Primordial black hole formation from inflation

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DSU 2024, Corfú

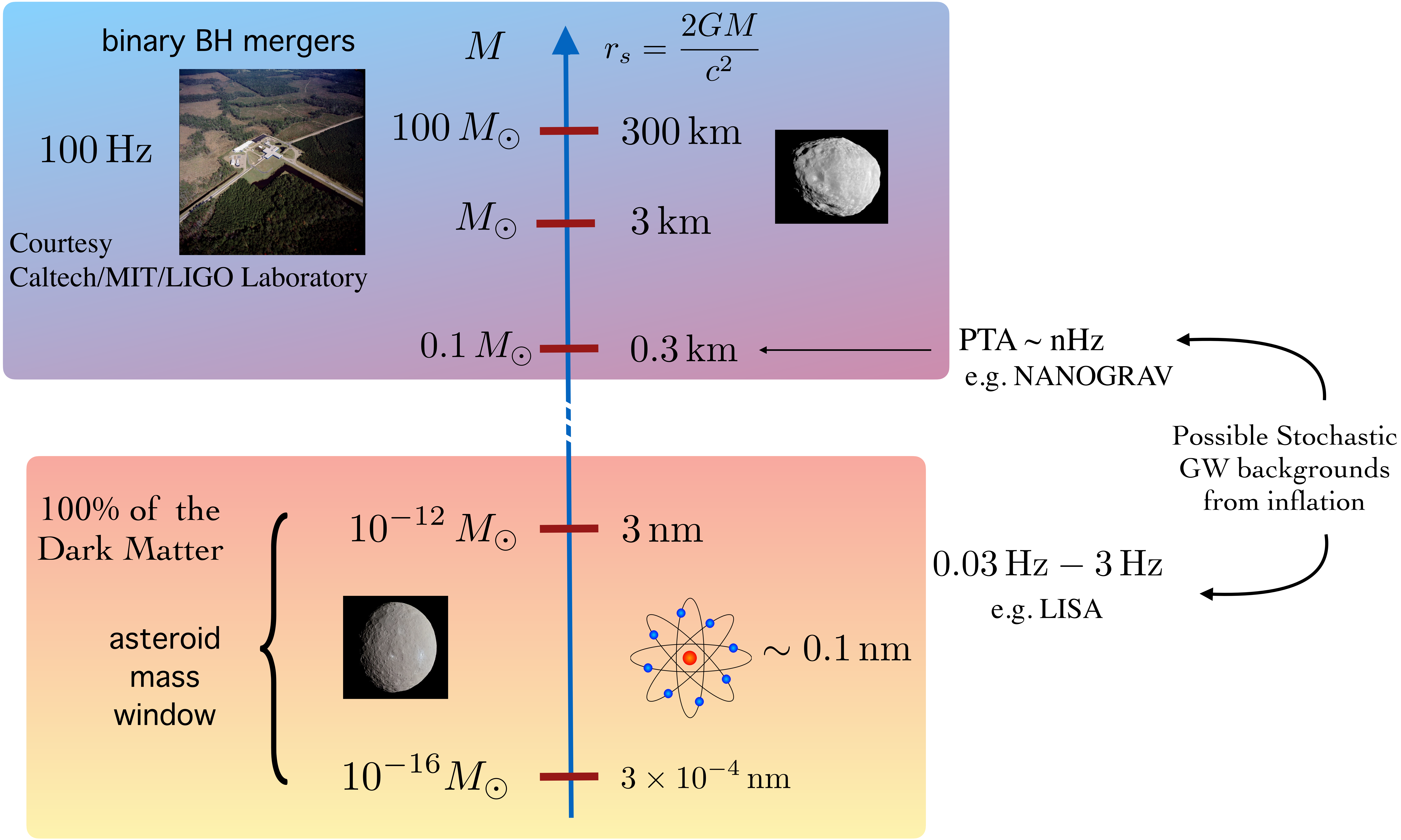
Primordial black holes

Zeldovich and Novikov 1967, Hawking1971, Hawking & Carr 1974, etc…

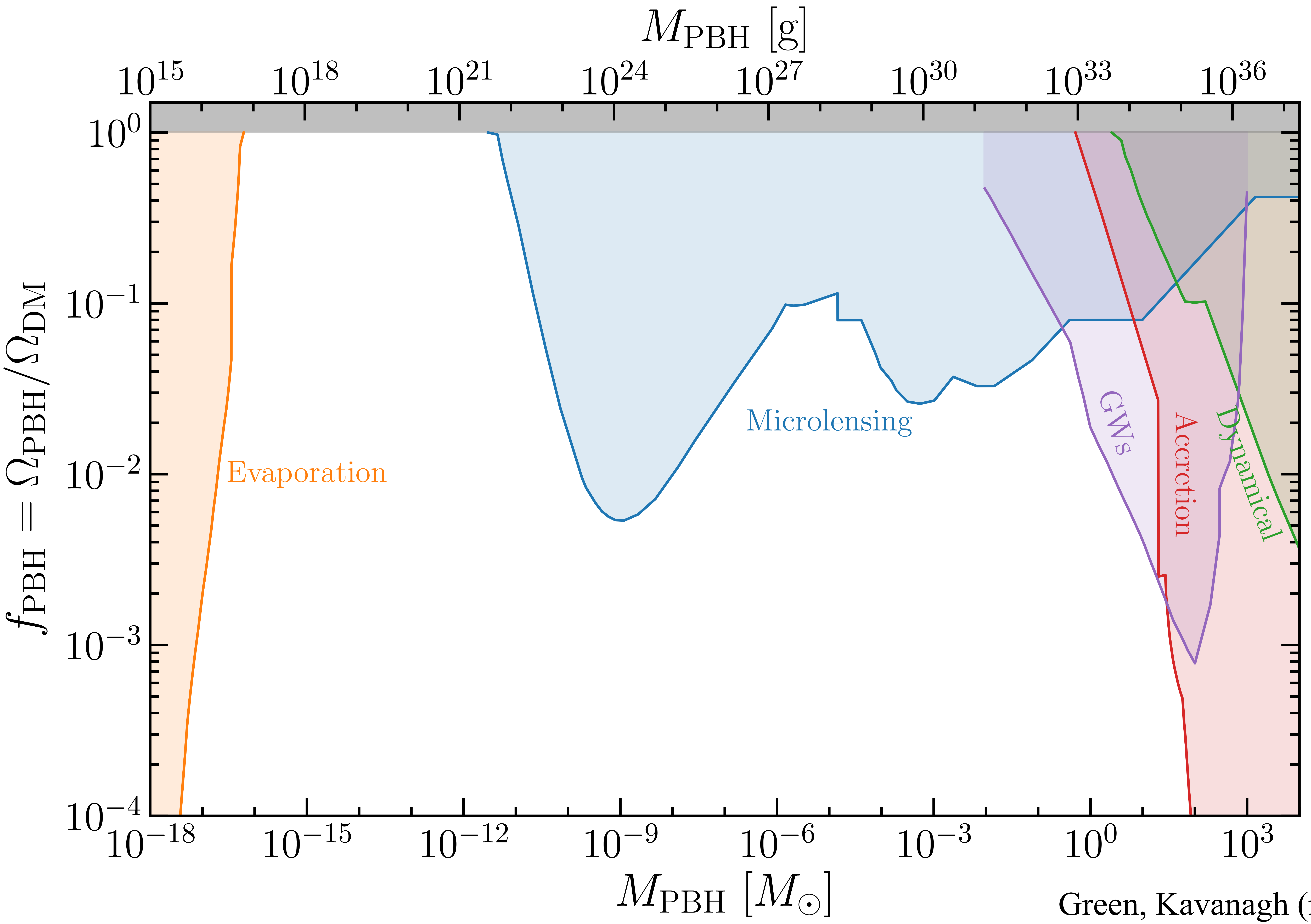
- **Dark matter candidate**
- Reheat the universe after inflation
- Catalysers for particle dark matter production
- Baryogenesis
- Seeds of supermassive black holes
- Possible BH merger outliers
- PTA indications of a stochastic GW background
- Window into the early universe

Outline

- Bounds on the PBH abundance
- Basic formation mechanism
- Most studied single-field inflation scenario
- Breakdown of perturbation theory?
- Other inflationary possibilities
- (The problem of the abundance)



binary BH mergers
100 Hz
Courtesy
Caltech/MIT/LIGO Laboratory
M
$r_s = \frac{2GM}{c^2}$
$100\,M_\odot$
$300\,\mathrm{km}$
$M_\odot$
$3\,\mathrm{km}$
$0.1\,M_\odot$
$0.3\,\mathrm{km}$
PTA ~ nHz
e.g. NANOGRAV
Possible Stochastic
GW backgrounds
from inflation
100% of the
Dark Matter
asteroid
mass
window
$10^{-12}\,M_\odot$
$3\,\mathrm{nm}$
$0.03\,\mathrm{Hz} - 3\,\mathrm{Hz}$
e.g. LISA
$\sim 0.1\,\mathrm{nm}$
$10^{-16}\,M_\odot$
$3 \times 10^{-4}\,\mathrm{nm}$

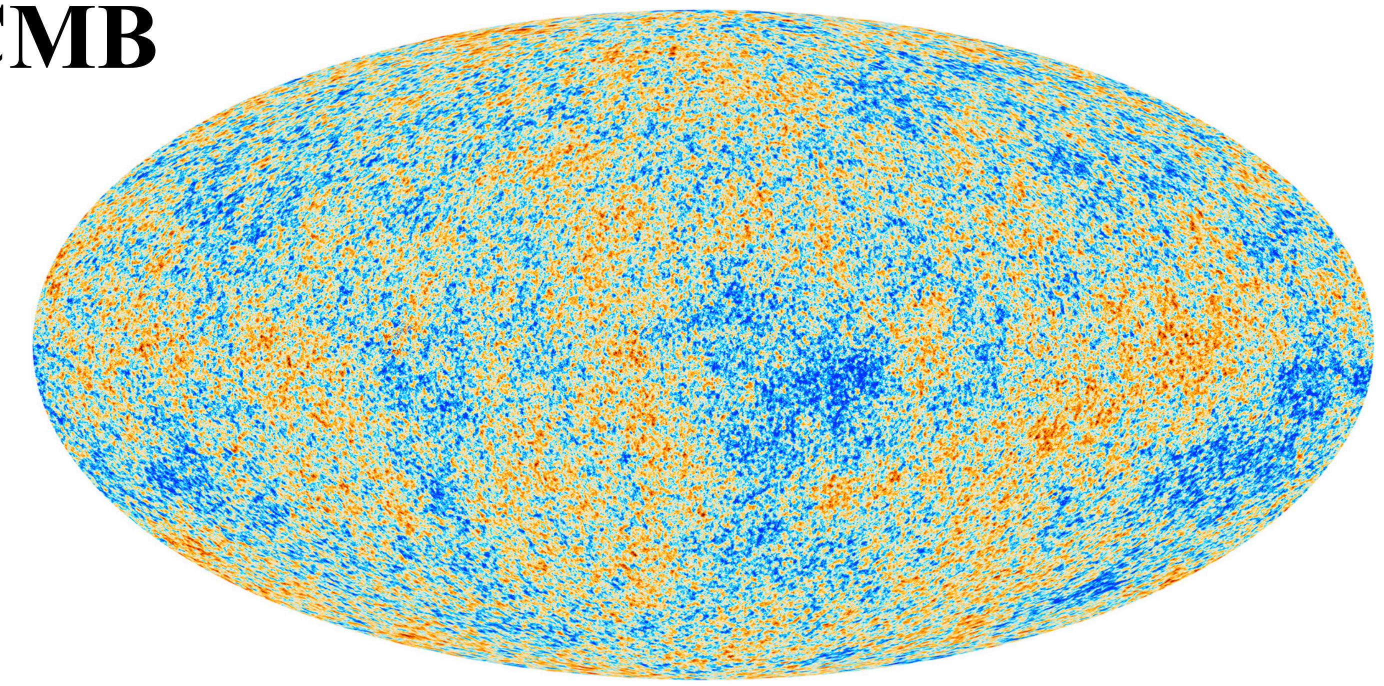


Green, Kavanagh (review). 2020

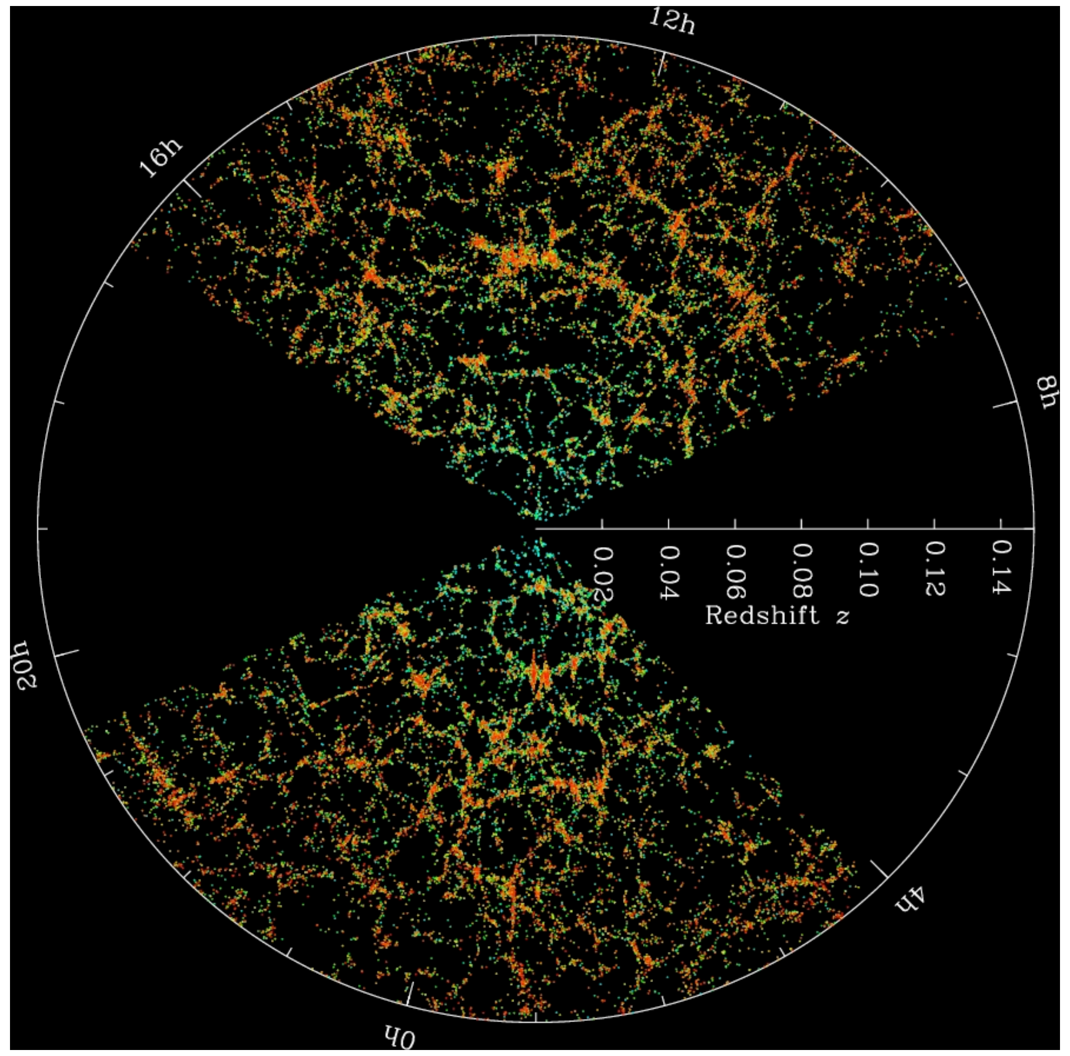
PBH dark matter (from inflation)

