MYTILENE 2007, FOURTH AEGEAN SUMMER SCHOOL: BLACK HOLES

Title: Linearized Gravity for Brane Models with a Non-Minimally coupled bulk scalar field

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arXiv:0705.2364
Title: Graviton localization and Newton's law for brane models with a non-minimally coupled bulk scalar field
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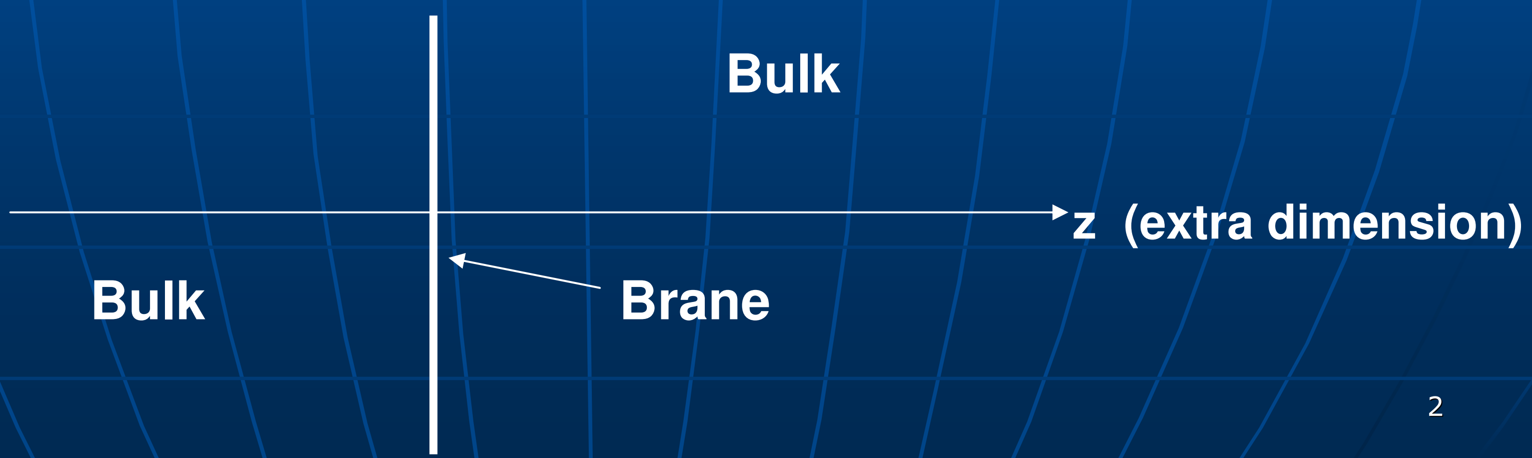
The article has been accepted for publication in Physical Review D

BRANE WORLD SCENARIO

- In the case of brane world scenario, ordinary matter is trapped into a three dimensional submanifold (brane world) that is embedded in a fundamental multidimensional manifold (bulk).
- Only gravitons can propagate in the bulk.
- This scenario allows the extra dimensions to be large or even infinite.
- Brane world models predict new phenomenology even at TeV scale, and put on a new basis fundamental problems such as the hierarchy problem and the cosmological constant problem.



There are two standard Brane world models with gravity

- FLAT EXTRA DIMENSIONS: ADD Model (Arkani-Hammed, Dimopoulos and D'vali). Large compact extra dimensions (R~160μm).

- WARPED EXTRA DIMENSIONS: First (Large compact extra dimensions) and Second (Infinite extra dimension) RANDALL-SUNDRUM Model

There are many generalizations of the RS-Model

- Models with more than five dimensions (e.g. six dimensions)
- Models with topological defects toward the extra dimensions (i.e. kink, vortex, monopole)
- Models with higher order curvature corrections (i.e. Gauss-Bonnet Gravity)

RS2-Model: is a single-brane model with an **infinite extra dimension z**. We have a brane with positive tension σ, and a bulk with a negative five-dimensional cosmological constant Λ.

$$L_{gravity} = \frac{1}{2}\sqrt{|g|}\left(R - 2\Lambda\right) - \sqrt{\left|g^{(brane)}\right|}\sigma\ \delta(z)$$

$$\Lambda = -\frac{1}{6}\sigma^2, \qquad \ell^{-1} = \frac{\sigma}{6}$$

Effective size of the extra dimension

$$ds^2 = e^{-2\frac{|z|}{\ell}}(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2) + dz^2$$

