

Muon $g-2$ and lepton flavor violation in SUSY-GUT theories

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Colaboration with A. Tiwari, Q. Shafi, C. Ün, J. Ellis, S. Lola, Ruiz de Austri, JHEP 07 (2020) 07, 096 JHEP 09 (2020), Eur. Phys. J. C (2022) 82:561 + arXiv:2404.02337 [hep-ph]

OUTLINE

-GUT's and SUSY.

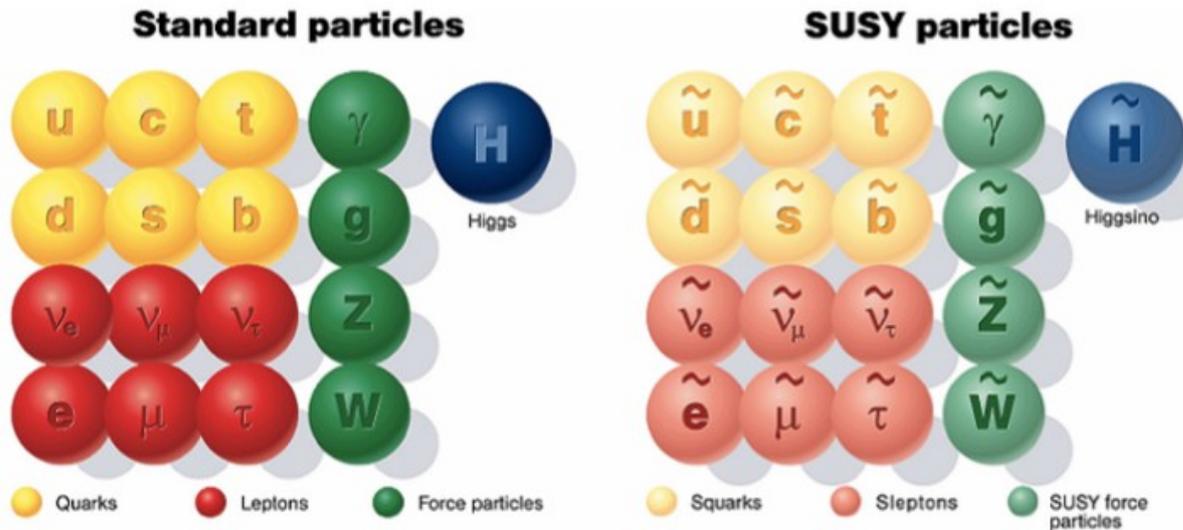
-PS: $SU(4) \times SU(2) \times SU(2)$

-Fitting muon $(g-2)$ in $SU(4) \times SU(2) \times SU(2)$ models

- *Muon $(g-2)$ vs Neutralino relic density and DM detection.*

- LFV vs LHC signals.

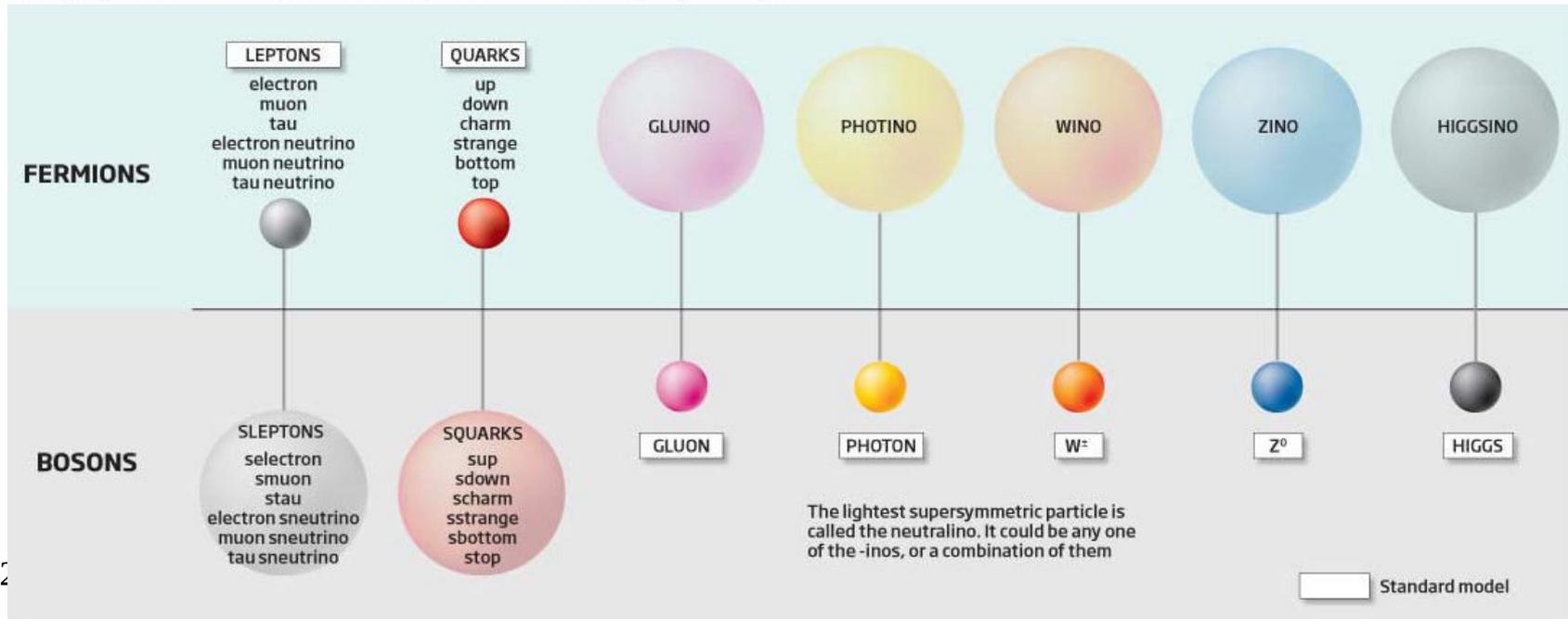
Conclusions.



Particle zoo

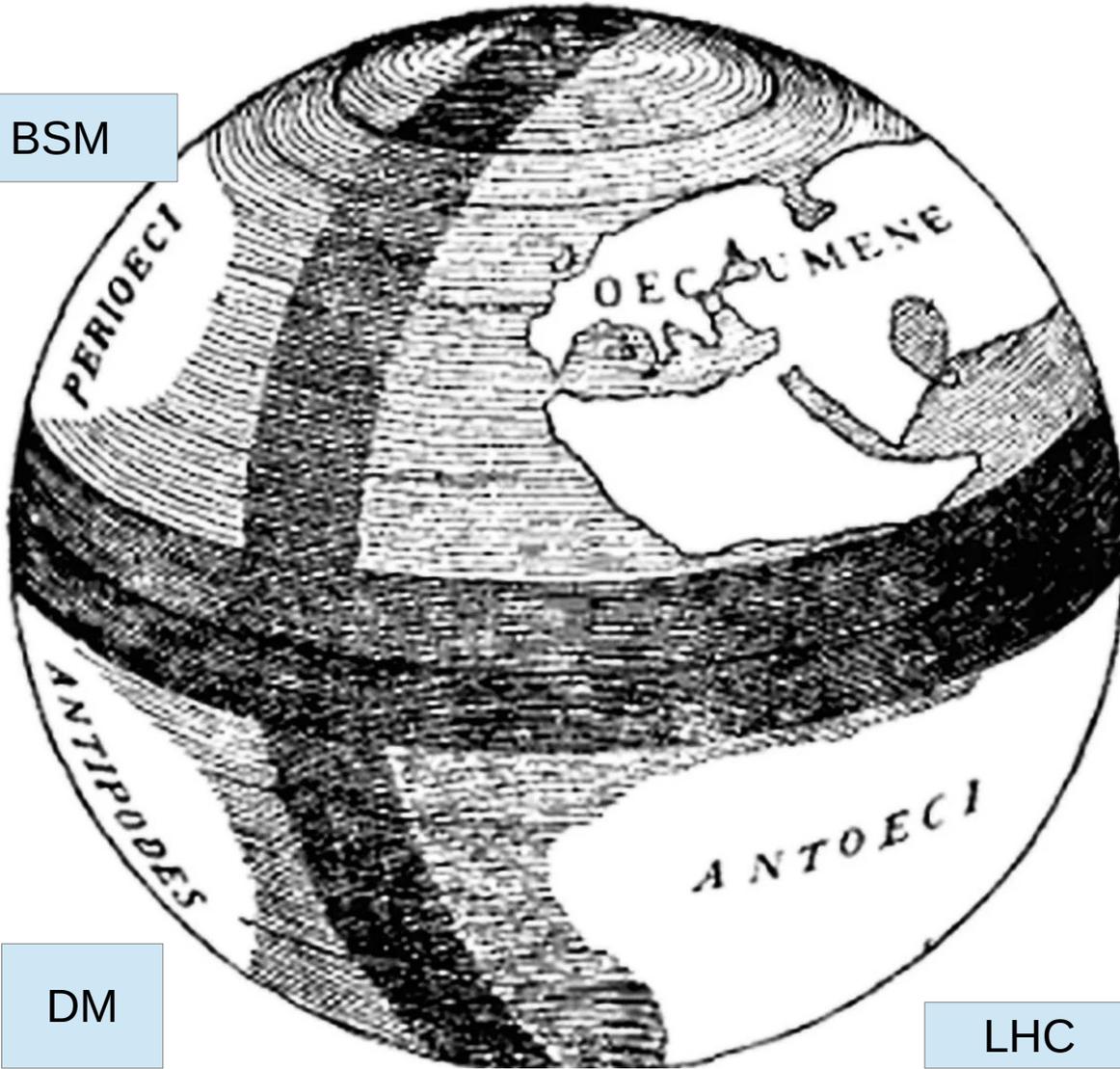
©NewScientist

Particles are divided into two families called bosons and fermions. Among them are groups known as leptons, quarks and force-carrying particles like the photon. Supersymmetry doubles the number of particles, giving each fermion a massive boson as a super-partner and vice versa. The LHC is expected to find the first supersymmetric particle



Crates of Mallus 150

BSM



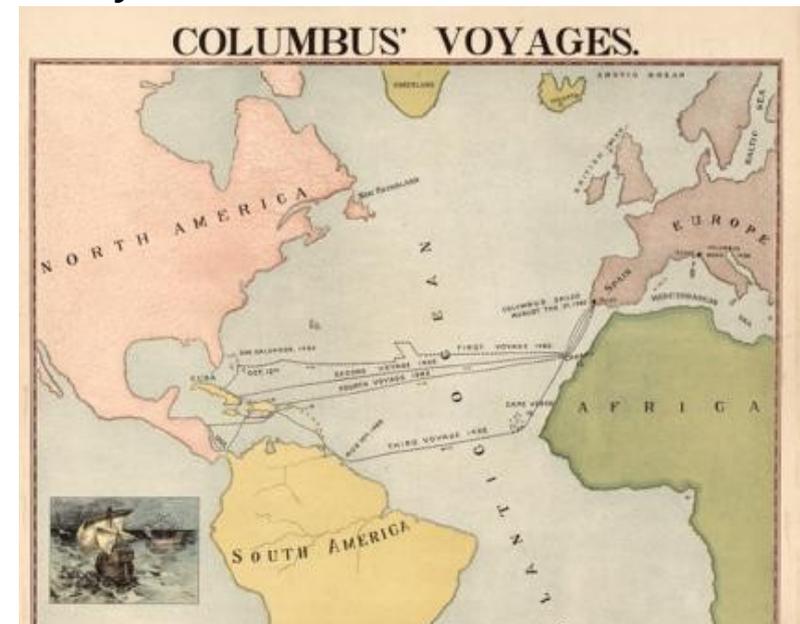
DM

LHC

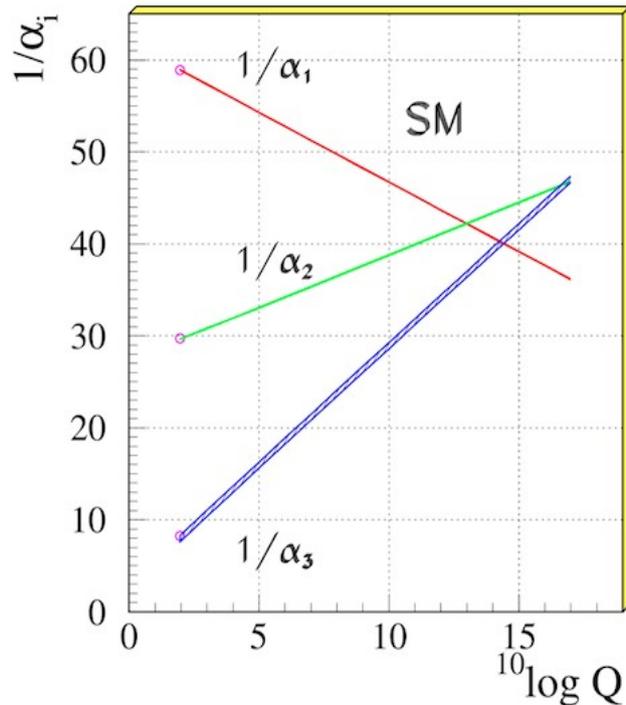
Antipodes theory:

Popular debate in the Middle age.

Probably referred in all explation trip proposals until it was eliminated by direct observation.