Credit: ESA/Planck

CMB

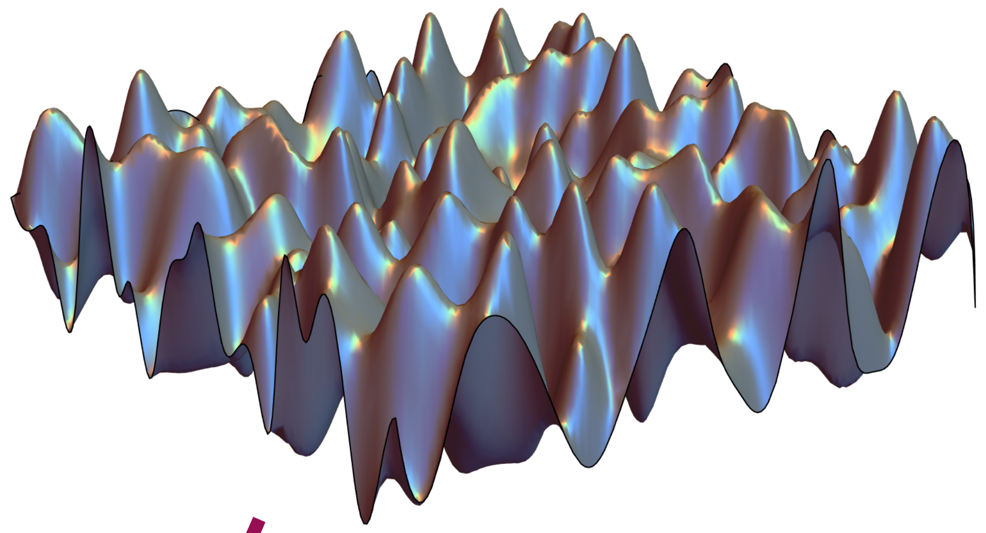


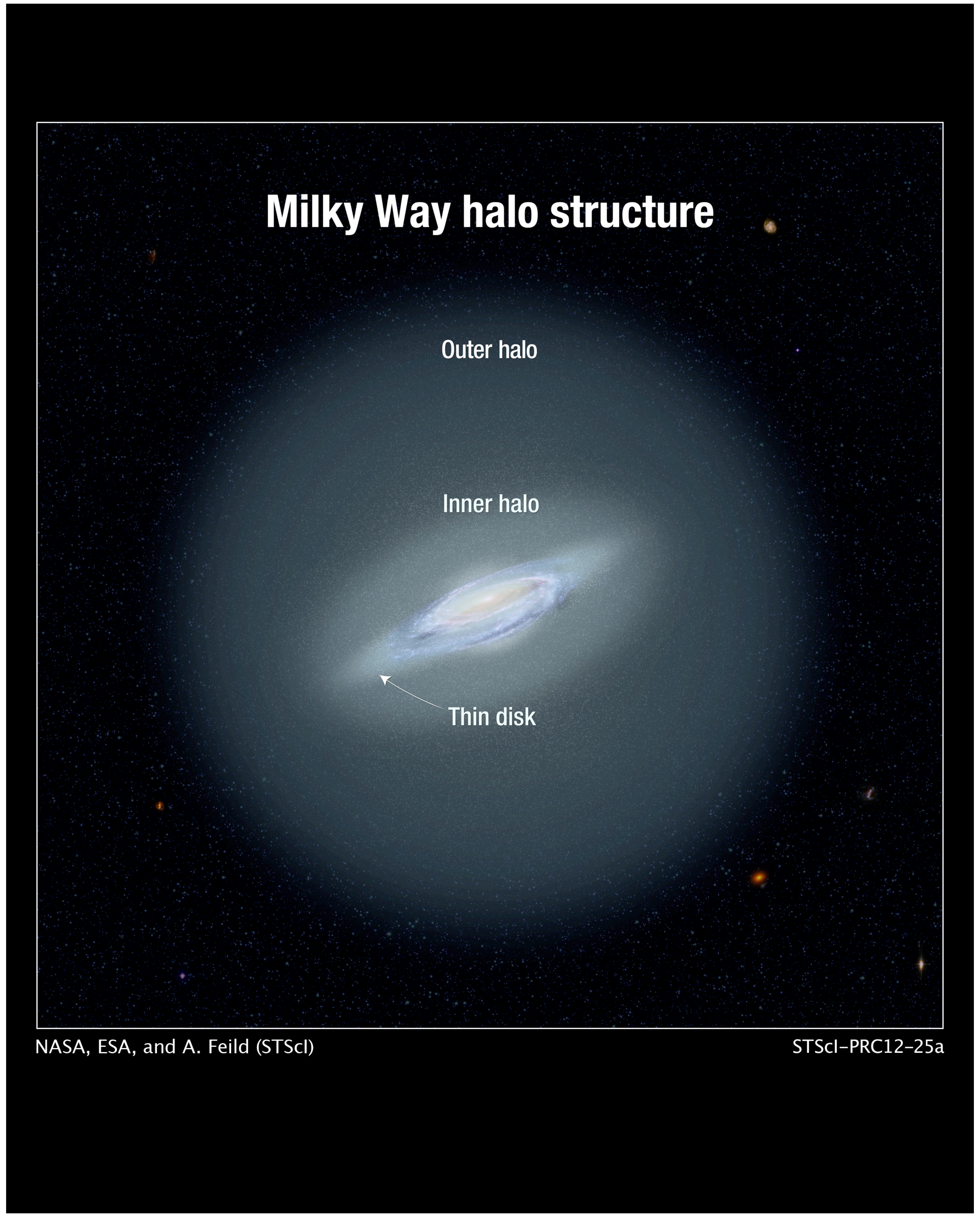
LSS

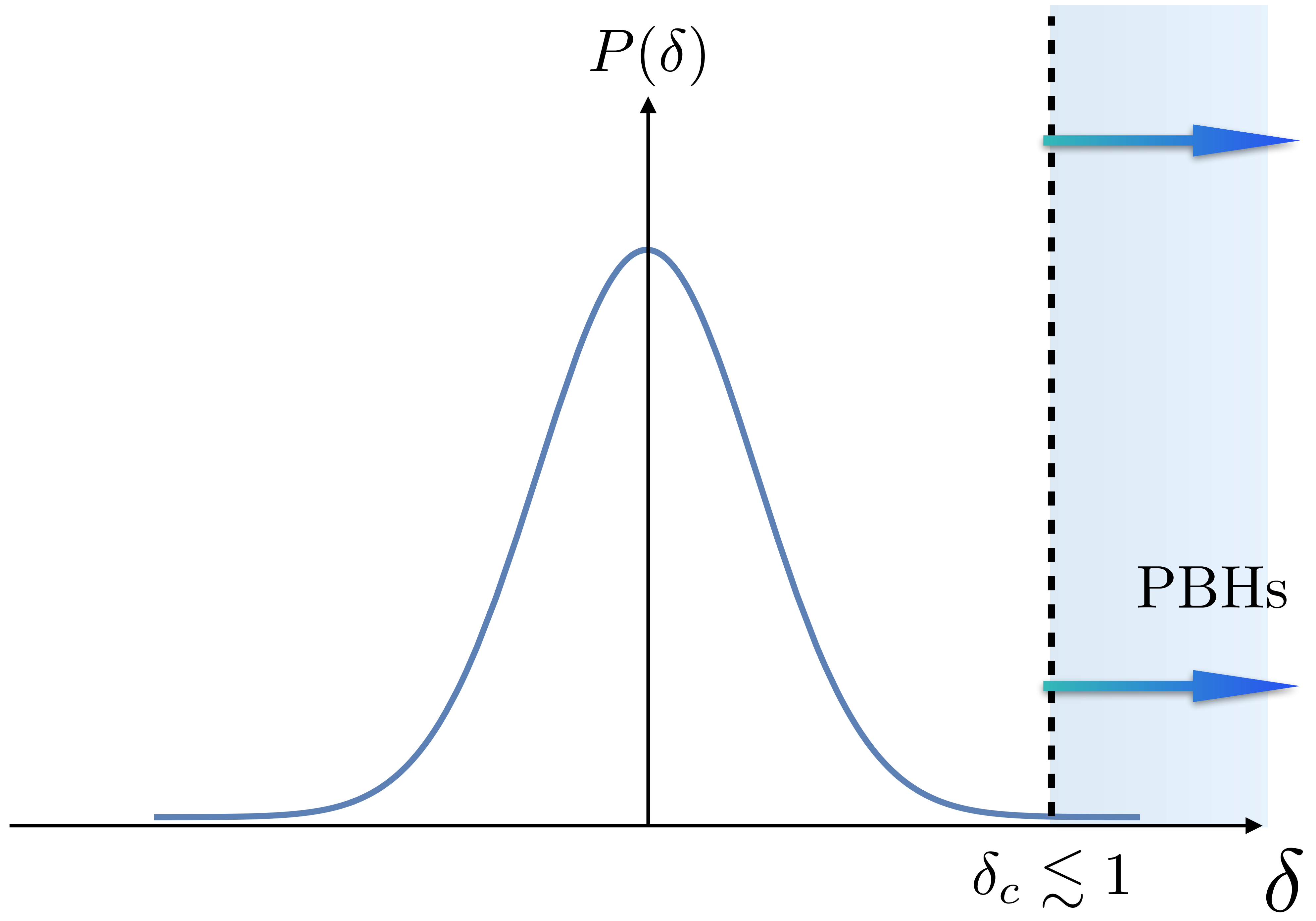


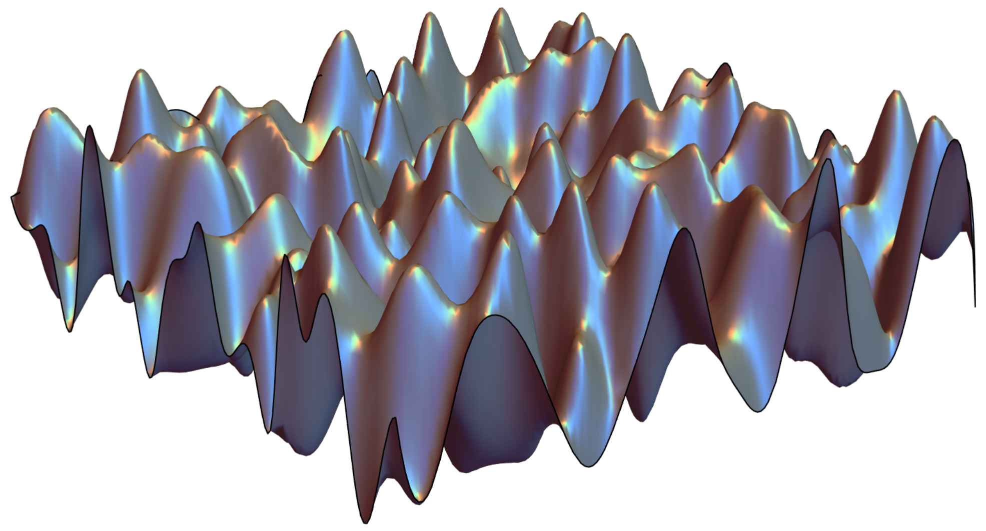
Credit: M Blanton and SDSS

Quantum fluctuations of space-time









Mass:

$$M \sim 10^{-14} \left(\frac{10^{13}\,\mathrm{Mpc}^{-1}}{k} \right)^2 M_\odot$$

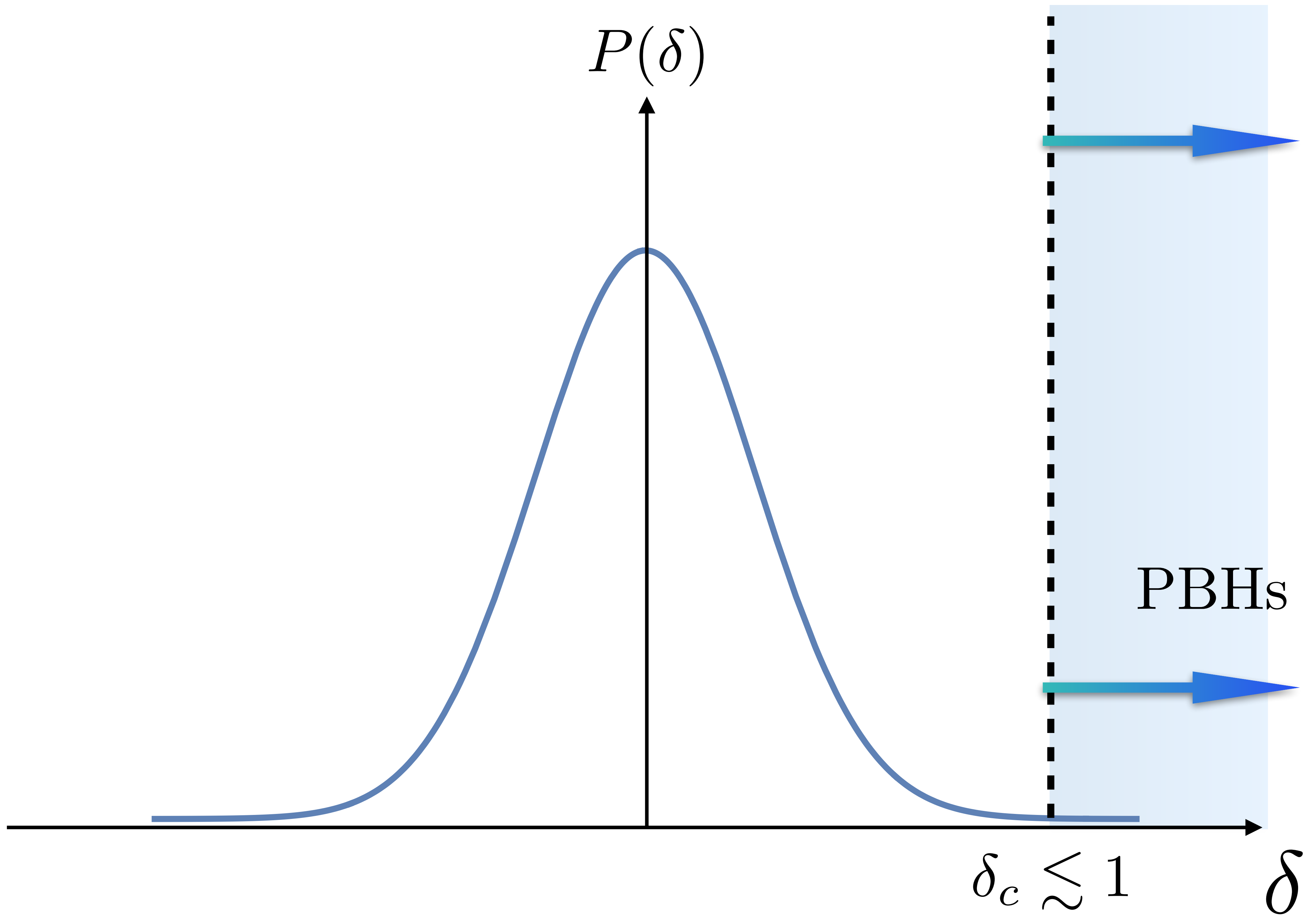
$$N_e \simeq 18 - \frac{1}{2} \log \frac{M}{M_\odot}$$