Corrections

$$V(r) = G_4 \frac{m_1 m_2}{r}\left(1 + \frac{2}{3}\frac{\ell^2}{r^2}\right)$$

Newtonian potential on the brane

Zero mode + brane bending

Continuum spectrum of positive energy states

We have considered a generalization of RS2-model which involves a Nonminimally Coupled Bulk Scalar Field

$$S=\int d^5x(L_{gravity}+L_{scalar}+L_{int})$$

$$L_{gravity}=\sqrt{|g|}\frac{1}{16\pi G_5}R-\sqrt{\left|g^{(brane)}\right|}\sigma(\phi)\ \delta(z)$$

$$L_{scalar}=\sqrt{|g|}\left(-\frac{1}{2}g^{MN}\nabla_M\phi\ \nabla_N\phi-V(\phi)\right)$$

Non-minimal Coupling interaction term

$$L_{int}=-\frac{1}{2}\xi\phi^2R$$

$\xi=0$ minimal coupling

$\xi=\frac{1}{4}\left[(n-2)/(n-1)\right]$, n=5, conformal coupling

$\xi\neq 0$ nonminimal coupling

Einstein Equations including the bulk field

$$F(\phi)G_{\mu\nu}-\nabla_\mu\nabla_\nu F(\phi)+g_{\mu\nu}\Box F(\phi)+\frac{1}{2}\sigma(\phi)\ \delta(z)\frac{\sqrt{\left|g^{(brane)}\right|}}{\sqrt{|g|}}g_{ij}\delta^i_\mu\delta^j_\nu=\frac{1}{2}T^{(\Phi)}_{\mu\nu}$$

$$\Box\phi+RF'(\phi)-V'(\phi)-\sigma'(\phi)\delta(z)=0,\ \ F(\phi)=\frac{1}{2}\left(1-\xi\phi^2\right)$$

$$T^{(\phi)}_{\mu\nu}=\nabla_\mu\phi\nabla_\nu\phi-g_{\mu\nu}\left(g^{\rho\sigma}\nabla_\rho\phi\nabla_\sigma\phi+V(\phi)\right)$$

Static Solutions of the Einstein Equations

$$ds^2=e^{2A(z)}\eta_{ij}dx^i dx^j+dz^2,\ \ \phi=\phi(z)$$

MOTIVATION:
A GRAVITY INDUCED MECHANISM FOR THE GAUGE FIELD LOCALIZATION ON THE BRANE

1) K. Farakos and P. Pasipoularides, Phys. Lett. B621 (2005) 224-23.

2) O. Bertolami and C. Carvalho, Title: Spontaneous Symmetry Breaking in the Bulk as a Localization Mechanism of Fields on the Brane, arXiv:0705.1923 .

NUMERICAL STATIC SOLUTIONS

3) K. Farakos and P. Pasipoularides, Phys.Rev. D73 (2006) 084012.

4) K. Farakos and P. Pasipoularides, Phys.Rev. D75 (2007) 024018.

ANALYTICAL STATIC SOLUTIONS

5) C. Bogdanos, A. Dimitriadis, K. Tamvakis, Phys.Rev. D74 (2006) 045003.

Metric fluctuations around the static solutions of brane models with a non-minimally coupled scalar field

0705.2364

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$$ds^2 = e^{2A(\bar{z})}\left(\eta_{ij} + \bar{h}_{ij}\right) d\bar{x}^i d\bar{x}^j + d\bar{z}^2, \ \phi(z) + \delta\phi \ \left(\delta\phi = 0\right)$$

Gaussian normal coordinates: $\bar{x}_i, \bar{z}$

The brane is located at $\bar{z} = 0$

$$\bar{h}_{i5} = \bar{h}_{55} = 0, \ i=0,1,2,3$$

Linearized Equations in a free source space-time

In order to achieve decoupling of the Einstein Equation we perform the following Gauge Transformation.

$$\bar{x}_i, \bar{z} \rightarrow x_i, z \qquad \bar{h}_{ij} \rightarrow h_{ij}$$

$$h = h_{ij}\eta^{ij} = 0 \quad \text{Traceless}$$

$$\partial^i h_{ij} = 0 \quad \text{Transverse}$$

$$F(\phi)\left[-e^{2A}\left(\partial_z^2 + 4A'\partial_z\right) - \Box^{(4)}\right]h_{ij} = e^{2A}F'(\phi)\partial_z h_{ij}$$

