Type II seesaw: orbit space, minima of the potential & vacuum stability

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2 Neutrino Mass in Type II Seesaw

Besides the Higgs doublet $H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$,

introduce a $Y = 1$ Higgs triplet

$$\Delta = \Delta^i \frac{\sigma^i}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}$$

allowing to write

$$\mathrm{L}_Y = (Y_\nu)_{\alpha\beta} \overline{\ell^c}_\alpha \varepsilon \Delta \ell_\beta + \mathrm{h.c.},$$

which, with $\langle \Delta^0 \rangle = v_\Delta / \sqrt{2}$, yields the neutrino mass matrix

$$(m_\nu)_{\alpha\beta} = \sqrt{2} (Y_\nu)_{\alpha\beta} \, v_\Delta$$

3 Type II Seesaw Scalar Potential

The scalar potential is

$$V = \mu_H^2 H^\dagger H + \mu_\Delta^2 \,\mathrm{tr}(\Delta^\dagger \Delta) + \frac{1}{2}\mu_{H\Delta}[H^T \varepsilon \Delta^\dagger H + \mathrm{h.c.}]$$
$$+ \lambda_H (H^\dagger H)^2 + \lambda_\Delta [\mathrm{tr}(\Delta^\dagger \Delta)]^2 + \lambda'_\Delta \,\mathrm{tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta)$$
$$+ \lambda_{H\Delta} H^\dagger H \,\mathrm{tr}(\Delta^\dagger \Delta) + \lambda'_{H\Delta} H^\dagger \Delta \Delta^\dagger H$$

- Electroweak VEV $v^2 = v_H^2 + 2v_\Delta^2$
- SM-like Higgs h^0 with $m_{h^0} = 125.1$ GeV
- Heavy H^0, A^0, H^+ and $H^{++} \equiv \Delta^{++}$ with small mass splitting

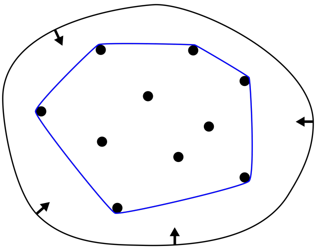
Orbit Space

- Potential is a function of gauge-invariant polynomials p_i with a finite basis: $V(\Phi) = V(p_i(\Phi))$
- Fewer invariants than real fields: removes redundancy
- Trade-off: fewer parameters but non-trivial shape
- Orbit space boundary more symmetric
- Down from vertices, edges,faces, … gauge group completely broken inside the orbit space

Abud & Sartori, Phys. Lett. B 104 (1981) 147; Kim, Nucl. Phys. B196 (1982) 285; Abud & Sartori, Annals Phys. 150 (1983) 307; Sartori & Valente, Annals Phys. 319 (2005) 286

5 Orbit Space

- 'Angular' orbit space parameters and 'radial' field norms
- Potential depends *linearly* on orbit space parameters
- Potential minima lie on the *convex hull* of the orbit space



6 Orbit Space

- Shape determined by the P-matrix

$$P_{ij} = \frac{\partial p_i}{\partial \Phi_a^\dagger} \frac{\partial p_j}{\partial \Phi^a}$$

- P_{ij} are also invariants: $P_{ij} = P_{ij}(p)$
- Orbit space boundary given by

$$\det(P) = 0$$

- Principal minors ($\text{rank}\, P = k$)
  give vertices, edges, faces, …

7 Type II Seesaw Orbit Space

Gauge invariants in the potential:

1. $H^\dagger H$
2. $\mathrm{tr}(\Delta^\dagger \Delta)$
3. $\mathrm{tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta) = \mathrm{tr}(\Delta^\dagger \Delta)^2 - 2|\det(\Delta)|^2$
4. $H^\dagger \Delta \Delta^\dagger H$
5. $\frac{1}{2}[H^T \varepsilon \Delta^\dagger H + \mathrm{h.c.}]$

8 Type II Seesaw Orbit Space

Dimensionless orbit space parameters

$$\xi = \frac{H^\dagger \Delta \Delta^\dagger H}{H^\dagger H \, \mathrm{tr}(\Delta^\dagger \Delta)}, \quad \zeta = \frac{\mathrm{tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta)}{\mathrm{tr}(\Delta^\dagger \Delta)^2} = 1 - \frac{2|\det(\Delta)|^2}{\mathrm{tr}(\Delta^\dagger \Delta)^2},$$

$$\chi = \frac{\frac{1}{2}[H^T \varepsilon \Delta^\dagger H + \mathrm{h.c.}]}{H^\dagger H \sqrt{\mathrm{tr}(\Delta^\dagger \Delta)}}$$

