The Price of Abandoning Dark Matter is Nonlocality

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What is the price of abandoning dark matter? Cosmological constraints on alternative gravity theories Kris Pardo, David N. Spergel (2007.00555)

"Any successful alternative gravity theory that obviates the need for dark matter must fit our cosmological observations. Measurements of microwave background polarization trace the large-scale baryon velocity field at recombination and show very strong, $O(1)$ baryon acoustic oscillations. Measurements of the large-scale structure of galaxies at low redshift show much weaker features in the spectrum. If the alternative gravity theory's dynamical equations for the growth rate of structure are linear, then the density field growth can be described by a Green's function: $\delta(\vec{x},t) = \delta(\vec{x},t')G(x,t,t')$. We show that the Green function, $G(x,t,t')$ must have dramatic features that erase the initial baryon oscillations. This implies an acceleration law that changes sign on the $\sim$ 150 Mpc scale. On the other hand, if the alternative gravity theory has a large nonlinear term that couples modes on different scales, then the theory would predict large-scale non-Gaussian features in large-scale structure. These are not seen in the distribution of galaxies nor in the distribution of quasars. No proposed alternative gravity theory for dark matter seems to satisfy these constraints."

Easy to get CDM contribution to $a(t)$ (1 DoF) Problem is perturbations (∞DoF)

- $ds^2 = -[1+2\Psi(t,\vec{x})]dt^2 + 2\partial_i B(t,\vec{x})dtdx^i + a^2(t)d\vec{x}\cdot d\vec{x}$
- CDM stress tensor characterized by
  - Background density: $\bar{\rho}(t)$
  - Density contrast: $\delta(t,\vec{x}) = \frac{\delta\rho(t,\vec{x})}{\bar{\rho}(t)}$
  - Momentum divergence: $\theta(t,\vec{x})$
- 0th Order: $\bar{T}_{\mu\nu} = \delta_\mu^0\delta_\nu^0\bar{\rho}$
- 1st Order:
  - $\delta T_{00} = \delta\rho + 2\Psi\bar{\rho}$
  - $\delta T_{0i} = -\bar{\rho}\partial_i\theta$
  - $\delta T_{ij} = 0$

Now use separate conservation: $D^{\mu}T_{\mu\nu} = 0$

- At 0th Order
  - $\nu = 0$ ➔ $-(\partial_t + 3H)\bar{\rho} = 0$ ➔ $\bar{\rho}(t) = \frac{\rho_0}{a^3(t)}$
  - $\nu = i$ ➔ $0 = 0$
- At 1st Order
  - $\nu = 0$ ➔ $\bar{\rho}\left\{-\dot{\delta} + \frac{\nabla^2}{a^2}[B - \theta]\right\} = 0$
  - $\nu = i$ ➔ $\bar{\rho}\,\partial_i\{\dot{\theta} + \Psi\} = 0$
- Solve for the density contrast & the momentum divergence
  - $\delta = \frac{1}{\partial_t}\left[\frac{\nabla^2}{a^2}\left(B + \frac{1}{\partial_t}\Psi\right)\right]$
  - $\theta = -\frac{1}{\partial_t}\Psi$

Construct $\Delta S[g] \ni -\frac{2}{\sqrt{-g}}\frac{\delta \Delta S}{\delta g^{\mu\nu}} = T_{\mu\nu}$ for CDM

- What we want
  - $T_{00} = \bar{\rho}\left\{1 + 2\Psi + \frac{1}{\partial_t}\left[\frac{\nabla^2}{a^2}\left(B + \frac{1}{\partial_t}\Psi\right)\right] + \cdots\right\}$
  - $T_{0i} = \bar{\rho}\left\{0 + \frac{\partial_i}{\partial_t}\Psi + \cdots\right\}$
  - $T_{ij} = \bar{\rho}\{0 + 0 + \cdots\}$
- What we need to get by varying wrt Ψ & B
  - $\frac{\delta \Delta S}{\delta \Psi} = -\sqrt{-g}\, g^{0\mu} g^{0\nu} T_{\mu\nu} = -a^3\bar{\rho}\left\{1 - \Psi + \frac{1}{\partial_t}\left[\frac{\nabla^2}{a^2}\left(B + \frac{1}{\partial_t}\Psi\right)\right] + \cdots\right\}$
  - $\frac{\delta \Delta S}{\delta B} = -\partial_i\left(\sqrt{-g}\, g^{0\mu} g^{i\nu} T_{\mu\nu}\right) = -a^3\bar{\rho}\left\{-\frac{\nabla^2}{a^2}\left[B + \frac{1}{\partial_t}\Psi\right] + \cdots\right\}$
  - NB $a^3\bar{\rho} = \rho_0$

