# Phantom Field Dynamics in Loop Quantum Cosmology

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***Contributed Talks***
*@ The 3rd Naresuan Research Conference, Phitsanulok, Thailand*
*@ The 4th Aegean School: "Black Holes" and the 1st Annual School of the EU Network "UniverseNet"-Origin of the Universe, Mytilini, Island of Lesvos, Greece*

# Outline

1. The accelerating universe
2. Phantom and the Big Rip
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5. Dynamical Analysis
    - *Autonomous system*
    - *Fixed points*
    - *Stability analysis*
6. Numerical results
7. Conclusions/Comments
8. Acknowledgements

# 1. The accelerating universe

Type IA supernovae (discovered in 1998)

A. G. Riess *et al.*

[Supernova Search Team Collaboration],

*Astrophys. J. 607, 665 (2004)*

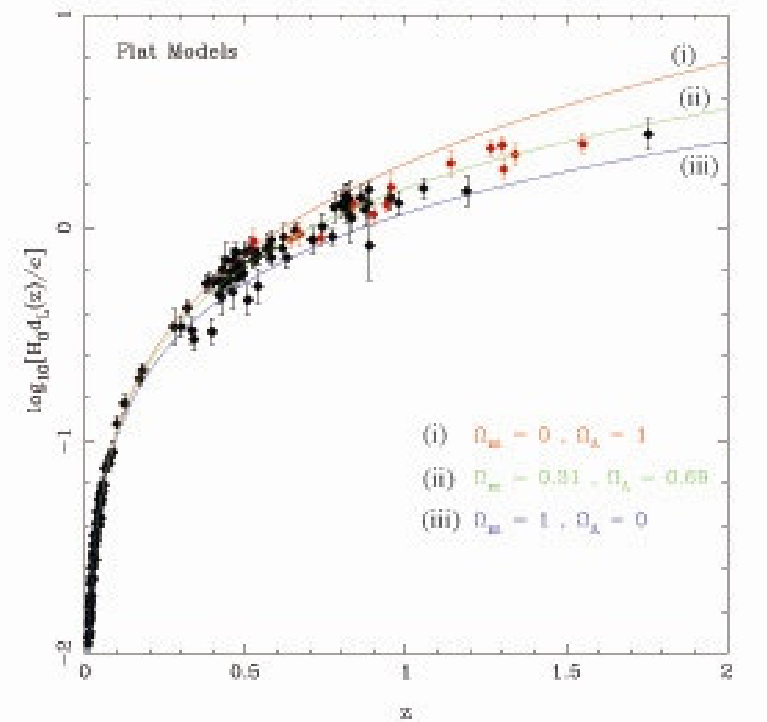



FIG. 2: The luminosity distance $H_0 d_L$ (log plot) versus the redshift $z$ for a flat cosmological model. The black points come from the "Gold" data sets by Riess *et al.* [85], whereas the red points show the recent data from HST. Three curves show the theoretical values of $H_0 d_L$ for (i) $\Omega_m^{(0)} = 0$, $\Omega_\Lambda^{(0)} = 1$, (ii) $\Omega_m^{(0)} = 0.31$, $\Omega_\Lambda^{(0)} = 0.69$ and (iii) $\Omega_m^{(0)} = 1$, $\Omega_\Lambda^{(0)} = 0$. From Ref. [86].

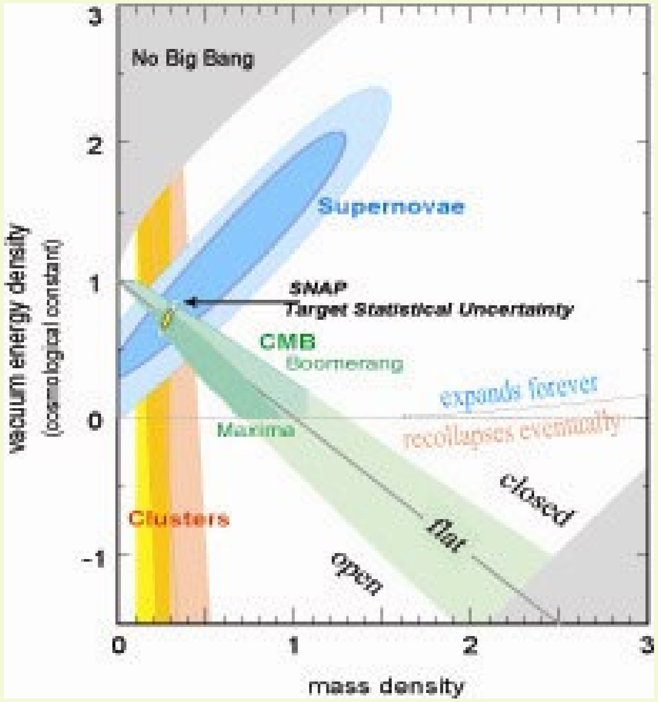
Cosmic Microwave Background anisotropies (WMAP) combined with Hubble Space Telescope

