Is M-theory Emergent?

Testing the M-theoretic Emergence Proposal

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Based on: **2404.01371** and **2404.05801** with R. Blumenhagen, N. Cribiori and A. Gligovic

The Swampland Program

Conceptual framework

Consistent set of conjectures motivated mainly (but not exclusively) by string theory.
Example reviews: [Palti '18, van Beest, Calderón-Infante, Mirfendereski, Valenzuela '22].

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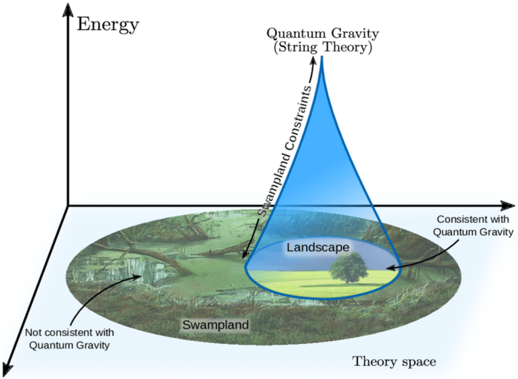

- No Global Symmetries Conjecture
- **Distance Conjecture**
- **Emergent String Conjecture**
- Weak Gravity Conjecture
- (A)dS Distance Conjecture
- Gravitino Conjecture

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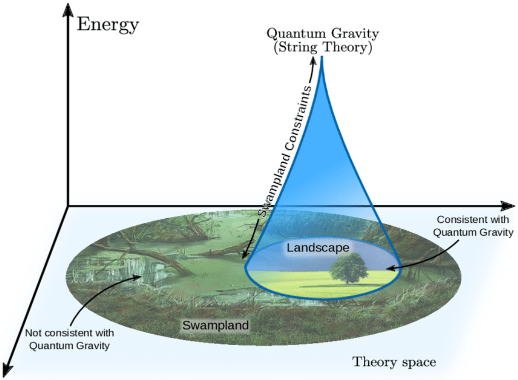

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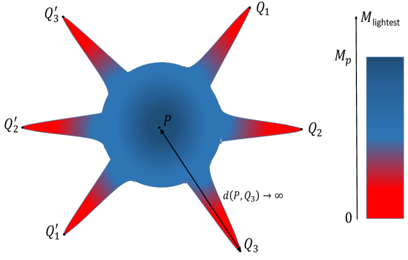
- **The Emergence Proposal**

Introduction

Swampland Considerations



Graphic taken from [Palti '18].

- **Swampland Distance Conjecture:** At an infinite distance in moduli space, a tower of exponentially light states appears [Ooguri, Vafa '06]

$$M(p) \sim M(p_0)\, e^{-\alpha d(p_0,p)}\,, \quad \alpha \sim \mathcal{O}(1)\,. \tag{1}$$

- **Emergent String Conjecture:** [Lee, Lerche, Weigand '18]

$$\to \begin{cases} \text{decompactification} \\ \text{emergent string limit} \end{cases}$$

Species Scale The UV cut-off in the presence of many light fields is [Dvali '08]

$$\Lambda_{\text{sp}} = \frac{M_{\text{pl}}^{(d)}}{N_{\text{sp}}^{1/(d-2)}}\,. \tag{2}$$

Introduction

Swampland Considerations

More recently, [van de Heisteeg, Vafa, Wiesner, Wu '22-'23]

$$S_{\text{corr.},d} \subset \frac{M_{\text{pl}}^{(d)\,d-2}}{2} \int d^d x \sqrt{-g} \left[\sum_n a_n(\phi_i) \frac{\mathcal{O}_n(\mathcal{R})}{M_{\text{pl}}^{(d)\,2n-2}} \right], \qquad \frac{1}{\Lambda_{\text{sp}}(\phi_i)^{2n-2}} \simeq \frac{a_n(\phi_i)}{M_{\text{pl}}^{(d)\,2n-2}}. \quad (3)$$