Two Ways to Do It

(1) Nonlocal Effective Action

$$\Delta\mathcal{L} = -\rho_0 \left\{ 1 + \Psi - \tfrac{1}{2}\Psi^2 - \left[B + \frac{1}{\partial_t}\Psi \right] \frac{\nabla^2}{2a^2} \left[B + \frac{1}{\partial_t}\Psi \right] + \cdots \right\}$$

(2) Using Auxiliary Scalars $\delta(t, \vec{x})$ and $\theta(t, \vec{x})$

$$\Delta\mathcal{L} = -\rho_0 \left\{ 1 + \Psi - \tfrac{1}{2}\Psi^2 - [B - \theta] \frac{\nabla^2}{2a^2} [B - \theta] + \delta(\dot{\theta} + \Psi) + \cdots \right\}$$

- $\frac{\delta \Delta S}{\delta \delta} = -\rho_0 \{ \dot{\theta} + \Psi + \cdots \} = 0$ ➔ $\theta = \frac{1}{\partial_t}\Psi + \cdots$
- $\frac{\delta \Delta S}{\delta \theta} = -\rho_0 \left\{ \frac{\nabla^2}{a^2}[B - \theta] - \dot{\delta} + \cdots \right\} = 0$ ➔ $\delta = \frac{1}{\partial_t}\left[\frac{\nabla^2}{a^2}\left(B + \frac{1}{\partial_t}\Psi \right) \right] + \cdots$
- $\frac{\delta \Delta S}{\delta \Psi} = -\rho_0 \{ 1 - \Psi + \delta + \cdots \}$
- $\frac{\delta \Delta S}{\delta B} = -\rho_0 \left\{ \frac{\nabla^2}{a^2}[B - \theta] + \cdots \right\}$

Extending to a general metric

- Pressureless CDM depends on $\rho(t,\vec{x})$ & $u_\mu(t,\vec{x})$

$$T_{\mu\nu} = \rho u_\mu u_\nu \qquad\qquad g^{\mu\nu}u_\mu u_\nu = -1$$

- Separate conservation ($D^\mu T_{\mu\nu} = 0$) requires

1) $\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}u_\nu\, \rho\right) = 0$
2) $g^{\alpha\beta}u_\alpha D_\beta u_\nu = 0$

First focus on the 4-velocity u_μ

- Our linearized model has
  - $u_0 = -1 - \Psi + \cdots = -\partial_t \left[t + \frac{1}{\partial_t}\Psi + \cdots \right]$
  - $u_i = 0 + \partial_i \theta + \cdots = -\partial_i \left[t + \frac{1}{\partial_t}\Psi + \cdots \right]$
- This suggests we try $u_\mu = -\partial_\mu \phi$ with
  - $\phi = t + \frac{1}{\partial_t}\Psi + \cdots$
- Normalized, timelike implies 1st order, nonlinear ϕ equation
  - $g^{\mu\nu} u_\mu u_\nu = g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi = -1$
- NB this also implies relation 2) of conservation!

$$g^{\alpha\beta} u_\alpha D_\beta u_\nu = g^{\alpha\beta}\partial_\alpha \phi D_\beta \partial_\nu \phi = g^{\alpha\beta}\partial_\alpha \phi D_\nu \partial_\beta \phi = \frac{1}{2} D_\nu [g^{\alpha\beta}\partial_\alpha \phi \partial_\beta \phi] = 0$$

The energy density ρ is a Lagrange multiplier

- $\Delta\mathcal{L} = -\frac{1}{2}\rho\left[\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} + 1\right]\sqrt{-g}$
  - $\frac{\delta\Delta S}{\delta\phi} = \partial_\mu\left[\rho\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi\right] = 0$ ➔ relation 1) of conservation!
  - $\frac{\delta\Delta S}{\delta\rho} = -\frac{1}{2}\left[\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} + 1\right]\sqrt{-g} = 0$ ➔ the scalar equation, hence 2)
- This simple action gives the full CDM stress tensor
  - $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\Delta S}{\delta g^{\mu\nu}} = \rho\partial_\mu\phi\partial_\nu\phi$
- CDM people cannot criticize this model because it EXACTLY reproduces CDM

