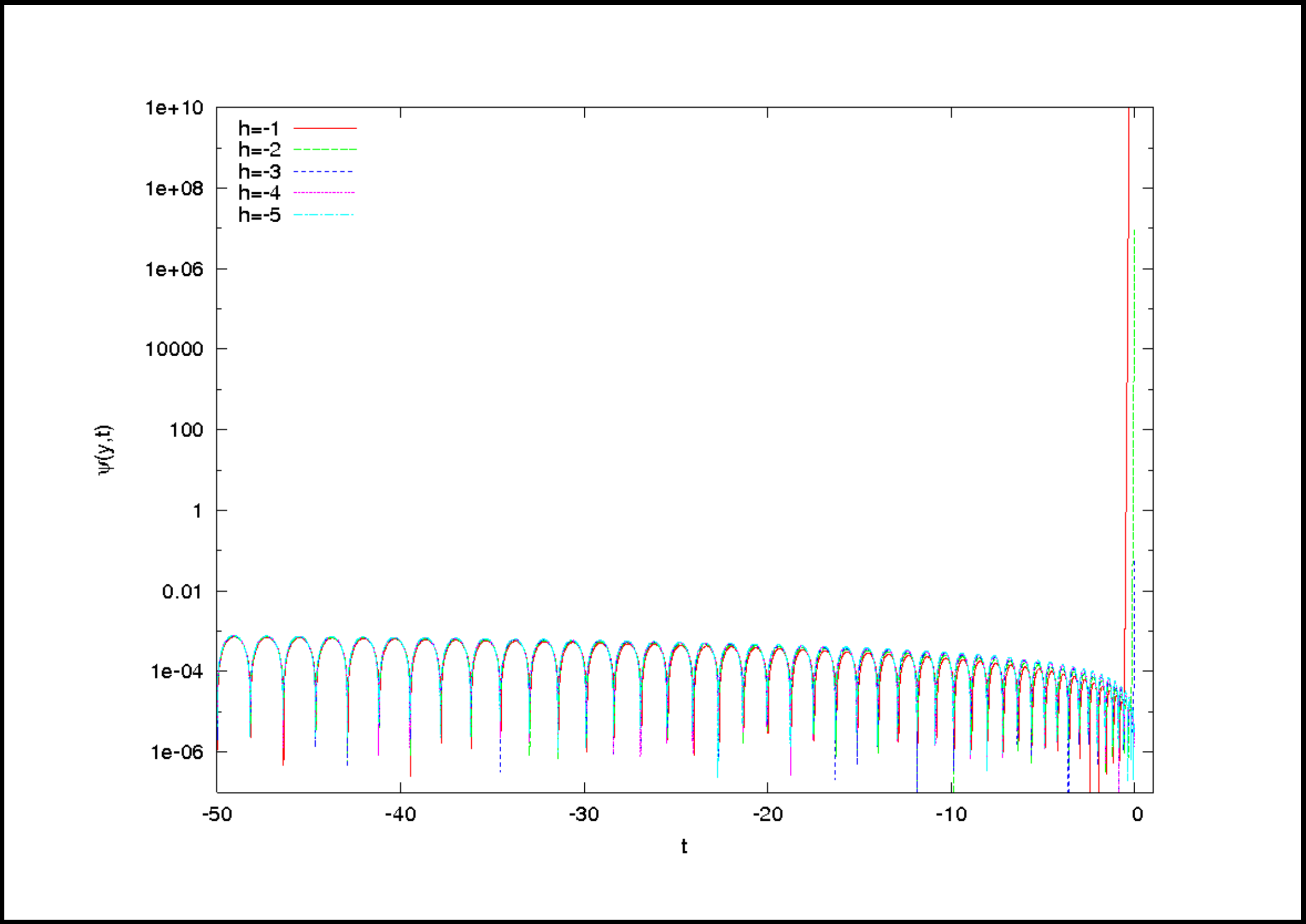