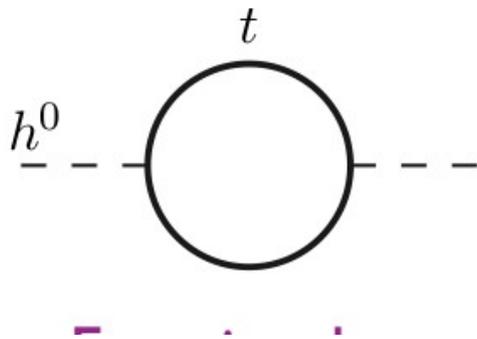
Hierarchy Problem



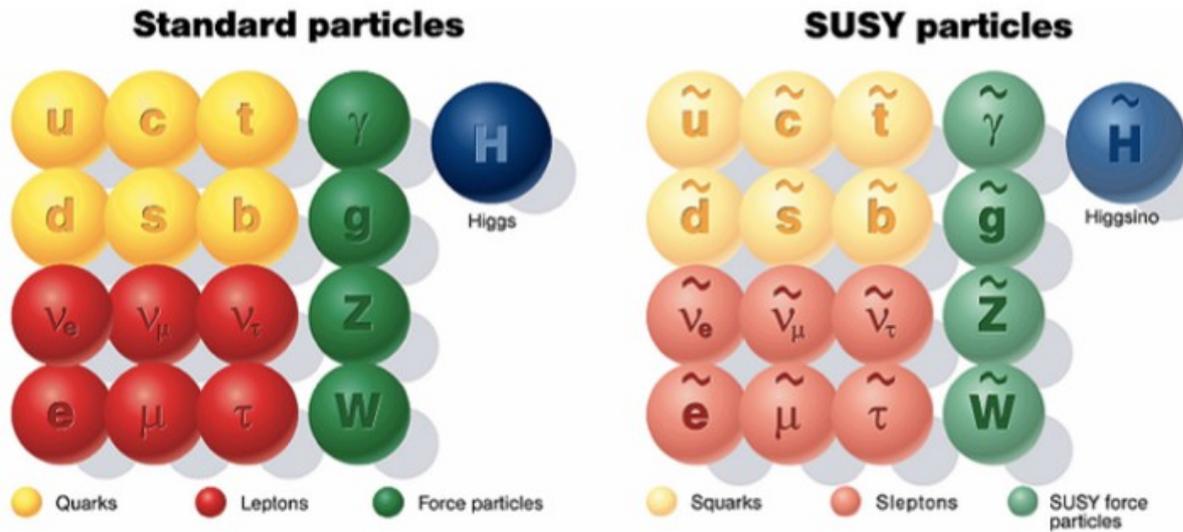
Almost unification at $\sim 10^{14}$ GeV

$$\mu \frac{d\alpha_i(\mu)}{d\mu} = -\frac{1}{2\pi} \left[b_i + \frac{1}{4\pi} \sum b_{ij} \alpha_j(\mu) \right] \alpha_i^2(\mu)$$

$$b_i = (0, -22/3, -11) + N_F(4/3, 4/3, 4/3) + N_H(1/10, 1/6, 0)$$



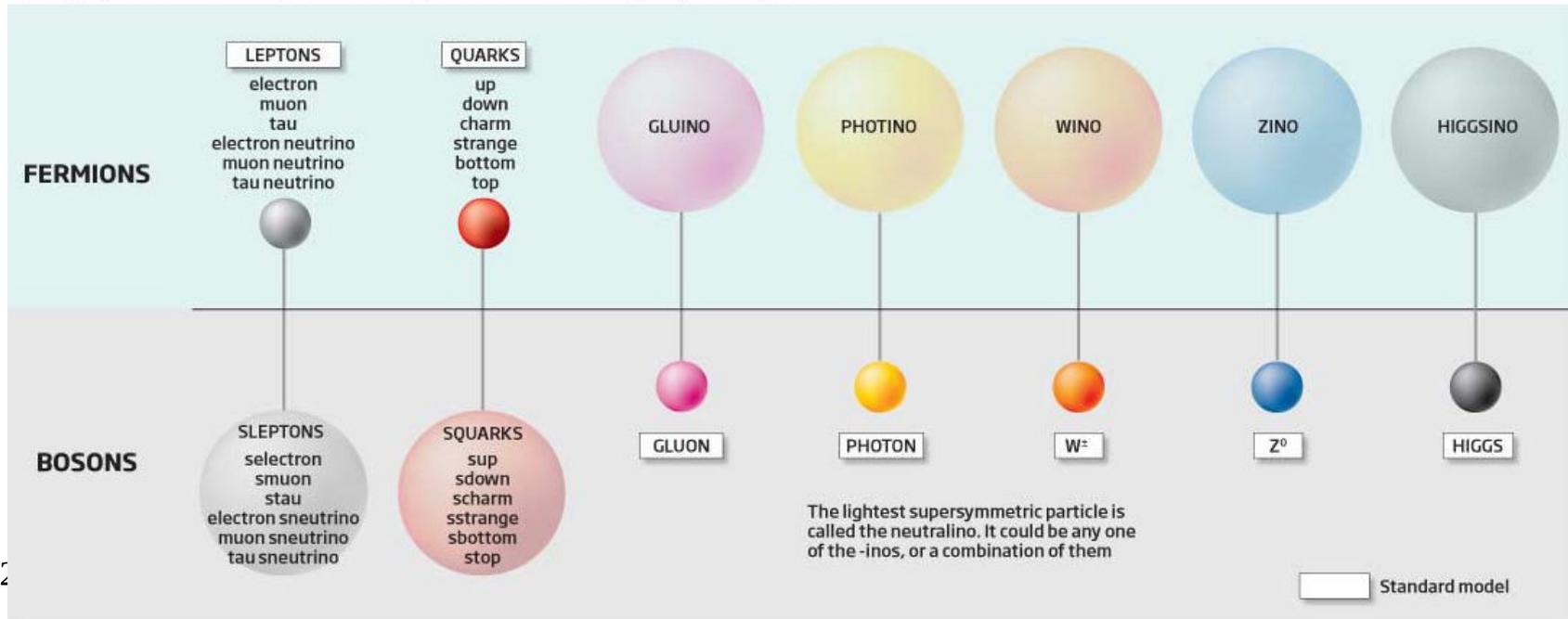
Divergent contribution to Higgs mass \uparrow with $(m_{\text{scale}})^2$



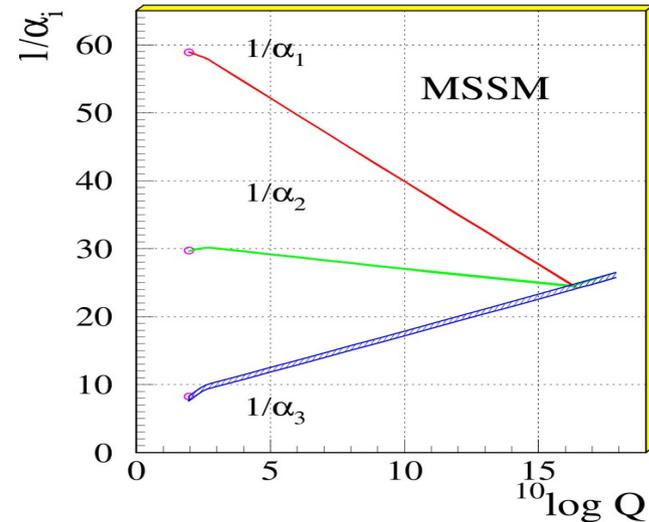
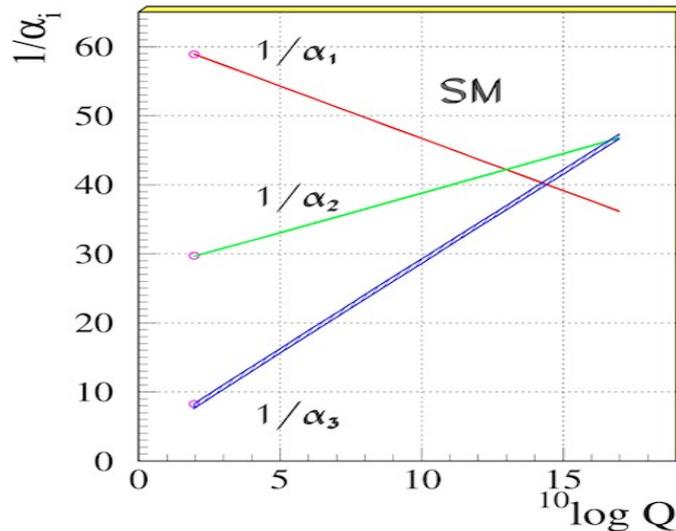
Particle zoo

©NewScientist

Particles are divided into two families called bosons and fermions. Among them are groups known as leptons, quarks and force-carrying particles like the photon. Supersymmetry doubles the number of particles, giving each fermion a massive boson as a super-partner and vice versa. The LHC is expected to find the first supersymmetric particle

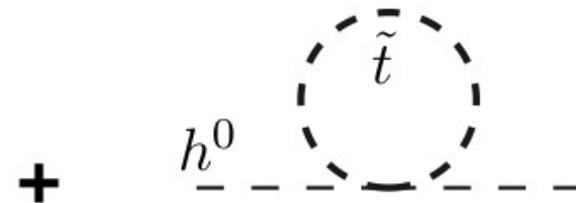
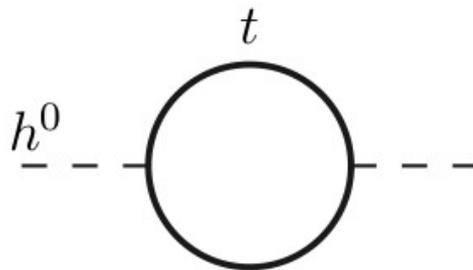


Hierarchy Problem



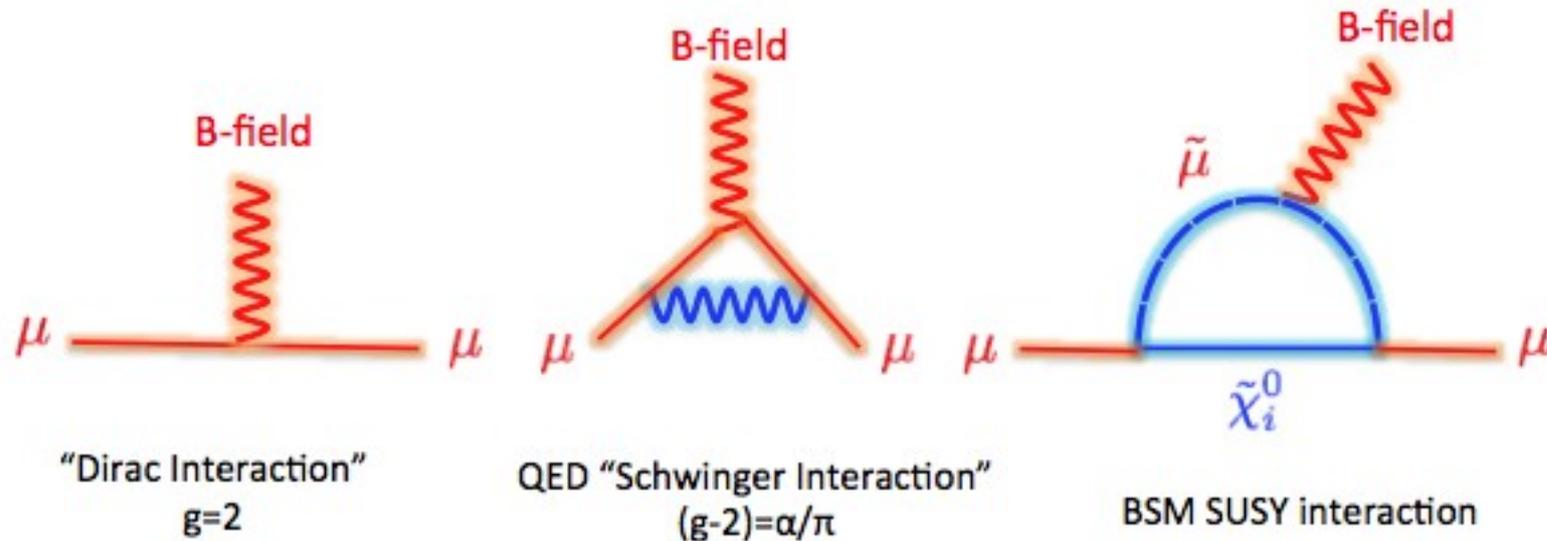
Unification at $\sim 10^{16}$ GeV

$$b_i = (0, -6, -9) + N_F(2, 2, 2) + N_H(3/10, 1/2, 0)$$



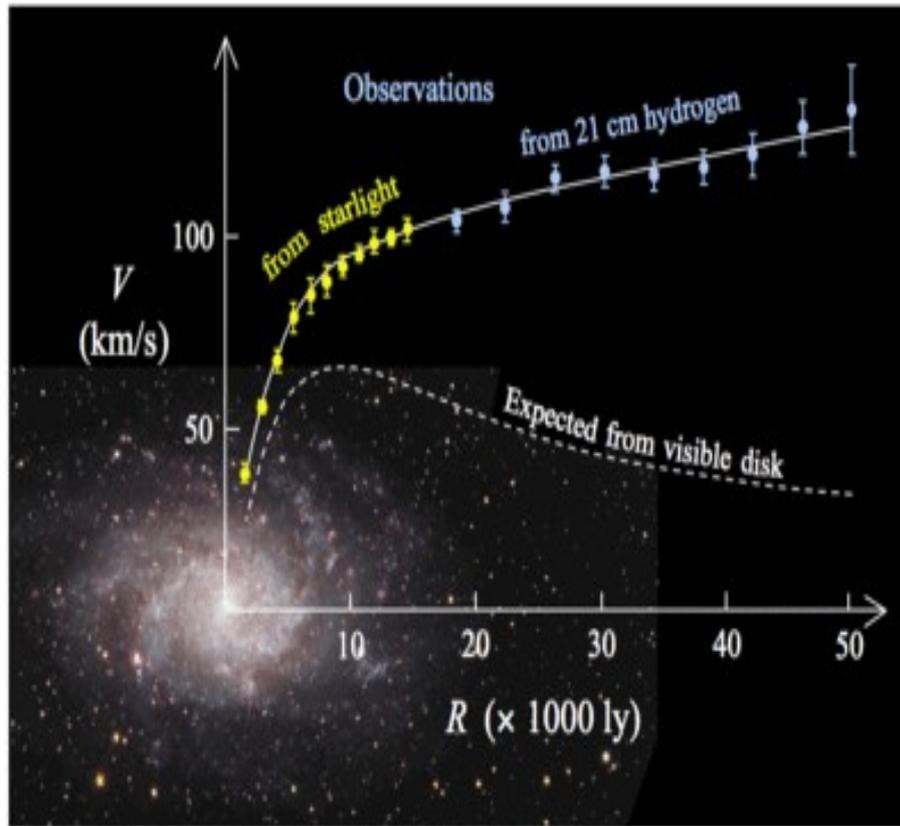
Cancelation of quadratic divergences

One loop contribution to SM proc.