Arhrib &c l, Phys. Rev. D 84 (2011) 095005 [1105.1925];

Bonilla, Fonseca, Valle, Phys. Rev. D 92 (2015) 075028 [1508.02323]

- Orbit space lies within
  $0 \leq \xi \leq 1$, $1/2 \leq \zeta \leq 1$, $-1 \leq \chi \leq 1$
- $1 - 2\xi + 2\xi^2 \leq \zeta \leq 1$

Type II Seesaw Orbit Space

$$V = \frac{1}{2}\mu_H^2 h^2 + \frac{1}{2}\mu_\Delta^2 \delta^2 + \frac{1}{2\sqrt{2}}\mu_{H\Delta}\chi h^2\delta + \frac{1}{4}\lambda_H h^4 + \frac{1}{4}(\lambda_\Delta + \lambda'_\Delta\zeta)\delta^4 + \frac{1}{4}(\lambda_{H\Delta}h^2 + \lambda'_{H\Delta}\xi)h^2\delta^2,$$

where $\frac{h^2}{2} \equiv H^\dagger H$ and $\frac{\delta^2}{2} \equiv \mathrm{tr}(\Delta^\dagger\Delta)$

- Norms $h \geqslant 0$, $\delta \geqslant 0$
- In our vacuum, $h = v_H$ and $\delta = v_\Delta$

Type II Seesaw Orbit Space

- Full 7×7 P-matrix given via seven gauge invariants p_i (two with $d > 4$)
- Much simpler to work with $d \leqslant 4$
  5×5 P-matrix via field components
- We consider real ϕ^0 and $\Delta^0, \Delta^+, \Delta^{++}$ and solve $\det(P) = 0$
- P-matrix can be calculated algebraically or via birdtracks
  Cvitanovic, Phys. Rev. D 14 (1976) 1536;
  Cvitanovic, Group theory: Birdtracks, Lie's and exceptional groups (2008)

Type II Seesaw Orbit Space

For real ϕ^0 and $\Delta^0, \Delta^+, \Delta^{++}$, we have

$$\xi = \frac{1}{2}\frac{2(\Delta^0)^2 + (\Delta^+)^2}{(\Delta^0)^2 + (\Delta^+)^2 + (\Delta^{++})^2},$$

$$\zeta = 1 - \frac{1}{2}\frac{[(\Delta^+)^2 + 2\Delta^0\Delta^{++}]^2}{[(\Delta^0)^2 + (\Delta^+)^2 + (\Delta^{++})^2]^2},$$

$$\chi = -\frac{\Delta^0}{\sqrt{(\Delta^0)^2 + (\Delta^+)^2 + (\Delta^{++})^2}}$$

Type II Seesaw Orbit Space

$\Delta^+ = 0$ yields curved edge(s)

$$\xi = \chi^2, \quad \zeta = 1 - 2\xi + 2\xi^2, \quad -1 \leq \chi \leq 1$$

bounded by the *neutral EWSB* vertices ($\Delta^{++} = 0$)

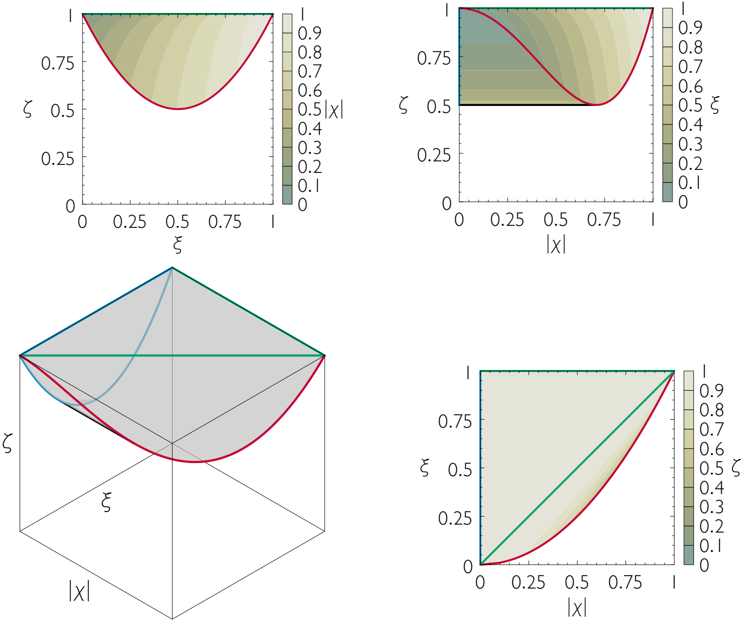
$$\xi = 1, \quad \zeta = 1, \quad \chi = \pm 1$$