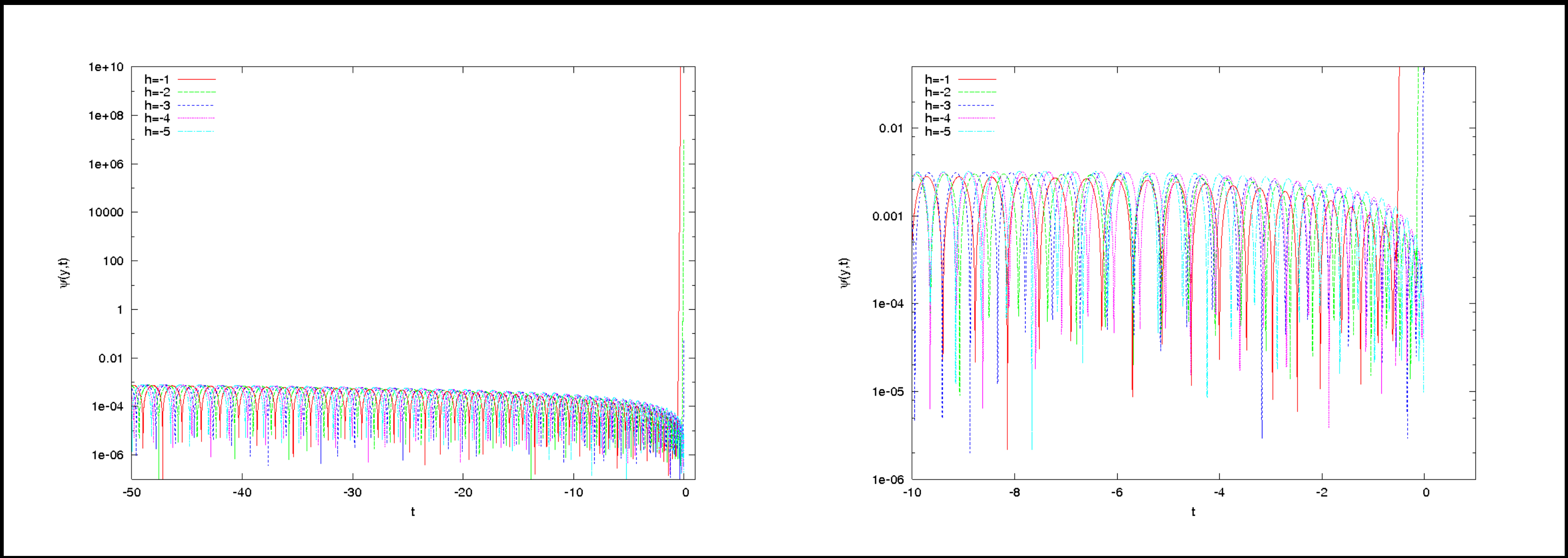


