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Threshold Anomaly in Quantum Space

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Paper

Kováčik, Samuel, Michaela Ďuríšková, and Patrik Rusnák. *Phenomenology of the dispersion law in three-dimensional quantum space.* preprint arXiv:2402.05832 (2024).

Quantum gravity (QG)

Direct testing of QG theories is VERY DIFFICULT!

Energy comparison.

$\frac{E_{LHC}}{E_{Planck}} \approx \frac{10^{13}\ \mathrm{eV}}{10^{28}\ \mathrm{eV}} = 10^{-15}$

We are very far from generating these kinds of energies.

Quantum gravity (QG)

Some phenomena give us a way to achieve something measurable.

If a discrepancy of 10^{-15} accumulates over a period of 10^{15} seconds, the final effect will be significant (in seconds).

Quantum gravity (QG)

Direct testing of QG theories $\rightarrow$ VERY DIFFICULT

Testing some phenomena of QG $\rightarrow$ Less than VERY DIFFICULT

Lorentz Invariance Violation (LIV)

LIV and modified dispersion. $c = 1$

LIV is caused by modified dispersion relation.

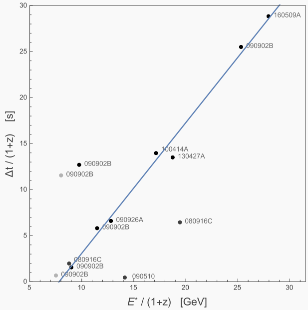
$$E^2 = p^2 \qquad \rightarrow \qquad E^2 = p^2 - a_1 \xi p^3 - a_2 \xi^2 p^4 - \dots$$

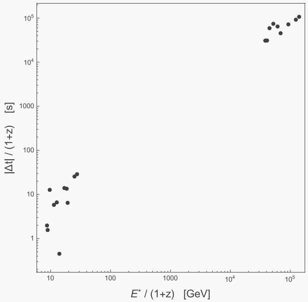
Amelino-Camelia, G., Ellis, J., Mavromatos, N. E., Nanopoulos, D. V. (1997). Distance measurement and wave dispersion in a Liouville-string approach to quantum gravity. International Journal of Modern Physics A, 12(03), 607-623.

Amelino-Camelia, G. (2002). Relativity in spacetimes with short-distance structure governed by an observer-independent (Planckian) length scale. International Journal of Modern Physics D, 11(01), 35-59.

Quantum gravity (QG)

Modified dispersion relation $\implies$ slowed high energy photons





Amelino-Camelia, G., D'Amico, G., Rosati, G., Loret, N. (2017). In vacuo dispersion features for gamma-ray-burst neutrinos and photons. Nature Astronomy, 1(7), 0139.

Observational considerations

	Temporal resolution	Energy range
GRBAlpha	1 s	80 keV – 950 keV
Fermi GBM	2 μs	8 keV – 40 MeV
Fermi LAT	1 μs	20 MeV – 300 GeV
Hermes	250 ns	5 keV – 0.5MeV

Pál, A., Ohno, M., Mészáros, L., Werner, N., Řípa, J., Csák, B., ... Uchida, Y. (2023). GRBAlpha: the smallest astrophysical space observatory–Part 1: Detector design, system description and satellite operations. arXiv preprint arXiv:2302.10048.

Thompson, D. J., & Wilson-Hodge, C. A. (2022). Fermi gamma-ray space telescope. In Handbook of X-ray and Gamma-ray Astrophysics (pp. 1-31). Singapore: Springer Nature Singapore.

Fiore, F., Burderi, L., Lavagna, M., Bertacin, R., Evangelista, Y., Campana, R., ... & Zanotti, G. (2020, December). The HERMES-technologic and scientific pathfinder. In Space Telescopes and Instrumentation 2020: Ultraviolet to Gamma Ray (Vol. 11444, pp. 214-228). SPIE.

Evangelista, Y., Fiore, F., Fuschino, F., Campana, R., Ceraudo, F., Demenev, E., ... & Manca, A. (2020, December). The scientific payload on-board the HERMES-TP and HERMES-SP CubeSat missions. In Space Telescopes and Instrumentation 2020: Ultraviolet to Gamma Ray (Vol. 11444, pp. 241-256). SPIE.

Doubly Special Relativity (DSR)

Postulates.

1. The laws of physics involve a fundamental velocity scale c and fundamental length scale l_F.
2. Each inertial observer can establish the value of l_F (same value for all inertial observers) by determining the dispersion relation for photons, which takes the form $E^2 - c^2p^2 + f(E, p, l_F) = 0$, where f is the same for all inertial observers and in particular all inertial observers agree on the leading dependence of f: $f(E, p, l_F) \simeq Ecp^2 l_F$.