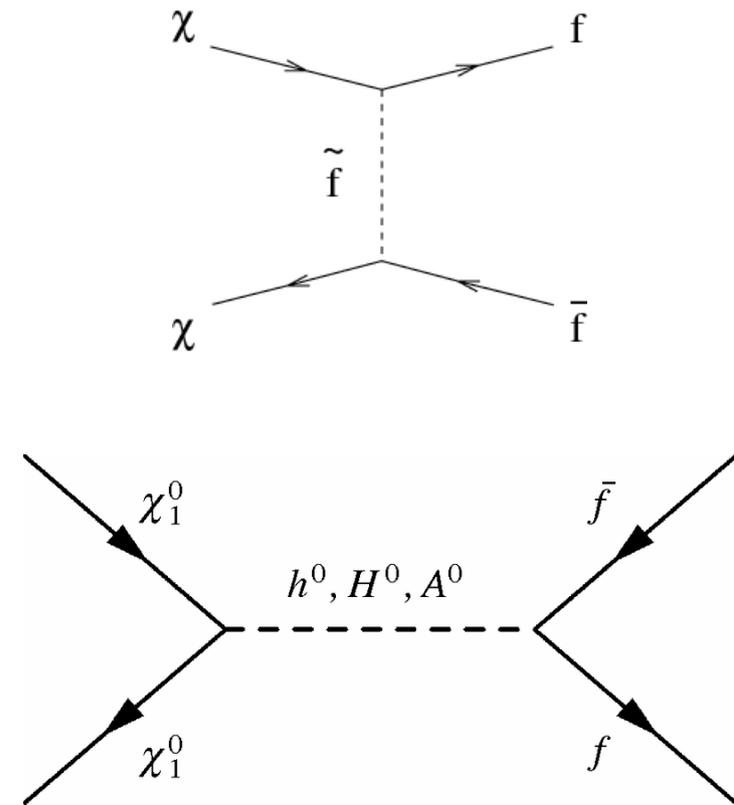


SM vs Experiment Discrepancies
anomalous magnetic dipole moment ($g_\mu-2$)

Dark Matter problem



Rotation curve of spiral galaxy M 33 (yellow and blue points with error bars), and a predicted one from distribution of the visible matter (white line). The discrepancy between the two curves can be accounted for by adding a dark matter halo surrounding the galaxy



$$\Omega h^2 \sim \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma_{\text{ann}} v_{\text{Mol}} \rangle}$$

Soft SUSY Breaking Terms

The soft SUSY breaking masses

$$\begin{aligned}
 -\mathcal{L}_{\text{soft}} = & -\frac{1}{2} \left(M_3 \lambda_g^a \lambda_g^a + M_2 \lambda_{\tilde{W}}^i \lambda_{\tilde{W}}^i + M_1 \lambda_{\tilde{B}} \lambda_{\tilde{B}} + \text{h.c.} \right) \\
 & + M_L^2 \tilde{L}^\dagger \tilde{L} + M_Q^2 \tilde{Q}^\dagger \tilde{Q} + M_U^2 \tilde{U}^* \tilde{U} + M_D^2 \tilde{D}^* \tilde{D} + M_E^2 \tilde{E}^* \tilde{E} + \\
 & m_{H_d}^2 \tilde{H}_d^\dagger \tilde{H}_d + m_{H_u}^2 H_u^\dagger H_u - \left(B\mu \tilde{H}_d^T H_u + \text{h.c.} \right) \\
 & + \left(y_\ell A_\ell H_d^\dagger \tilde{L} \tilde{E} + y_d A_d H_d^\dagger \tilde{Q} \tilde{D} - y_u A_u H_u^T \tilde{Q} \tilde{U} + \text{h.c.} \right),
 \end{aligned}$$

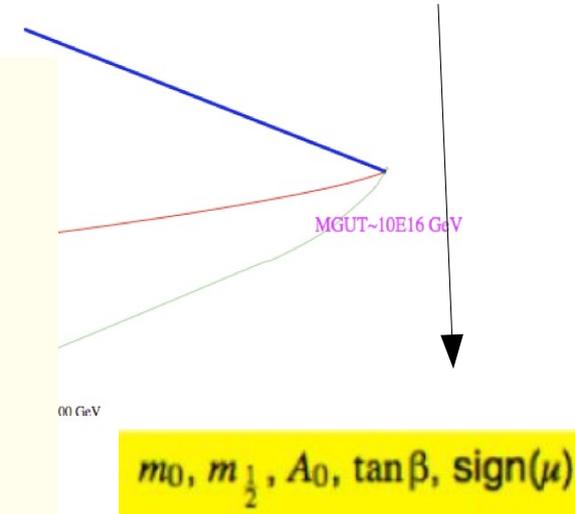
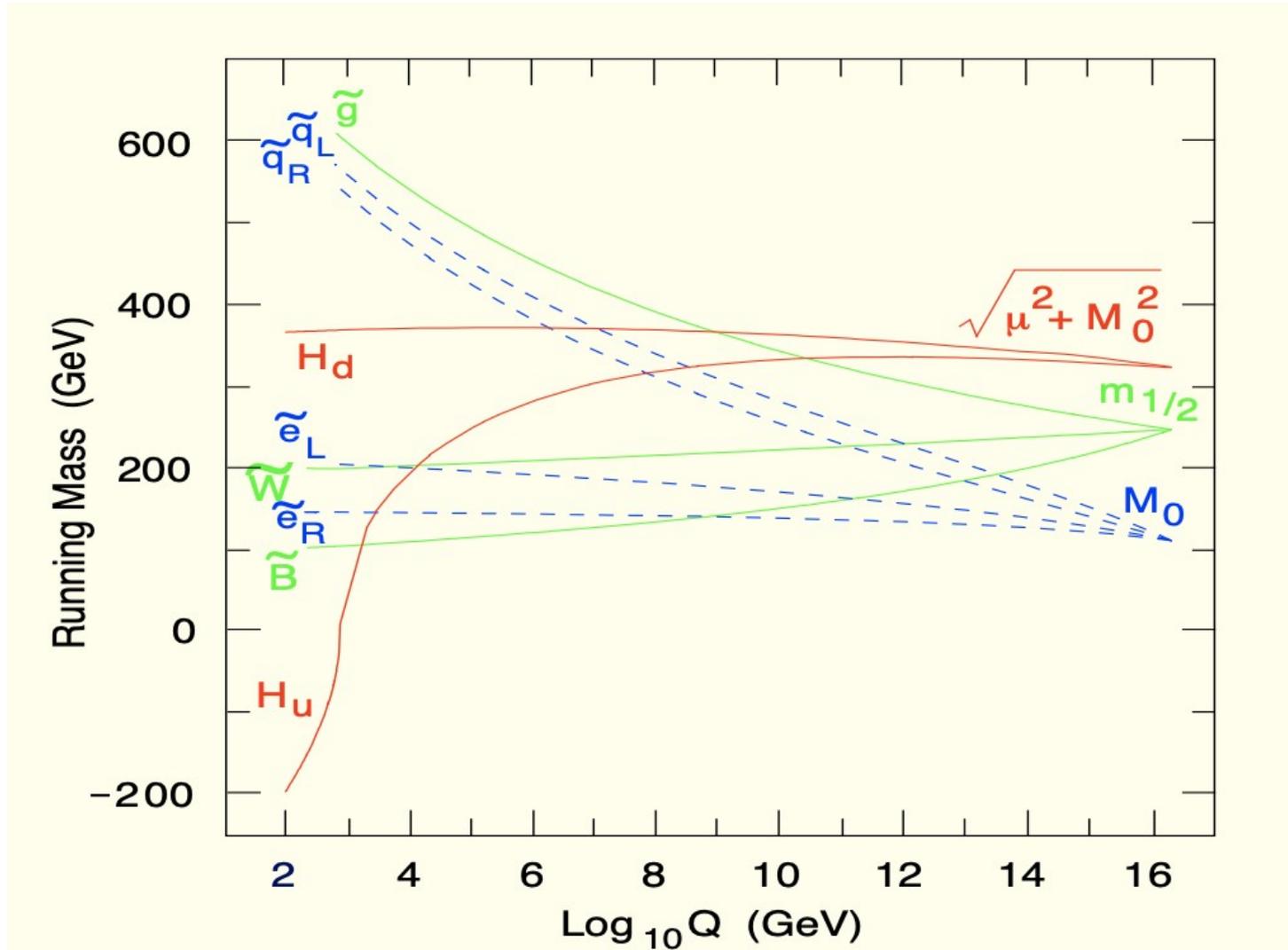
Inspired from supergravity assume universal soft breaking, $\mathcal{L}_{\text{soft}}$:

$$\sum_{f,H} m_0^2 \tilde{f} \tilde{f} + \sum_{\lambda} m_{\frac{1}{2}} \lambda \lambda + \sum_f A_0 Y_f \tilde{f} \tilde{F} H_f + B\mu H_u H_d$$

$$m_0, m_{\frac{1}{2}}, A_0, \tan\beta, \text{sign}(\mu)$$

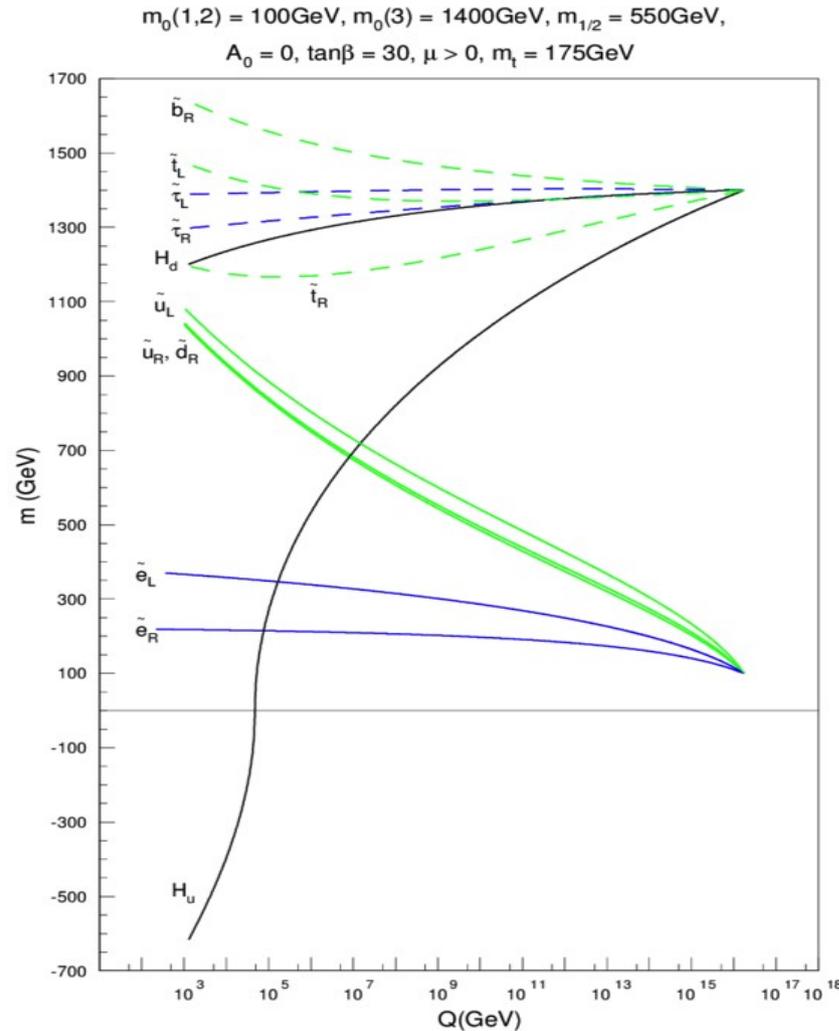
μ and A_0 can be complex, however their phases constraint to be $< 0,2$ rad by the bounds c the fermion EDM.

GUT initial conditions

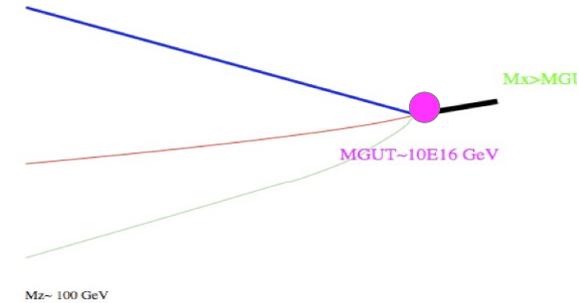


Gunion,
Int. J. Mod. Phys. A 2010

Non Universal scenarios



Baer et al., HEP 06 (2004) 044



CMSSM choice:

- m_0 Universal soft masses.
- $m_{1/2}$ Universal gaugino masses.
- A_0 Universal Trilinear terms.

Representation-dependent choice

$$m_r = x_r m_0$$

$$A_r = Y_r A_0, \quad A_0 = a_0 m_0$$



SM

MSSM+neutrino masses

GUT Sacale

Family Symmetries
Additional Fields

Planck Sacale

PATI-SALAM Unification

$$G_{PS} \equiv SU(4) \times SU(2)_L \times SU(2)_R$$

$4_c 2_L 2_R$

MATTER FIELDS

$$F_r \quad \begin{pmatrix} d_r & -u_r \\ e_r & -\nu_r \end{pmatrix} \quad (4, 2, 1)$$

$$F_r^c \quad \begin{pmatrix} u_r^c \\ d_r^c \end{pmatrix}, \begin{pmatrix} \nu_r^c \\ e_r^c \end{pmatrix} \quad (\bar{4}, 1, 2)$$

$$\langle \tilde{\nu}_H^c \rangle = \langle \tilde{\nu}_H^c \rangle \sim M$$

HIGGS FIELDS

$$H^c \quad \begin{pmatrix} u_H^c \\ d_H^c \end{pmatrix}, \begin{pmatrix} \nu_H^c \\ e_H^c \end{pmatrix} \quad (\bar{4}, 1, 2)$$

$$\bar{H}^c \quad \begin{pmatrix} \bar{u}_H^c & \bar{d}_H^c \\ \bar{\nu}_H^c & \bar{e}_H^c \end{pmatrix} \quad (4, 1, 2)$$

$$h \quad \begin{pmatrix} h_2^+ & h_1^0 \\ h_2^0 & h_1^- \end{pmatrix} \quad (1, 2, 2)$$

$$G_{PS} \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

$$M_4 = M_3; M_2 = y_{LR} M_{2R}$$

$$M_4 = M_3; M_2 = y_{LR} M_{2R}$$

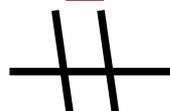
Condition for gaugino masses.

$$m_{H_{u,d}}^2 = m_{10}^2 \mp 2M_D^2$$

PS(4-2-1) *LR Asymmetry*

MATTER FIELDS

$$F_r \quad \begin{pmatrix} d_r & -u_r \\ e_r & -\nu_r \end{pmatrix} \quad (4, 2, 1)$$

$$m_L$$


$$F_r^c \quad \begin{pmatrix} u_r^c \\ d_r^c \end{pmatrix}, \begin{pmatrix} \nu_r^c \\ e_r^c \end{pmatrix} \quad (\bar{4}, 1, 2)$$

