Dark Energy from topology change at the foam level

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Tensions in Cosmology, September 2023 Corfu

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- The GB term appears in the effective low energy limit of string theory.
- The GB term is considered a topological invariant in 4D, thus it dosen't contribute to the equations of motion.
- Euclidean Quantum Gravity (EQG) predicts instatons, solutions at the foam level, of distinct topology from the background [Hawking, 1978].

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The main idea

- Take EQG topology changing instatons seriously.
- Calculate the variation of the GB term during topology change.

Topological significance of the GB term

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- The GB curvature polynomial

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$$\mathcal{G} = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}, \qquad (1)$$

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- is the euler density in 4D, thus according to the Chern-Gauss-Bonnet theorem, its integral over a 4D manifold M, without boundary, yields the euler characteristic χ

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$$\chi(M) = \frac{1}{32\pi^2}\int_M d^4x\sqrt{g}\,\mathcal{G} \tag{2}$$

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- the essense of the theorem is that despite any deformations caused by smooth variations of the metric, the topology of the manifold remains constant, as it is characterized by the topological index χ

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    1. The time dimension is Wick rotated $t \rightarrow i\tau$
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- Thus, the spacetime manifold is Euclideanized and its signature changes from (-,+,+,+) to (+,+,+,+)
- then instatons, saddle point solutions, appear with different topology from the background.

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Table: Euler character χ of spacetime manifolds as it has been calculated in [Gibbons and Hawking, 1979]

Spacetime	χ
Minkowski	0
Extreme BHs	0
Self-dual Taub-NUT	1
Schwarchild and Kerr BHs	2
Nariai $S_2 \times S_2$	4
Euclidean Wormhole $S_1 \times S_3$	0

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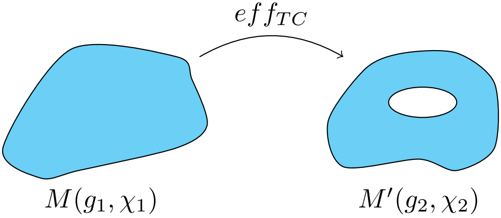

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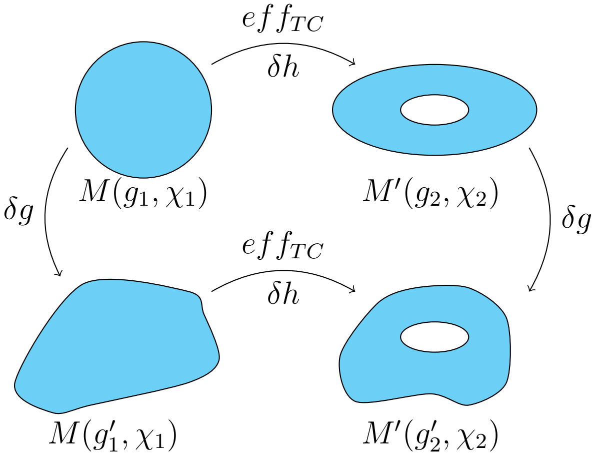

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- the background field approximation
- the expansion of the EH action
- the expansion of the GB action under the ETVC
- the semiclassical approach, where the effective action will be the sum of EH and the GB action
- the extremization of the effective action by the tadpole condition

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Background independence

- Physical observables must be background independent and the action must be diffeomorphic invariant [Pawlowski and Reichert, 2021]

$$g(\tilde{g}, h) \rightarrow g(\tilde{g} + \delta\tilde{g}, h + \delta h) = g(\tilde{g}, h). \qquad (8)$$

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- The tadpole condition provides a bridge for energy transfer between the quantum and the classical scale.

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$$S_{EH} = -\int d^4x\sqrt{g}R \tag{11}$$

$$S_{GB} = -\alpha \int d^4x\sqrt{g}\left(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\right), \tag{12}$$

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- and Γ_{1L} corresponds to the gauge fixing and ghost terms