(for PBHs formed during radiation domination)

Abundance (naive Gaussian estimate):

$$f_{\mathrm{PBH}} = \frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{DM}}} \propto \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma^2} \right) d\delta$$

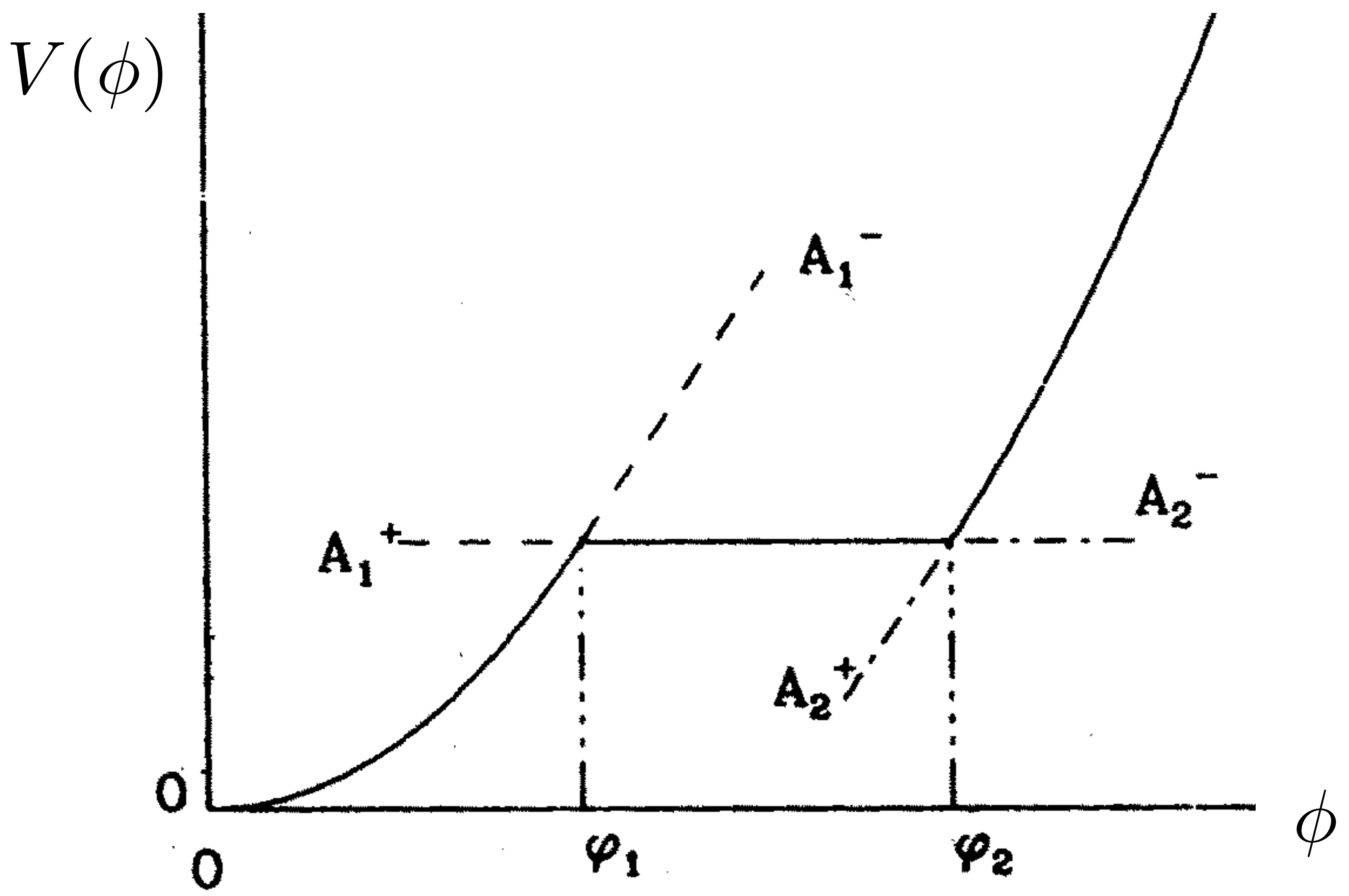
$$\sigma \sim \mathcal{P}_{\mathcal{R}} \sim 10^{-2} \quad \Longrightarrow \quad \frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{DM}}} \sim 1$$



$$\mathcal{P}_{\mathcal{R}} \sim \left(\frac{H}{m_P}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2 \sim \frac{1}{m_P^2}\left(\frac{V}{V'}\right)^2 \frac{V}{m_P^4}$$

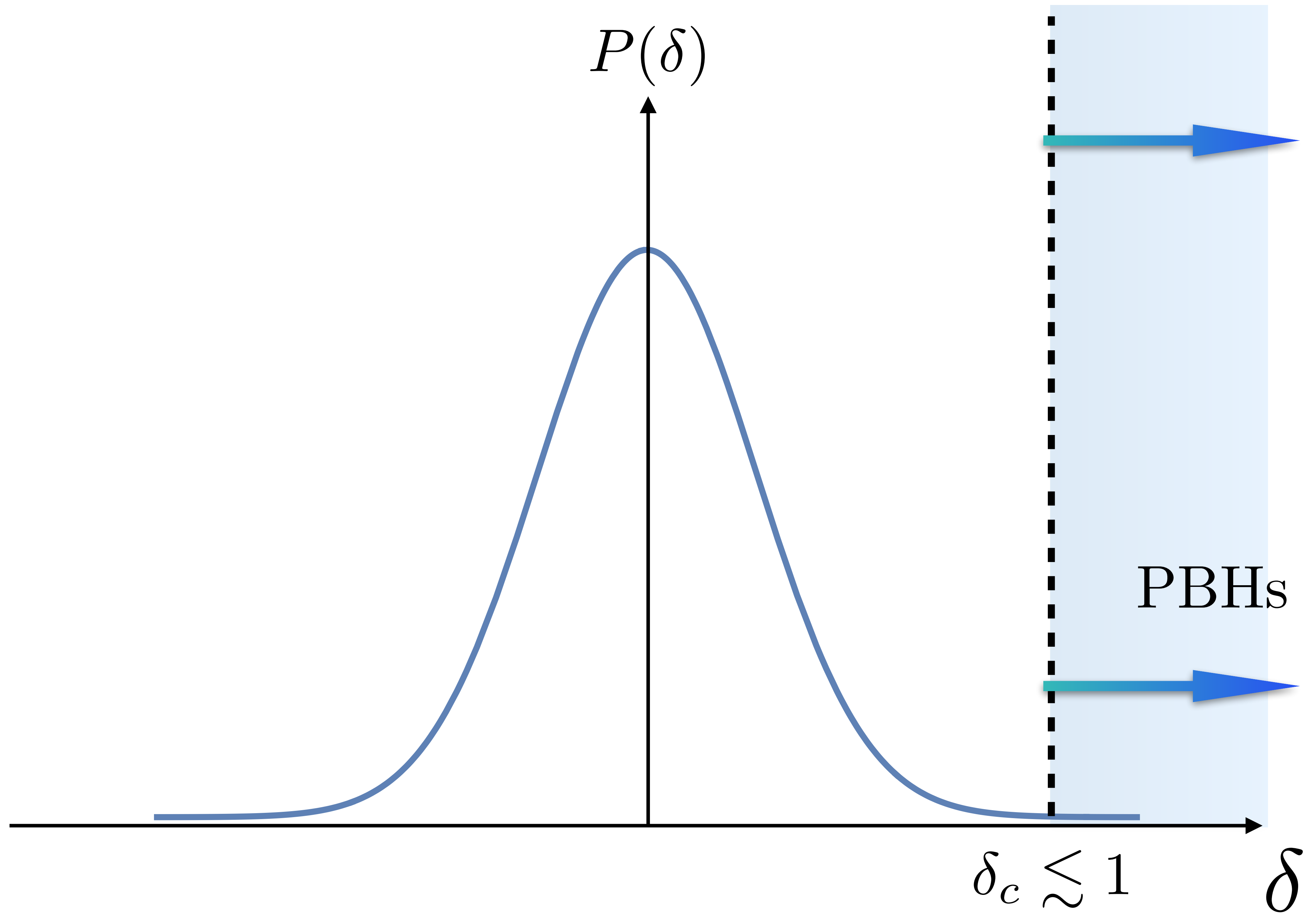
Inflation and primordial black holes as dark matter

Ivanov, Naselsky, Novikov. 1994



$$V(\phi) = \begin{cases} V_0 + A_+(\phi - \phi_0) & \text{for } \phi > \phi_0 \\ V_0 + A_-(\phi - \phi_0) & \text{for } \phi < \phi_0 \end{cases}$$

Starobinsky 1994



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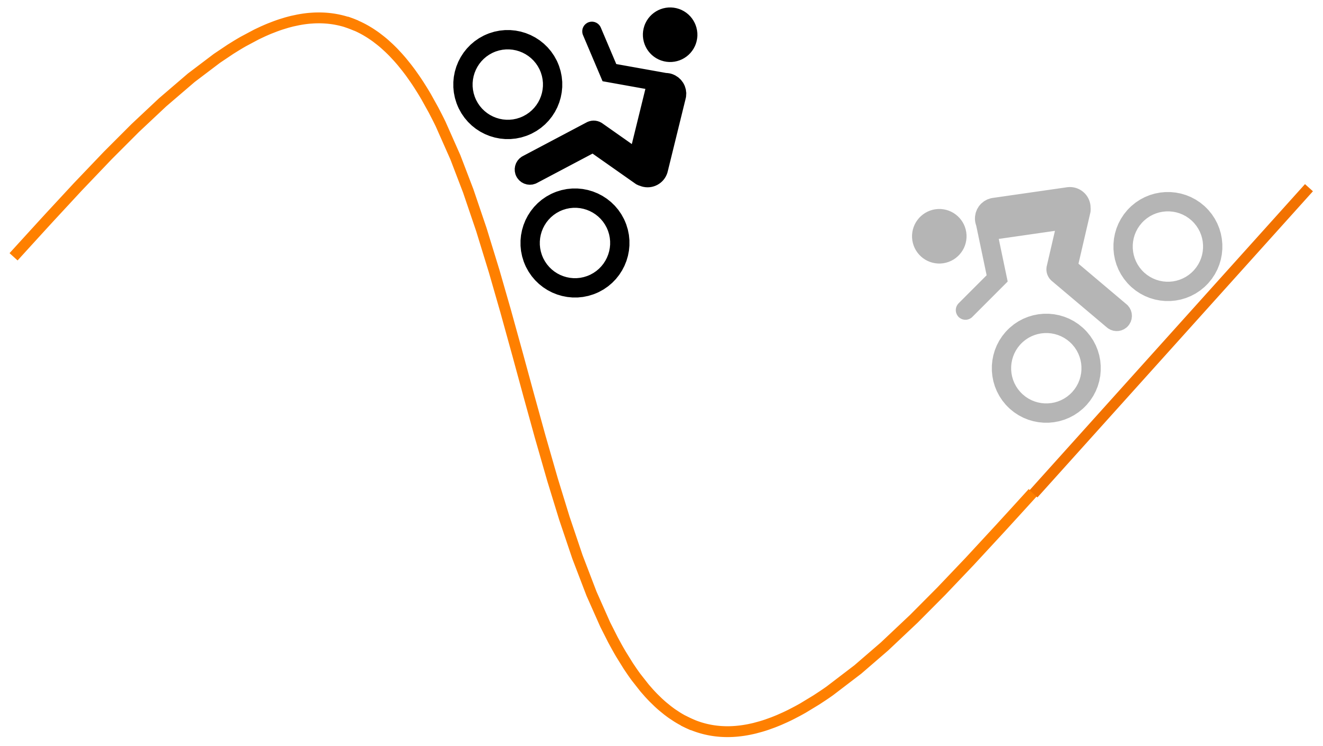
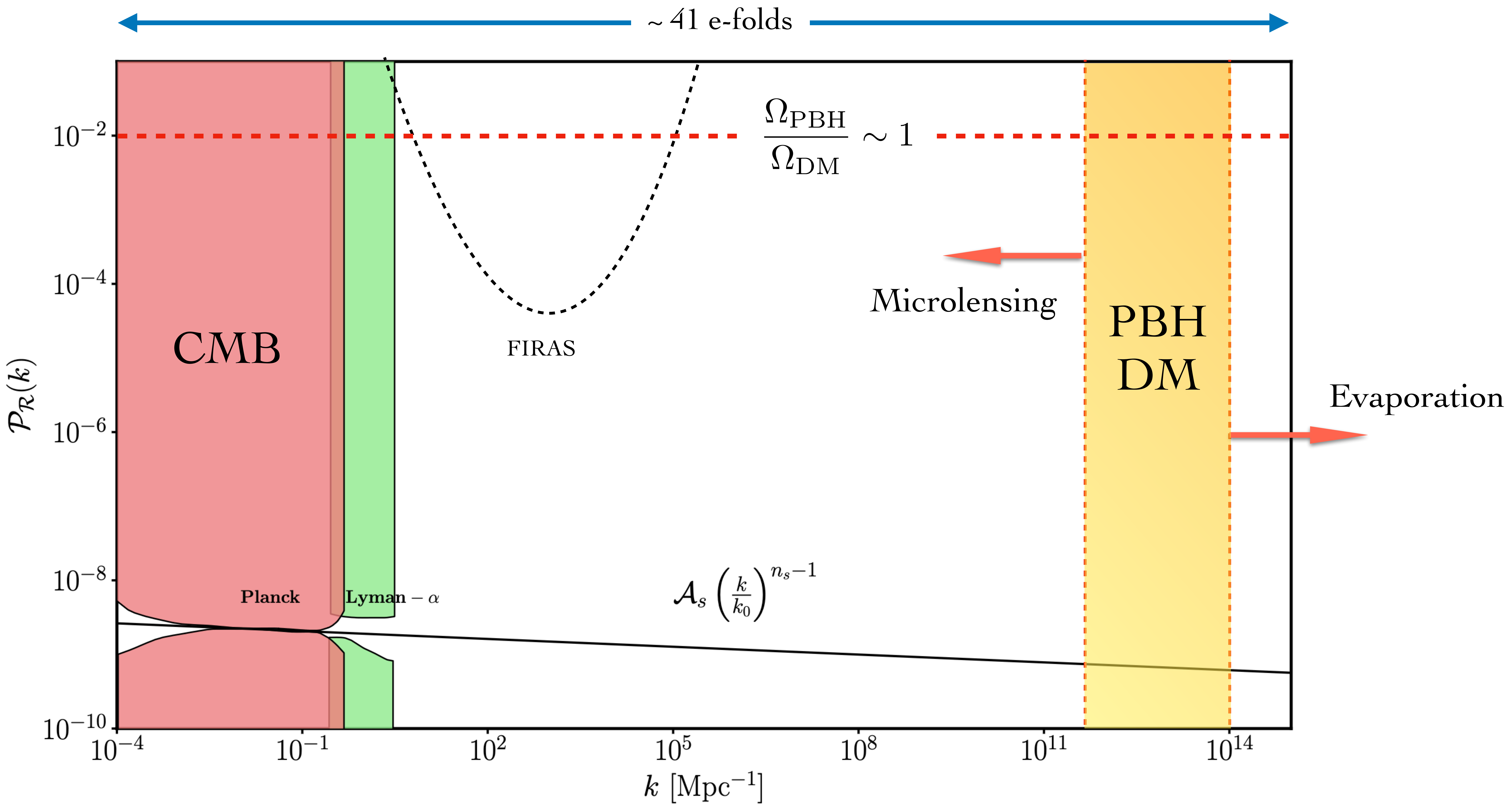
Ultra Slow-Roll (USR)

