

Beyond the Standard Model:

from axions to nonHermiticity

Sarben Sarkar

Department of Physics
King's College London, UK

August 29, 2023

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Overview

Talk based on: ArXiv hep-th 2306.02122 with N Mavromatos

1. General features of baryogenesis
2. Related approaches to Beyond the Standard Model physics
3. Leptogenesis through CPT Violation
4. String-inspired models and torsion
5. Chern-Simons gravity
6. Einstein-Cartan theory
7. Stability of gravitational axion vacuum

Asymmetry between matter and antimatter [1]

- Matter-antimatter asymmetry basics in the Sakharov perspective
- The role of CPT symmetry
- Why CPT symmetry may be violated
- Nonhermiticity and CPT violation (explicit and spontaneous)

The observed **baryon asymmetry** (i.e. the fact that there is hardly any primordial antimatter) in the current cosmological era of the Universe implies that, at an earlier era (for times $t < 10^{-6}\text{s}$),

$$\Delta n_B (T \sim 1 \text{ GeV}) = \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \sim (8.4 - 8.9) \times 10^{-11} \quad (1)$$

where n_B is the baryon number density and $n_{\bar{B}}$ is the anti-baryon number density at this earlier epoch of the universe.

Sakharov conditions in Minkowski space

Prerequisites: Lorentz invariant theory: CPT invariance.

The existence of baryon-number violating processes in the SM (which happen to be non-perturbative)

Explicit violation of symmetry

C and CP violation should be present so that the amplitudes for the processes $X \rightarrow Y + b$ and $\bar{X} \rightarrow \bar{Y} + \bar{b}$ are unequal (where X, Y and b denote particles and the barred quantities their antiparticles)

Existence of processes out of chemical equilibrium

Quantitatively above does not work (not enough CP violation observed): seek ideas beyond Standard Model

Beyond the Standard Model (BSM)

Disparate examples

1. Neutrino oscillations
2. Quantum gravity, black holes and information loss
3. Gravitation multiplet in string theory contains the *Kalb-Ramond* field and *dilaton* in addition to the *graviton*

These are examples of physics beyond the SM.
Do they imply a model?

Approaches to BSM [2, 3, 4]

Examples	Model	Modification 1	Modification 2
Neutrino osc	Seesaw mechanism	Kalb Ramond (KR), CPT	Spont broken
Quantum gravity	non-unitary evolution	CPT symmetry broken	Decoherence
Strong CP problem	Axions	String theory	Dark energy?
Gravitational Leptogenesis	sterile neutrinos, axions	Kalb-Ramond quanta	Dark energy

Table: Pot pourri of connections

CPT is generically broken for non flat space-time and nonHermitian Hamiltonians.

Phenomenological Leptogenesis model [6, 8] I

The Lagrangian is given by:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{N}\not{\partial}N - \frac{m_N}{2}(\bar{N}^c N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - \sum_k y_k \bar{L}_k \tilde{\varphi} N + h.c. \quad (2)$$

where

- \mathcal{L}_{SM} denotes the SM Lagrangian
- B_μ is a CPTV background c-number BSM field
- N is the RHN field with (Majorana) mass m_N , N^c denotes charge conjugate
- $\tilde{\varphi}$ is the adjoint ($\tilde{\varphi}_i = \varepsilon_{ij}\varphi_j$) of the Higgs field φ
- L_k is a lepton (doublet) field of the SM sector with k a generation index
- y_k is a non-zero Yukawa coupling provides a non-trivial (“Higgs portal”) interaction between the RHN and the SM sectors
- Fits with phenomenological SME (Standard model extension) ([9, 10])

Phenomenological Leptogenesis model [6, 8] II

Phenomenologically adequate to

- take $k = 1$, and
- set $y_1 = y$.
- B_μ is assumed to have only a non-zero temporal component, which is taken to be constant in the Robertson-Walker frame of the early universe,

$$B_0 = \text{const} \neq 0, B_i = 0, i = 1, 2, 3.$$

A lepton asymmetry is generated due to tree level decays

$$\begin{aligned} \text{Channel I} & : N \rightarrow l^- h^+, \nu h^0, \\ \text{Channel II} & : N \rightarrow l^+ h^-, \bar{\nu} h^0. \end{aligned} \tag{3}$$

where ℓ^\pm are charged leptons, ν ($\bar{\nu}$) are light, “active”, neutrinos (antineutrinos) in the SM sector, h^0 is the neutral Higgs field, and h^\pm are the charged Higgs fields¹.