Graviton Localization

$$h_{ij}(x,z) = e^{ipx}u(m,z) \qquad m^2 = -p^\mu p_\mu, \quad Q = 4A + \ln(F(\phi))$$

Non-Minimal Coupling

$$\left(\partial_z^2 + Q'\partial_z + m^2 e^{-2A}\right)u(m,z) = 0$$

Minimal Coupling

$$\left(\partial_z^2 + 4A'\partial_z + m^2 e^{-2A}\right)u(m,z) = 0$$

Zero Mode (4D-Graviton)

$$m^2 = 0 \Rightarrow \left(\partial_z^2 + Q'\partial_z\right)u(m,z) = 0 \Rightarrow u(m=0,z) = \text{constant}$$

The constant solution u(0,z) in normalizable with a weight factor

$$r(z) = F(\phi)e^{2A} > 0, \quad \int_{-\infty}^{+\infty} dz\, r(z)\, u(0,z)^2 < +\infty$$

Zero mode+ Continuum spectrum of positive energy states

$$ds^2 = e^{2\tilde{A}(w)}\left(\eta_{ij}dx^i dx^j + dw^2\right),\ \tilde{\phi} = \tilde{\phi}(w),\ w = \int_0^z e^{-A(z)}dz$$

$$\tilde{Q}(w) = 3\tilde{A}(w) + \ln\left(F\left(\tilde{\phi}(w)\right)\right),\ \tilde{V}(w) = \frac{1}{2}\tilde{Q}''(w) + \frac{1}{4}\left(\tilde{Q}'(w)\right)^2,\ \tilde{u}(m,w) = e^{-\frac{\tilde{Q}}{2}}\tilde{v}(m,w),$$

Schrodinger Equation

$$-\partial_z^2\tilde{v}(m,w) + \left(\tilde{V}(w) - m^2\right)\tilde{v}(m,w) = 0$$

$$L^{\dagger}L\tilde{v}(m,w) = m^2\tilde{v}(m,w) \Rightarrow m^2 \geq 0$$

$$L^{\dagger} = -\partial_w - \frac{1}{2}Q'(w),\ L = \partial_w - \frac{1}{2}Q'(w)$$

$$\lim_{w\to\pm\infty}\tilde{V}(w) = 0 \Rightarrow \text{Continuum spectrum}$$

"VOLCANO" FORM POTENTIAL

4D Gravity on the Brane in the case of a Non-Minimally coupled bulk scalar field

The proper approach to determine 4D-Gravity on the brane is the Bent Brane Formalism by J. Garriga and T. Tanaka

We put a matter source (point-like source) on the brane

$$T_{\mu\nu} = S_{\mu\nu}\left(\overline{x}\right)\delta\left(\overline{z}\right),\; S_{\mu\nu}\left(\overline{x}\right) = M\delta^{0}_{\mu}\delta^{0}_{\nu}\delta\left(\overline{x}\right)$$

The issue, is to determine the metric fluctuations by solving the Einstein Equations in the presence of the matter source on the brane

$$ds^2 = e^{-2k\overline{z}}\left(\eta_{ij} + \overline{h}_{ij}\right)d\overline{x}^i d\overline{x}^j + d\overline{z}^2$$

Bent Brane Formalism

In order to achieve decoupling of the Einstein Equation we consider the following Gauge transformation (point transformation)

$$x^i = \bar{x}^i + \hat{\xi}^i(\bar{x}), \quad z = \bar{z} + \hat{\xi}^5(\bar{x}) \qquad \bar{h}_{5\mu} = h_{5\mu} = \bar{h}_{55} = h_{55} = 0$$