$$\ddot{\phi} + 3H\dot{\phi} \simeq 0$$

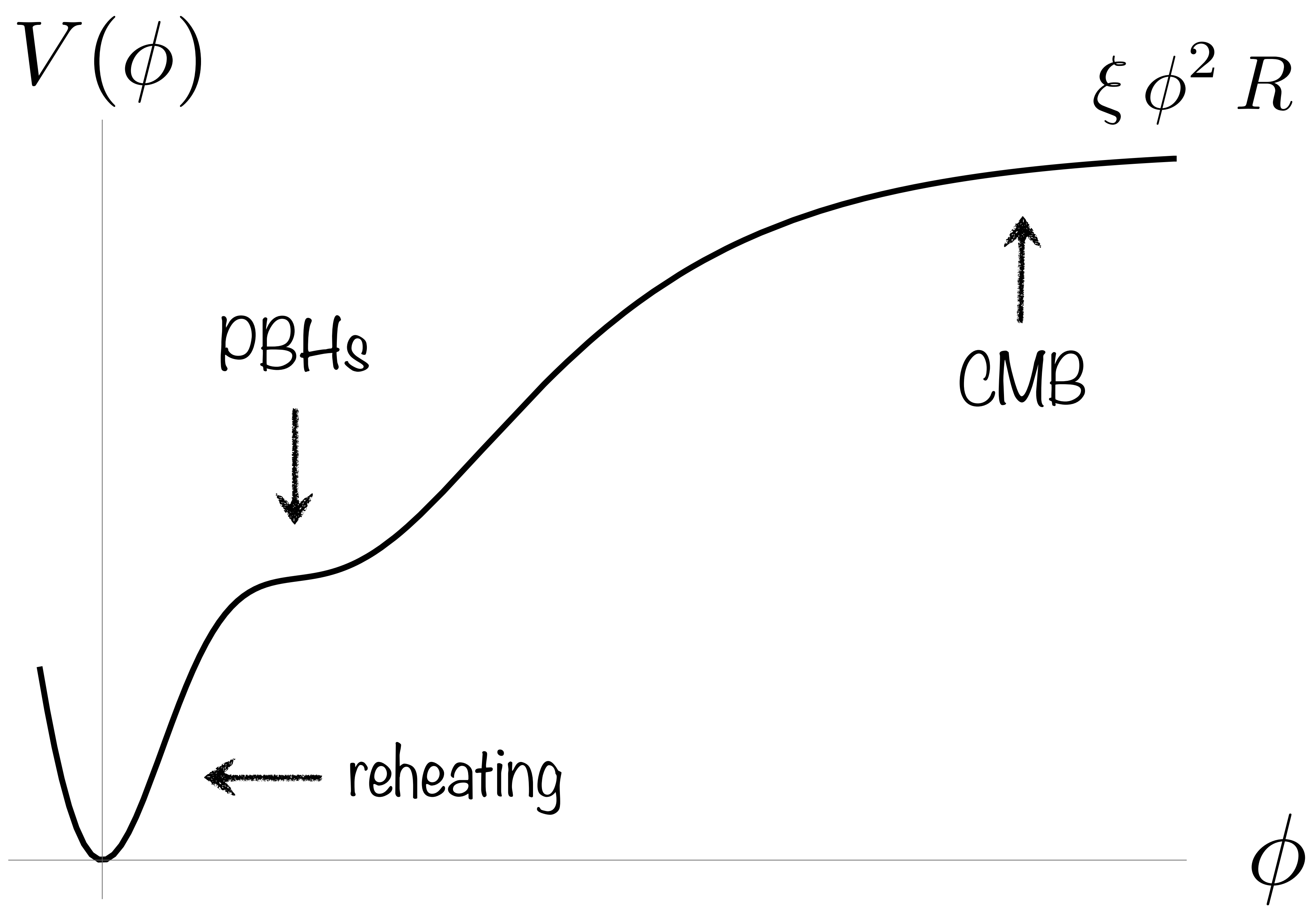


$$\eta \sim \frac{\ddot{\phi}}{\dot{\phi}\, H} \sim -3$$



Franco-Abellán, EUCAPT symposium 2023 (modified)

+ enough inflation & successful reheating



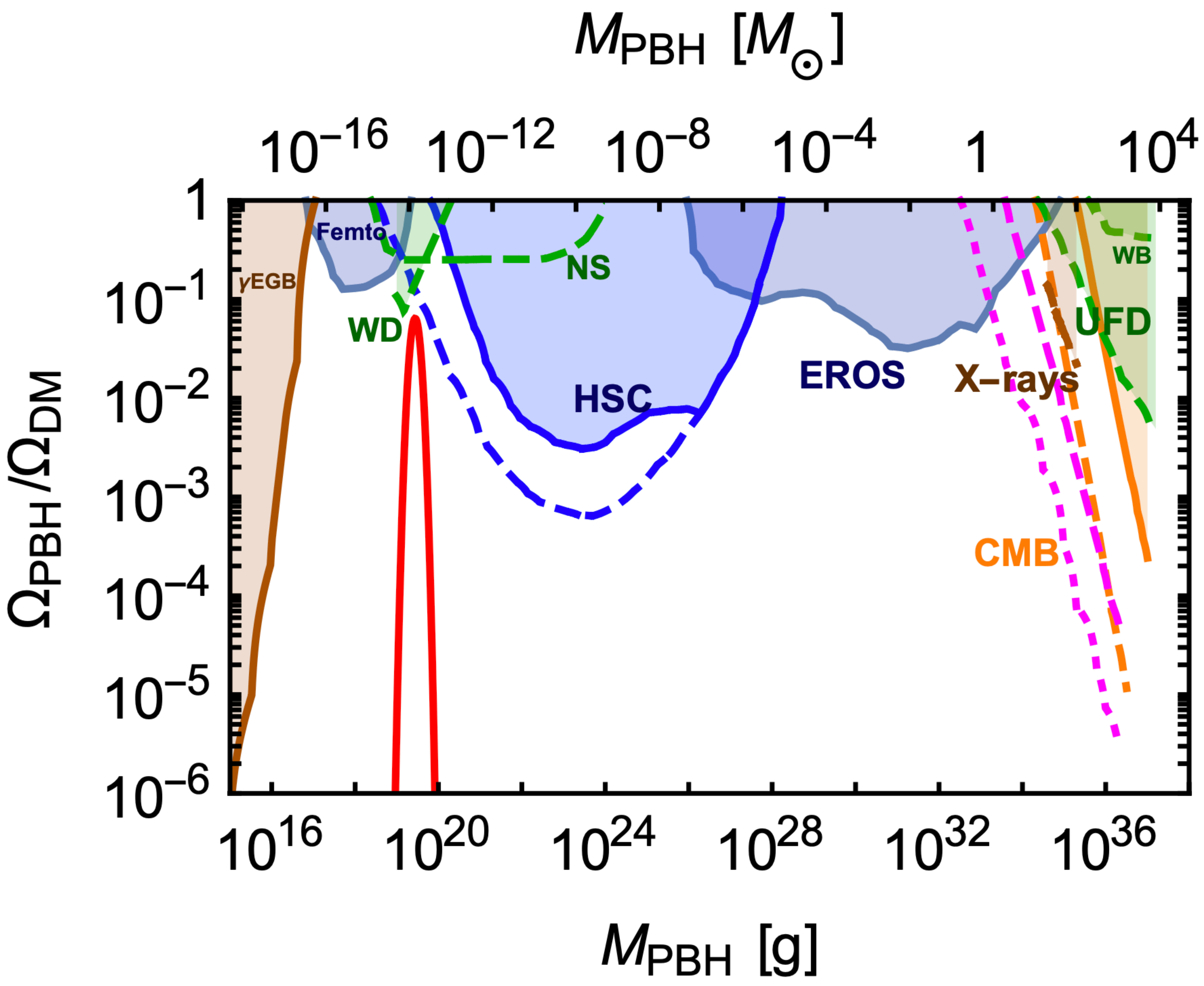
$$V = \lambda(\phi)\,\phi^4$$

$$\lambda(\phi) = \lambda(\phi_0) + \frac{1}{2}\beta_\lambda(\phi_0)\log\frac{\phi^2}{\phi_0^2} + \frac{1}{8}\beta'_\lambda(\phi_0)\left(\log\frac{\phi^2}{\phi_0^2}\right)^2 + \cdots$$

GB, Taoso 2017

- Enough inflation
- Agreement with the CMB (n_s within 3σ)
- $\frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}} \sim 1$ (Gaussian, RD)

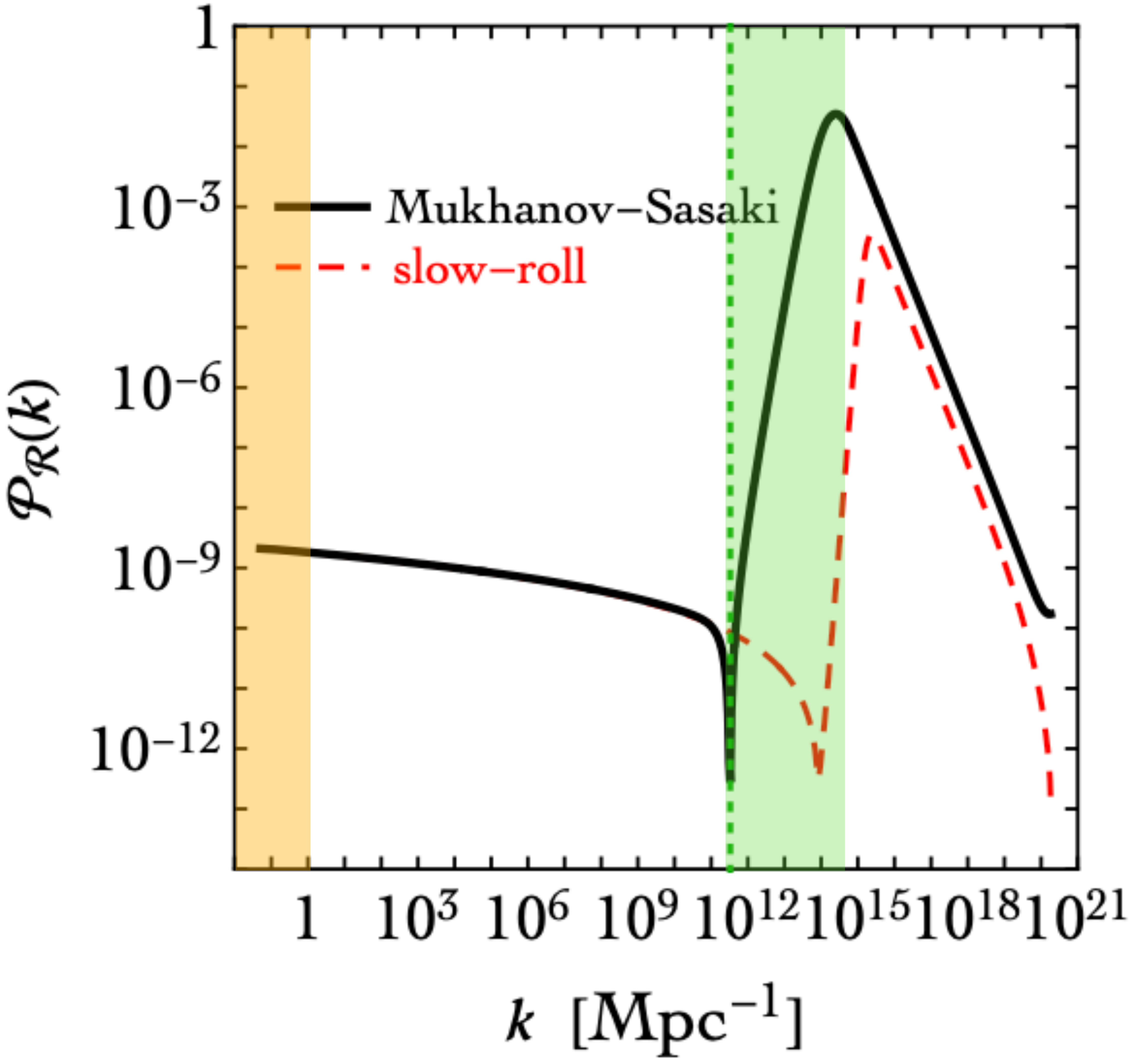
$$10^{-16} M_\odot \longleftrightarrow 10^{-12} M_\odot$$



GB, Taoso 2017

$$V = \sum_{n=2}^{4+\epsilon\,\mathcal{O}(5)} a_n \phi^n \qquad (+\,\xi\,\phi^2\,R)$$

GB, Taoso 2017
GB, Rey, Taoso, Urbano 2020



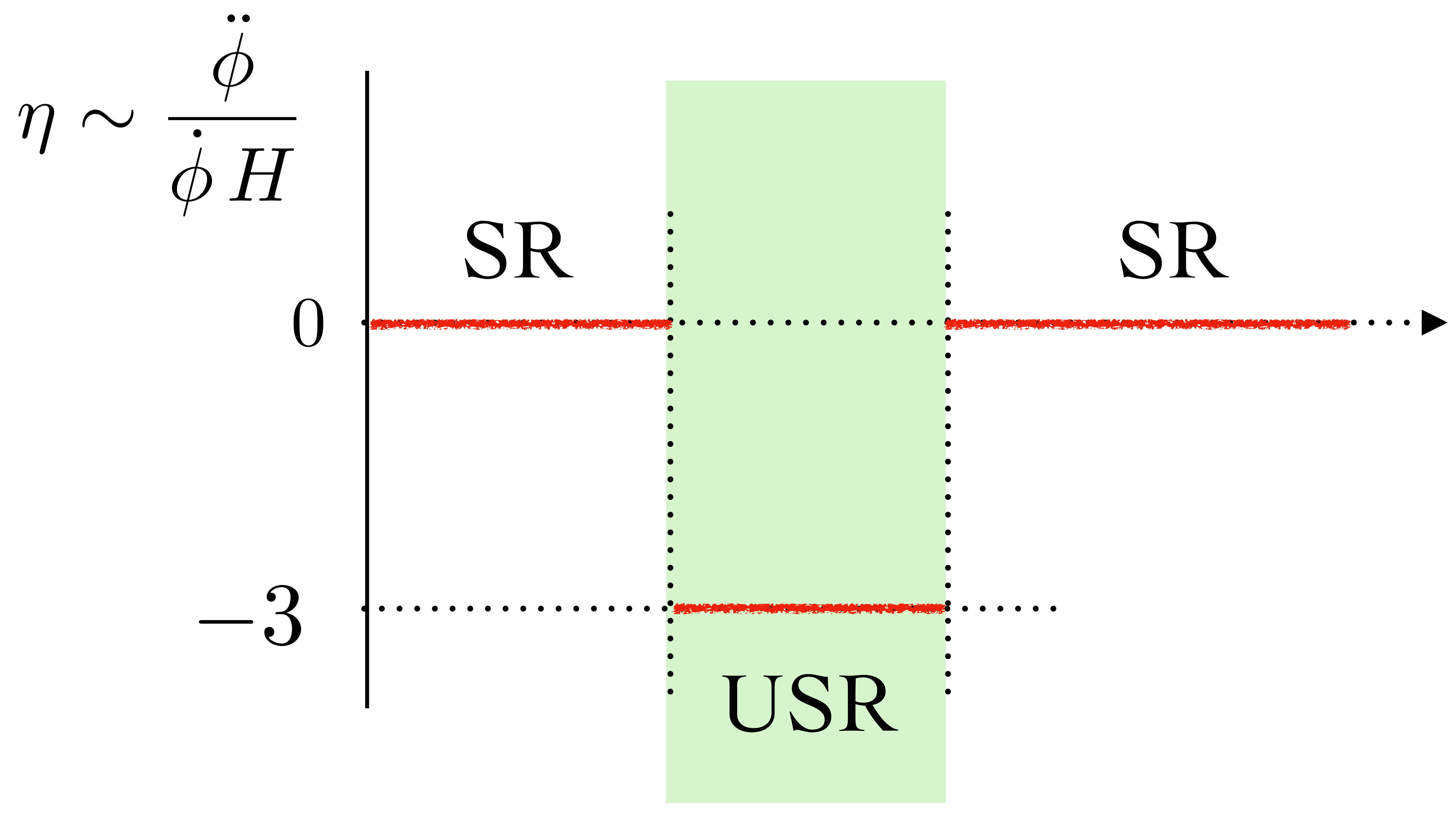
Breakdown of perturbation theory in USR inflation?