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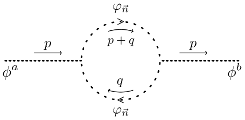
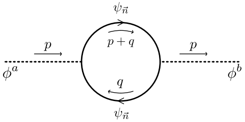
Emergence Proposal (Strong): The dynamics (kinetic terms) for all fields are emergent in the infrared by integrating out towers of states down from an ultraviolet scale Λ, which is below the Planck scale. [Palti ’19]

See also: [Heidenreich, Reece, Rudelius ’18, Grimm, Palti, Valenzuela ’18, Lee, Lerche, Weigand ’21, Castellano, Herráez, Ibáñez ’22, Blumenhagen, Gligovic, AP ’23]





$$G_{\phi\phi}^{1-\mathrm{loop}} \simeq \frac{\Lambda_{\mathrm{sp}}^{d-1}}{M_{\mathrm{pl}}^{(d)\,d-2}} \frac{(\partial_\phi \Delta m(\phi))^2}{(\Delta m(\phi))^3} + \ldots . \tag{4}$$

An M-theoretic Emergence Proposal

Overview

Emergence Proposal (Strong): The dynamics (kinetic terms) for all fields are emergent in the infrared by integrating out towers of states down from an ultraviolet scale Λ. [Palti '19]

An M-theoretic Emergence Proposal

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Emergence Proposal (M-theory): In the M-theory limit $M_* R_{11} \gg 1$ with the Planck scale kept fixed, a perturbative QG theory arises whose low energy effective description is emerging by integrating out the *full infinite* towers of states with a mass scale parametrically not larger than M_*. [Blumenhagen, Cribiori, Gligovic, AP '24]

Light States: Transverse M2, M5 branes carrying KK momentum

$$R_{11} \to \lambda R_{11}\,, \quad M_* \to \frac{M_*}{\lambda^{\frac{1}{d-1}}}\,, \quad R_I \to \lambda^{\frac{1}{d-1}} R_I\,, \quad M_{D0} \sim \frac{M_{\rm pl}^{(d)}}{\lambda}\,, \quad M_{D2,NS5} \sim \frac{M_{\rm pl}^{(d)}}{\lambda^{1/(d-1)}}\,. \tag{5}$$

Independent Motivation: BFSS matrix model [Banks, Fischler, Shenker, Susskind '97].

R^4 couplings

GGV approach

M-theory on a $(k+1)$ torus of volume $r_{11}\mathcal{V}_k$

$$S_{R^4} \simeq M_*^{d-8} \int d^d x \sqrt{-g}\, r_{11} \mathcal{V}_k\, a_d\, t_8 t_8\, R^4\,, \quad k = 10 - d. \tag{6}$$

The **1/2 BPS saturated** coefficient a_d can be determined via [Green,Gutperle,Vanhove '97]

$$a_{10-k} \simeq \frac{2\pi}{r_{11}\mathcal{V}_k} \sum_{m_I \in \mathbb{Z}} \int_0^\infty \frac{dt}{t^{\frac{4-k}{2}}} e^{-\pi t \sum_{I,J=1}^k m_I G^{IJ}_{(k+1)} m_J}\,. \tag{7}$$

Regularization method: Poisson resummation over all integers and T-duality for $\hat{m}_I = 0$.

What more can we learn?

R^4 couplings

GGV approach

$$d = 10: \qquad a_{10} \simeq \underbrace{\mathbf{C}}_{\text{one-loop}} + \underbrace{\frac{2\zeta(3)}{r_{11}^3}}_{\text{tree level}}, \tag{8}$$

$$d = 9: \qquad a_9 \simeq \frac{2\zeta(3)}{r_{11}^3} + \mathbf{C} + \frac{8\pi}{r_{11}^2 r_1} \sum_{m\neq 0} \sum_{m_1>0} \left|\frac{m}{m_1}\right| K_1\left(2\pi|m|m_1 \frac{r_1}{r_{11}}\right). \tag{9}$$

$$\text{T-duality} \rightarrow \mathbf{C} = \tfrac{2\pi^2}{3}.$$

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$$\text{T-duality} \to \mathbf{C} = \frac{2\pi^2}{3}.$$