$$m_R$$

New Parameter

$$X_{LR} = \frac{m_R}{m_L}$$

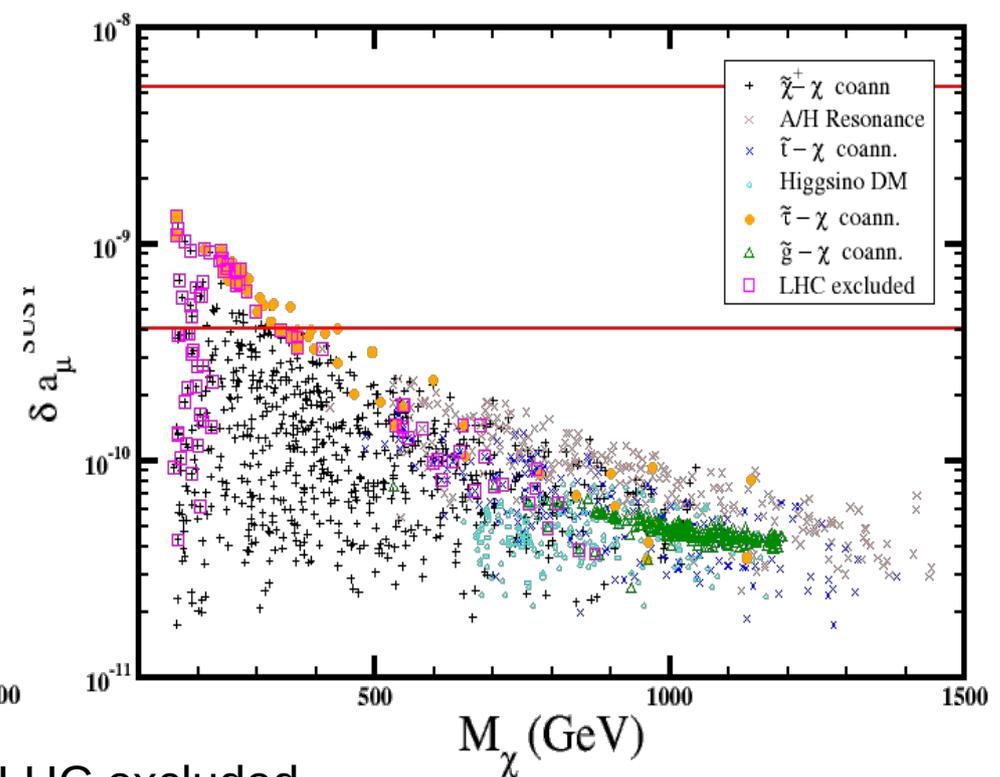
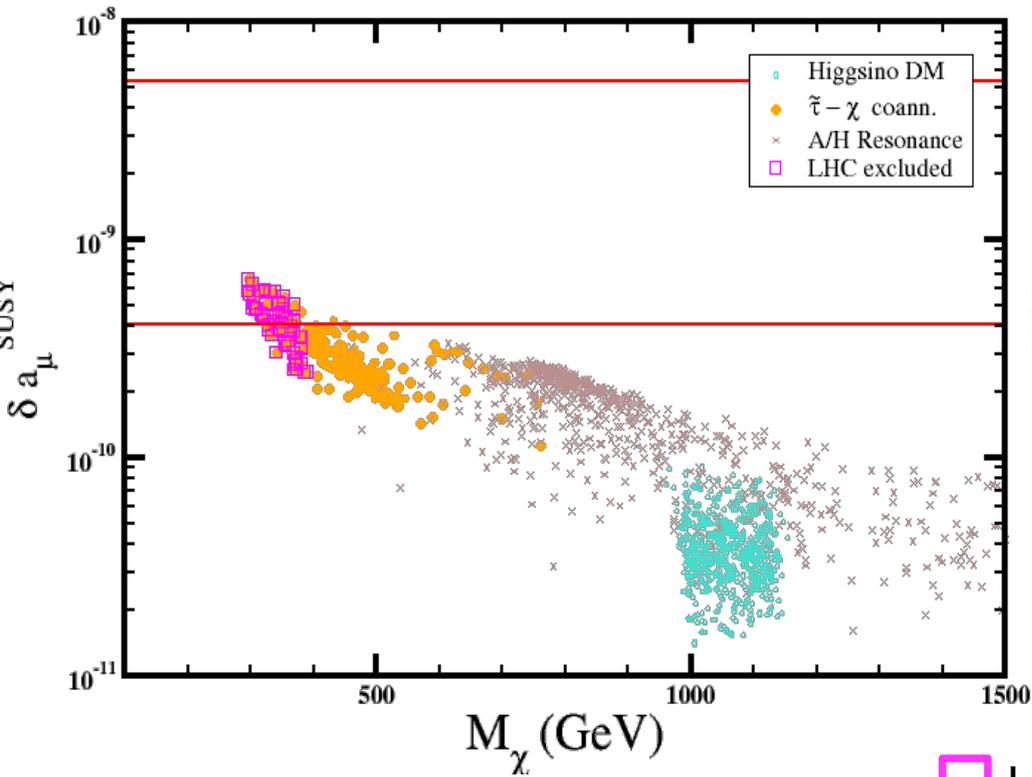
Gaugino Masses

$$M_1 = \frac{3}{5}M_{2R} + \frac{2}{5}M_4,$$

$$M_4 = M_3; M_2 = y_{LR} M_{2R}$$

SO(10)

PS(4-2-2)



 LHC excluded.

Higgsino DM

A/H Resonances

$h_f > 0.1, |m_A - 2m_\chi| > 0.1 m_\chi.$ 

$|m_A - 2m_\chi| \leq 0.1 m_\chi$ 

$h_f \equiv |N_{13}|^2 + |N_{14}|^2,$

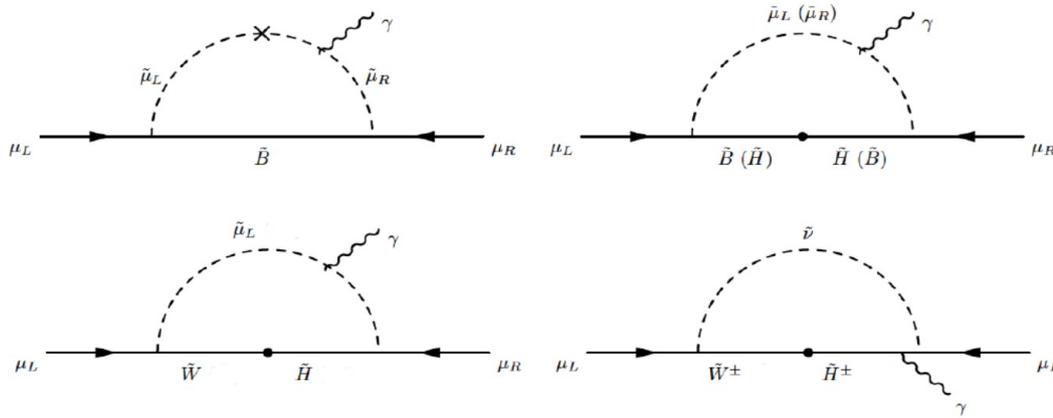
Coannihilations: $(m_z - m_{LSP}) < 0.1 m_{LSP}$



Muon g-2 combining Fermilab + BNL data

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.1 \pm 5.9) \times 10^{-10} .$$

SUSY Contribution to Muon g-2



Low Scale	GUT Scale
$m_{\tilde{\mu}_L}, m_{\tilde{\nu}}$	m_L
$m_{\tilde{\mu}_R}$	m_R
$M_{\tilde{B}}$	M_1
$M_{\tilde{W}}$	M_2
μ	m_{H_u}, m_{H_d}
A_μ	A_0
$\tan \beta$	$\tan \beta$

$$m_h = 123 - 127 \text{ GeV}$$

$$m_{\tilde{g}} \geq 2.1 \text{ TeV (800 GeV if it is NLSP)}$$

$$0.8 \times 10^{-9} \leq \text{BR}(B_s \rightarrow \mu^+ \mu^-) \leq 6.2 \times 10^{-9} \text{ (} 2\sigma \text{)}$$

$$2.99 \times 10^{-4} \leq \text{BR}(B \rightarrow X_s \gamma) \leq 3.87 \times 10^{-4} \text{ (} 2\sigma \text{)}$$

$$0.114 \leq \Omega_{\text{CDM}} h^2 \leq 0.126 .$$

$$0 \leq m_L \leq 5 \text{ TeV}$$

$$0 \leq M_{2L} \leq 5 \text{ TeV}$$

$$-3 \leq M_3 \leq 5 \text{ TeV}$$

$$-3 \leq A_0/m_L \leq 3$$

$$1.2 \leq \tan \beta \leq 60$$

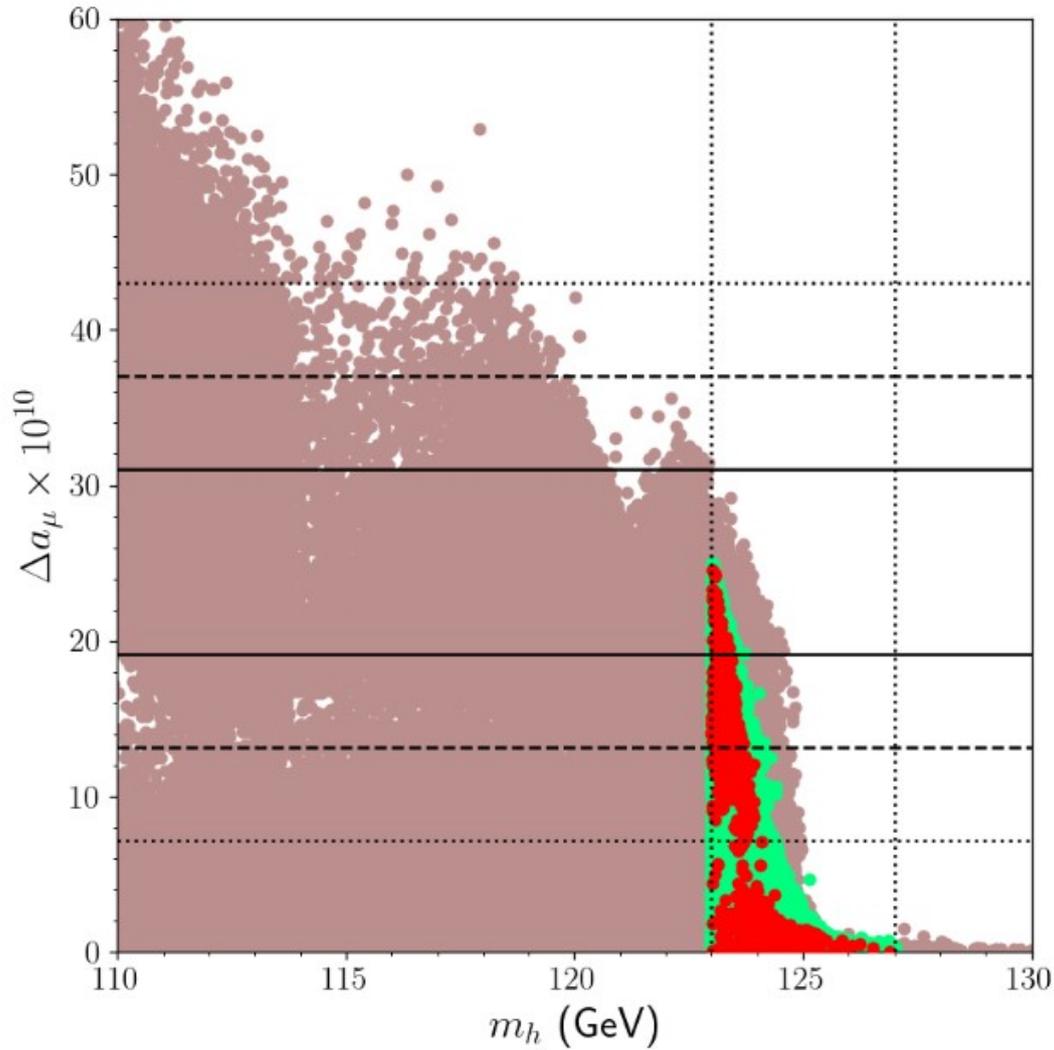
$$0 \leq x_{\text{LR}} \leq 3$$

$$-3 \leq y_{\text{LR}} \leq 3$$

$$0 \leq x_{\text{d}} \leq 3$$

$$-1 \leq x_{\text{u}} \leq 2 .$$

Important contribution is in tension with the Higgs mass



$$\Delta a_\mu^{\tilde{B}\tilde{\mu}_L\tilde{\mu}_R} \simeq \frac{g_1^2}{16\pi^2} \frac{m_\mu^2 M_{\tilde{B}} (\mu \tan \beta - A_\mu)}{m_{\tilde{\mu}_L}^2 m_{\tilde{\mu}_R}^2} F_N \left(\frac{m_{\tilde{\mu}_L}^2}{M_{\tilde{B}}^2}, \frac{m_{\tilde{\mu}_R}^2}{M_{\tilde{B}}^2} \right)$$

$$\Delta m_h^2 \simeq \frac{m_t^4}{16\pi^2 v^2 \sin^2 \beta} \frac{\mu A_t}{M_{\text{SUSY}}^2} \left[\frac{A_t^2}{M_{\text{SUSY}}^2} - 6 \right]$$

$$\begin{aligned}
\frac{dm_{\tilde{l}_{R_i}}^2}{dt} &= -\frac{1}{16\pi^2} \left[2(Y_l^i)^2 P_{\tilde{l}}^i + g_1^2 \text{Tr}(Y m^2) - 4g_1^2 M_1^2 \right] \\
\frac{dm_{\tilde{L}_i}^2}{dt} &= -\frac{1}{16\pi^2} \left[(Y_l^i)^2 P_{\tilde{l}}^i - \frac{1}{2} g_1^2 \text{Tr}(Y m^2) - (g_1^2 M_1^2 + 3g_2^2 M_2^2) \right] \\
\frac{dm_{\tilde{d}_{R_i}}^2}{dt} &= -\frac{1}{16\pi^2} \left[2(Y_d^i)^2 P_{\tilde{d}}^i + \frac{1}{3} g_1^2 \text{Tr}(Y m^2) - \left(\frac{4}{9} g_1^2 M_1^2 + \frac{16}{3} g_3^2 M_3^2 \right) \right] \\
\frac{dm_{\tilde{u}_{R_i}}^2}{dt} &= -\frac{1}{16\pi^2} \left[2(Y_u^i)^2 P_{\tilde{u}}^i - \frac{2}{3} g_1^2 \text{Tr}(Y m^2) - \left(\frac{16}{9} g_1^2 M_1^2 + \frac{16}{3} g_3^2 M_3^2 \right) \right] \\
\frac{dm_{\tilde{Q}_i}^2}{dt} &= -\frac{1}{16\pi^2} \left[(Y_u^i)^2 P_{\tilde{u}}^i + (Y_d^i)^2 P_{\tilde{d}}^i + \frac{1}{6} g_1^2 \text{Tr}(Y m^2) - \left(\frac{1}{9} g_1^2 M_1^2 + 3g_2^2 M_2^2 + \frac{16}{3} g_3^2 M_3^2 \right) \right]
\end{aligned}$$

$$m_{\tilde{u}_{iL}}^2 = m_{\tilde{q}_{i0}}^2 + m_{u_i}^2 + \tilde{\alpha}_G \left(\frac{8}{3} f_3 m_3^2 + \frac{3}{2} f_2 m_2^2 + \frac{1}{30} f_1 m_1^2 \right)$$

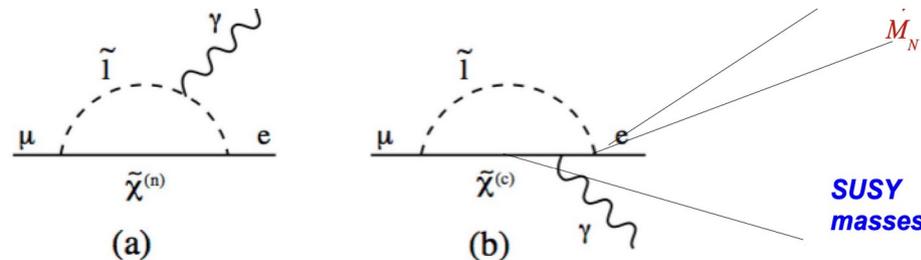
$$m_{\tilde{u}_{iR}}^2 = m_{\tilde{u}_{i0}}^2 + m_{u_i}^2 + \tilde{\alpha}_G \left(\frac{8}{3} f_3 m_3^2 + \frac{8}{15} f_1 m_1^2 \right)$$

$$m_{\tilde{e}_{iL}}^2 = m_{\tilde{\ell}_{i0}}^2 + m_{e_i}^2 + \tilde{\alpha}_G \left(\frac{3}{2} f_2 m_2^2 + \frac{3}{10} f_1 m_1^2 \right)$$

$$m_{\tilde{e}_{iR}}^2 = m_{\tilde{e}_{i0}}^2 + m_{e_i}^2 + \tilde{\alpha}_G \frac{6}{5} f_1 m_1^2.$$

LFV violating through soft masses.