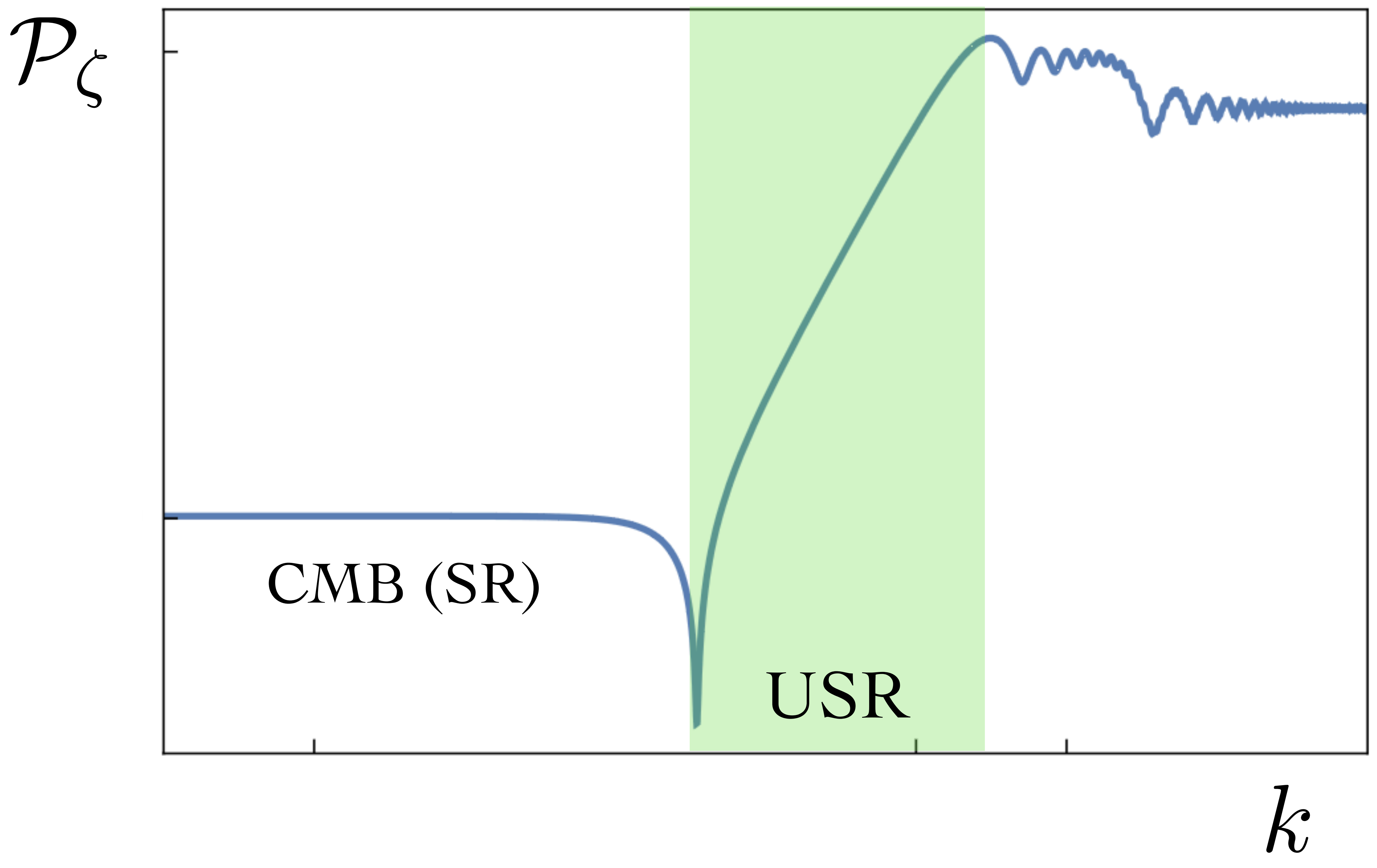
Claim: *a large enough tree-level primordial spectrum for PBH DM implies perturbation theory breaks at CMB scales.*

Kristiano and Yokoyama, 2022 & 2023

Toy model: SR —> USR —> SR

(with sharp transitions)





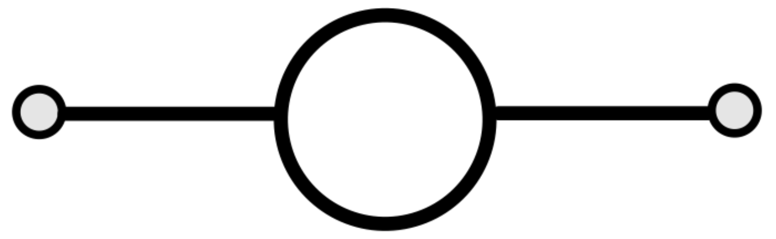
Breakdown of perturbation theory in USR inflation?

Claim: *a large enough tree-level primordial spectrum for PBH DM implies perturbation theory breaks at CMB scales*

Kristiano and Yokoyama, 2022 & 2023

Method: Primordial spectrum at one-loop with the in-in formalism. SR approximation.

A single cubic interaction: $$H_{\rm int} = -\frac{M_P^2}{2}\int d^3x\,\epsilon\,\eta'\,a^2\,\zeta'\,\zeta^2$$



UV divergence: Cut-off given by location of spectral peak

$$\mathcal{P}_\zeta \ll \frac{1}{(\Delta\eta)^2} \simeq 0.03$$ (for perturbation theory to hold)

Breakdown of perturbation theory in USR inflation?

It has been argued that the one-loop power spectrum at large scales is small (even zero)

Riotto 2023. Firouzjahi and Riotto 2023. Iacconi, Mulryne and Seery 2023. Inomata 2024.

E.g. Tada, Terada, Tokuda 2023:

"We find that Maldacena's consistency relation is satisfied and guarantees the cancellation of contributions from the short-scale modes."

E.g. Franciolini, Iovino, Taoso, Urbano 2023

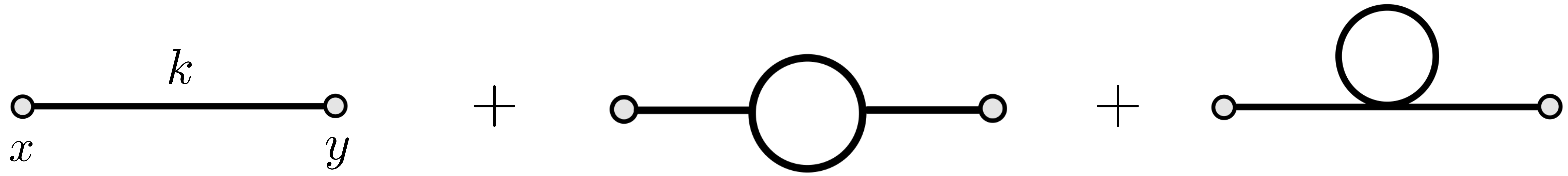
"Loop corrections of short modes on the power spectrum of long modes are not large enough to violate perturbativity, but remain appreciable."

$\delta\phi$ – gauge at lowest order in ϵ

$$S = \int \mathrm{d}\tau \, \mathrm{d}^3\mathbf{x} \left[\frac{a^2}{2} \left((\partial_\tau \delta\phi)^2 - (\partial_i \delta\phi)^2 \right) - a^4 \sum_{n\geq 2} \frac{V_n \, \delta\phi^{\,n}}{n!} \right]$$

$$a^2 V_2 = -(aH)^2(\nu^2 - 9/4)\,, \quad a^2 V_3 = -\frac{aH(\nu^2)'}{\sqrt{2\epsilon} M_P}\,, \quad a^2 V_4 = -\frac{1}{2\epsilon M_P^2}\left((\nu^2)'' - aH(\nu^2)' \left(1 + \frac{\eta}{2} \right) \right)$$

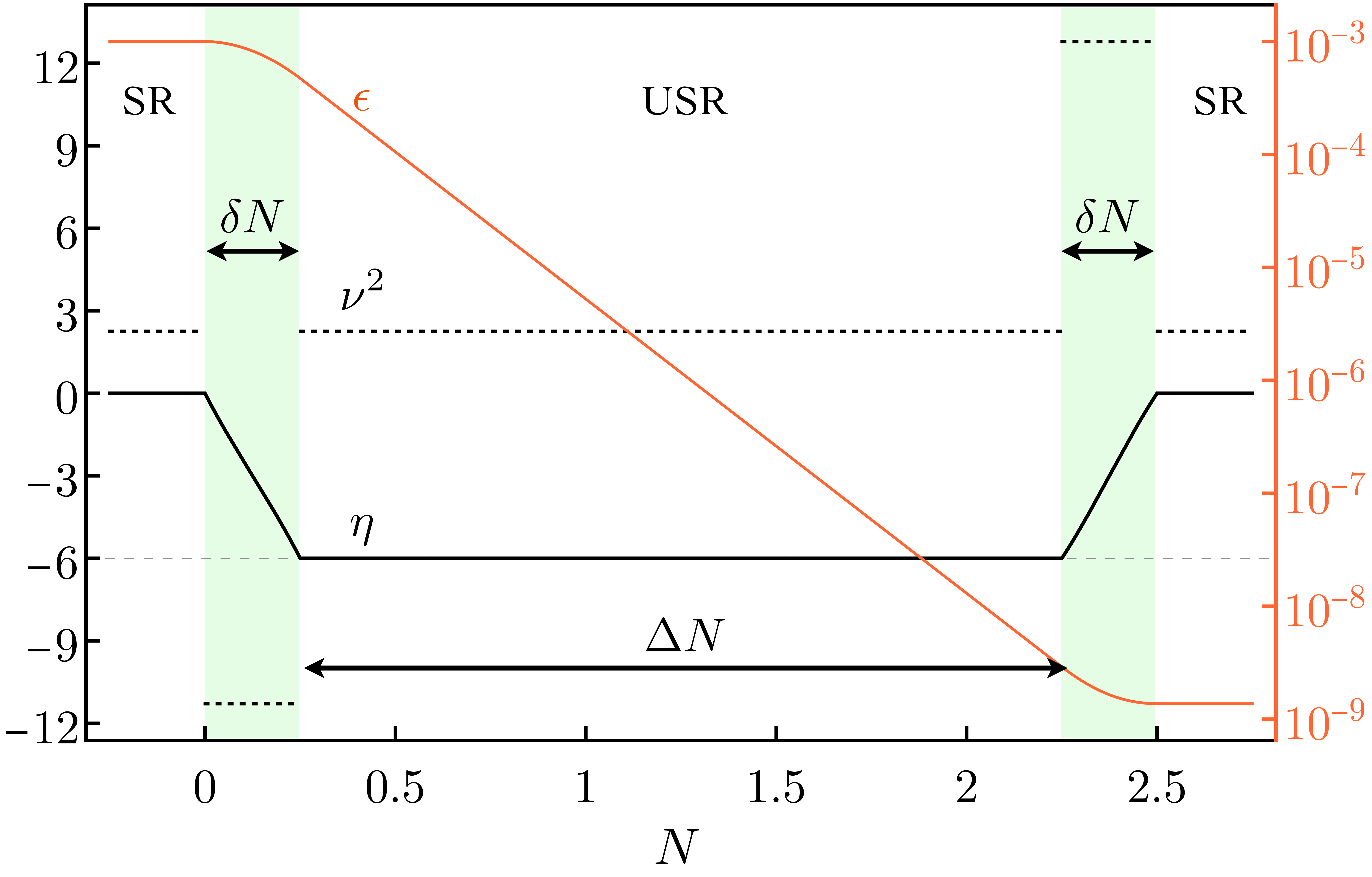
$$\nu^2 = \frac{9}{4} + \frac{1}{2}\left(3\eta + \frac{\eta^2}{2} + \frac{\eta'}{aH} \right)$$



$$\mathcal{P}_\zeta = \frac{1}{2\epsilon \, M_P^2} \mathcal{P}_{\delta\phi}$$

GB, Gambín Egea, 2024

Two-parameter model. $\nu^2 = \frac{9}{4} + \frac{1}{2}\left(3\eta + \frac{\eta^2}{2} + \frac{\eta'}{aH}\right)$ is piece-wise constant.



$$V_{3,4} \sim \Delta\nu^2 \delta(N - N_*)$$

$$|\nu^2| \to \frac{3}{\delta N} \quad \text{when} \quad \delta N \to 0$$

GB, Gambín Egea, 2024

$$S = \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x}\, M_P^2 a^2 \epsilon \left((\zeta')^2 - (\partial\zeta)^2 + \frac{\eta'}{2}\zeta'\zeta^2 \right)$$

Interaction Hamiltonian *in the interaction picture:*
(in which the conjugate momentum only sees the free action)

$$H_I(\tau) = \int \mathrm{d}^3\mathbf{x}\, M_P^2 a^2 \epsilon \left(-\frac{\eta'}{2}\zeta'\zeta^2 + \frac{(\eta')^2}{16}\zeta^4 \right)$$

(See also Firouzjahi, 2023)

1. Same two-point function computed in both gauges in the limit of instantaneous transitions.
2. The consistency relation holds (checked at tree-level) by direct calculation in both gauges:

$$B_\zeta(\tau; \mathbf{k}, \mathbf{p}, -\mathbf{p}) = -\frac{\mathrm{d}\log \mathcal{P}_\zeta(\tau, p)}{\mathrm{d}\log p} P_\zeta(\tau, p) P_\zeta(\tau, k)$$