Dispersion law in DSR. $l_F = 1/E_F$

$$E = \frac{-p^2/E_F + p\sqrt{4 + p^2/E_F^2}}{2} \implies E^2 = p^2 - \frac{p^3}{E_F} + \frac{1}{2}\frac{p^4}{E_F^2} + \mathcal{O}(E_F^{-3})$$

Amelino-Camelia, G. (2002). Relativity in spacetimes with short-distance structure governed by an observer-independent (Planckian) length scale. International Journal of Modern Physics D, 11(01), 35-59.

R^3_λ Quantum Space (QS)

Noncommutative R^3_λ QS.

$$\left[\hat{X}_i, \hat{X}_j\right] = 2i\lambda\varepsilon_{ijk}\hat{X}_k$$

$\hat{X}$ is a position operator
λ is a scale parameter

Consequence. Velocity operator

$$\frac{1}{2}\hat{V}^2 = \hat{H}\left(1 - \frac{\lambda^2}{2}\hat{H}\right) \implies H(V) = \frac{1}{\lambda^2} \pm \frac{1}{\lambda^2}\sqrt{1 - \lambda^2 V^2}$$

Kováčik, S., Prešnajder, P. (2013). The velocity operator in quantum mechanics in noncommutative space. Journal of Mathematical Physics, 54(10).

Legendre transformation

Legendre transformation.

$$p(V) = \frac{\partial H}{\partial V} = \mp \frac{V}{\sqrt{1-\lambda^2 V^2}} \implies V(p)$$

$$H(V(p)) \equiv H(p) = \frac{1}{\lambda^2}\left(1 - \sqrt{\frac{1}{1+\lambda^2 p^2}}\right)$$

Taylor series.

$$H(p) = \frac{1}{2}p^2 - \frac{3}{8}\lambda^2 p^4 + \frac{5}{16}\lambda^4 p^6 + \mathcal{O}(\lambda^6)$$

Relativistic dispersion law.

We have a non-relativistic dispersion law

$$H(p) = \frac{1}{\lambda^2}\left(1 - \sqrt{\frac{1}{1+\lambda^2 p^2}}\right) = \frac{1}{2}p^2 - \frac{3}{8}\lambda^2 p^4 + \frac{5}{16}\lambda^4 p^6 + \mathcal{O}(\lambda^6)$$

The relativistic relation must satisfy two conditions.

1. If $\lambda \to 0 \implies H = pc = p$
2. Quantum structure affects fast and slow particles similarly.

Solution.

Let's make the substitution $\frac{p^2}{2} \to p$ and $\lambda^2 \to 1/E_F$

$$H^2(p) = E_F^2\left(1 - \sqrt{\frac{E_F}{E_F+2p}}\right)^2 = p^2 - \frac{3p^3}{E_F} + \frac{29p^4}{4E_F^2} + \mathcal{O}(E_F^{-3})$$

Dispersion relations

Dispersion law in different theories.

Special Relativity $\rightarrow E^2 = p^2$

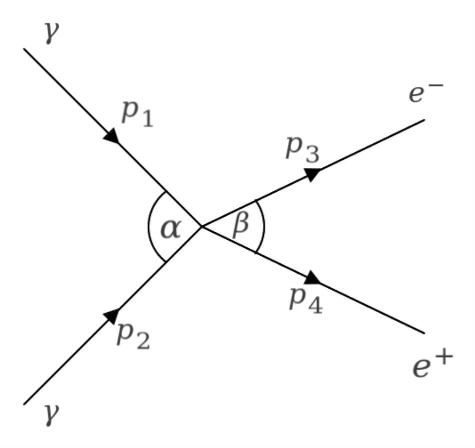
R^3_λ Quantum Space $\rightarrow E^2 = p^2 - \frac{3p^3}{E_F} + \frac{29p^4}{4E_F^2} + \mathcal{O}(E_F^{-3})$

Doubly Special Relativity $\rightarrow E^2 = p^2 - \frac{p^3}{E_F} + \frac{1}{2}\frac{p^4}{E_F^2} + \mathcal{O}(E_F^{-3})$

General structure of the dispersion law.