- LFV can be induced by a misalignment of leptons and sleptons



- Flavor dependence on soft terms can be induced:
 - Above GUT by extra fields needed to explain fermion hierarchy.
 - Below GUT. Radiatively generated by the same mechanism that explain neutrino oscillations

MSSM extended by seesaw mechanism

- The superpotential for MSSM-Seesaw I can be written as

$$W = W_{\text{MSSM}} + Y_{\nu}^{ij} \epsilon_{\alpha\beta} H_2^{\alpha} N_i^c L_j^{\beta} + \frac{1}{2} M_N^{ij} N_i^c N_j^c, \quad (5)$$

- The full set of soft SUSY-breaking terms is given by,

$$\begin{aligned} -\mathcal{L}_{\text{soft,SI}} = & -\mathcal{L}_{\text{soft}} + (m_{\tilde{\nu}}^2)_j^i \tilde{\nu}_{Ri}^* \tilde{\nu}_R^j + \left(\frac{1}{2} B_{\nu}^{ij} M_N^{ij} \tilde{\nu}_{Ri}^* \tilde{\nu}_{Rj}^* \right. \\ & \left. + A_{\nu}^{ij} h_2 \tilde{\nu}_{Ri}^* \tilde{l}_{Lj} + \text{h.c.} \right), \end{aligned} \quad (6)$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^{D^T} & M_R \end{pmatrix}$$

“See-Saw” explanation for tiny masses.

• The physical masses are:

1. $m_1 \equiv m_{light} \simeq \frac{(m_\nu^D)^2}{M_R}$

2. $m_2 \simeq M_R$

• For $(m_\nu^D)_{33} \approx (200 \text{ GeV})$ ($\lambda_\nu \approx \lambda_t$) and $M_{N_3} \approx O(10^{14} \text{ GeV})$, $m_{eff} \approx 0.05 \text{ eV}$

$$W = W_{\text{MSSM}} + \frac{1}{2} (Y_\nu L H_2)^T M_N^{-1} (Y_\nu L H_2).$$

$$m_{\text{eff}} = -\frac{1}{2} v_u^2 Y_\nu \cdot M_N^{-1} \cdot Y_\nu^T, \quad m_\nu^\delta = U^T m_{\text{eff}} U$$

Slepton flavor mixings

$$(m_{\tilde{L}}^2)_{ij} \sim \frac{1}{16\pi^2} (6m_0^2 + 2A_0^2) (Y_\nu^\dagger Y_\nu)_{ij} \log \left(\frac{M_{\text{GUT}}}{M_N} \right)$$

$$(m_{\tilde{e}}^2)_{ij} \sim 0$$

$$(A_l)_{ij} \sim \frac{3}{8\pi^2} A_0 Y_{li} (Y_\nu^\dagger Y_\nu)_{ij} \log \left(\frac{M_{\text{GUT}}}{M_N} \right)$$

Orthogonal matrix

$$Y_\nu = \frac{\sqrt{2}}{v_u} \sqrt{M_R^\delta} R \sqrt{m_\nu^\delta} U^\dagger$$

Casas + Ibarra

Diagonal Universal
1E14 GeV

Limit case of degenerate MR

Using neutrino data LFV depends is controlled by MR

Order 1

$$Y_\nu^\dagger Y_\nu = \frac{2}{v_u^2} M_R U m_\nu^\delta U^\dagger$$

Slepton flavor mixing above GUT.

Generation of Yukawa textures using family symmetries, for example Abelian U(1)'s

$$\Phi_i \Phi_J^c h \frac{\theta^{(q_i - q_j + q_h)}}{M}, \quad \epsilon = \frac{\langle \theta \rangle}{M} \quad Y_{IJ} \Phi_I \Phi_J^c h$$

$$Y_{IJ} \sim \epsilon^{(q_i - q_j + q_h)}$$

Froggatt-Nilsen 1979

Soft terms: In SUGRA models, redefinition of fields due to flavons results in non universal soft masses.

$$m_f^2 = \begin{pmatrix} 1 & \epsilon^{q^{12}} & \epsilon^{q^{13}} \\ \epsilon^{q^{12}} & 1 & \epsilon^{q^{23}} \\ \epsilon^{q^{12}} & \epsilon^{q^{23}} & 1 \end{pmatrix} \times m_{f0}^2,$$

S. F. King et al 2005,
Olive+Velasco-Sevilla 2005
Das et al 2017



GUT

$$m_f^2 = \begin{pmatrix} 1 & \epsilon^{q_L^{12}} & \epsilon^{q_L^{13}} \\ \epsilon^{q_L^{12}} & 1 & \epsilon^{q_L^{23}} \\ \epsilon^{q_L^{12}} & \epsilon^{q_L^{23}} & 1 \end{pmatrix} \times m_L^2,$$

$$m_{fc}^2 = \begin{pmatrix} 1 & \epsilon^{q_R^{12}} & \epsilon^{q_R^{13}} \\ \epsilon^{q_R^{12}} & 1 & \epsilon^{q_R^{23}} \\ \epsilon^{q_R^{12}} & \epsilon^{q_R^{23}} & 1 \end{pmatrix} \times m_R^2,$$



M_SUSY scale

$$m_{\tilde{Q}}^2 \sim m_L^2 + (k_3 \cdot M_3^2 + k_2 \cdot M_1^2 + \frac{1}{36} k_1 \cdot M_1^2) \times I,$$

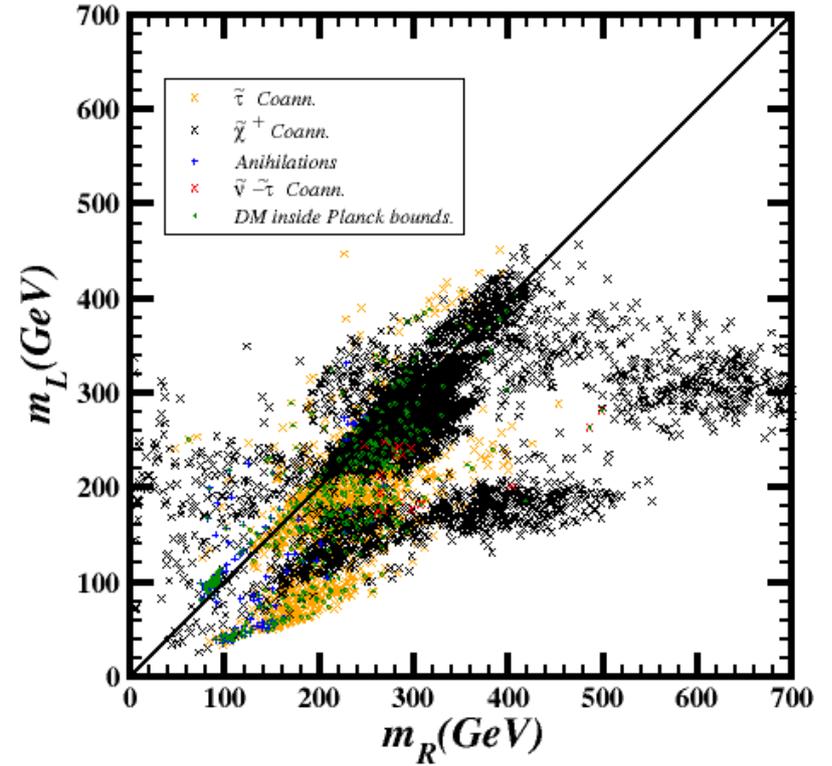
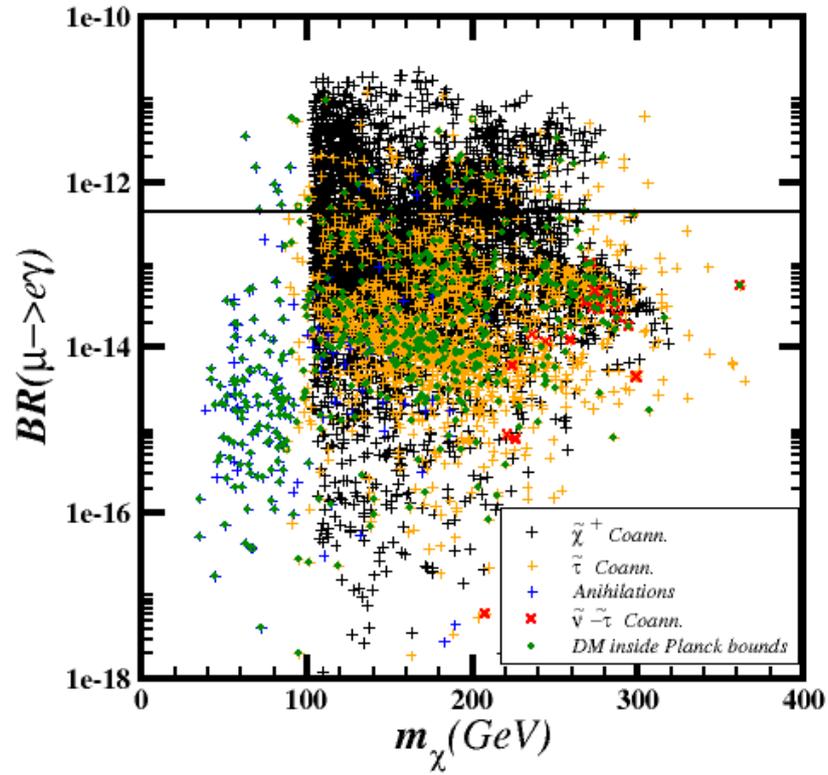
$$m_{\tilde{U}}^2 \sim m_R^2 + (k_3 \cdot M_3^2 + \frac{4}{9} k_1 \cdot M_1^2) \times I,$$

$$m_{\tilde{D}}^2 \sim m_R^2 + (k_3 \cdot M_3^2 + \frac{1}{9} k_1 \cdot M_1^2) \times I,$$

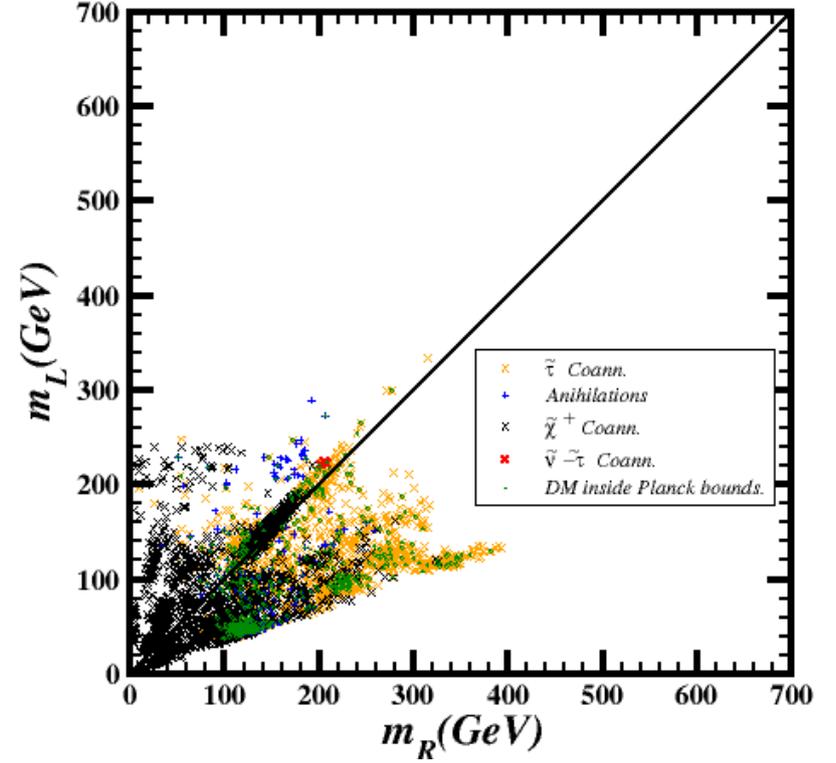
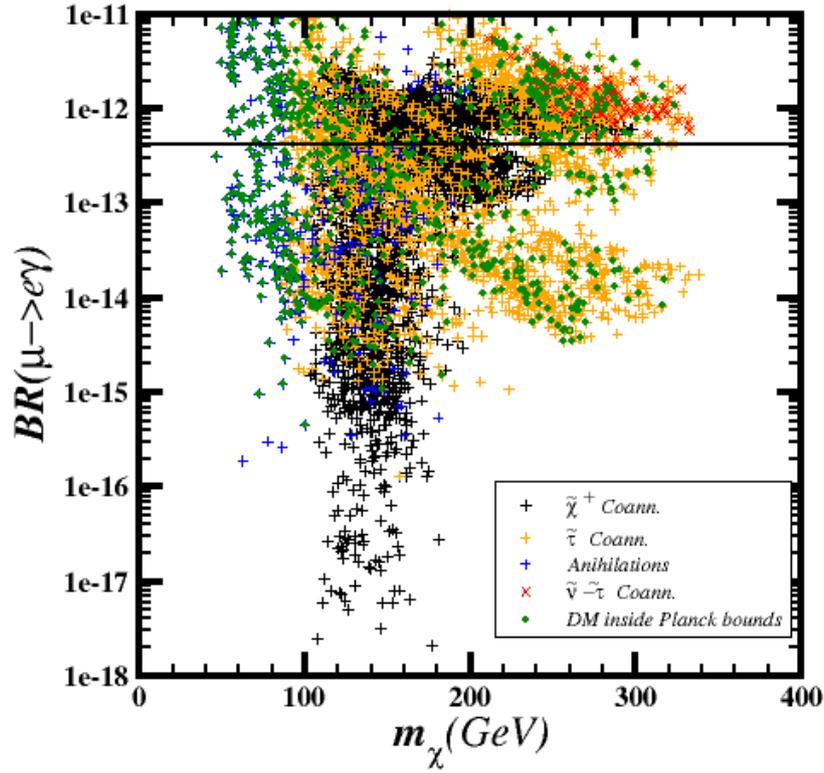
$$m_{\tilde{e}_L}^2 \sim m_L^2 + (k_2 \cdot M_2^2 + \frac{1}{4} k_1 \cdot M_1^2) \times I,$$