Figure 3: Quasinormal modes at $y = 0$ for $m = 0.1, 2.0$ and different values of k.



Quasinormal Frequencies

Table 1: Quasinormal Frequencies at $y = 0$.

$\|\mathbf{h}\|$	$m = 0$		$m = 0.1$		$m = 2$	
k	$\omega_{\mathbf{R}}$	$\omega_{\mathbf{I}}$	$\omega_{\mathbf{R}}$	$\omega_{\mathbf{I}}$	$\omega_{\mathbf{R}}$	$\omega_{\mathbf{I}}$
1	1.428	-0.0023	3.452	0.0015	6.411	0.0010
2	1.428	-0.0021	4.028	0.0016	7.662	0.0025
3	1.428	-0.0021	4.363	0.0014	8.491	0.0026
4	1.428	-0.0020	4.689	0.0017	8.976	0.0022
5	1.428	-0.0020	4.909	0.0016	9.520	0.0028

Figure 4: Quasinormal modes at $y = 50$ for $m = 0, 0.1, 2.0$ and different values of k.

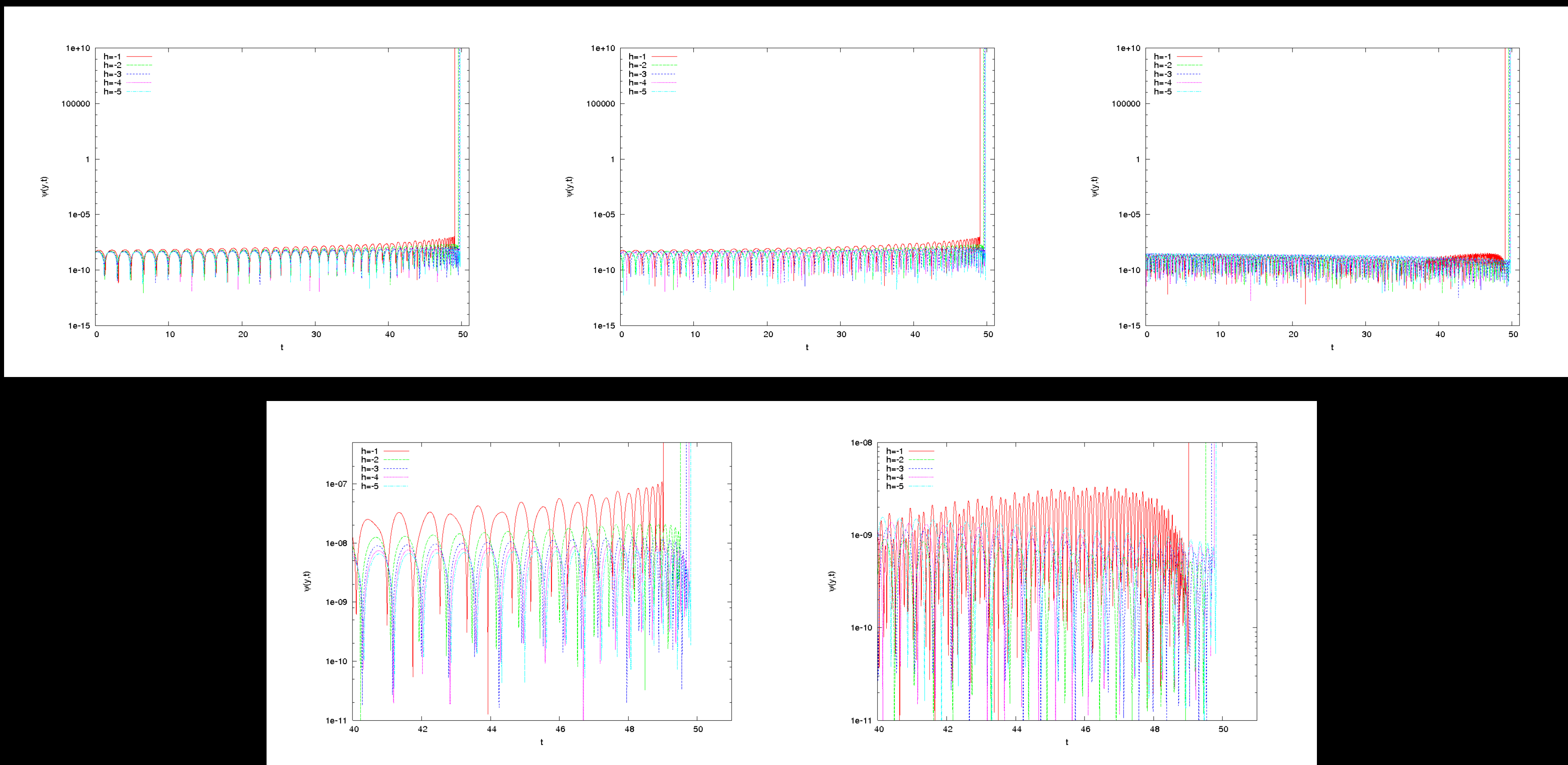


Table 2: Quasinormal Frequencies at $y = 50$.

$\|\mathbf{h}\|$	$m = 0$		$m = 0.1$		$m = 2$	
$\mathbf{k}$	$\omega_{\mathbf{R}}$	$\omega_{\mathbf{I}}$	$\omega_{\mathbf{R}}$	$\omega_{\mathbf{I}}$	$\omega_{\mathbf{R}}$	$\omega_{\mathbf{I}}$
1	1.293	-0.0005	3.740	0.0006	7.140	0.0013
2	1.288	-0.0005	4.363	0.0008	8.491	0.0010
3	1.293	-0.0006	4.760	0.0008	9.240	0.0017
4	1.293	-0.0006	5.150	0.0003	10.134	-0.0025
5	1.293	-0.0006	5.417	0.0007	10.472	0.0022

Figure 5: Quasinormal modes at $y = 100$ for $m = 0, 0.1, 2.0$ and different values of k.