Phenomenological Leptogenesis model [6, 8] III

Since $B_0 \neq 0$ background the decay rates of the Majorana RHN between the channels I and II are different, giving a Lepton asymmetry, ΔL^{TOT} .

$$\boxed{\frac{\Delta L^{TOT}}{s} \simeq (0.016, 0.019) \frac{B_0}{m_N}}, \quad (4)$$

at the freezeout temperature $T = T_D$: $m_N/T_D \simeq (1.44, 1.77)$, where s is the entropy density of the universe (and the numbers inside the brackets distinguish different Padé approximations).

¹At high temperatures, above the spontaneous electroweak symmetry breaking, the charged Higgs fields h^\pm do not decouple from the physical spectrum, and play an important rôle in leptogenesis.

Microscopic Leptogenesis model [2] I

A microscopic origin of B_μ can arise in two ways:

- as *string-inspired effective gravitational theories* (SEG)
- as generic Einstein-Cartan theories with condensates of (the totally antisymmetric part of the) torsion, which we describe briefly.

Effective low energy gravitational theories

- $S_B = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - e^{-4\Phi} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - \Omega \right) + \dots,$
- $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$, $B_{\mu\nu}$ is a gauge field
- $\kappa^2 = 8\pi G_N$, and $G_N = M_P^{-2}$ is the (3+1)-dimensional Newton constant (with $M_P = 1.22 \times 10^{19}$ GeV the four-dimensional Planck mass); $\alpha' = M_s^{-2}$; M_s is the string mass scale

Microscopic Leptogenesis model [2] II

- Ω is related to the cosmological constant
- KR field strength terms H^2 can be absorbed into a generalised curvature with a “torsionful connection (Christoffel symbol)”, with the contorsion proportional to the field strength $H_{\mu\nu}^\rho$
- $\bar{\Gamma}_{\mu\nu}^\rho = \Gamma_{\mu\nu}^\rho + H_{\mu\nu}^\rho \neq \bar{\Gamma}_{\nu\mu}^\rho$, where $\Gamma_{\mu\nu}^\rho = \Gamma_{\nu\mu}^\rho$ is the torsion-free Christoffel symbol.
- In the string effective action this is only an approximation since there are higher order corrections in α'
- In (Heterotic) string theory, in the presence of gauge and gravitational fields, the Bianchi identity is modified by appropriate (parity-violating) Chern–Simons three-forms, which lead to a non-zero right-hand side of the Bianchi identity, expressing gauge and gravitational anomalies

$$\varepsilon_{\alpha\beta\sigma}{}^\mu \mathcal{H}^{\alpha\beta\sigma}{}_{;\mu} = \frac{\alpha'}{32\kappa} \sqrt{-g} \left(R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - \mathbf{F}_{\mu\nu} \tilde{\mathbf{F}}^{\mu\nu} \right), \quad (5)$$

Microscopic Leptogenesis model [2] III

- Distinguish Levi-Civita symbols

$$\varepsilon_{\mu\nu\rho\sigma} = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma}, \quad \varepsilon^{\mu\nu\rho\sigma} = \frac{\text{sgn}(g)}{\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma}, \quad (6)$$

- The semicolon “;” denotes the gravitational covariant derivative with respect to the standard (torsion-free) Christoffel connection, $\mathbf{F}_{\mu\nu}$ is the non-Abelian gauge field strength, $R_{\mu\nu\rho\sigma}$ is the four-dimensional Riemann tensor, and $\tilde{R}^{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta}{}^{\rho\sigma}$, $\tilde{\mathbf{F}}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} \mathbf{F}_{\alpha\beta}$ are the corresponding dual tensors
- The torsion interpretation of the field strength \mathcal{H} implies a *linear coupling* of \mathcal{H} with the *axial fermion* current $\sum_i \bar{\psi}_i \gamma^\mu \gamma^5 \psi$, where the sum is over *all* fermion species of the theory:
- The kinetic terms of the fermions contain the contorted gravitational covariant derivative which is *linear* in the contorsion.