$$\Omega_K^{(0)} = -0.010^{+0.016}_{-0.009}$$

J. L. Sievers et al.,
*Astrophys. J.* 591, 599 (2003)

WMAP combined
with large scale galaxy surveys

N. A. Bahcall, J. P. Ostriker,
S. Perlmutter and P. J. Steinhardt,
*Science* 284, 1481 (1999)

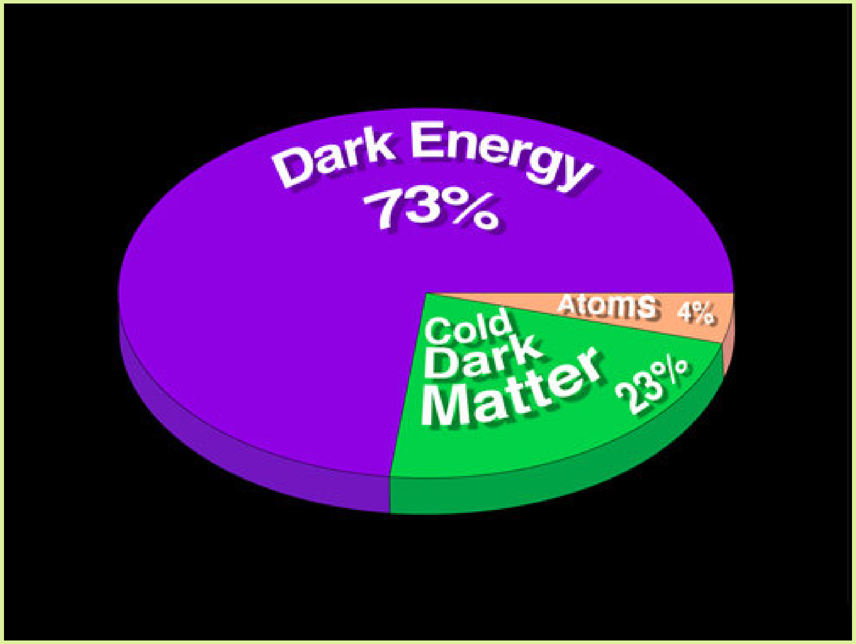


# 2. Phantom and the Big Rip

## What causes the acceleration?

- Dark energy (scalar field & cosmological constant)
  (For review see E. J. Copeland, M. Sami and S. Tsujikawa, hep-th/0603057)
- Modification of gravity (including braneworld models)
  (many authors)
- Backreaction of cosmological perturbations
  E. W. Kolb, S. Matarrese, A. Notari and A. Riotto, hep-th/0503117

If we are in the Dark cult., then...

# Most recent observation results

Assuming flat universe, the first result from ESSENCE Supernova Survey Ia combined with SuperNova Legacy Survey Ia gives a constraint of

$$w = -1.06^{+0.13}_{-0.08}$$

[W. M. Wood-Vasey *et al.*, arXiv:astro-ph/0701041]

Without assumption of flat universe but only combined WMAP, large scale structure and supernova data implies strong constraint

$$w = -1.07 \pm 0.09$$

[D. N. Spergel *et al.*, arXiv:astro-ph/0603449]

This could be any types of scalar field including Phantom! $w < -1$

Phantom field in the standard cosmology finally results in

The Big Rip singularity!

$a \to \infty, \rho \to \infty$ and $|p| \to \infty$ when $t \to t_s$ in future

To avoid Bip Rip singularity, loop quantum cosmology can help!