$E(p)^2 = p^2 - \frac{a_1}{E_F}p^3 - \frac{a_2}{E_F^2}p^4 + \mathcal{O}(E_F^{-3})$

Annihilation $\gamma\gamma \to e^+e^-$



Annihilation $\gamma\gamma \rightarrow e^+e^-$

Minimum energy threshold. $\alpha = \pi$ a $\beta = 0$

$$p_1 = (E, 0, 0, E)$$
$$p_2 = (\varepsilon, 0, 0, -\varepsilon)$$
$$p_3 = p_4 = \left(\tfrac{E+\varepsilon}{2}, 0, 0, \tfrac{E-\varepsilon}{2}\right)$$

$$m^2 \equiv p_3^2 = \left(\tfrac{E+\varepsilon}{2}\right)^2 - \left(\tfrac{E-\varepsilon}{2}\right)^2 \implies E \geq E_{th} = \tfrac{m^2}{\varepsilon}$$

Consequence.

If the energy of a photon is $E \geq E_{th}$, the photon will annihilate and we will not detect it.

Sources of photons in the Universe

Sources of the high energy photons.

Gamma Ray Burst (GRB), Pulsars

The strongest GRB so far has been GRB20221009A $\rightarrow 18$ TeV

Sources of background photons in the Universe.

Cosmic Microwave Background (CMB) $\rightarrow E_{th} = 411$ TeV

Extragalactic Background Light (EBL) $\rightarrow E_{th} \in (261 \text{ GeV}, 261 \text{ TeV})$

We can notice that $261 \text{ GeV} \ll 18 \text{ TeV}$, which is indeed a problem.

Li, H., Ma, B. Q. (2023). Lorentz invariance violation induced threshold anomaly versus very-high energy cosmic photon emission from GRB 221009A. Astroparticle Physics, 148, 102831.

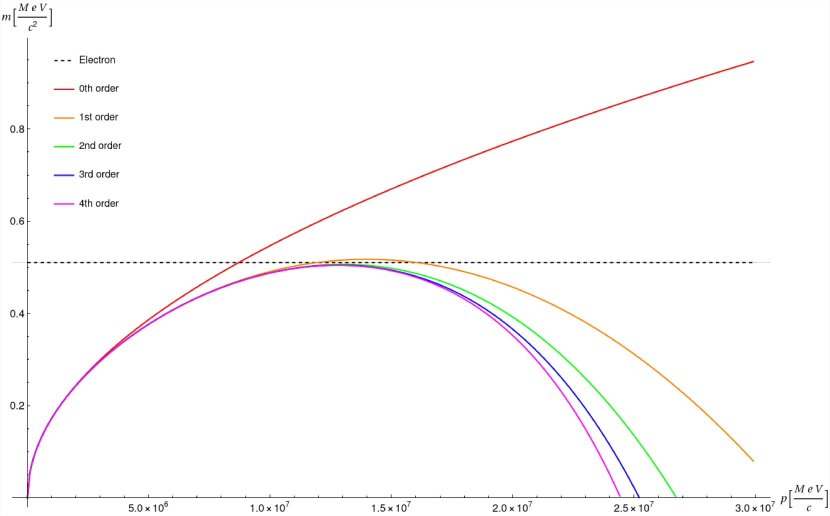
Annihilation $\gamma\gamma \to e^+e^-$

Modified minimum energy threshold. $\alpha = \pi$ a $\beta = 0$

$$p_1 = (E(p), 0, 0, p)$$
$$p_2 = (\varepsilon, 0, 0, -\varepsilon)$$
$$p_3 = p_4 = \left(\frac{E(p)+\varepsilon}{2}, 0, 0, \frac{p-\varepsilon}{2}\right)$$

$$m^2 \equiv p_3^2 = \left(\frac{E(p)+\varepsilon}{2}\right)^2 - \left(\frac{p-\varepsilon}{2}\right)^2 \implies \text{minimum mass equation}$$

Orders of minimum mass equation



Energy cut-off scale E_F

Mass of a particle produced by an interaction between an $18\ \mathrm{TeV}$ photon with a background photon of energy ε with various values of the fundamental energy scale E_F as measured in MeV/c^2.

$\varepsilon \setminus E_F$ [MeV]	∞	E_{Planck}	$10^{-1}E_{Planck}$	$10^{-2}E_{Planck}$
1 eV	4.2	4.2	3.8	0.019
10^{-1} eV	1.3	1.2	0.0061	< 0
10^{-2} eV	0.42	0.0019	< 0	< 0
10^{-3} eV	0.13	< 0	< 0	< 0

Summary

Key takeaways.

It is useful to study modified dispersion relation effects.

Different theories have a similar dispersion relations.

Telescopes are continuously improving.

Thank you for your attention!