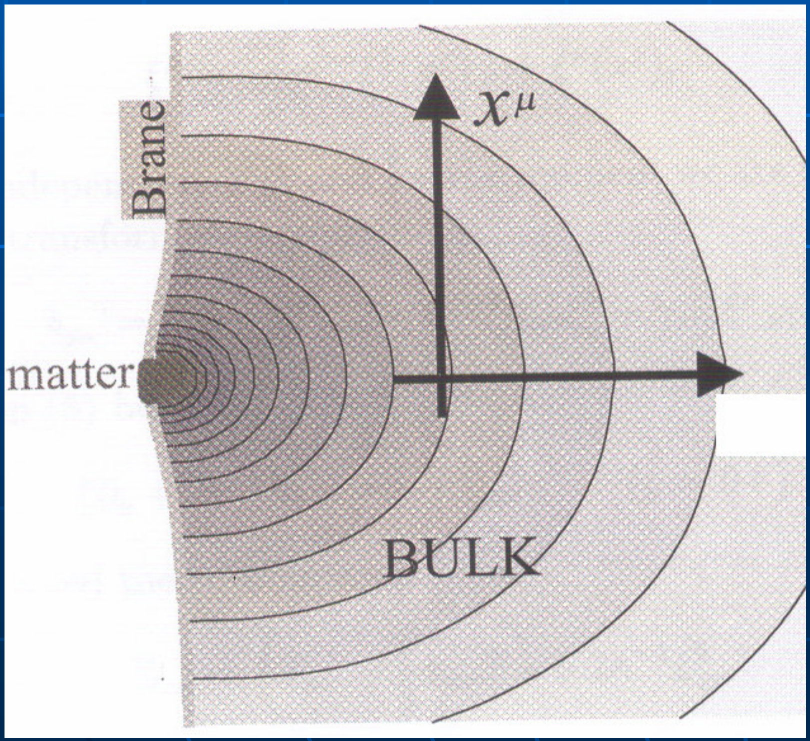
$$h_{ij} = \bar{h}_{ij} - \partial_i \hat{\xi}_j - \partial_j \hat{\xi}_i + 2\int d\bar{z} e^{2A(\bar{z})} \partial_i \partial_j \hat{\xi}_5 - 2\eta_{ij} A'(\bar{z}) \hat{\xi}_5$$

$$\Box^{(4)} \hat{\xi}^5 = -\frac{1}{12} F(\phi(0)) S \quad \Rightarrow \quad h = h_{ij}\eta^{ij} = 0, \quad \partial^i h_{ij} = 0,$$

Bent Brane Formalism

$$\Box^{(4)}\hat{\xi}^5 = -\frac{1}{12}F(\phi(0))S \Rightarrow \hat{\xi}^5(x) \neq 0$$

$$z = \bar{z} + \hat{\xi}^5(\bar{x}), \quad \bar{z} = 0 \Rightarrow z = \hat{\xi}^5(x)$$



In the case of a mass point source, the shape of the brane (in the coordinate system where we achieve decoupling of the Einstein Equations) is not flat but appears to be bent.

Decoupled Linearized Equations with a point-like source on the brane

$$F(\phi)\left(\partial_z^2 + Q'\partial_z + e^{-2A}\Box^{(4)}\right)h_{ij} = -\Sigma_{ij}(x)\delta(z) \qquad \Sigma_{ij}(x) = \tilde{S}_{ij}(x) - 4F(\phi(0))\partial_i\partial_j\hat{\xi}_5$$

$$h_{ij}(x,z) = -\int d^4x'\, G_5^{(R)}(x,z;x',0)\,\Sigma_{ij}(x')$$

$$G_5^{(R)}(x,z;x',z) = -\int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')}\left[\frac{u(0,z)u(0,z')}{p^2} + \sum_{m>0}\frac{u(m,z)u(m,z')}{p^2+m^2}\right]$$

METRIC FLUCTUATIONS FOR $z=0$

$$\bar{h}_{ij}(x,0) = -\int d^3x'\, G_5(x,0;x',0)\,\tilde{S}_{ij}(x') + \eta_{ij}A'(0)\hat{\xi}_5$$

Newton's Law

$$V(r) = \frac{\bar{h}_{00}}{2} = \frac{MG_4}{r} + \frac{4}{3}\frac{MG_4}{r(1+\alpha_F)}\sum_{m>0}\frac{u(m,0)^2}{u(0,0)^2}e^{-mr}$$

Zero mode+ Brane bending

Tower of the massive states

$$\alpha_F = \frac{1}{3}\left(1 - \frac{\int_{-\infty}^{+\infty} d\hat{z} e^{2A} F(\phi)}{k^{-1} F(\phi(0))}\right)$$

Zero mode truncation

If $r=|x-x'|\gg \ell$ the main contribution to Green function is due to the zero mode

Hence for large distances we can neglect the contributions of the massive states, then metric fluctuations on the brane obey a differential equation which is equivalent with a Brans-Dicke theory.