Using e.g. [Green, Gutperle '97]

$$\begin{aligned} d = 8: \qquad a_8 \simeq& \frac{2\zeta(3)}{r_{11}^3} - \frac{2\pi}{r_{11} r_1 r_2} \log\left(\mathbf{r_2^2\, r_{11}} |\eta(iu)\eta(ir_{11}r_1r_2)|^4\right) \\ &+ \frac{8\pi}{r_{11}^2 r_1} \sum_{\substack{m>0 \\ (m_1,m_2)\neq(0,0)}} \frac{m}{|m_1 + im_2 u|} K_1\left(2\pi \frac{r_1}{r_{11}} m|m_1 + im_2 u|\right), \end{aligned} \tag{10}$$

Mathematical Object: Eisenstein series → Generalization to lower dimensions: [Kiritsis, Pioline '97, Obers, Pioline '99], but also e.g. [Green, Russo, Vanhove '10].

R^4 couplings

Exploring the perturbative string theory limit

Usual starting point:

$$a_{10-k}^{1-\text{loop}} \simeq 2\pi \int_{\mathcal{F}} \frac{d^2\tau}{\tau_1^2} \sum_{m^I, n^I \in \mathbb{Z}^k} e^{-\frac{\pi}{\tau_1} \sum_{I,J=1}^k (m^I + n^I \tau) G_{IJ} (m^J + n^J \bar{\tau})} , \tag{11}$$

is also an Eisenstein series [Obers, Pioline '99, Angelantonj, Florakis, Pioline '12]

$$\mathcal{E}^{SO(k,k)}_{V; s=\frac{k}{2}-1} \simeq \frac{2\pi}{\vartheta_{(k)}} \int_0^\infty \frac{dt}{t^{(k/2-1)+1}} \hat{\sum_{m_I, n^I \in \mathbb{Z}^k}} \delta(\text{BPS})\, e^{-\frac{\pi}{t} M^2} , \tag{12}$$

with the BPS conditions taking the form of a simple Diophantine equation

$$\sum_{I=1}^k m_I\, n^I = 0 \,. \tag{13}$$

Regularization: Introducing UV regulator, minimal subtraction and ζ-function regularization following [Blumenhagen, Cribiori, Gligovic, AP '23].

R^4 couplings

Exploring the perturbative string theory limit

- $d = 10$: Our method seems to not work? $\int_\epsilon^\infty dt/t^2 = 1/\epsilon$.
- $d = 9$:

$$\int_\epsilon^\infty \frac{dt}{t^{\frac{3}{2}}} e^{-tA} = \frac{2}{\sqrt{\epsilon}} - 2\sqrt{\pi A} + \mathcal{O}(\sqrt{\epsilon})\,, \tag{14}$$

$$\text{BPS conditions} \rightarrow \begin{cases} a_{9,m=0}^{1-\text{loop}} \simeq \frac{2\pi}{\rho_1} \sum_{n\neq 0} \int_0^\infty \frac{dt}{t^{3/2}} e^{-\pi t \rho_1^2 n^2} = \frac{2\pi^2}{3}\,, & \text{winding,} \\ a_{9,n=0}^{1-\text{loop}} \simeq \frac{2\pi}{\rho_1} \sum_{m\neq 0} \int_0^\infty \frac{dt}{t^{3/2}} e^{-\pi t \frac{m^2}{\rho_1^2}} = \frac{2\pi^2}{3} \frac{1}{\rho_1^2}\,, & \text{KK.} \end{cases} \tag{15}$$

  We can get the full **one-loop** result and decompactify to get the 10D one.