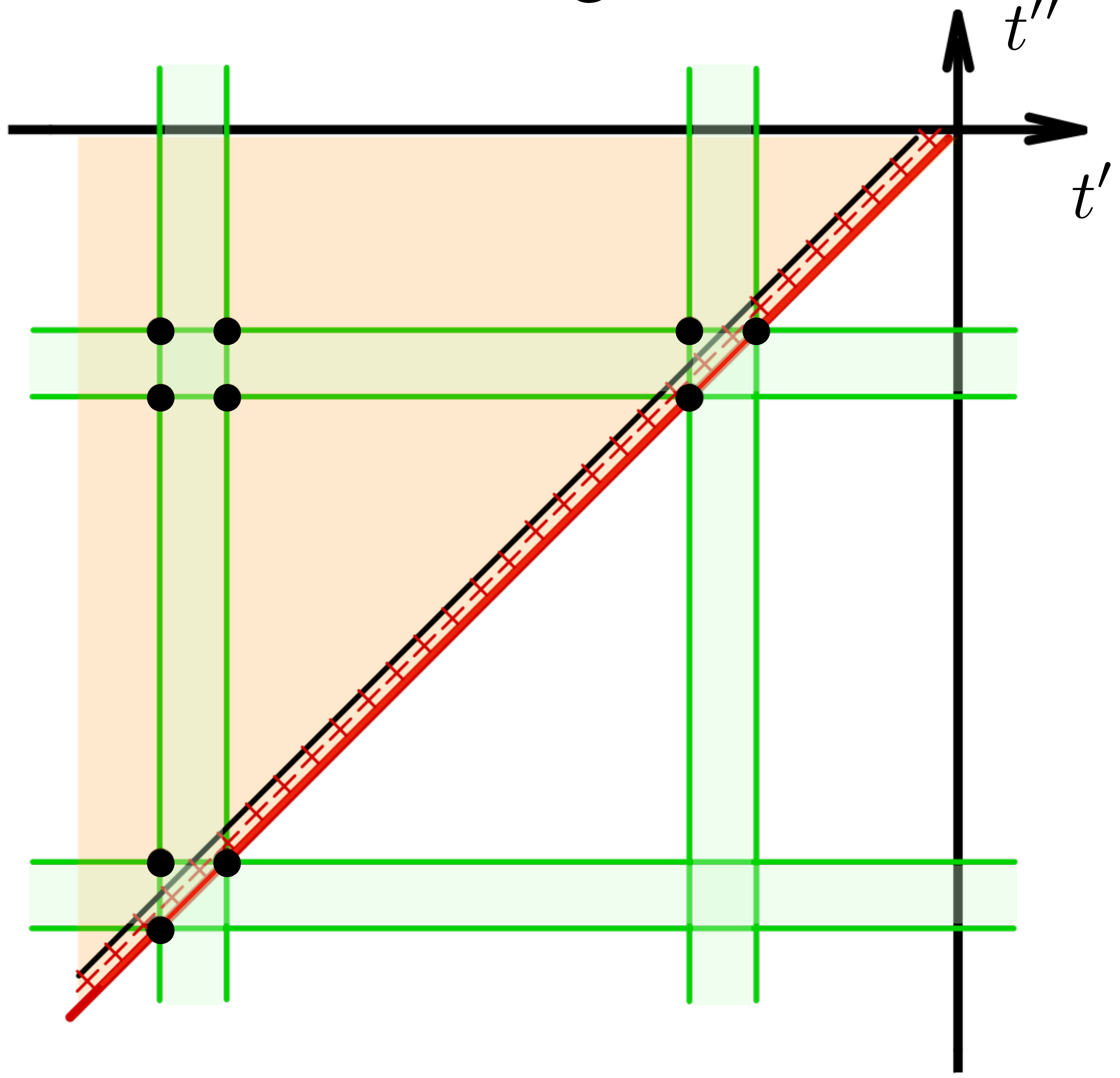
(Finite tree-level bispectrum for instantaneous transitions)

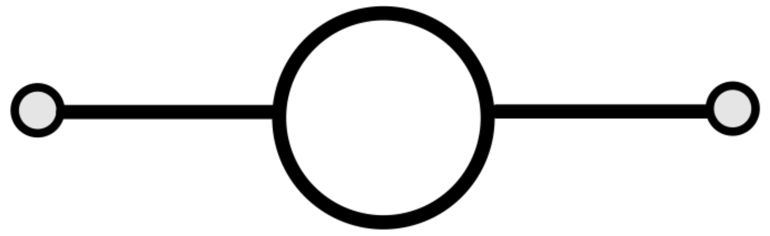
Two point function in the In-In Formalism (at 1-loop) in flat gauge

One momentum integral:

$$\int^{\infty} \mathrm{d}^3\mathbf{p} \to \int^{\Lambda} \mathrm{d}^3\mathbf{p}$$

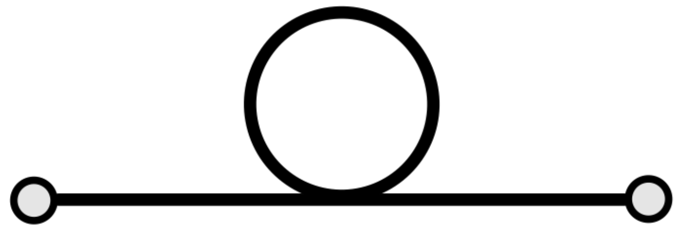
Two time integrals:





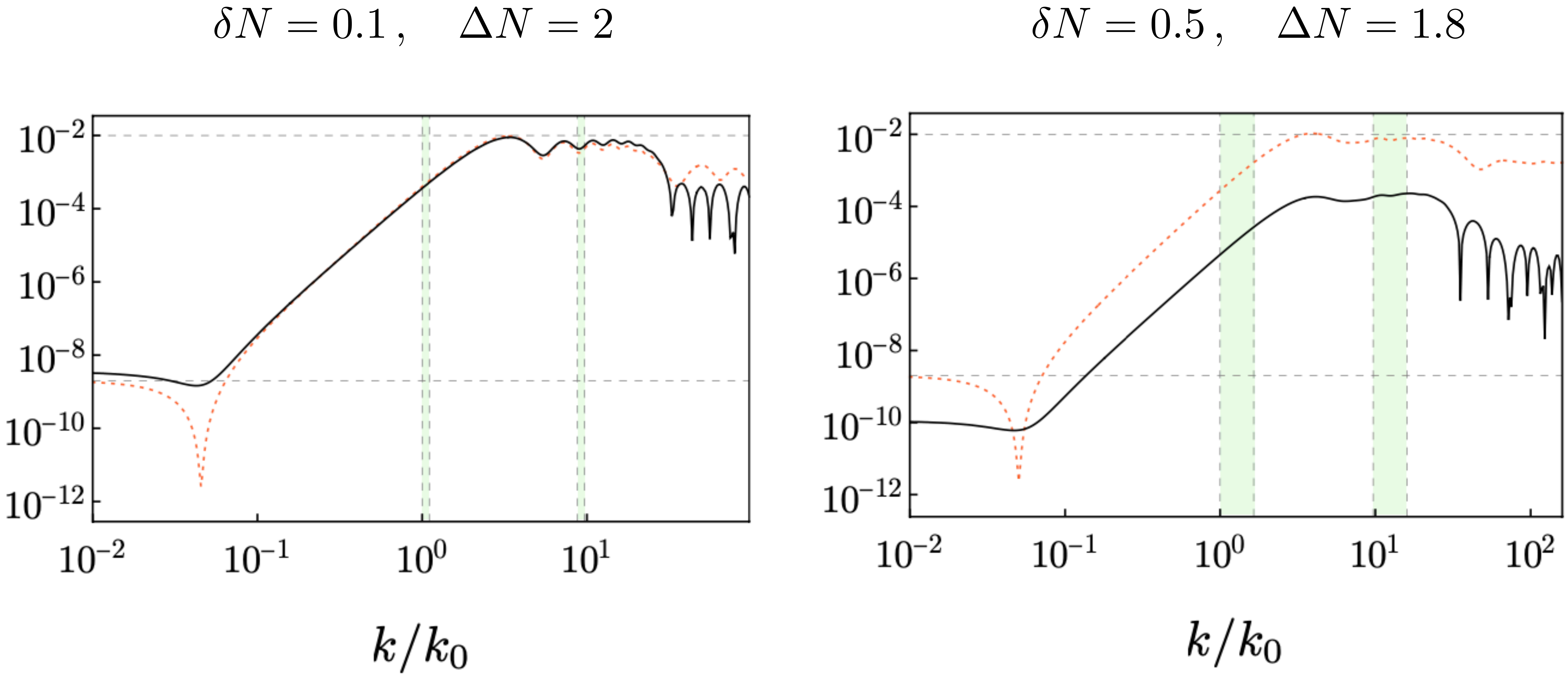
is *not* UV divergent

$\tau_\pm \equiv \tau(1 \pm i\omega)$ prescription is key

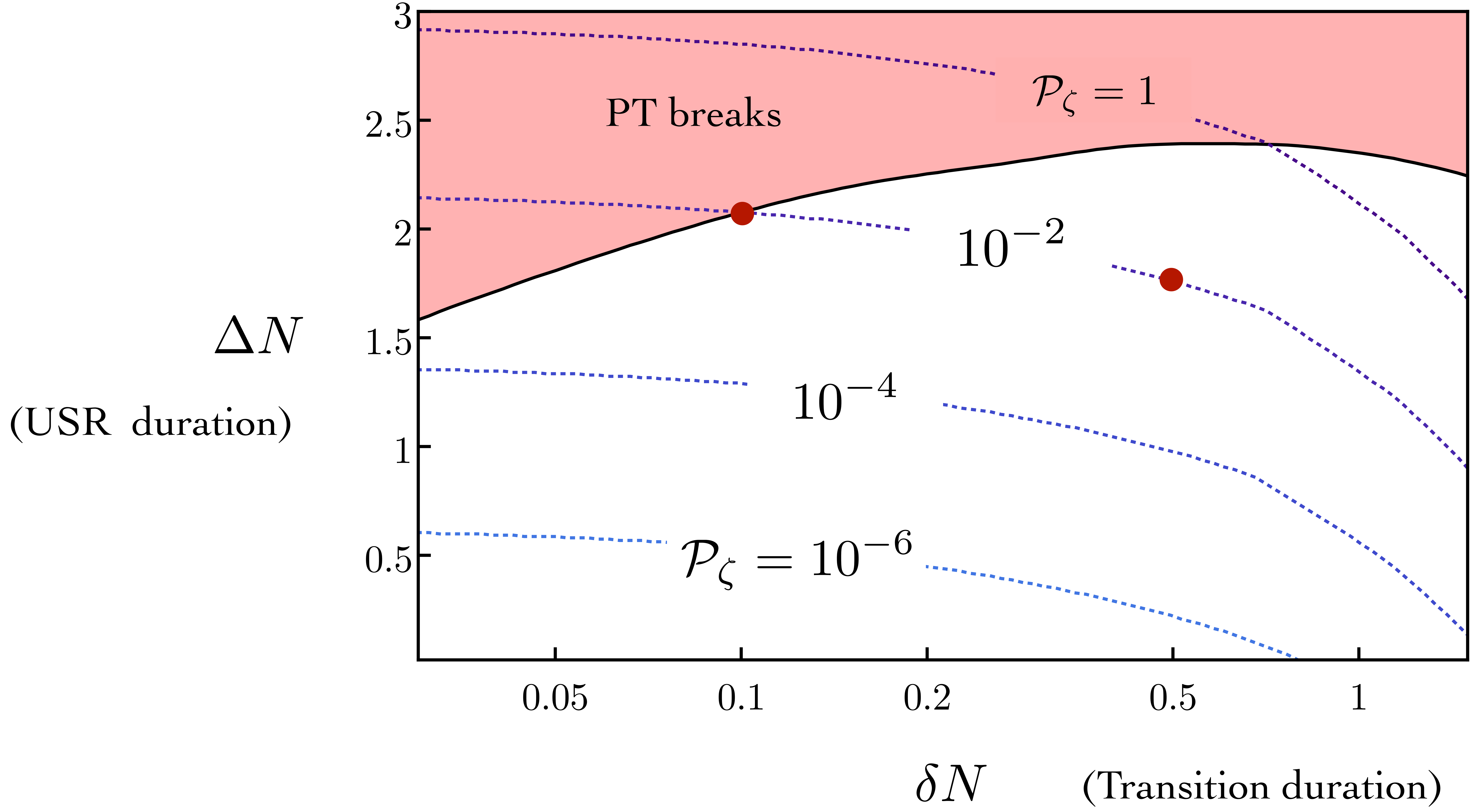


can be completely absorbed by

(at large scales)



$\delta N = 0.1\,, \quad \Delta N = 2$
$\delta N = 0.5\,, \quad \Delta N = 1.8$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
k/k_0
k/k_0



$\mathcal{P}_\zeta = 1$
PT breaks
10^{-2}
10^{-4}
$\mathcal{P}_\zeta = 10^{-6}$
ΔN
(USR duration)
δN
(Transition duration)
3
2.5
2
1.5
1
0.5
0.05
0.1
0.2
0.5
1

Other mechanisms to form PBH

* Single-field inflation other than USR
* Transient Dissipation during inflation

✴ Single-field inflation other than USR

$$\mathcal{S} = \int d^4x\, M\, \frac{a^3 \epsilon}{c_s^2} \left(\dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2} |\nabla \mathcal{R}|^2 \right) \qquad \mathcal{R} \simeq C_1 + C_2 \int \frac{c_s^2}{a^3\, M^2\, \epsilon\, H} dN$$

Rate of change of ϵ

$$\frac{d\mathcal{R}}{dN} \propto \exp\left[- \int (3 + \epsilon - 2\eta - 2s + \mu) \right] dN$$

Rate of change of c_s

Rate of change of M

GB, Beltrán Jiménez, Pieroni. 2018
GB, Céspedes, Santoni. 2021

✴ Dissipation during inflation

Background: $\ddot{\phi} + (3H + \boldsymbol{\Gamma})\dot{\phi} + V' = 0$

Fluctuations (schematically): $\delta\ddot{\phi}_{\boldsymbol{k}} + (3H + \Gamma)\delta\dot{\phi}_{\boldsymbol{k}} + \left(\frac{k^2}{a^2} + \dot{\phi}\,\Gamma_\phi\right)\delta\phi_{\boldsymbol{k}} \propto \sqrt{\frac{2\Gamma T}{a^3}}\,\xi_{\boldsymbol{k}}(t)$

(Stochastic thermal noise)

✴ Dissipation during inflation

Langevin approach

$$\mathbf{\Phi} \equiv \left(\psi, \delta\rho_r, \frac{\mathrm{d}\delta\phi}{\mathrm{d}N}, \delta\phi\right)^{\mathrm{T}}$$

$$\mathrm{d}\mathbf{\Phi} + \mathbf{A}\,\mathbf{\Phi}\,\mathrm{d}N = \mathbf{B}\,\xi\,\mathrm{d}N$$

System of <u>stochastic differential equations</u>

10000 realizations required for a ~1% relative error in the primordial spectrum

~1 day (in a 50-core cluster)

<u>Matrix formalism</u>

$$\boldsymbol{Q} \equiv \langle \mathbf{\Phi}\mathbf{\Phi}^{\dagger}\rangle_{\mathrm{S}}$$

$$\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}N} = -\boldsymbol{A}\boldsymbol{Q} - \boldsymbol{Q}\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}$$