and charged vertices ($\Delta^0 = 0$)

$$\xi = 0, \quad \zeta = 1, \quad \chi = 0$$

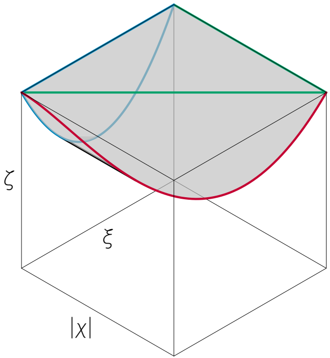
- The type II seesaw orbit space is the *convex hull* of this edge

Type II Seesaw Orbit Space



Potential Extrema

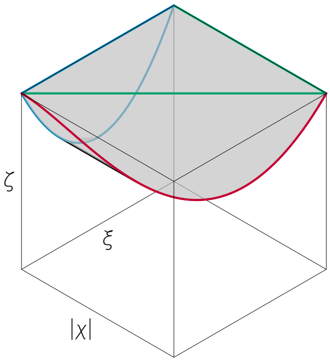
- Any minimum must lie on the curved edge due to the $\mu_{H\Delta}$ term proportional to χ
- No CP-breaking minima
- For $h \neq 0$, $\delta \neq 0$, always three extremum solutions for δ (can be spurious)



15 Potential Extrema

Orbit space classification

- Origin O
- Our neutral vacuum $N_{H\Delta}$ (minimum by parametrisation)
- Other neutral extrema $N'_{H\Delta}$ or 'panic vacua'
- Charged extremum $CB_{H\Delta}^{\chi=0}$
- Charged extrema $CB_{H\Delta}^{\chi\neq 0}$
- Continuous extrema CB_{Δ}, N_{Δ} with $\langle H \rangle = 0$



Similar to Ferreira & Gonçalves, JHEP 02 (2020) 182 [1911.09746]

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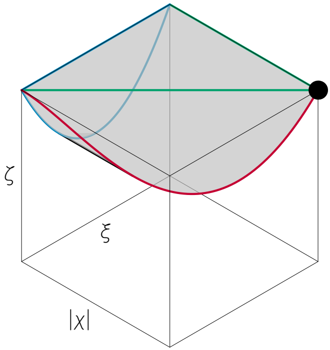

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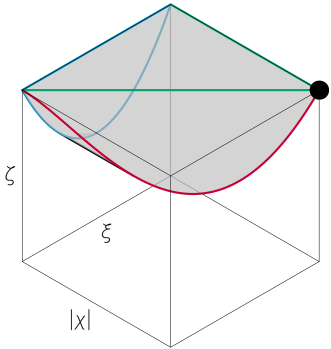

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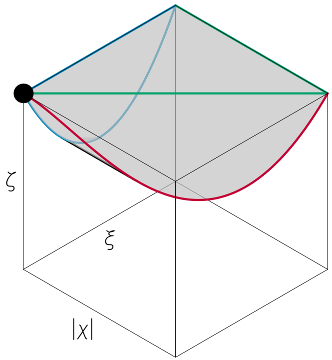

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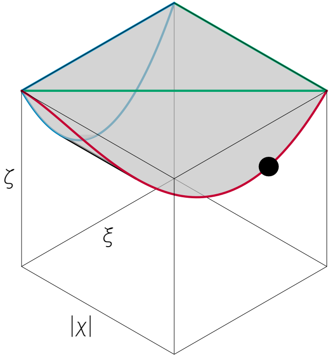

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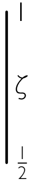

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16 Continuous Extrema with $\langle H \rangle = 0$