# 3. Cosmological Eqs. in LQC

- In LQC, space-time is quantized
- with FRW symmetry the effective Friedmann Eq. is (notice the cutoff density)

$$H^2 = \frac{\rho_{\rm t}}{3M_{\rm P}^2}\left(1 - \frac{\rho_{\rm t}}{\rho_{\rm lc}}\right)$$

- Within the framework then:

$$\dot{H} = -\frac{(\rho + p)}{2M_{\mathrm{P}}^2}\left(1 - \frac{2\rho}{\rho_{\mathrm{lc}}}\right).$$

$$H^2 = \frac{1}{3M_{\mathrm{P}}^2}\left(-\frac{\dot{\phi}^2}{2} + V\right)\left(1 - \frac{\rho}{\rho_{\mathrm{lc}}}\right),$$

$$\dot{\rho} = -3H\rho\left(1 + \frac{\dot{\phi}^2 + 2V}{\dot{\phi}^2 - 2V}\right),$$

$$\dot{H} = \frac{\dot{\phi}^2}{2M_{\mathrm{P}}^2}\left(1 - \frac{2\rho}{\rho_{\mathrm{lc}}}\right).$$

# 4. Canonical phantom field

- Negative kinetic energy!
- Violation of dominant energy condition
- Motivations: Braneworlds, Scalar-Tensor theory of gravitation, higher order gravity

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial^a \phi)(\partial_a \phi) - V(\phi) \right]$$

$$\rho = -\frac{1}{2}\dot{\phi}^2 + V(\phi),$$
$$p = -\frac{1}{2}\dot{\phi}^2 - V(\phi).$$

$$w = \frac{p}{\rho} = \frac{\dot{\phi}^2 + 2V}{\dot{\phi}^2 - 2V}$$

- The fluid equation: $\ddot{\phi} + 3H\dot{\phi} - V' = 0,$

# 5. Dynamical analysis

## 5.1 Autonomous system

New variables

$$X \equiv \frac{\dot{\phi}}{\sqrt{6} M_{\rm P} H}\,, \quad Y \equiv \frac{\sqrt{V}}{\sqrt{3} M_{\rm P} H}\,, \quad Z \equiv \frac{\rho}{\rho_{\rm lc}}\,,$$

$$\lambda \equiv -\frac{M_{\rm P} V'}{V}\,, \quad \Gamma \equiv \frac{V V''}{(V')^2}\,, \quad \frac{\rm d}{{\rm d}N} \equiv \frac{1}{H}\frac{\rm d}{{\rm d}t}\,,$$

Autonomous system

$$\frac{{\rm d}X}{{\rm d}N} = -3X - \sqrt{\frac{3}{2}}\,\lambda Y^2 - 3X^3\,(1-2Z)\,,$$

$$\frac{{\rm d}Y}{{\rm d}N} = -\sqrt{\frac{3}{2}}\,\lambda XY - 3X^2 Y\,(1-2Z)\,,$$

$$\frac{{\rm d}Z}{{\rm d}N} = -3Z\left(1 + \frac{X^2+Y^2}{X^2-Y^2}\right),$$

$$\frac{{\rm d}\lambda}{{\rm d}N} = -\sqrt{6}(\Gamma-1)\lambda^2 X\,.$$

Potential

$$V(\phi) = V_0 \exp\left(-\frac{\lambda}{M_{\rm P}}\phi\right)$$

# 5.2 Fixed points

Fixed points $f \equiv \mathrm{d}X/\mathrm{d}N, g \equiv \mathrm{d}Y/\mathrm{d}N$ and $h \equiv \mathrm{d}Z/\mathrm{d}N$

$$(f, g, h)|_{(X_c, Y_c, Z_c)} = 0 .$$

- Point (a) : $(\frac{-\lambda}{\sqrt{6}}, \sqrt{1+\frac{\lambda^2}{6}}, 0),$
- Point (b) : $(\frac{-\lambda}{\sqrt{6}}, -\sqrt{1+\frac{\lambda^2}{6}}, 0).$

# 5.3 Stability analysis

## Eigenvalues

- At point (a):

$$\mu_1 = \lambda^2\,, \quad \mu_2 = -\lambda^2\,, \quad \mu_3 = -3 - \frac{\lambda^2}{2}$$

- At point (b):