$$\Gamma_{1L} = \Gamma_{GF} + \Gamma_{Fgh}, \tag{13}$$

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\begin{aligned}
S_0 &= -\int d^4x\sqrt{\tilde{g}}\,\tilde{R} \\
S_1 &= \int d^4x\sqrt{\tilde{g}}\left(\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R}\right)h^{\mu\nu} \\
S_2 &= -\int d^4x\sqrt{\tilde{g}}\Bigg\{\frac{1}{4}h^{\mu\nu}\nabla^2 h_{\mu\nu} - \frac{1}{8}h\nabla^2 h \\
&\quad + \frac{1}{2}\left(\nabla^\nu h_{\nu\mu} - \frac{1}{2}\nabla_\mu h\right)^2 + \frac{1}{2}h^{\mu\lambda}h^{\nu\sigma}\tilde{R}_{\mu\lambda\nu\sigma} \\
&\quad + \frac{1}{2}\left(h^{\mu\lambda}h^\nu_\lambda - hh^{\mu\nu}\right)\tilde{R}_{\mu\nu} + \frac{1}{8}\left(h^2 - 2h^{\mu\nu}h_{\mu\nu}\right)\tilde{R}\Bigg\}.
\end{aligned}
\tag{14}
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$$\begin{aligned} &\delta_h S_{GB} = \\ &\delta_h \left(-\alpha \int_M d^4x \sqrt{g} \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right) \right) \\ = &- 32\pi^2 \alpha \frac{\delta \chi}{\delta h^{\mu\nu}} \delta h^{\mu\nu}, \end{aligned} \tag{15}$$

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$$\delta_h S_{GB} = -16\pi^2\alpha \frac{\partial\chi}{\partial V}\frac{\delta V}{\delta h^{\mu\nu}}\delta h^{\mu\nu} = -16\pi^2\alpha\frac{\partial\chi}{\partial V}\delta_h\left(\int_M d^4x\sqrt{g}\right)$$

$$= -16\pi^2\alpha\frac{\partial\chi}{\partial V}\int_M d^4x\frac{\delta\sqrt{g}}{\delta h^{\mu\nu}}\delta h^{\mu\nu}. \qquad (16)$$

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$$\frac{\delta\sqrt{g}}{\delta h^{\mu\nu}} = \frac{\delta}{\delta h^{\mu\nu}}\Bigg(\sqrt{\tilde{g}} + \frac{1}{2}\sqrt{\tilde{g}}\tilde{g}_{\mu\nu}h^{\mu\nu} - \frac{1}{4}\sqrt{\tilde{g}}h^{\mu\nu}h_{\mu\nu}$$
$$+ \frac{1}{8}\sqrt{\tilde{g}}(h^{\mu}_{\mu})^2 + \mathcal{O}(h^3)\Bigg)$$
$$= 0 + \frac{1}{2}\sqrt{\tilde{g}}\tilde{g}_{\mu\nu} + \mathcal{O}(h), \tag{17}$$

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- Equivalently, one can express the topological variation of the Gauss-Bonnet term as

$$\frac{1}{\sqrt{\tilde{g}}} \frac{\delta S_{GB}}{\delta h^{\mu\nu}} = -16\pi^2 \alpha \frac{\partial \chi}{\partial V} \tilde{g}_{\mu\nu} + \mathcal{O}(h). \qquad (20)$$

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$$\frac{1}{\sqrt{\tilde{g}}}\frac{\delta \Gamma}{\delta h^{\mu\nu}} = \frac{1}{\sqrt{\tilde{g}}}\frac{\delta S_{EH}}{\delta h^{\mu\nu}} + \frac{1}{\sqrt{\tilde{g}}}\frac{\delta S_{GB}}{\delta h^{\mu\nu}} + \frac{1}{\sqrt{\tilde{g}}}\frac{\delta \Gamma_{1L}}{\delta h^{\mu\nu}}$$
$$= \tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} - 16\pi^2\alpha\frac{\partial\chi}{\partial V}\tilde{g}_{\mu\nu} + \frac{1}{\sqrt{\tilde{g}}}\frac{\delta \Gamma_{1L}}{\delta h^{\mu\nu}} + \mathcal{O}(h), \tag{21}$$

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$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} + \tilde{g}_{\mu\nu}\Lambda_{eff} = 8\pi G T_{\mu\nu}, \tag{22}$$

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- where the stress tensor originates from the one loop part of the effective action, containing the matter like correction terms in the right side of the equation.

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- and the dynamical topological term appears in the equation of motion as an effective cosmological constant

$$\Lambda_{eff} = -16\pi^2\alpha\frac{\partial\chi}{\partial V}. \qquad (24)$$

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- The upper bound for the density will be for one wormhole per Planck volume, that is $\rho_{Mw} = \frac{1}{l_p^4} \sim 10^{140}$
- Thus, the range of the theoretical value of Λ_{eff} can be $0 \leq \Lambda_{eff} \leq \Lambda_M$ and the ratio $\frac{\Lambda_M}{\Lambda_{obs}} = 10^{124}$ of the model spans the 120 orders of magnitude, of the known discrepancy.

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2. the creation of Nariai instatons corresponds to a negative component of the net effective cosmological constant, absorbing up to 120 orders of vacuum energy, like a sponge absorbing water [Padmanabhan, 2003], thus providing a mechanism for the ending of the inflation period.

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Thank you for your attention!!!

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Thank you for your attention!!!

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