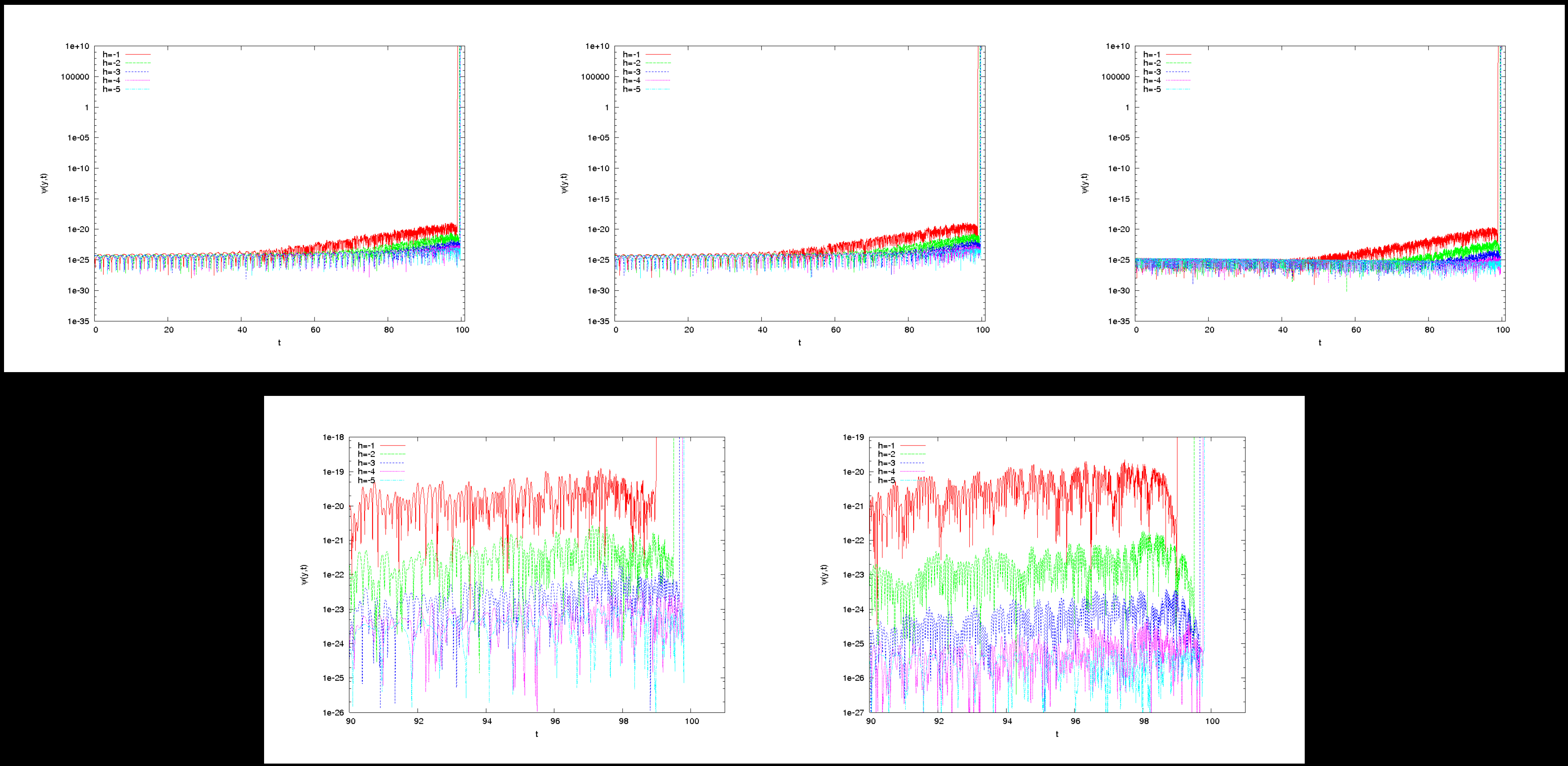


Table 3: Quasinormal Frequencies at $y = 100$.

$\|\mathbf{h}\|$	$m = 0$		$m = 0.1$		$m = 2$	
$\mathbf{k}$	$\omega_{\mathbf{R}}$	$\omega_{\mathbf{I}}$	$\omega_{\mathbf{R}}$	$\omega_{\mathbf{I}}$	$\omega_{\mathbf{R}}$	$\omega_{\mathbf{I}}$
1	1.199	0.0076	3.927	0.0773	7.854	0.0863
2	1.200	0.0639	4.620	0.0642	9.240	0.0799
3	1.204	0.0582	5.150	0.0590	10.472	0.0838
4	1.213	0.0548	5.512	0.0551	10.833	0.0232
5	1.213	0.0519	5.818	0.0524	11.220	0.0271

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♣ The quasinormal frequencies calculated at the stable period show a strong dependence on the parameters of the model for $m \neq 0$ as it is expected.

♣ Between $t = 0$ and $t = kL/(-h)$ the scalar field oscillations increase frequency and amplitude showing the instability generated by the curvature singularity at the negative tension brane that finally envelopes all the spacetime.