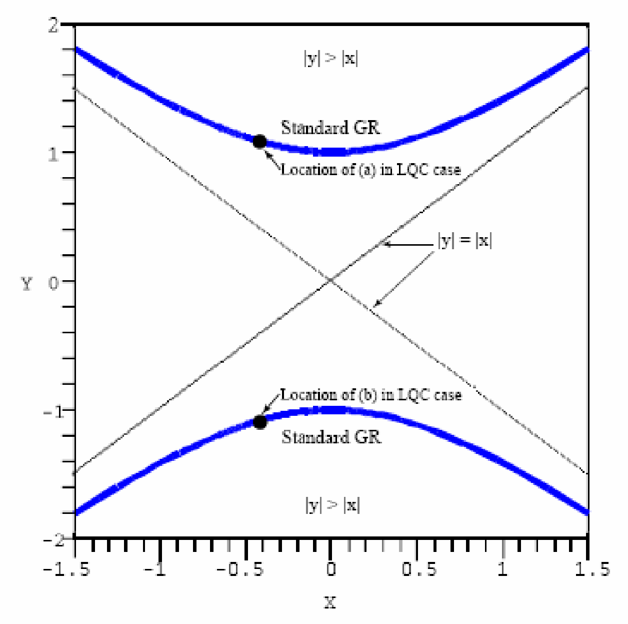
$$\mu_1 = \lambda^2\,, \quad \mu_2 = -\lambda^2\,, \quad \mu_3 = -3 - \frac{\lambda^2}{2}$$

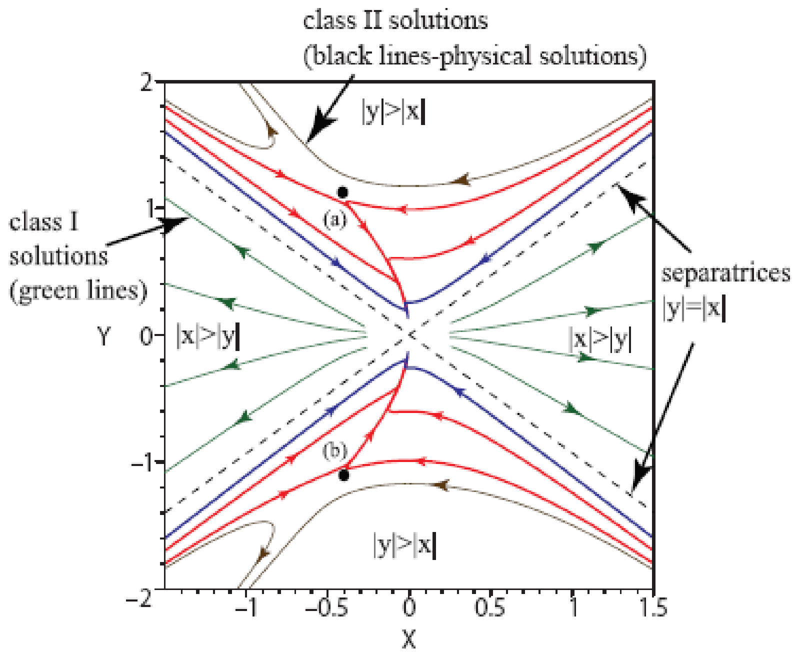
| Name | $X$ | $Y$ | $Z$ | Existence | Stability | $w$ | Acceleration |
|---|---|---|---|---|---|---|---|
| (a) | $-\frac{\lambda}{\sqrt{6}}$ | $\sqrt{1+\frac{\lambda^2}{6}}$ | 0 | All $\lambda$ | Saddle point for all $\lambda$ | $-1-\frac{\lambda^2}{3}$ | For all $\lambda$ (i.e. $\lambda^2 > -2$) |
| (b) | $-\frac{\lambda}{\sqrt{6}}$ | $-\sqrt{1+\frac{\lambda^2}{6}}$ | 0 | All $\lambda$ | Saddle point for all $\lambda$ | $-1-\frac{\lambda^2}{3}$ | For all $\lambda$ (i.e. $\lambda^2 > -2$) |

TABLE I: Properties of fixed points of phantom field dynamics in LQC background under the exponential potential.