$$m_{\tilde{e}_R}^2 \sim m_R^2 + (k_1 \cdot M_1^2) \times I.$$

See-Saw with MR scale at $2.5 \cdot 10^{12}$ GeV



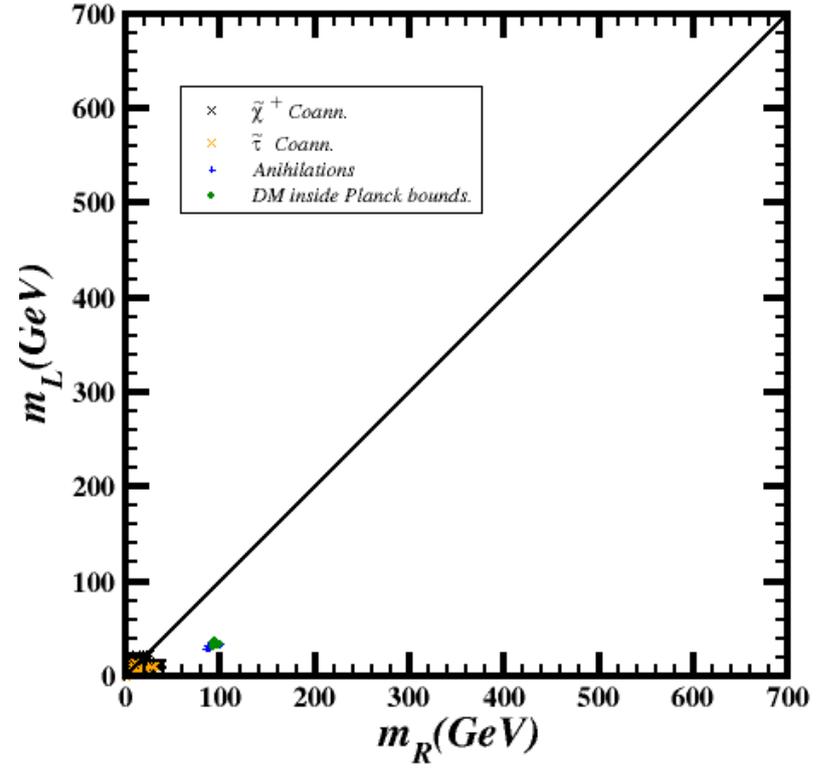
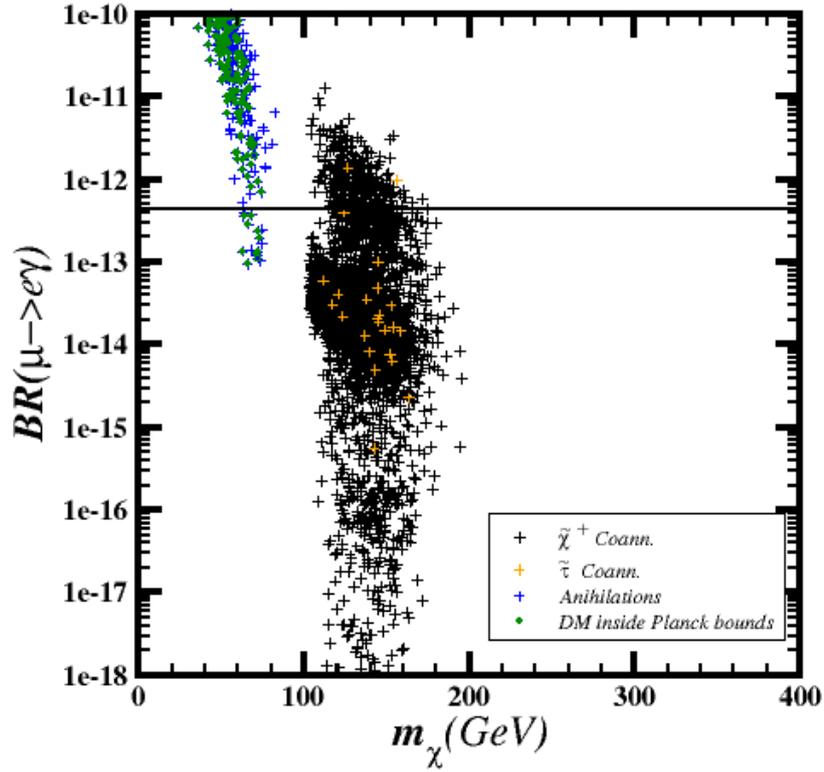
$$\varepsilon = 0.05$$



$$m_f^2 = \begin{pmatrix} 1 & \epsilon^{q_L^{12}} & \epsilon^{q_L^{13}} \\ \epsilon^{q_L^{12}} & 1 & \epsilon^{q_L^{23}} \\ \epsilon^{q_L^{12}} & \epsilon^{q_L^{23}} & 1 \end{pmatrix} \times m_L^2,$$

$$m_{fc}^2 = \begin{pmatrix} 1 & \epsilon^{q_R^{12}} & \epsilon^{q_R^{13}} \\ \epsilon^{q_R^{12}} & 1 & \epsilon^{q_R^{23}} \\ \epsilon^{q_R^{12}} & \epsilon^{q_R^{23}} & 1 \end{pmatrix} \times m_R^2,$$

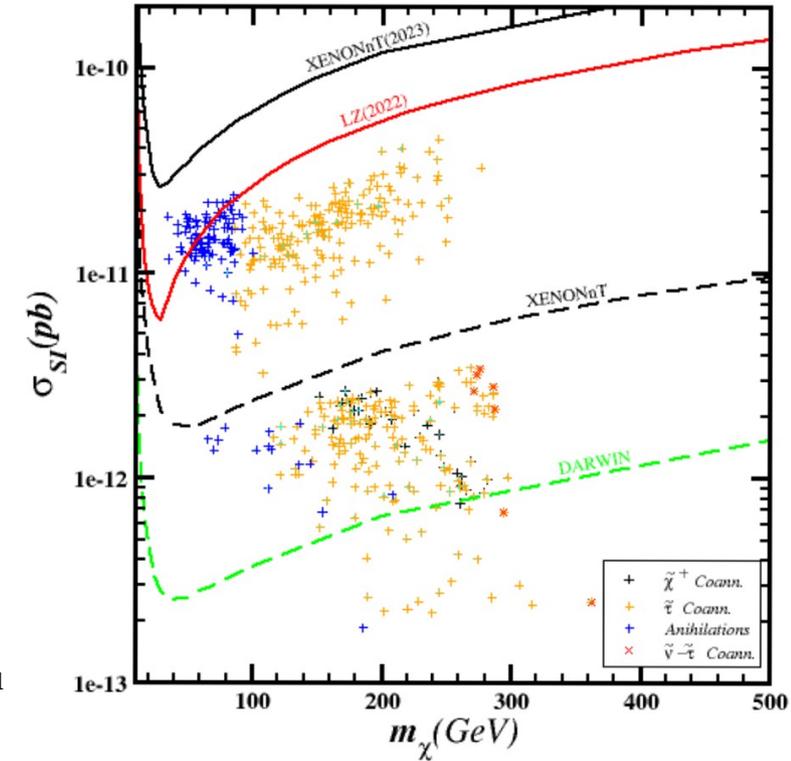
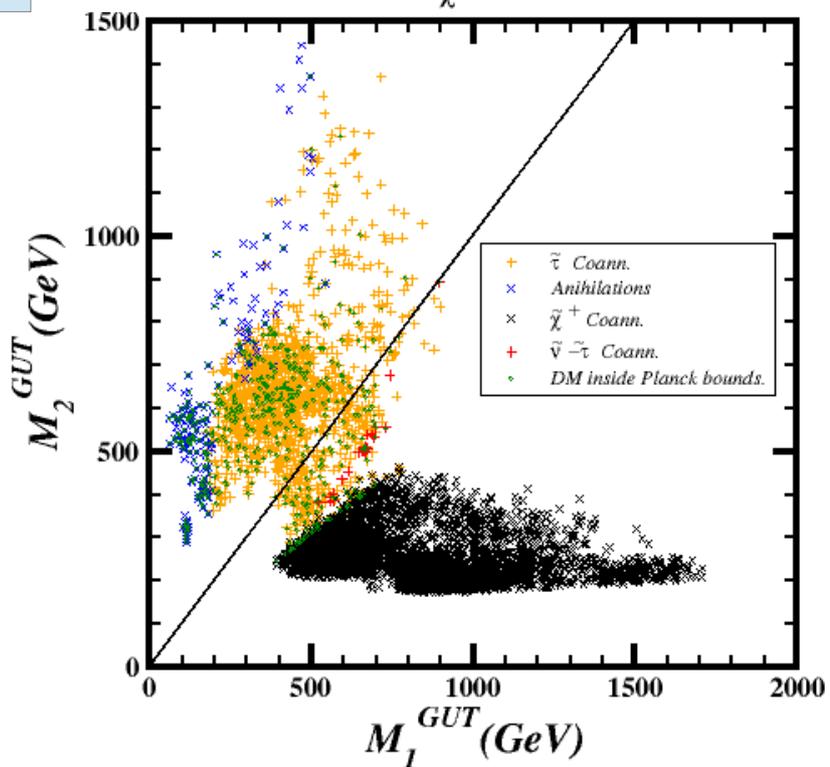
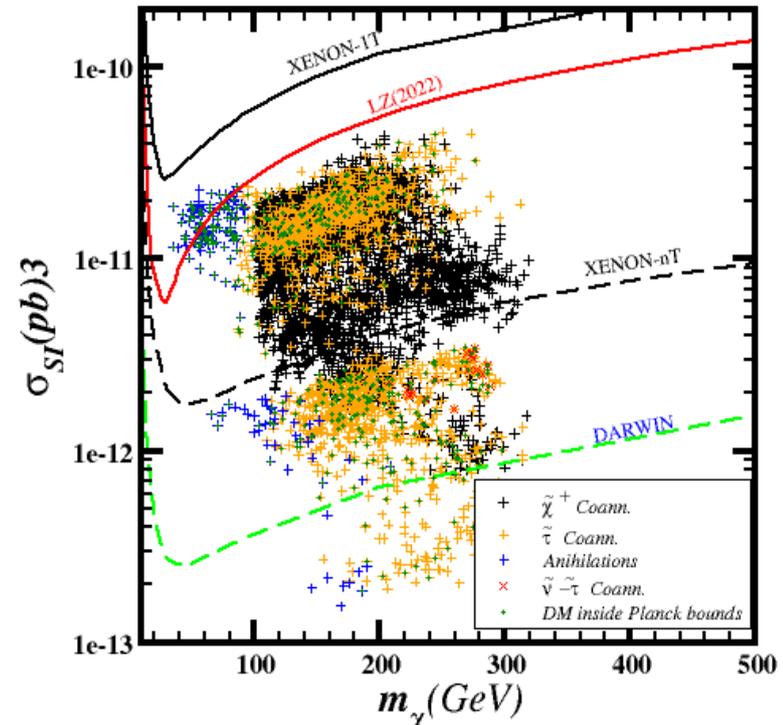
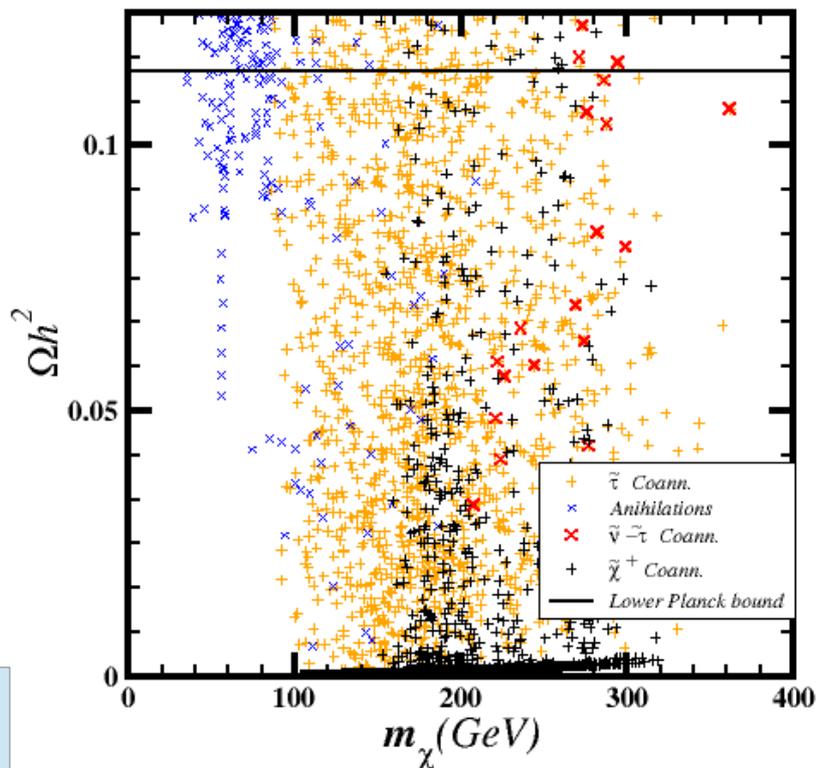
$$\varepsilon = 0.2$$