Potential with $\langle H \rangle = 0$ is

$$V_{\langle H \rangle = 0} = \frac{1}{2}\mu_\Delta^2 \delta^2 + \frac{1}{4}(\lambda_\Delta + \lambda'_\Delta \zeta)\delta^4,$$

whose extrema are

$$\delta^2 = -\frac{\mu_\Delta^2}{\lambda_\Delta + \zeta\lambda'_\Delta}, \qquad V_\Delta = \frac{1}{4}\frac{\mu_\Delta^4}{\lambda_\Delta + \zeta\lambda'_\Delta}$$

- In the minimum either $\zeta = \frac{1}{2}$ or $\zeta = 1$
- ζ depends on $|\det(\Delta)|^2$: $|\det(\Delta)|^2 = 0$ gives $\zeta = 1$
- Invariant under $\Delta \to U\Delta U^\dagger$: a circular flat direction and a physical Goldstone boson
- Neutral point on a charge-breaking circle

Absolute Stability

- Which vacua can be lower than ours?
- Can show analytically that $CB_{H\Delta}^{\chi=0}$ cannot
- Numerically, 'panic vacua' cannot

  Ferreira & Gonçalves, JHEP 02 (2020) 182 [1911.09746]
- In practice, only the $\zeta = 1$ continuous minimum with $\langle H \rangle = 0$ endangers absolute stability

Metastability

- Bubble nucleation rate

$$\Gamma \approx R_0^4 e^{-S_E},$$

- Vacuum is metastable if

$$\Gamma VT \approx 0.15 H_0^{-4} \Gamma < 1$$

where $H_0 = 1.44 \times 10^{-42}$ GeV

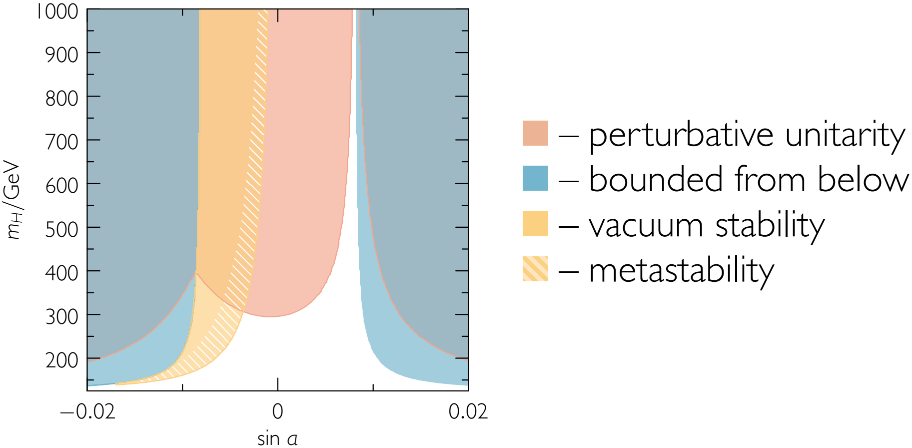
- $\zeta = 1$ continuous minimum with $\langle H \rangle = 0$: tunnelling into the neutral N_Δ gives the highest rate

Other Constraints

- ρ parameter gives $0 \leq v_\Delta \leq 2.58$ GeV at 3σ level
- Perturbative unitarity
- Potential bounded from below
- H^{++} decays give
  $m_{H^{++}} \geqslant 220$ GeV for $v_\Delta > 10^{-4}$ GeV and
  $m_{H^{++}} \geqslant 870$ GeV for $v_\Delta < 10^{-4}$ GeV
- Electroweak precision parameters give
  $|m_{H^+} - m_{H^0}| \approx |m_{H^{++}} - m_{H^+}| \leq 45.5$ GeV at 90% C.L.

Parameter Space

Degenerate $m_{H^{\pm\pm}} = m_{H^{\pm}} = m_{H^0} = m_{A^0}$
with $v_\Delta = 1$ GeV



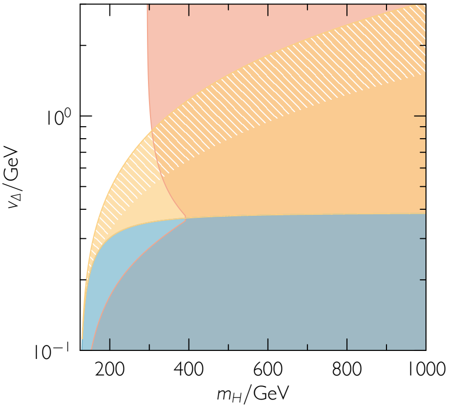
- Similar results for the non-degenerate case