- $d \leq 8$: The pattern remains the same, we can get the full **one-loop** result, as long as we are able to solve the BPS conditions.

- **Key Observation**: constant term $\leftrightarrow$ extended objects (strings).

Emergence of R^4 couplings

Testing the Emergence Proposal in M-theory

Extend the Ansatz of GGV to include the **full spectrum** of **light** 1/2 BPS particle states

$$a_d \simeq \frac{2\pi}{r_{11}\mathcal{V}_k} \hat{\sum}_{N^A, m\in\mathbb{Z}} \int_0^\infty \frac{dt}{t^{\frac{d-6}{2}}}\, \delta(\mathrm{BPS})\, \exp\left(-\pi t\, N^A \mathcal{M}_{AB} N^B - \pi t\, \frac{m^2}{r_{11}^2}\right). \tag{16}$$

Regularization: UV regulator + ζ-functions [Blumenhagen, Cribiori, Gligovic, AP '23].

- $d = 10$: We can only have particle-like KK modes, just like GGV.

$$a_{10} \simeq \frac{2\pi}{r_{11}} \sum_{m\neq 0} \int_0^\infty \frac{dt}{t^2}\, e^{-\pi t \frac{m^2}{r_{11}^2}} \simeq \frac{2\zeta(3)}{r_{11}^3}, \tag{17}$$

using

$$\int_\epsilon^\infty \frac{dt}{t^2}\, e^{-\pi t A} = \frac{1}{\epsilon} + \pi A\Big(\log(\pi A \epsilon) + \gamma_E - 1\Big) + \mathcal{O}(\epsilon). \tag{18}$$

Downside: The (constant) one-loop term is missing.

Emergence of R^4 couplings

Testing the Emergence Proposal in M-theory

- $d = 9$: Similar behavior (no **light** wrapped branes)

$$a_9 \simeq \frac{2\zeta(3)}{r_{11}^3} + \frac{8\pi}{r_{11}^2 r_1} \sum_{m \neq 0} \sum_{m_1 > 0} \left| \frac{m}{m_1} \right| K_1 \left(2\pi |m| m_1 \frac{r_1}{r_{11}} \right) . \tag{19}$$

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- $d = 8$: We have light wrapped $M2$-branes with 1/2 BPS conditions [Obers, Pioline '99]

$$\sum_J n_{IJ}\, m_J = 0 \;\to \mathcal{M}^2 = n_{12}^2 t_{12}^2 + \frac{m^2}{r_{11}^2} . \tag{20}$$

Our ansatz implies that the contribution to the R^4 coupling is

$$a_{8,D2/D0} \simeq \frac{2\pi}{r_{11}t_{12}} \sum_{n_{12}\neq 0} \sum_{m\in\mathbb{Z}} \int_0^\infty \frac{dt}{t}\, e^{-\pi t\left(n_{12}^2 t_{12}^2 + \frac{m^2}{r_{11}^2}\right)} \to \tag{21}$$

$$a_{8,D2/D0} \simeq \frac{2\pi}{r_{11}t_{12}} \left(\frac{\pi}{3} r_{11}\, t_{12} + 4 \sum_{n_{12},m>0} \frac{1}{n_{12}} e^{-2\pi n_{12} m r_{11} t_{12}}\right) = -\frac{2\pi}{r_{11}t_{12}} \log\left(|\eta(ir_{11}t_{12})|^4\right) .$$

Emergence of R^4 couplings

Testing the Emergence Proposal in M-theory

Adding the KK contribution

$$a_{8,D0} \simeq \frac{2\zeta(3)}{r_{11}^3} - \frac{2\pi}{r_{11}t_{12}} \log\left(r_{11}r_2^2|\eta(iu)|^4\right) + \frac{8\pi}{r_{11}^2} \sum_{m>0} \sum_{(m_1,m_2)\neq(0,0)} \frac{m\, K_1\left(2\pi \frac{m}{r_{11}}\sqrt{m_1^2 r_1^2 + m_2^2 r_2^2}\right)}{\sqrt{m_1^2 r_1^2 + m_2^2 r_2^2}}\,, \tag{23}$$

where we have used

$$\int_\epsilon^\infty \frac{dt}{t} e^{-tA} = -\gamma_E - \log(\epsilon A) + \mathcal{O}(\epsilon)\,. \tag{24}$$

We thus obtain the **complete** result in 8d.