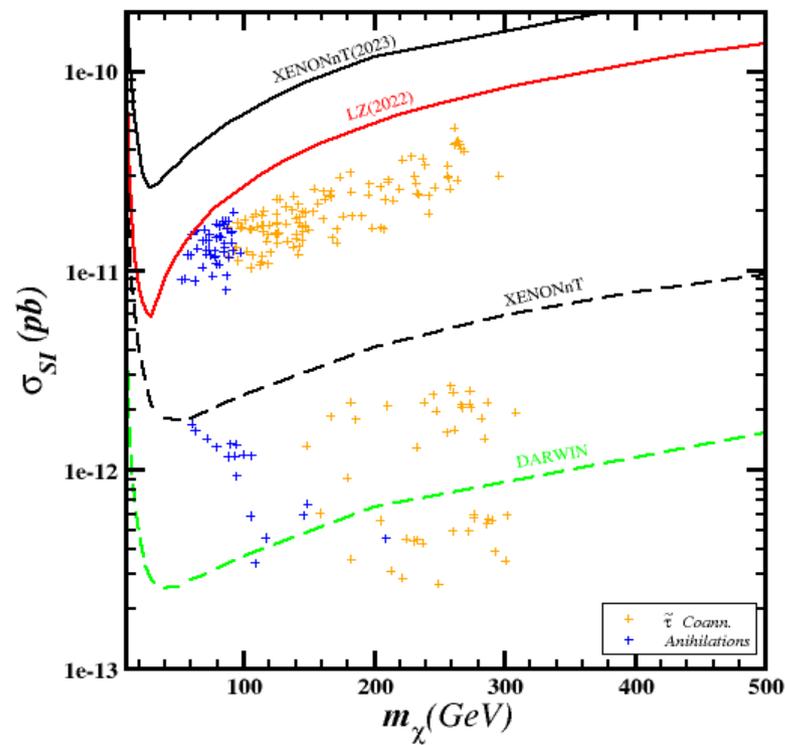
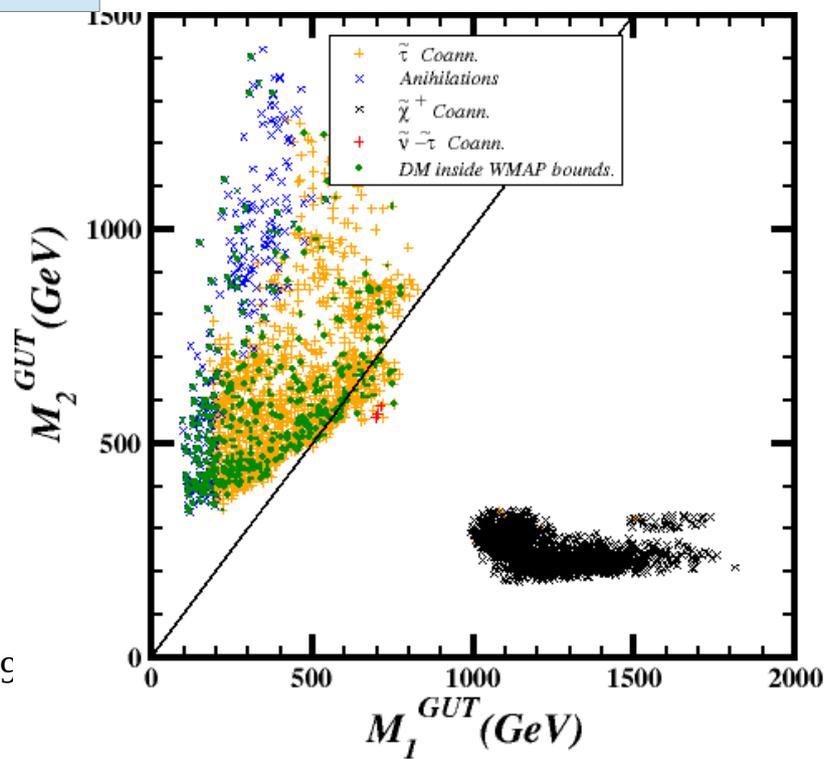
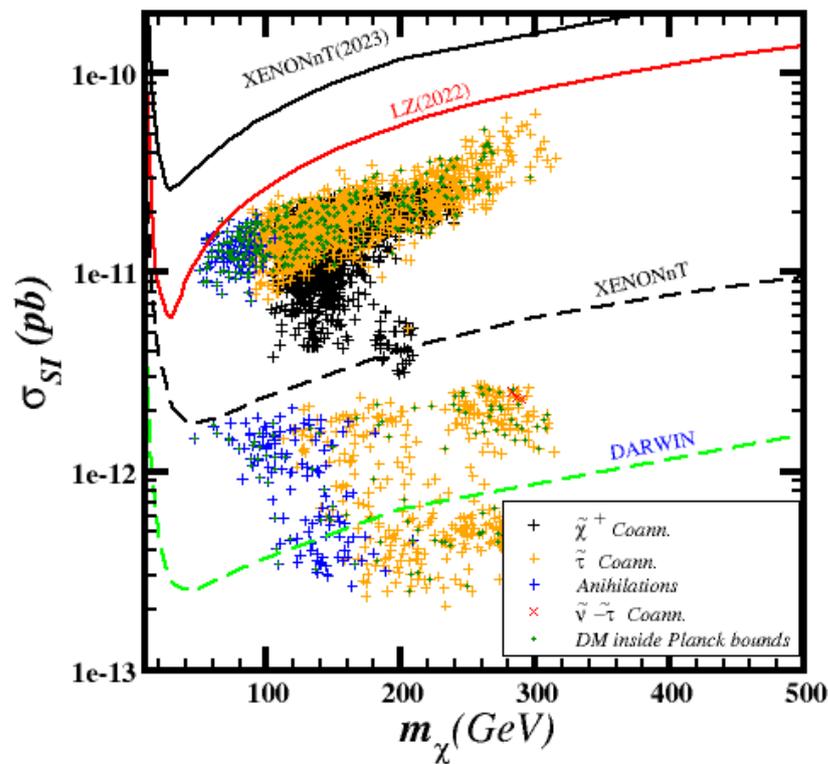
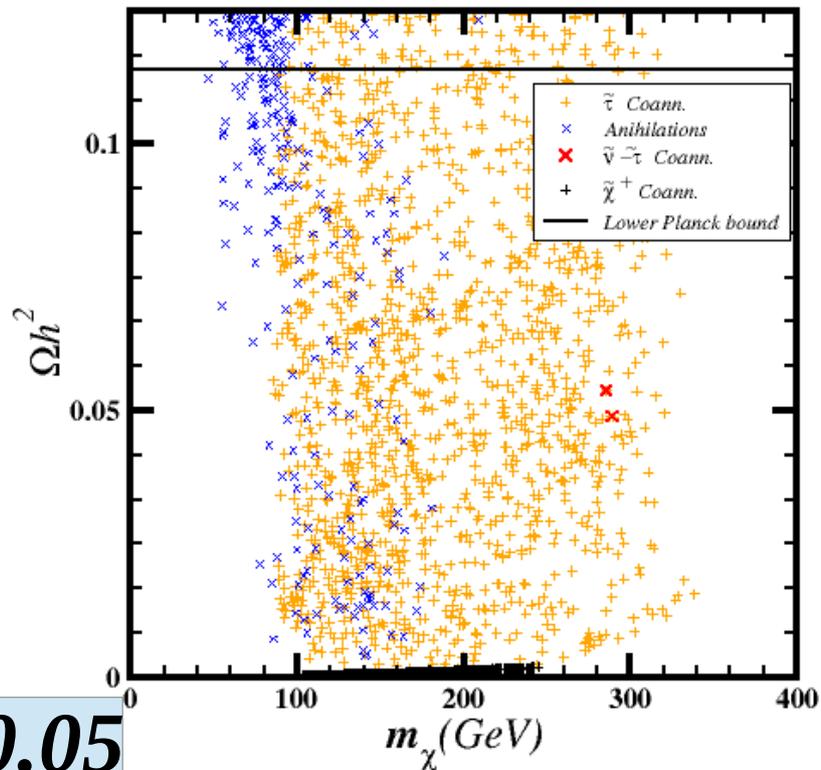
$$m_f^2 = \begin{pmatrix} 1 & \epsilon^{q_L^{12}} & \epsilon^{q_L^{13}} \\ \epsilon^{q_L^{12}} & 1 & \epsilon^{q_L^{23}} \\ \epsilon^{q_L^{12}} & \epsilon^{q_L^{23}} & 1 \end{pmatrix} \times m_L^2,$$

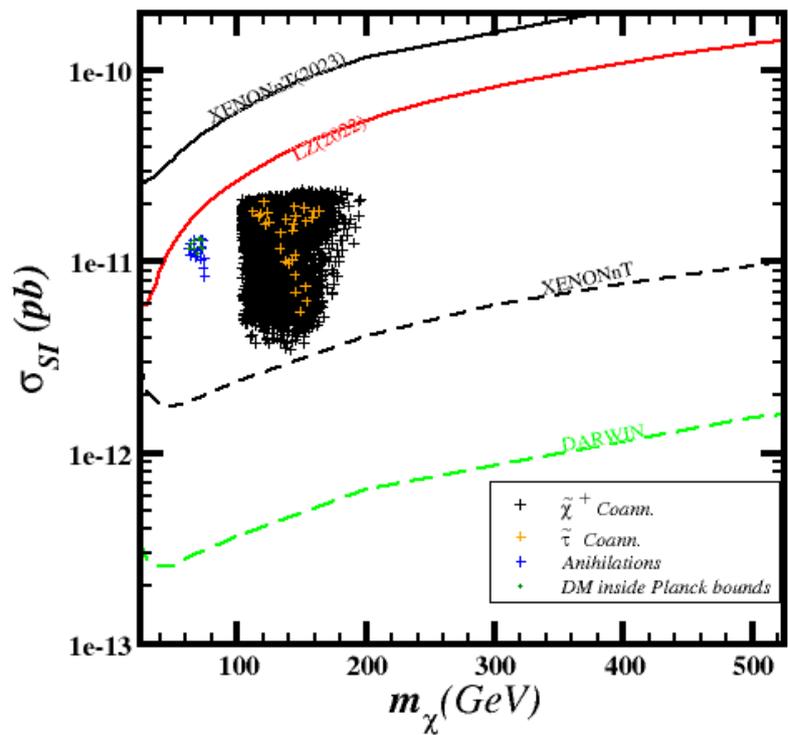
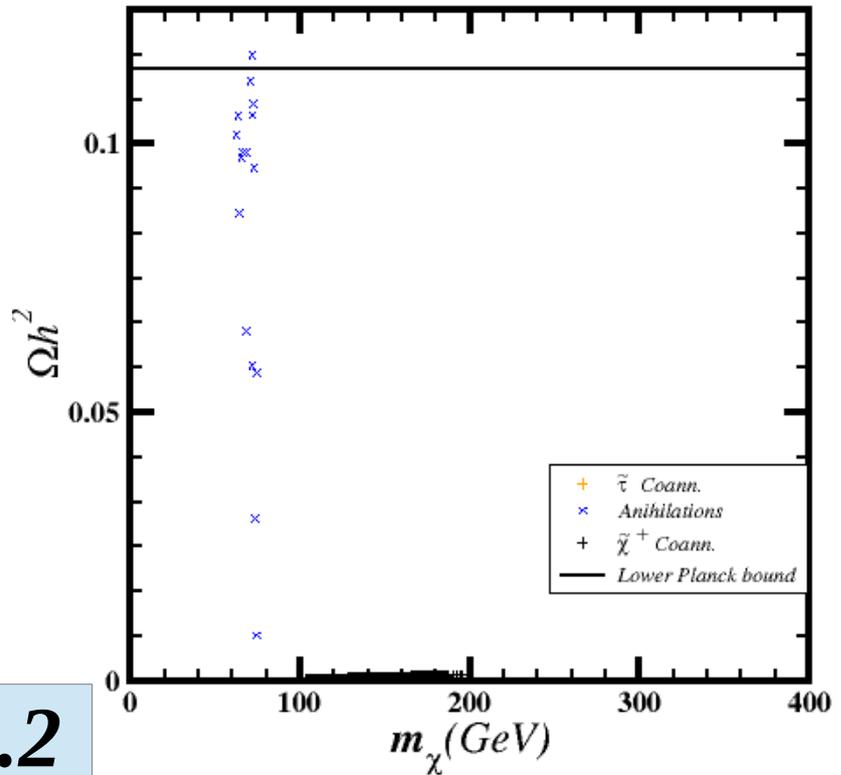
$$m_{fc}^2 = \begin{pmatrix} 1 & \epsilon^{q_R^{12}} & \epsilon^{q_R^{13}} \\ \epsilon^{q_R^{12}} & 1 & \epsilon^{q_R^{23}} \\ \epsilon^{q_R^{12}} & \epsilon^{q_R^{23}} & 1 \end{pmatrix} \times m_R^2,$$

See-Saw

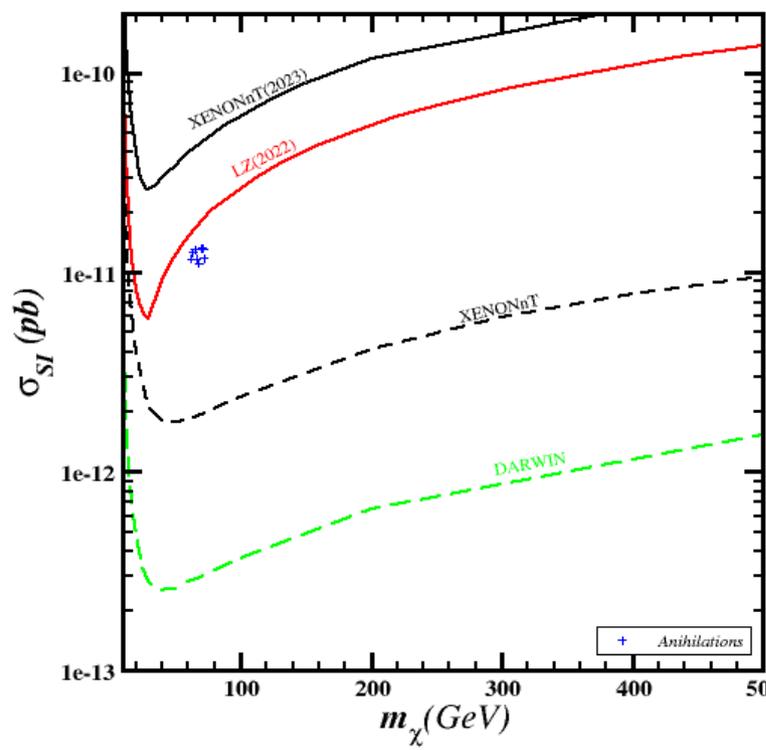
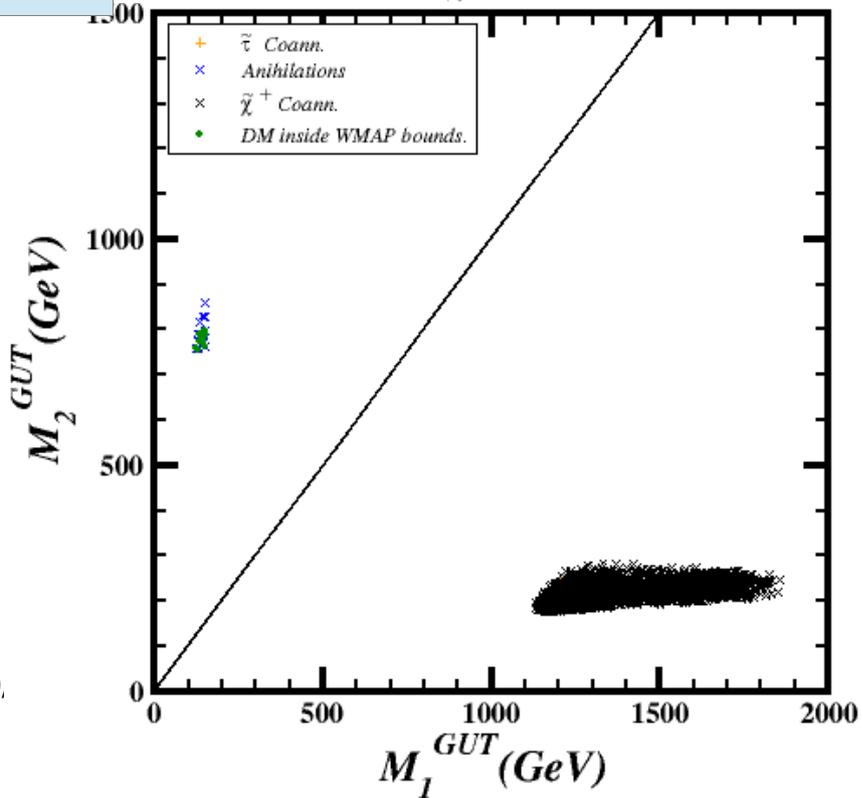