Parameter Space

Degenerate

$m_{H^{\pm\pm}} = m_{H^{\pm}} = m_{H^0} = m_{A^0}$

with $a = -10^{-3}\pi$



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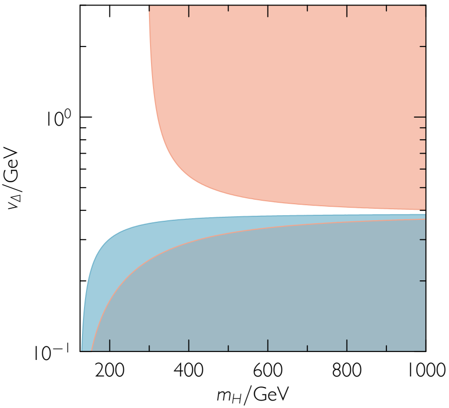


Conclusions

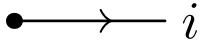
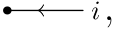
- Orbit space makes it easy to understand the minimum structure of the scalar potential
- Our minimum can be metastable (only the $\langle H \rangle = 0$ vacuum seems to endanger it)
- In progress: phase transitions and gravitational waves

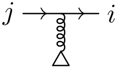
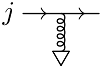
23 Type II Seesaw Orbit Space

Gauge invariants in the P-matrix:

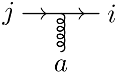
1. $H^\dagger H$
2. $\mathrm{tr}(\Delta^\dagger \Delta)$
3. $\mathrm{tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta)$
4. $H^\dagger \Delta \Delta^\dagger H$
5. $\frac{1}{2}[H^T \varepsilon \Delta^\dagger H + \text{h.c.}]$
6. $\frac{1}{2}(\mathrm{tr}\,\Delta^{\dagger 2}\, H^T \varepsilon \Delta H + \mathrm{tr}\,\Delta^2\, H^\dagger \Delta^\dagger \varepsilon^\dagger H^*)$
7. $H^\dagger \Delta^\dagger H H^\dagger \Delta H$

24 *P*-Matrix via Birdtracks

$H^i =$  $,\qquad H^\dagger_i =$  $,$

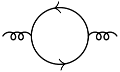
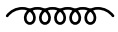
$\Delta^i_j = \Delta^a (T^a)^i_j =$  $,\quad (\Delta^\dagger)^i_j = \Delta^{*a}(T^a)^i_j =$  $,$

where

$$(T^a)^i_j =$$ 

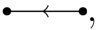
with $i, j, \ldots = 1, 2$ and $a, b, \ldots = 1, 2, 3$

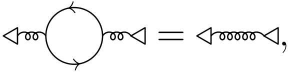
- With the normalisation

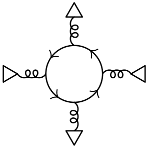
$$\operatorname{tr} T^a T^b = \delta_{ab} \quad \text{or}$$  $=$  $,$

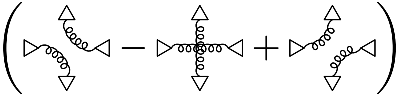
we have $T^a = \sigma_a / \sqrt{2}$

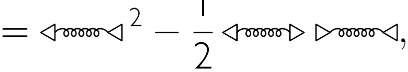
25 *P*-Matrix via Birdtracks

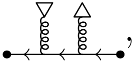
$H^\dagger H =$ 

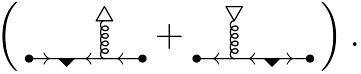
$\mathrm{tr}(\Delta^\dagger\Delta) =$ 

$\mathrm{tr}(\Delta^\dagger\Delta\Delta^\dagger\Delta) =$ 

$= \frac{1}{2}$ 



$H^\dagger\Delta\Delta^\dagger H =$ 

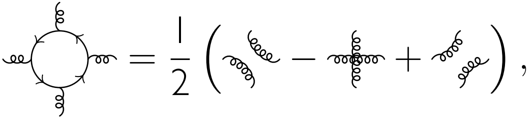
$\frac{1}{2}(H^T\varepsilon\Delta^\dagger H + \mathrm{h.c.}) = \frac{1}{2}$ 