$$\Box^{(4)}\bar{h}_{ij}=-16\pi G_4\left(S_{ij}-\frac{1}{2}\eta_{ij}S\right)-8\pi G_4\alpha_F\eta_{ij}S$$

Brans-Dicke parameter ⟶ $\omega_{BD}=\frac{1}{2\alpha_F}-\frac{3}{2}$

In the case of 4D general relativity the metric fluctuations obey an equation of the form

$$\Box^{(4)}\bar{h}_{ij}=-16\pi G_4\left(S_{ij}-\frac{1}{2}\eta_{ij}S\right)$$

Constraints on the parameter space of the model

For consistency with solar system measurements the Brans-Dicke parameter must obey the following inequality

$$\omega_{BD} \geq 500,\ \omega_{BD} = \frac{1}{2\alpha_F} - \frac{3}{2} \Rightarrow 0 \leq \alpha_F \leq 10^{-3}$$

1) $\phi(0) \neq 0$

$$0 \leq \frac{\xi k \phi(0)^2 \left(-1 + \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \frac{\phi(\hat{z})^2}{\phi(0)^2} \right)}{3\left(1 - \xi k \phi(0)^2\right)} \leq 10^{-3}$$

2) $\phi(0) = 0$

$$0 \leq \frac{\xi k}{3} \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \phi(\hat{z})^2 \leq 10^{-3}$$

Constraints on the parameter space of the model

1) $\phi(0) \neq 0$ $$0 \leq \frac{\xi k \phi(0)^2 \left(-1 + \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \frac{\phi(\hat{z})^2}{\phi(0)^2} \right)}{3\left(1 - \xi k \phi(0)^2\right)} \leq 10^{-3}$$

2) $\phi(0) = 0$ $$0 \leq \frac{\xi k}{3} \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \phi(\hat{z})^2 \leq 10^{-3}$$

A complete investigation of the above restriction equations is beyond our scope, as the analytical static solutions of the model depend on a large number of free parameters. However we can obtain a quite general conclusion:

$$1) \left| -1 + \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \frac{\phi(\hat{z})^2}{\phi(0)^2} \right| \sim 1, \quad 2) \; k\phi(0)^2 \sim 1$$

$$\Rightarrow |\xi| \sim 10^{-3}$$

Conclusions

- We studied gravitational perturbations for a category of brane models involving a scalar field non-minimally coupled with gravity.
- We showed that there is a zero mode, which corresponds to a localized 4D graviton, plus a continuum spectrum of positive energy states.
- We used the bent brane formalism, in order to reproduce the Newton’ s Law on the brane and the corrections terms due to the massive modes.
- However we found that, the 4D gravity on the brane does not coincide with standard general relativity, but rather with a Brans-Dicke scalar-tensor theory.
- In order to be consistent with solar system measurements we had to set severe restrictions to the parameter space of the models we examine.

EINSTEIN FRAME

$$\tilde{g}_{\mu\nu} = \omega^2(x)\, g_{\mu\nu}, \quad \omega^2(x) = \left(F(\phi)\right)^{\frac{3}{2}}$$

$$S = \int d^5x \sqrt{|g|}\,(F(\phi)R + \ldots) = \int d^5x \sqrt{|\tilde{g}|}\left(\tilde{R} + \ldots\right)$$

$$ds_{JF}^2 = e^{2\tilde{A}(w)}\left(\left(\eta_{ij} + \overline{h}_{ij}\right)dx^i dx^j + dw^2\right)$$

$$ds_{EF}^2 = \left(F(\phi)\right)^{\frac{3}{2}} e^{2\tilde{A}(w)}\left(\left(\eta_{ij} + \overline{h}_{ij}\right)dx^i dx^j + dw^2\right)$$

$$\left(\partial_w^2 + \tilde{Q}'(w)\,\partial_w + m^2 e^{-2\tilde{A}}\right)\tilde{u}(m, w) = 0$$

Sturm-Liouville form

$$-\partial_z\left(p(z)\partial_z u(\lambda,z)\right)+q(z)u(\lambda,z)=\lambda r(z)u(\lambda,z)$$

$$p(z)=F(\phi)e^{4A},\quad r(z)=F(\phi)e^{2A}$$

$$q(z)=0,\qquad ,\quad \lambda=m^2$$

$$p(z)=e^{4A},\quad r(z)=e^{2A}$$

$$q(z)=0,\qquad ,\quad \lambda=m^2$$