$\epsilon = 0.05$

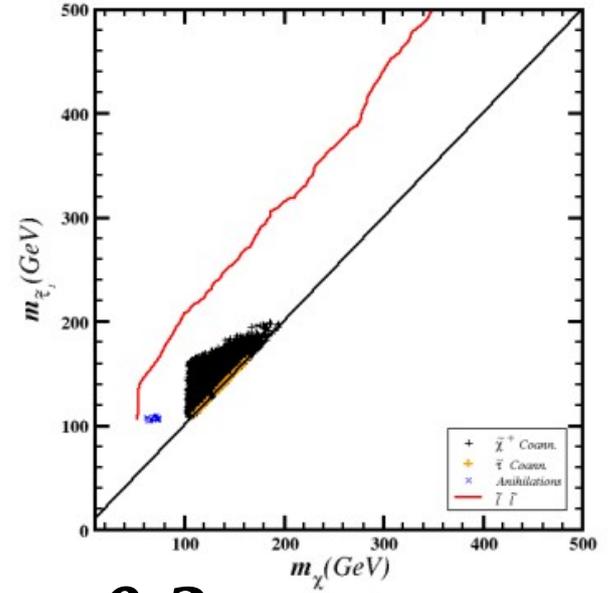
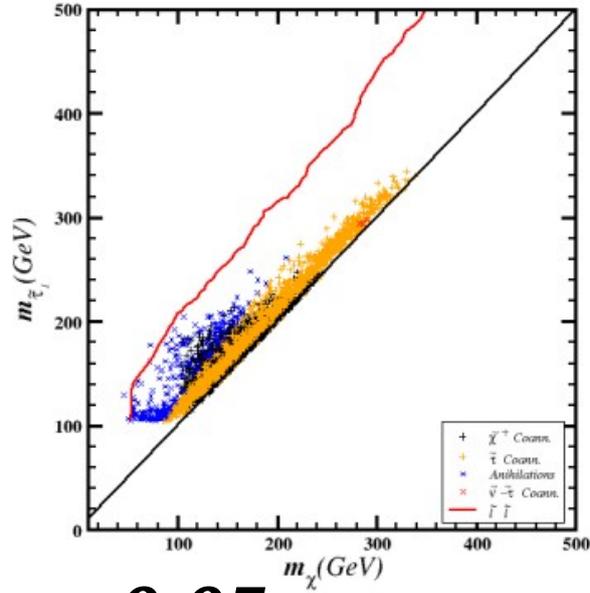
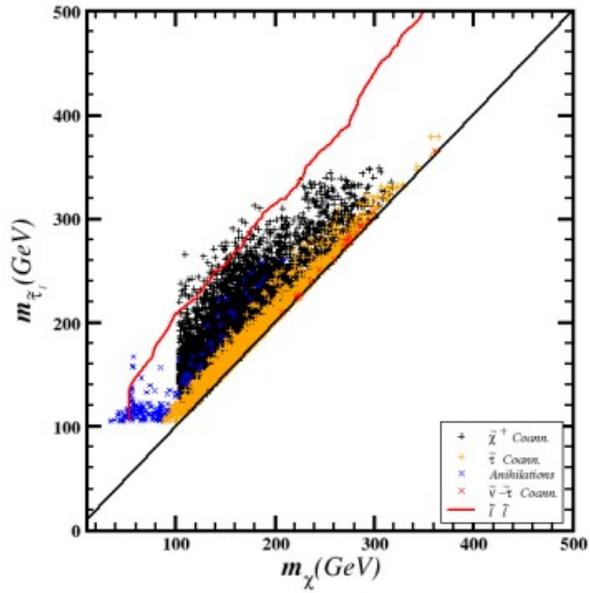




$\epsilon = 0.2$

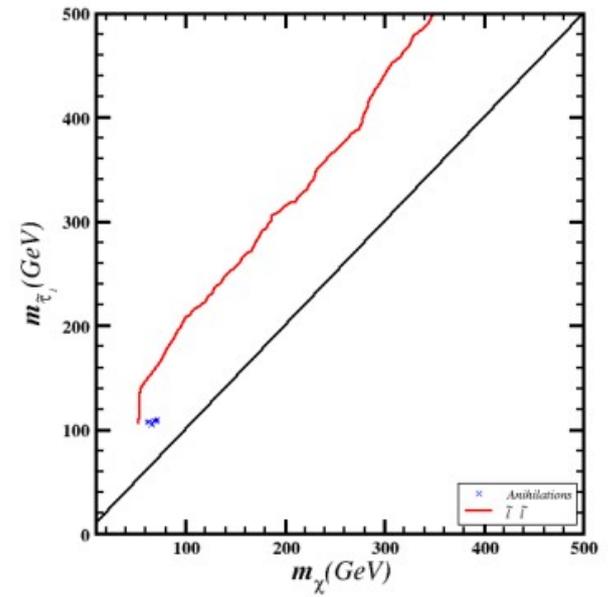
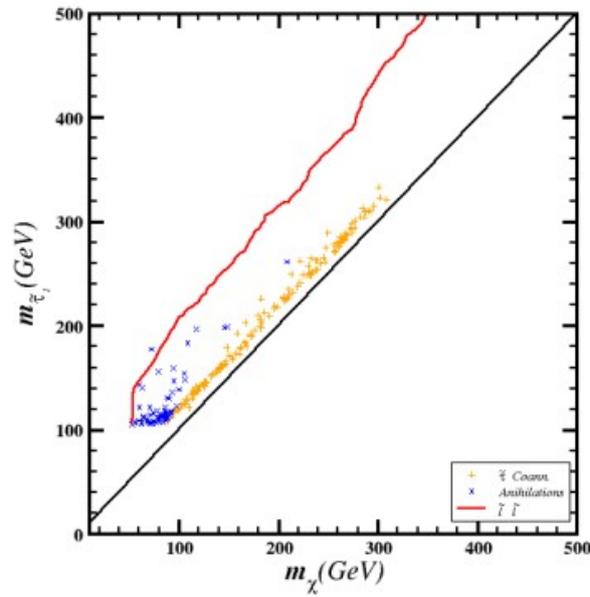
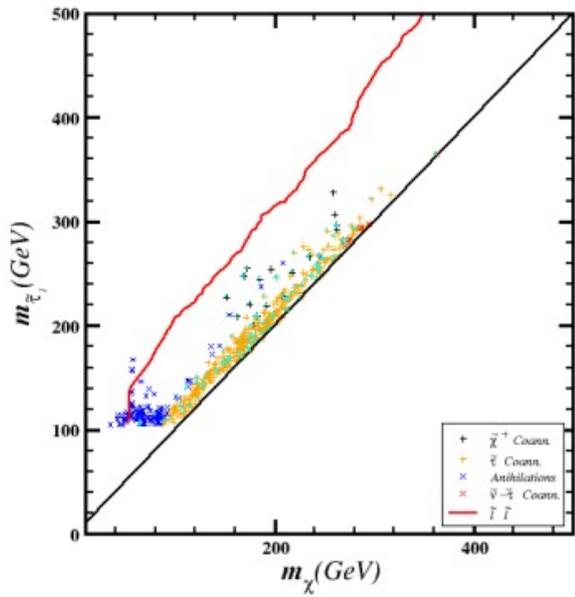


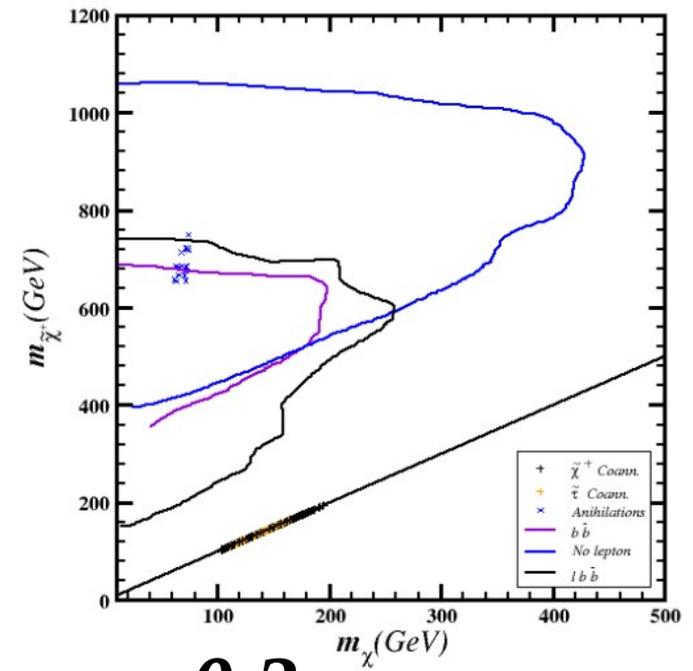
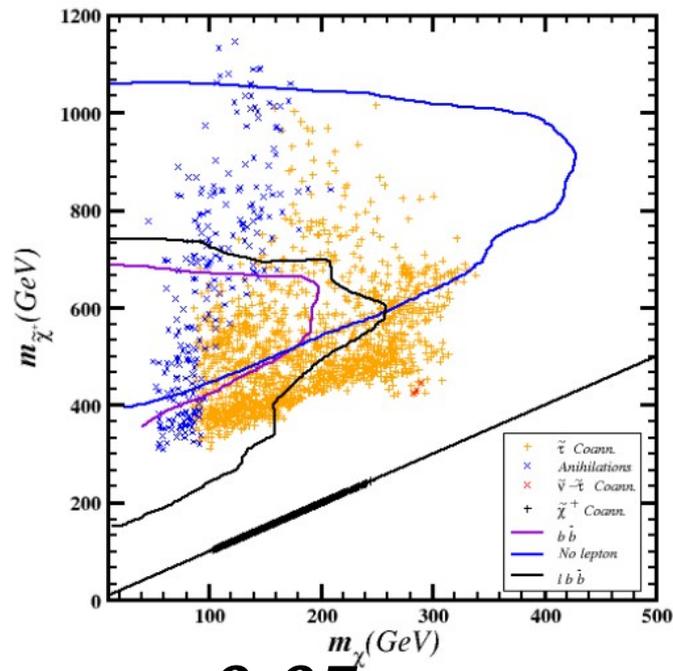
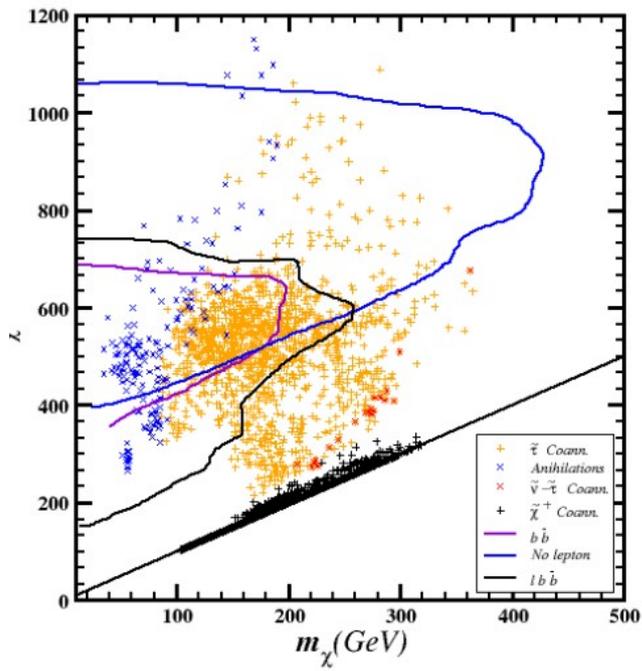
LHC vs LFV



$\varepsilon = 0.05$

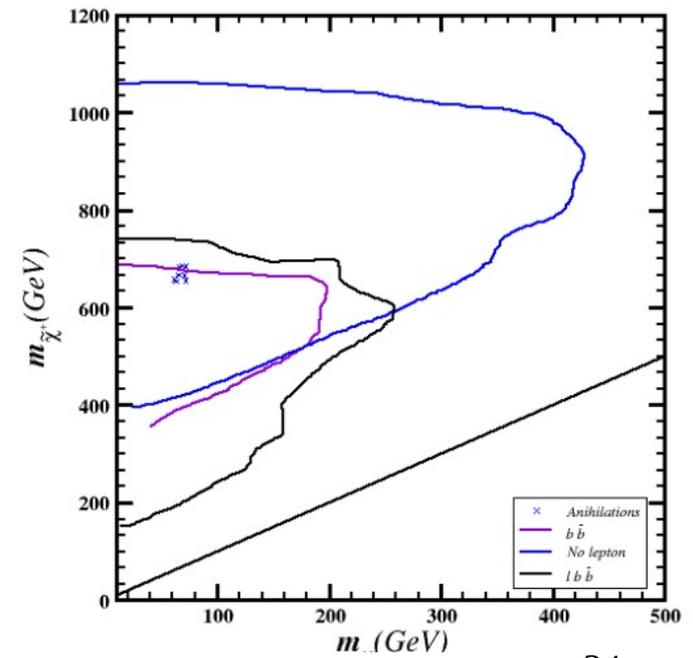
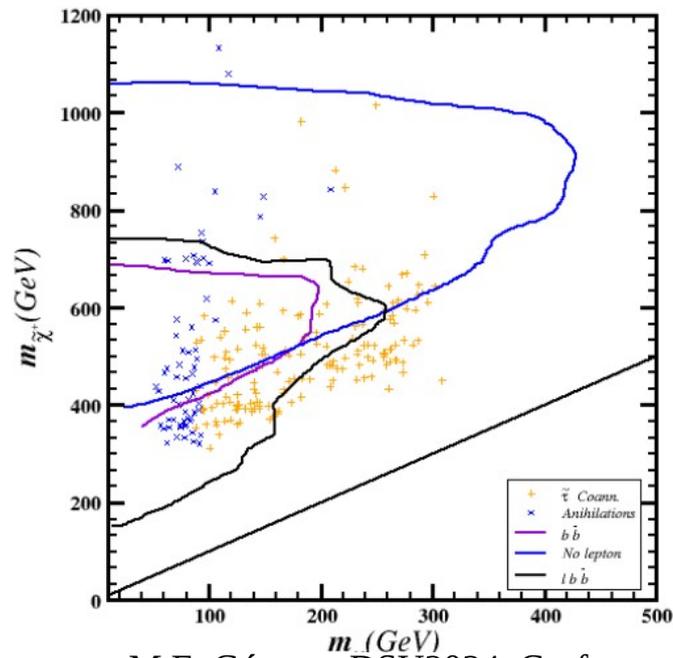
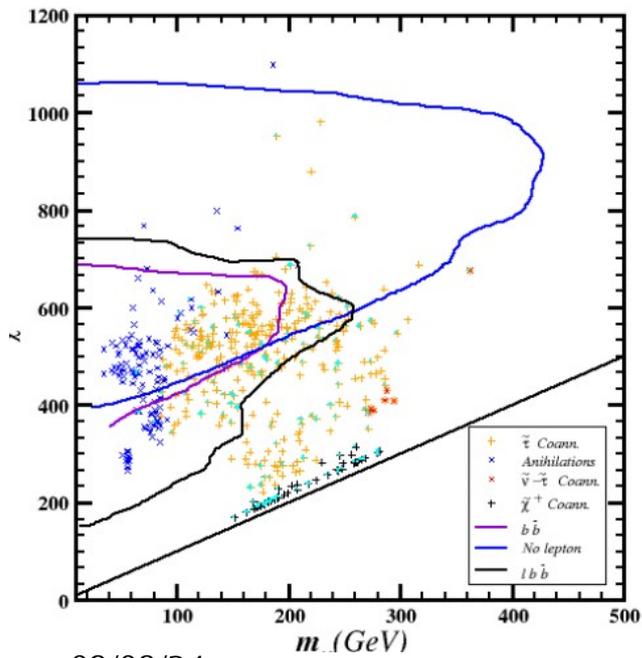
$\varepsilon = 0.2$





$\epsilon = 0.05$

$\epsilon = 0.2$



CONCLUSIONS

Susy Models with large gluino masses can explain the muon $(g-2)$ discrepancy via a SUSY contribution maintaining the prediction for the observed Higgs masses.

Models with Pati-Salam Unification, $SU(4) \times SU(2) \times SU(2)$ can motivate gaugino non-universality ($M_3 \gg M_2, M_1$) and a L/R asymmetry on the scalars such that muon $(g-2)$ can be explained while LSP that satisfies the relic abundance condition.

$(g-2) + \text{DM}$ satisfaction implies chargino and slepton masses below the TeV range. However, the small mass difference neutralino-stau makes difficult the identification of the signal at LHC. But that may be at the reach of planned linear colliders.

When the model is complemented with a mechanism to explain lepton mass hierarchy $\text{BR}(\mu \rightarrow e \gamma)$ falls in the experimental range (i.e. MEG projected bound). Therefore, the observation of LFV can indicate the presence of SUSY particles that can not be detected at the LHC.

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