26 *P*-Matrix via Birdtracks

Rules like

$$\operatorname{tr} T^a T^b T^c T^d = \frac{1}{2}(\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})$$

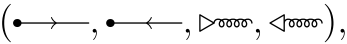
or



P-Matrix via Birdtracks

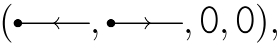
Using

$$\Phi = (H^i, H_j^\dagger, \Delta^a, \Delta^{*b}) =$$



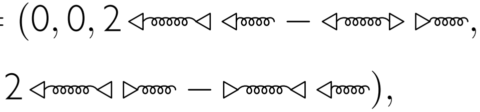
we calculate the derivatives by plucking off fields and using the Leibniz rule:

$$\frac{\partial p_1}{\partial \Phi} =$$



$$\frac{\partial p_2}{\partial \Phi} =$$

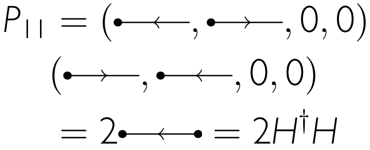
$$\frac{\partial p_3}{\partial \Phi} =$$



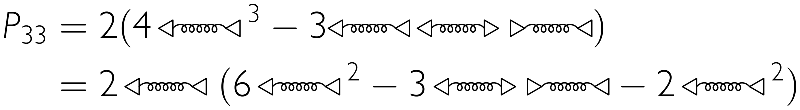
$$\ldots$$

P-Matrix via Birdtracks

We then calculate the *P*-matrix elements by joining the free lines in derivatives, for example



or



$$= 2p_2(6p_3 - 2p_2^2)$$

29 Perturbative Unitarity

$$
\begin{aligned}
&|\lambda_H| < 2\pi, \quad |\lambda_\Delta| < 4\pi, \quad |\lambda_\Delta + \lambda'_\Delta| < 2\pi,\\
&|2\lambda_\Delta + \lambda'_\Delta| < 4\pi, \quad |2\lambda_\Delta - \lambda'_\Delta| < 8\pi, \quad |\lambda_{H\Delta}| < 8\pi,\\
&|\lambda_{H\Delta} + \lambda'_{H\Delta}| < 8\pi, \quad |2\lambda_{H\Delta} \pm \lambda'_{H\Delta}| < 16\pi, \quad |2\lambda_{H\Delta} + 3\lambda'_{H\Delta}| < 16\pi,\\
&|\lambda_H + \lambda_\Delta + \lambda'_\Delta \pm \sqrt{(\lambda_H - \lambda_\Delta - 2\lambda'_\Delta)^2 + \lambda'^2_{H\Delta}}| < 8\pi,\\
&|6\lambda_H + 8\lambda_\Delta + 6\lambda'_\Delta\\
&\pm\sqrt{4(3\lambda_H - 4\lambda_\Delta - 3\lambda'_\Delta)^2 + 6(2\lambda_{H\Delta} + \lambda'_{H\Delta})^2}| < 16\pi
\end{aligned}
$$

Arhrib et al., Phys. Rev. D 84 (2011) 095005 [1105.1925]

30 Bounded-from-Below Conditions

$$
\begin{aligned}
\lambda_H > 0, \quad \lambda_\Delta + \lambda'_\Delta > 0, \quad \lambda_\Delta + \frac{1}{2}\lambda'_\Delta &> 0, \\
\lambda_{H\Delta} + 2\sqrt{\lambda_H(\lambda_\Delta + \lambda'_\Delta)} &> 0, \\
\lambda_{H\Delta} + \lambda'_{H\Delta} + 2\sqrt{\lambda_H(\lambda_\Delta + \lambda'_\Delta)} &> 0 \\
|\lambda'_{H\Delta}|\sqrt{\lambda_\Delta + \lambda'_\Delta} \geq 2\sqrt{\lambda_H}\lambda'_\Delta& \\
\vee\ 2\lambda_{H\Delta} + \lambda'_{H\Delta} + \sqrt{(8\lambda_H\lambda'_\Delta - \lambda'^2_{H\Delta})\left(2\frac{\lambda_\Delta}{\lambda'_\Delta} + 1\right)} &> 0.
\end{aligned}
$$

Bonilla, Fonseca & Valle, Phys. Rev. D 92 (2015) 075028 [1508.02323].