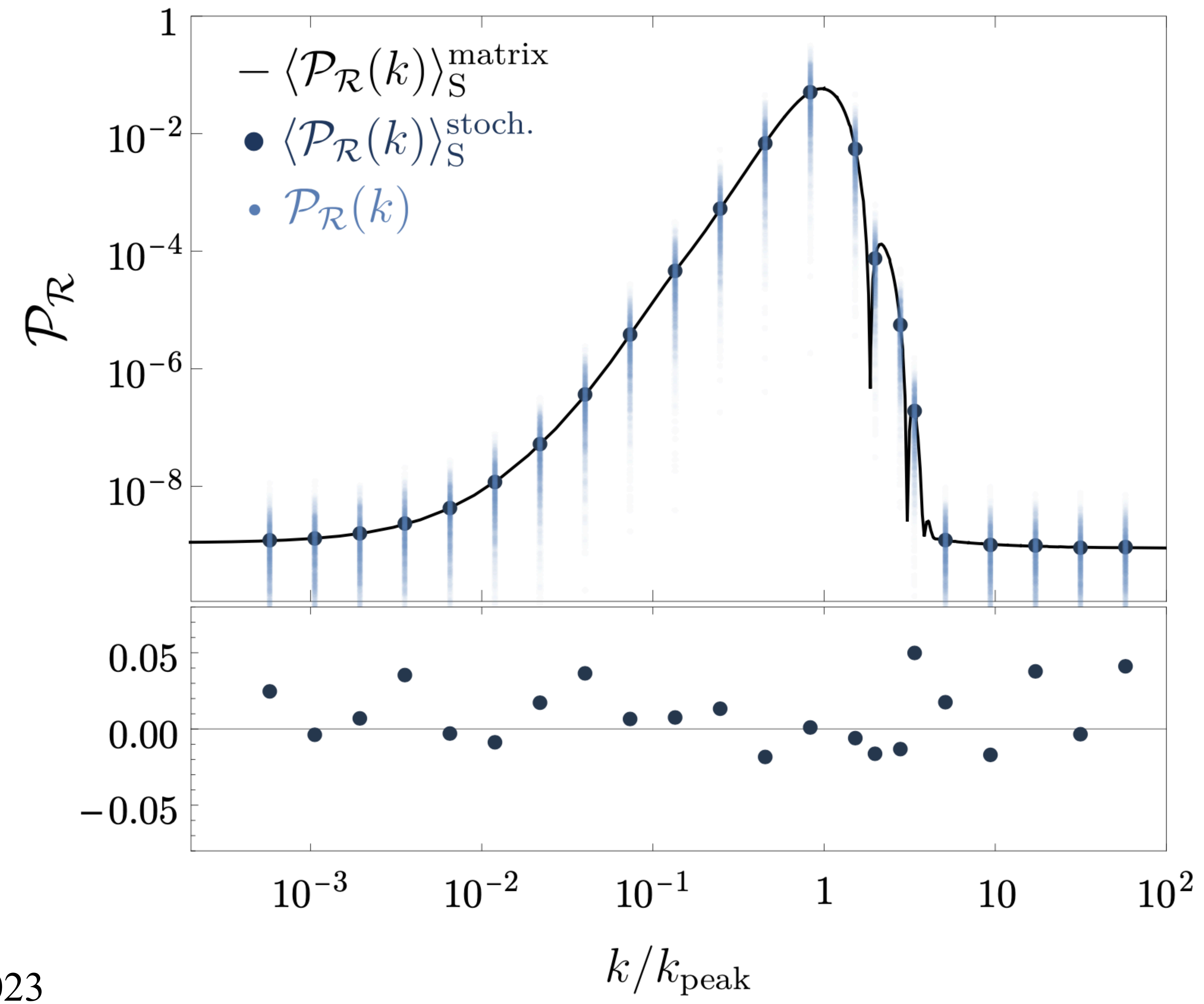
System of <u>ODE</u> for two-point functions

Exact solution (in a single "realization")

~1 minute (in a 4-core laptop)

✴ Dissipation during inflation

Each blue vertical band
=
About 2000 realizations per k-mode

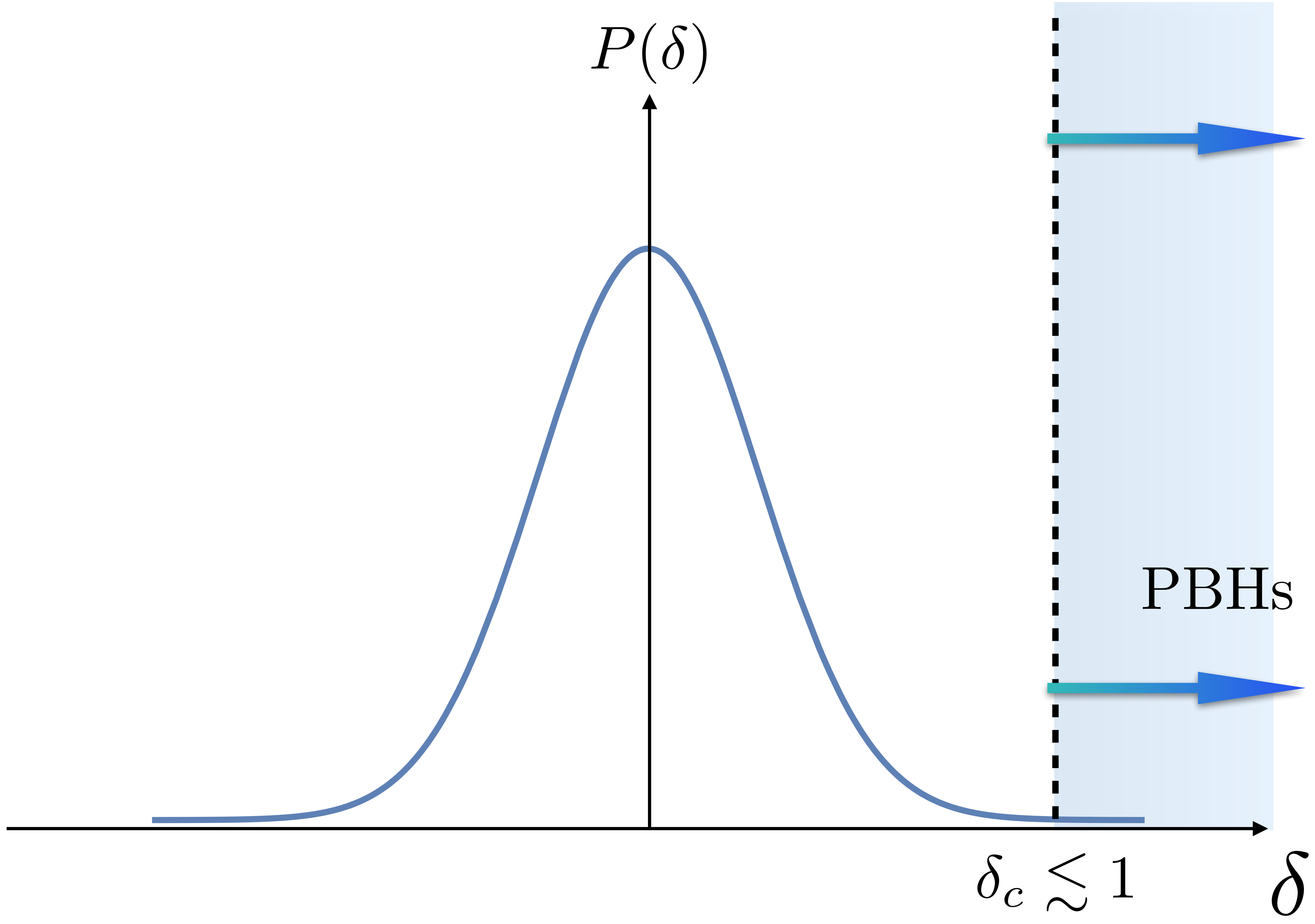


GB, A. Pérez Rodríguez, J. Rey, M. Pierre, 2023

Summary

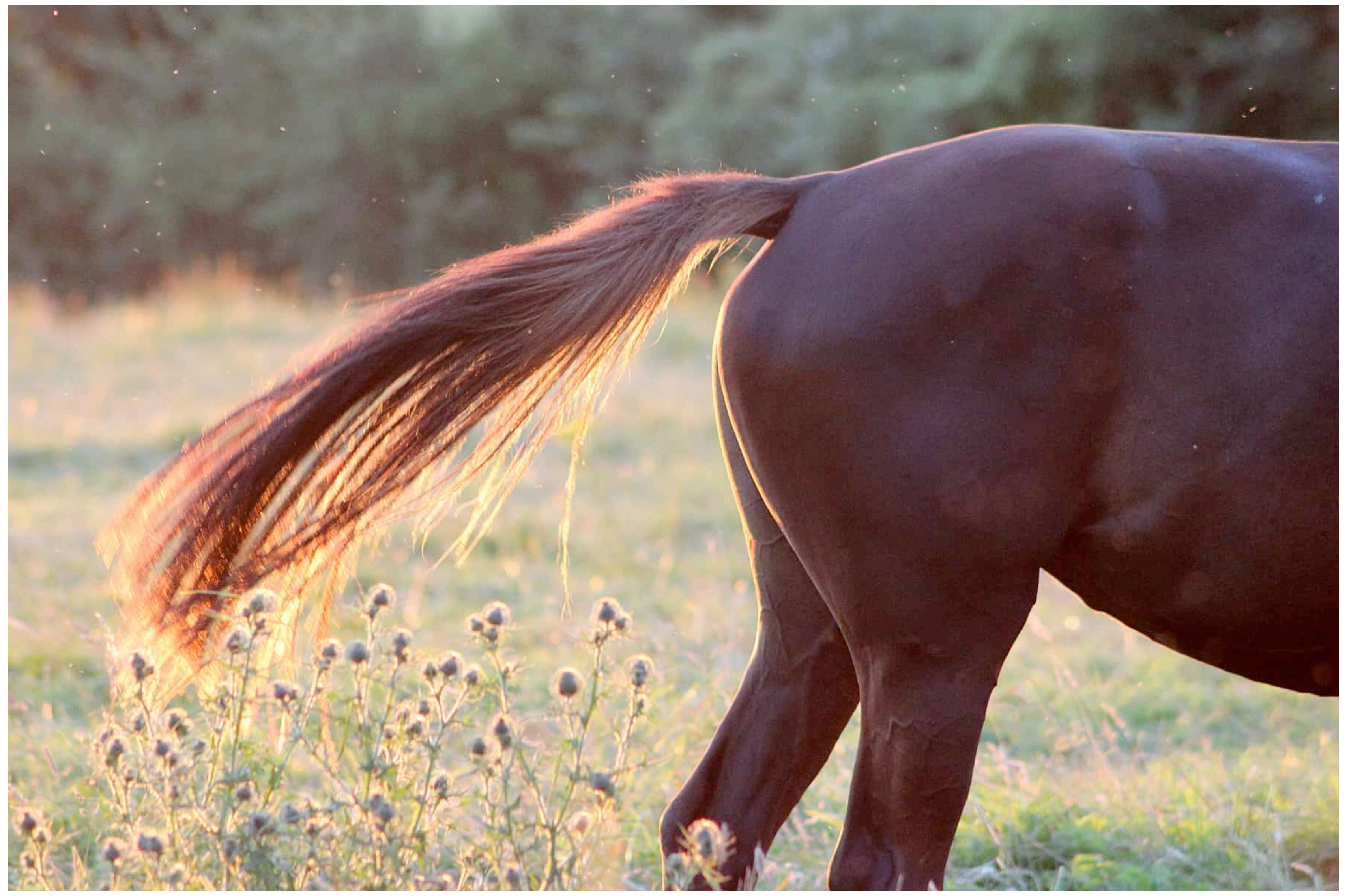
- Asteroid-mass PBHs are a strong contender to explain DM
- Ultra-slow roll inflation: alive
- Other ideas worth exploring (e.g. EFT of inflation, transient dissipation)
- PBH studies are leading to new insights and methods for inflation

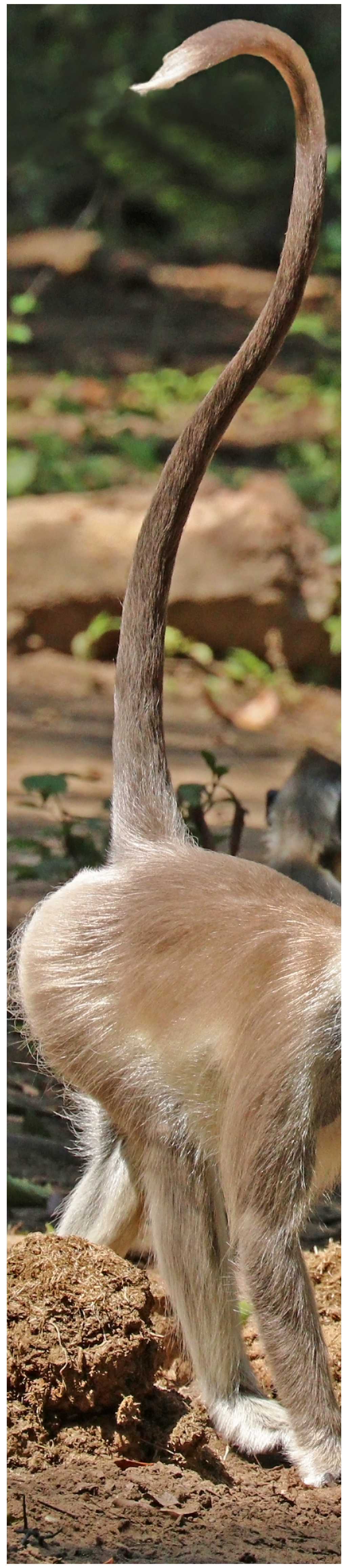
The problem of the abundance

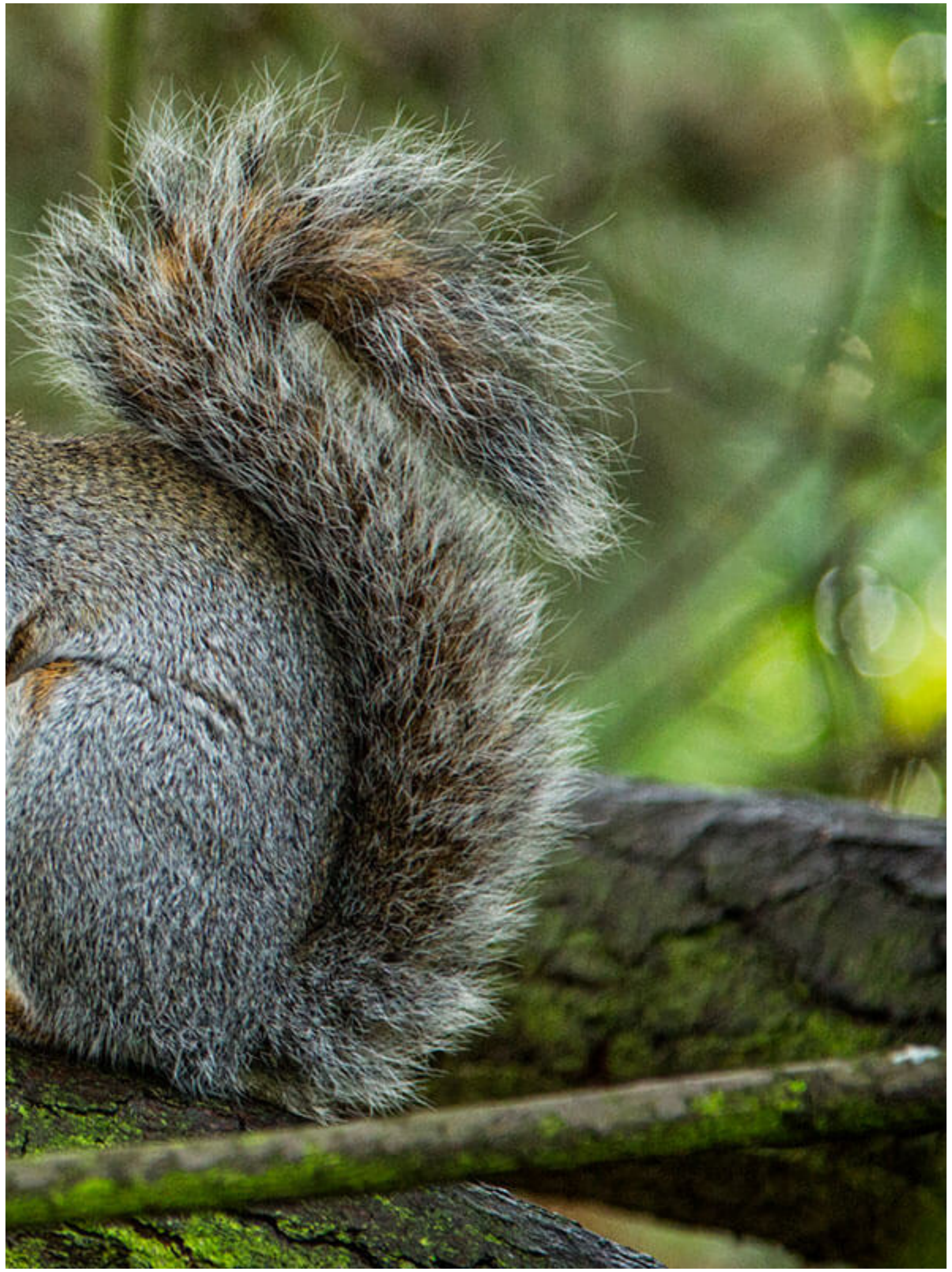


$$\frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}} = \int_{\delta_c}^{\infty} f(\delta)\, d\delta$$

How does the tail of the PDF look like?