The ϕ and ρ equations are both well-posed

- ADM: $ds^2 = -N^2dt^2 + \gamma_{ij}(dx^i - N^i dt)(dx^j - N^j dt)$
- ϕ equation ($g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -1$) is quadratic in $\dot{\phi}$

$$\dot{\phi} = N\sqrt{1+\gamma^{ij}\partial_i\phi\partial_j\phi} - N^i\partial_i\phi$$

- ρ equation ($\partial_\mu\left[\rho\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi\right] = 0$) with $\dot{\phi}$ from above

$$\partial_t\left[\rho\sqrt{\gamma}\sqrt{1+\gamma^{jk}\partial_j\phi\partial_k\phi}\right] = \partial_i\left[N\rho\sqrt{\gamma}\,\gamma^{ij}\partial_j\phi - N^i\rho\sqrt{\gamma}\sqrt{1+\gamma^{jk}\partial_j\phi\partial_k\phi}\right]$$

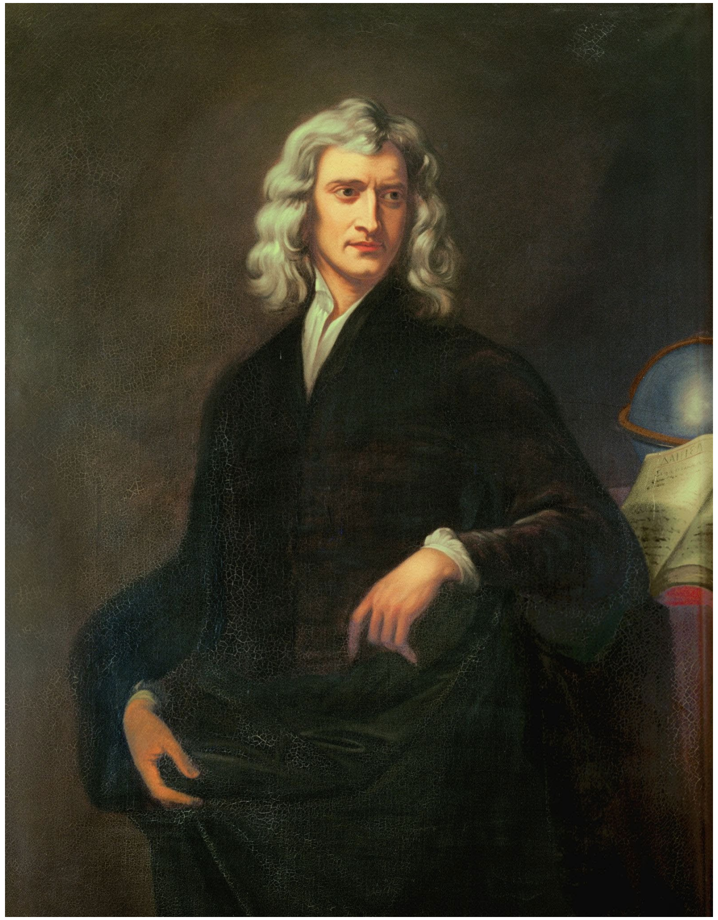
- Unique initial value data
  - $\phi(0,\vec{x}) = 0$
  - $\rho(0,\vec{x}) = \frac{\rho_0}{\sqrt{\gamma(0,\vec{x})}}$

"Who ordered that?"
We don't believe in fundamental nonlocality

- Newton called it:

  "so great an Absurdity that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it."

- We think fundamental theory is local
- But Effective Actions are nonlocal
  - Massless loops give big IR effects
  - Gravitons are massless
  - Inflationary particle production causes growing effects
  - Force of gravity gets nonperturbatively large changes



Conclusions about Modified Gravity

- Strategy
  - Use conservation to solve for CDM $T_{\mu\nu}$ in terms of GR potentials
  - Construct nonlocal effective action whose variation gives this $T_{\mu\nu}$
  - No new fields, no breaking of invariance, but nonlocality (answer to Pardo & Spergel)
- Basic strategy guaranteed to work
  - Not guaranteed to be pretty, but it is
  - Nonperturbative QG effects from primordial inflation might give it
  - Could be falsified by direct detection or by proving CDM interacts
- Issues
  - Finding an invariant effective action to give both cosmology & bound systems
  - Deriving it from first principles
  - What happens for the Bullet Cluster?
- But at least as good as CDM on a purely phenomenological level

Bottom Line: Modified Gravity is a Contender