Can this be checked further?

- The pattern: **constant terms** $\leftrightarrow$ **extended objects** persists in $d = 6, 7$.

Emergence of R^4 couplings

Testing the Emergence Proposal in M-theory

- The instanton contributions can be generally determined.

Particle states	Instantons
$(D0, \mathrm{KK}_{(K)})$	$ED0_{(K)}$
$(D2_{(IJ)}, \mathrm{KK}_{(K)})$	$ED2_{(IJK)}$
$(NS5_{(IJKLM)}, \mathrm{KK}_{(N)})$	$ENS5_{(IJKLMN)}$
$(D2_{(IJ)}, D0)$	$EF1_{(IJ)}$
$(NS5_{(IJKLM)}, D0)$	$ED4_{(IJKLM)}$
$(NS5_{(IJKLM)}, D2_{(LM)})$	$ED2_{(IJK)}$

$$M_{\rm pl} = \text{const}$$

Our result are consistent with more formal results [Obers, Pioline '99, [Green, Russo,Vanhove '10, Bossard, Kleinschmidt '15, Bossard, Pioline '16].

pert. string theory	desert	M-theory
$g_s \ll 1$	$g_s = \mathcal{O}(1)$	$g_s \gg 1$

$$a_d = \frac{c_0}{g_s^2} + \underbrace{\left(c_1 + \mathcal{O}(e^{-S_{\rm ws}})\right)}_{\text{one-loop}} + \mathcal{O}(e^{-S_{\rm st}}) \quad = \quad \mathcal{E}^{E_{k+1(k+1)}}_{\Lambda_{E_{k+1}}, s=\frac{k}{2}-1} \quad = \quad \mathcal{E}^{E_{k(k)}}_{\Lambda_{E_k}\oplus 1, s=\frac{k}{2}-1}$$

Summary and Outlook

Testing the **M-theoretic Emergence Proposal**, we extended the ansatz of [Green, Gutperle, Vanhove '97] to include the **full spectrum** of **light** 1/2 BPS particle states

$$a_d \simeq \frac{2\pi}{r_{11}\mathcal{V}_k} \hat{\sum_{N^A, m\in\mathbb{Z}}} \int_0^\infty \frac{dt}{t^{\frac{d-6}{2}}}\, \delta(\mathrm{BPS})\ \exp\left(-\pi t\, N^A \mathcal{M}_{AB} N^B - \pi t\, \frac{m^2}{r_{11}^2}\right). \tag{25}$$

Regularization Technique: UV cutoff and ζ-function regularization.

Upshots:

- Self-consistently getting full results in $d = 7, 8$. Precise instanton predictions.
- Providing physical interpretation for previously ambiguous terms.
- Connecting the work of [Obers, Pioline '99] to the swampland framework.

Future directions:

- Non 1/2 BPS quantities?
- Connections with other approaches e.g. [Hattab, Palti '23-24]?
- M(-atrix) model implications?

Thank you

Back-up Slide: 8d calculation in string theory

The Kaluza-Klein contribution is

$$a_8^{1-\text{loop},(1)} \simeq \frac{2\pi}{\vartheta_{12}} \sum_{(m_1,m_2)\neq(0,0)} \int_0^\infty \frac{dt}{t} e^{-\pi t\left(\frac{m_1^2}{\rho_1^2}+\frac{m_2^2}{\rho_2^2}\right)}\,, \tag{26}$$