# 6. Numerical results

## 6.1 Autonomous system-evolving with $N$




GR

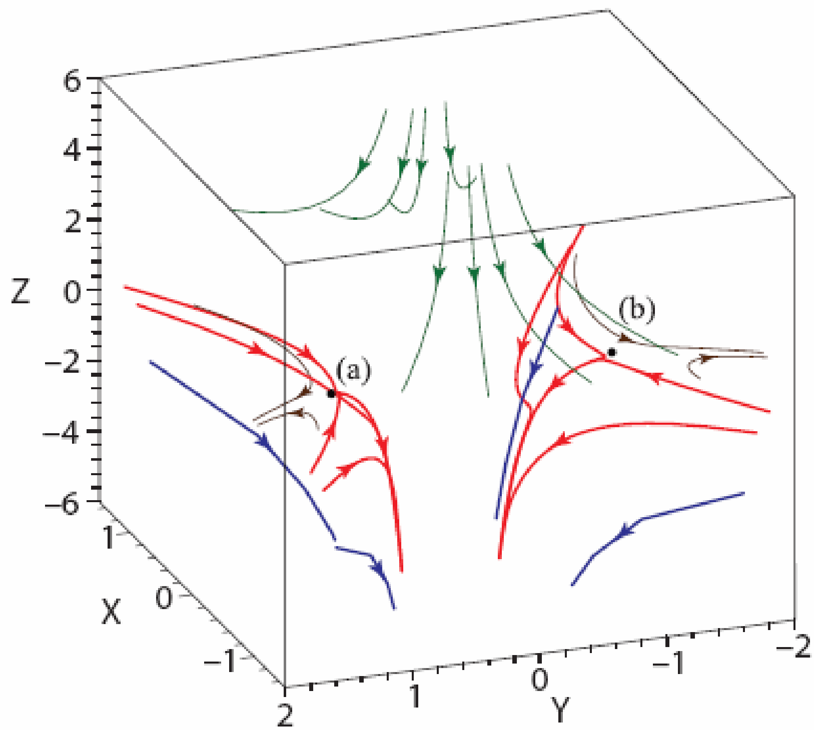
both points are attractors

LQC

both points are saddles.

Black lines are physical, i.e.

$|Y| > |X|$ and $0 < Z < 1$




3 Dim-LQC

both points are saddles.

Black lines are physical.

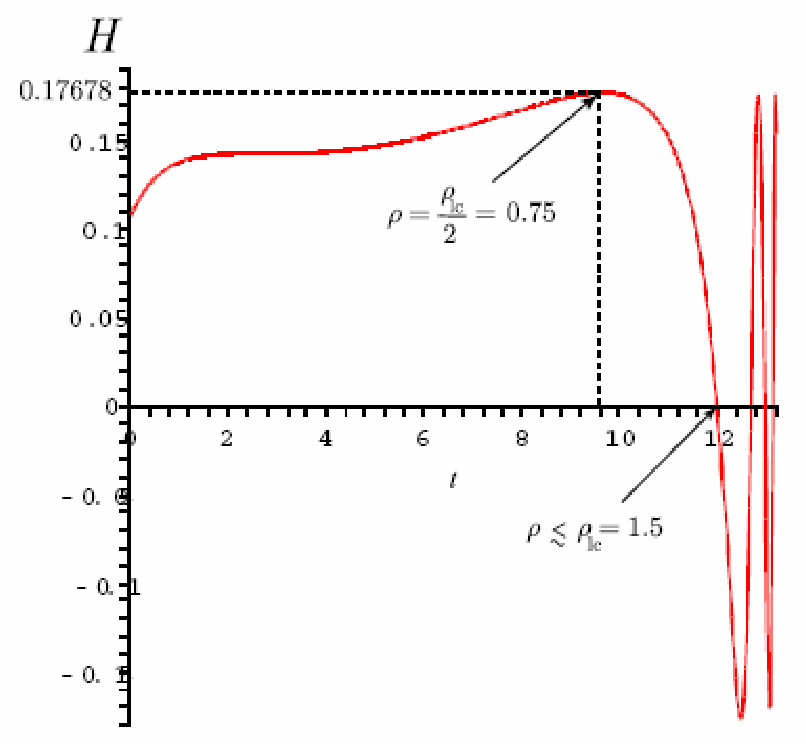
# 6.2 Autonomous system-evolving with $t$