- The relation between ζ and δ is non-linear

- ζ is, in general, intrinsically non-gaussian

$$f(\delta) \sim \exp\left(-\frac{\zeta}{2\,\mathcal{P}_\zeta} + \frac{\langle \zeta\zeta\zeta \rangle}{{\mathcal{P}_\zeta}^3}\zeta^3 + \dots \right)$$

Small corrections for small ζ

Several indications of non-gaussian tails, for large ζ

Non-linear saddle point for ζ^4 — Celoria, Creminelli, Tambalo, Yingcharoenrat 2021

USR:

- Stochastic inflation, numerically — Figueroa, Raatikainen, Rasanen, Tomberg 2020
- Stochastic δN formalism — Pattison, Vennin, Wands, Assadullahi 2021

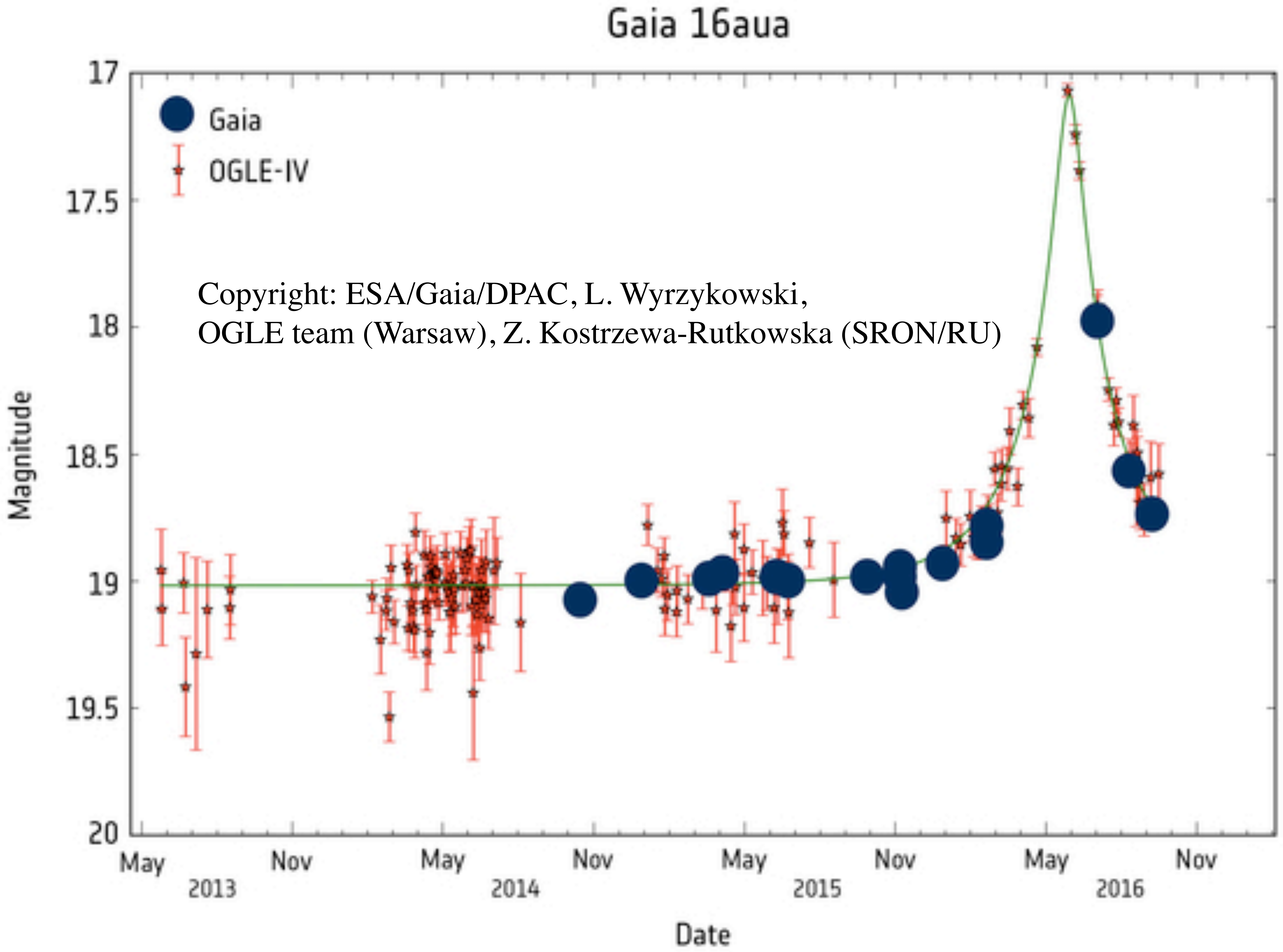
$$\frac{\dot\phi}{H} \gg \frac{H}{2\pi} \longrightarrow \mathcal{P}_\zeta \ll 1$$

Quantum diffusion

Non-Gaussian tails in the PDF of curvature perturbations arise in USR inflation without invoking stochastic inflation.

GB, Konstandin, Pérez Rodríguez, Pierre, Rey 2024

Microlensing



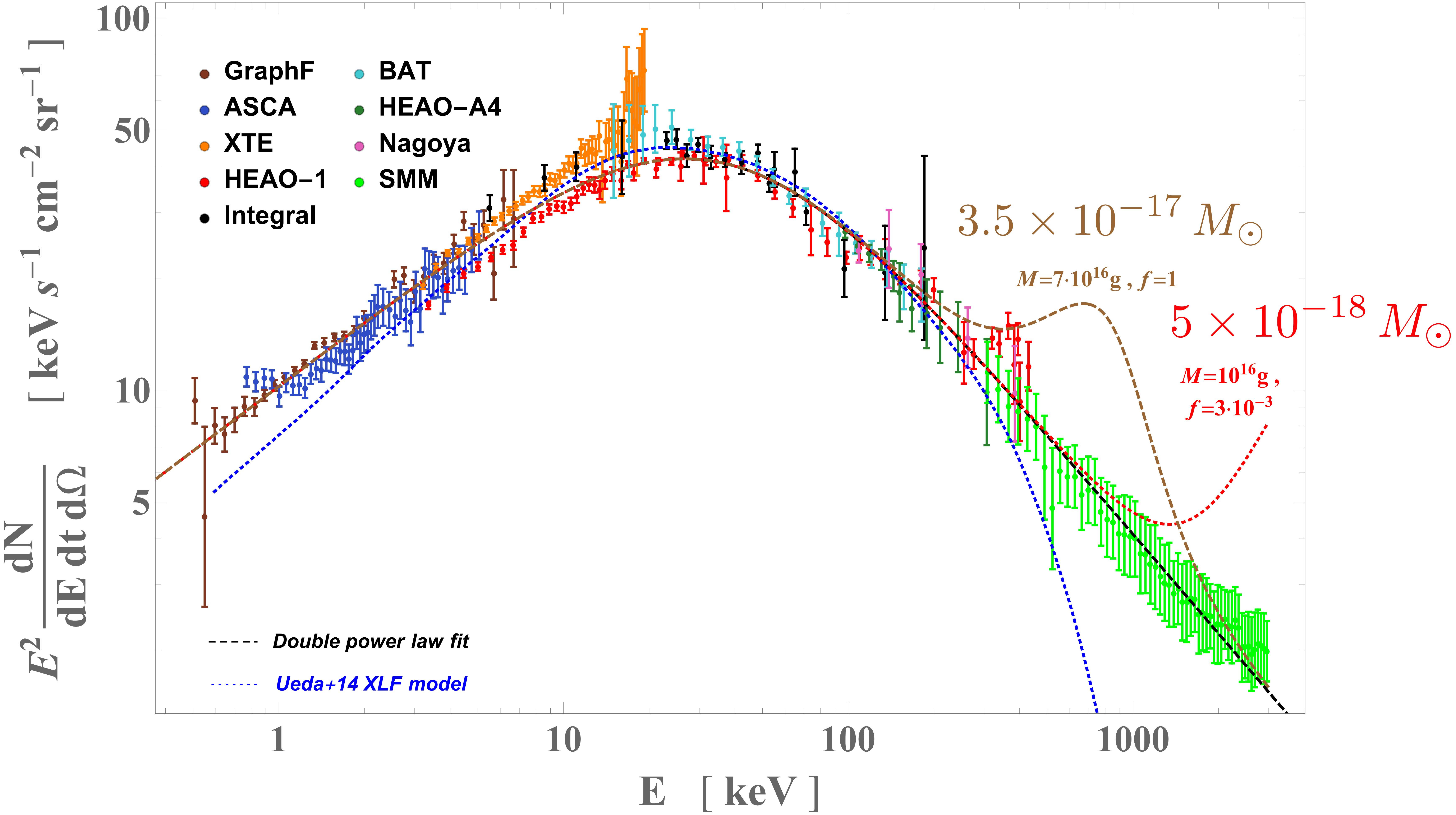
Eros, OGLE: Galactic Bulge and Magellanic Clouds.
Subaru HSC: M31 (finite source and wave optics effects)

For $M \sim 10^{-10} M_\odot \;\; , \quad r_s \sim \lambda$

Evaporation by Hawking radiation

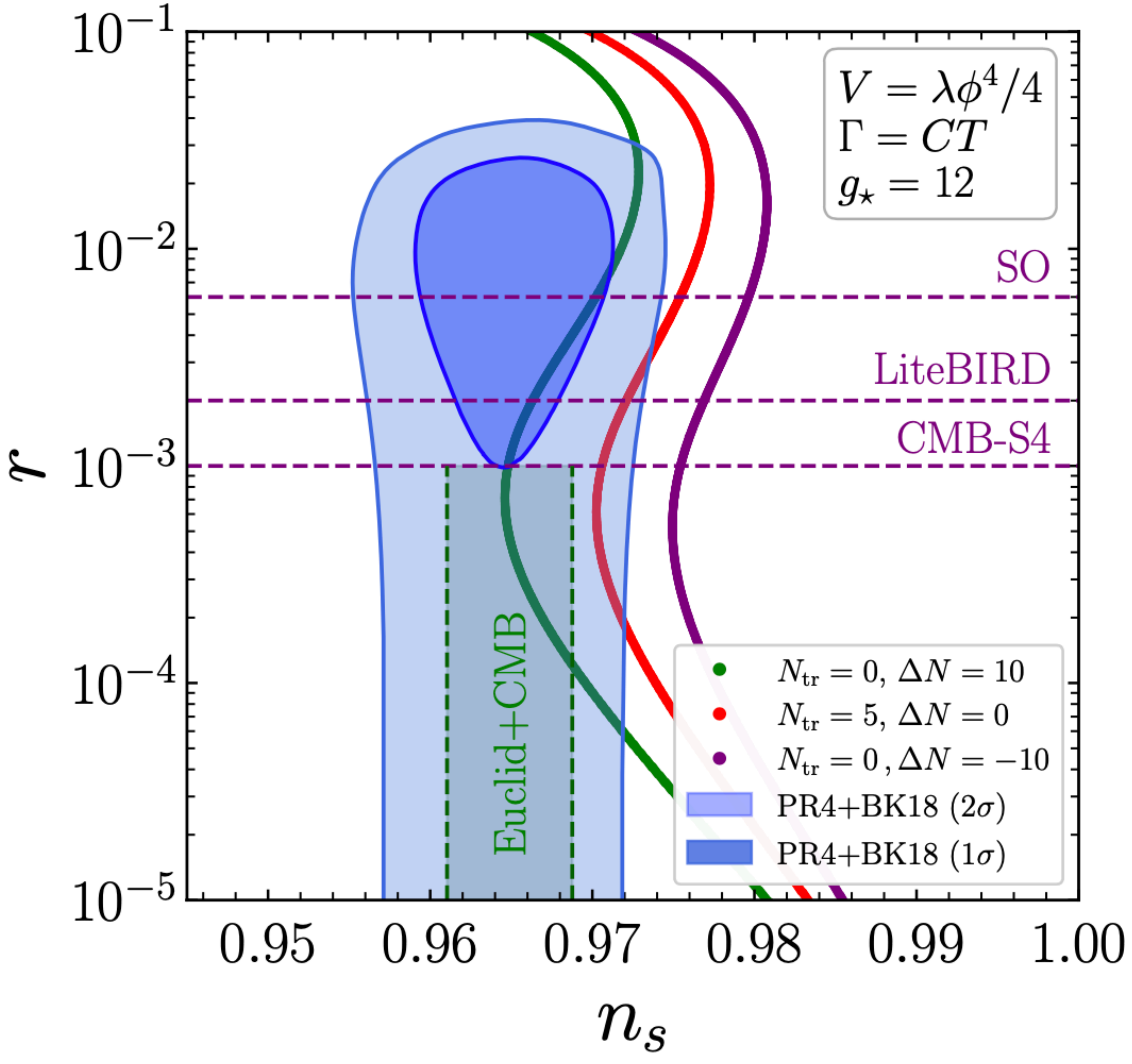
$$T = \frac{\hbar\, c^2}{8\pi G\, k_B M} = 6 \times 10^{-8} \frac{M_\odot}{M} K$$

Bound for $M \lesssim 10^{17}\, g \simeq 5 \times 10^{-17} M_\odot$



GB, Coronado-Blázquez, Gaggero. 1906.10113

Warm inflation



		ϕ^6	ϕ^4	ϕ^2
Warm little inflation Bastero-Gil et al. 2016	T	✔	✔	✖
Minimal warm inflation Berghaus et al. 2019	T^3	✖	✔	✖
Bastero-Gil et al. 2012	T^3/ϕ^2	✖	✖	✖

GB, A. Pérez Rodríguez, M. Pierre, 2023