which, using

$$\int_0^\infty \frac{dx}{x^{1-\nu}}\, e^{-\frac{b}{x}-cx} = 2\left|\frac{b}{c}\right|^{\frac{\nu}{2}} K_\nu\left(2\sqrt{|b\,c|}\right)\,, \qquad \int_\epsilon^\infty \frac{dt}{t} e^{-tA} = -\gamma_E - \log(\epsilon A) + \mathcal{O}(\epsilon) \tag{27}$$

and $\epsilon \to \tilde{\epsilon} = \epsilon\, 4\pi e^{-\gamma_E}$, gives rise to

$$a_8^{1-\text{loop},(1)} \simeq -\frac{2\pi}{\vartheta_{12}} \log\left(\rho_2^2 |\eta(iu)|^4\right)\,. \tag{28}$$

Meanwhile, defining $\alpha = (1,0), (0,1), (1,1)$, the winding sector contributes as

$$a_8^{1-\text{loop},(2)} = \sum_{|\alpha|=1}^{2} \binom{2}{|\alpha|} a_8^{1-\text{loop},\alpha} \frac{2\pi}{\vartheta_{12}} \cdot 2^{|\alpha|} \sum_{\tilde{\mathbf{n}}_\alpha>0} \sum_{M\in\mathbb{Z}} \sum_{N>0} \int_0^\infty \frac{dt}{t}\, e^{-\pi t\left(N^2 L_\alpha^2 + \frac{M^2 L_\alpha^2}{\vartheta_{12}^2}\right)} \tag{29}$$

$$= -\frac{2\pi}{\vartheta_{12}} \log\left(|\eta(i\vartheta_{12})|^4\right)\,. \tag{30}$$

Back-up Slide: Example of an $NS5$-brane contribution

he full set of BPS conditions is [Obers, Pioline '99]

$$\sum_J n_{IJ} m_J = 0\,, \quad n_{[IJ}\, n_{KL]} + \sum_P m_P\, n_{PIJKL} = 0\,, \quad n_{I[J}\, n_{KLMNP]} = 0\,. \tag{31}$$

An example of such a solution is the configuration

$$(n_{45}, m_1) = P(-\tilde{\nu}_5, \tilde{n}_{23})\,, \quad (n_{15}, m_4) = Q(\tilde{\nu}_5, \tilde{n}_{23})\,, \quad (n_{14}, m_5) = R(-\tilde{\nu}_5, \tilde{n}_{23})\,, \tag{32}$$

where $P, Q, R \in \mathbb{Z}$. This contributes as

$$a_5^{\text{typ}} \simeq \frac{2\pi}{r_{11} t_{12345}} \sum_{\tilde{n}_{23}, \tilde{\nu}_5 \in \mathbb{Z}} \sum_{N>0} \sum_{P,Q,R,m \in \mathbb{Z}} \int_0^\infty dt\, t^{\frac{1}{2}}\, e^{-\pi t \left(N^2 t_{23}^2 L^2 + \frac{m^2}{r_{11}^2} + \left(\frac{P^2}{r_1^2} + \frac{Q^2}{r_4^2} + \frac{R^2}{r_5^2} \right) L^2 \right)} \tag{33}$$

$$\simeq 2\pi \sum_{\tilde{n}_{23}, \tilde{\nu}_5 \in \mathbb{Z}} \sum_{N>0} \sum_{(P,Q,R,m) \neq (0,0,0,0)} \frac{1}{S\, L^2}\, e^{-2\pi N S}\,, \tag{34}$$

with $L = \sqrt{\tilde{\nu}_5^2 t_{145}^2 + \tilde{n}_{23}^2}$, $t_{12345} = r_1 r_2 r_3 r_4 r_5$ and

$$S = \sqrt{P^2 t_{123}^2 + Q^2 t_{234}^2 + R^2 t_{235}^2 + m^2 \left(\tilde{n}_{23}^2 (r_{11} t_{23})^2 + \tilde{\nu}_5^2 (r_{11} t_{12345})^2 \right)}\,. \tag{35}$$