$$\dot{\phi} = S$$

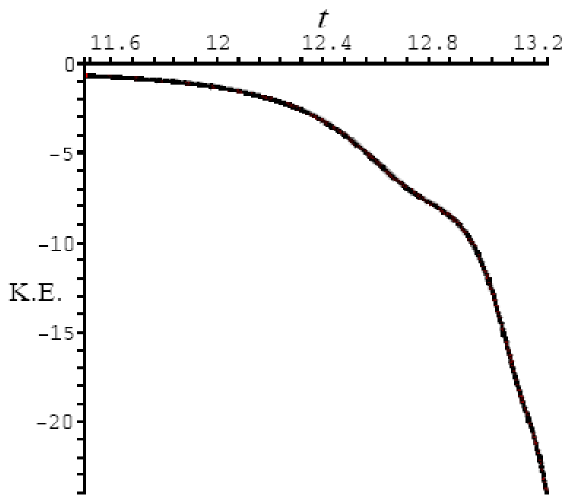
$$\dot{H} = \frac{S^2}{2M_P^2}\left[1 - \frac{2}{\rho_{lc}}\left(-\frac{S^2}{2} + V(\phi)\right)\right]$$

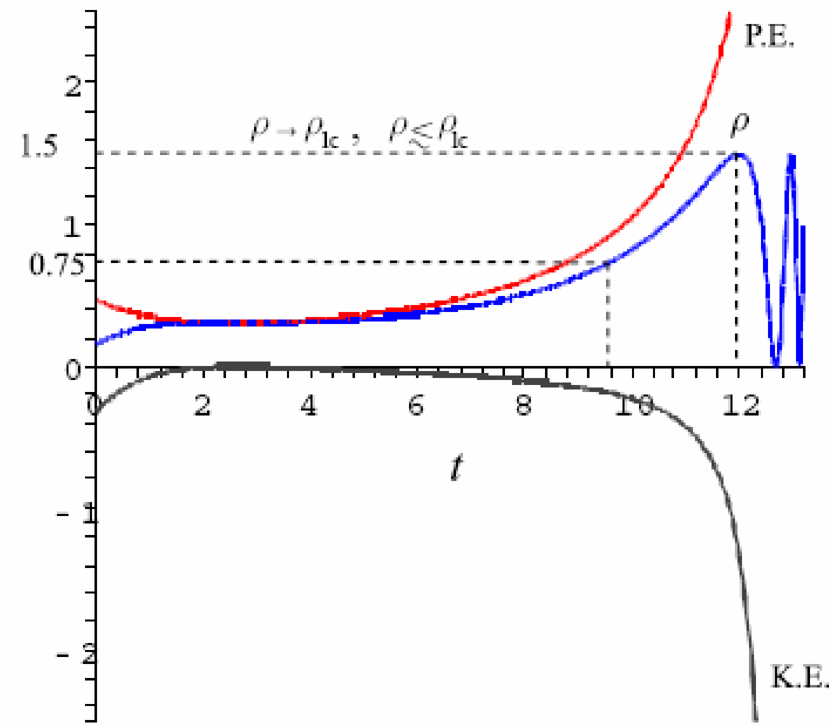
$$\dot{S} = -3HS + V'.$$

$$\left(\frac{d^2H}{d\rho^2}\right)_{\rho=\rho_{lc}/2} = \frac{-2}{M_P\sqrt{3\rho_{lc}^3}} < 0,$$




[Bouncing in $H$- the Big Rip avoidance]
+ [oscillation in $H$]




Oscillation in $\rho$ is a result of oscillation in negative kinetic energy hence this results in oscillation in $H$.

# 7. Conclusions/Comments

1. The scenario can avoid the Big Rip
2. Fixed points of GR cases are the same as in LQC case
3. We obtain that the attractors in GR case turns to saddle points in our scenario.
4. The universe will be in oscillating phase after the bounce.
5. Negative kinetic energy does oscillate and contributes to oscillation in total density and the Hubble parameter.
6. Further problem on singularity in oscillation frequency!

# 8. Acknowledgements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6. Picture credits: NASA
7. The host, Elefterios Papantonopoulos


# Thank you for your attention.