Microscopic Leptogenesis model [2] IV

- The Bianchi identity is implemented, in the effective action, by means of a (canonically normalised) pseudoscalar Lagrange multiplier field $b(x)$; the path-integration over the torsion H -field
- The effective action is the **Chern-Simons gravity model**

$$S^{\text{eff}} = \int d^4x \sqrt{-g} \left[-\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\kappa} \partial_\mu b(x) \mathcal{K}^\mu \right] \\ + S_{\text{Dirac or Majorana}}^{\text{Free}} + \int d^4x \sqrt{-g} \left[\left(\mathcal{F}_\mu + \frac{\alpha'}{2\kappa} \sqrt{\frac{3}{2}} \partial_\mu b \right) J^{5\mu} - \frac{3\alpha'^2}{16\kappa^2} J_\mu^5 J^{5\mu} \right] + \dots, \quad (7)$$

where the \dots indicate gauge field kinetic terms, as well as terms of higher order in derivatives.

Microscopic Leptogenesis model [2] V

- \mathcal{K}^μ , an anomalous current, is defined through

$$\begin{aligned}\sqrt{-g} \left(R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - \mathbf{F}_{\mu\nu} \tilde{\mathbf{F}}^{\mu\nu} \right) &= \sqrt{-g} \mathcal{K}^\mu(\omega)_{;\mu} = \partial_\mu \left(\sqrt{-g} \mathcal{K}^\mu(\omega) \right) \\ &= 2 \partial_\mu \left[\epsilon^{\mu\nu\alpha\beta} \omega_\nu^{ab} \left(\partial_\alpha \omega_{\beta ab} + \frac{2}{3} \omega_{\alpha a}{}^c \omega_{\beta cb} \right) - 2 \epsilon^{\mu\nu\alpha\beta} \left(A_\nu^i \partial_\alpha A_\beta^i + \frac{2}{3} f^{ijk} A_\nu^i A_\alpha^j A_\beta^k \right) \right],\end{aligned}\tag{8}$$

with Latin letters i, j, k being gauge group indices.

- $J^{5\mu} \equiv \sum_i \bar{\psi}_i \gamma^5 \gamma^\mu \psi_i$
- $\mathcal{F}^d = \varepsilon^{abcd} e_{b\lambda} \partial_a e_c^\lambda$ with e_c^μ the vielbeins
- From (7) the axion coupling is $f_b^{\text{str}} = 96 \sqrt{\frac{3}{2}} \frac{\kappa}{\alpha'} \simeq 2 \times 10^2 \frac{\kappa}{\alpha'}$.
- There is also a repulsive current-current interaction

Stringy vacuum [11] I

The Chern-Simons gravity action is the key to two developments:

Emergence of restricted Einstein-Cartan like theory

Absence of term proportional to \mathcal{K}^μ leads to Einstein-Cartan theory with purely antisymmetric torsion

Running vacuum model (RVM)

\mathcal{K}^μ leads to time-evolving b (linear in t) in the presence of a gravitational condensate of chiral gravitational waves:

$$\langle R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \rangle_{\text{condensate } \mathcal{N}} = \frac{\mathcal{N}(t)}{\sqrt{-g}} \frac{1.1}{\pi^2} \left(\frac{H}{M_{\text{Pl}}} \right)^3 \mu^4 \frac{\dot{b}(t)}{M_\xi^2} \equiv n_\star \frac{1.1}{\pi^2} \left(\frac{H}{M_{\text{Pl}}} \right)^3 \mu^4 \frac{\dot{b}(t)}{M_\xi^2}, \quad (9)$$

where H is the Hubble volume and remains approximately constant during inflation.

Stringy vacuum [11] II

RVM b background: compatible with Leptogenesis model

From equations of motion (EOM) (7)

$$\partial_\mu \left(\sqrt{-g} \left[\partial^\mu b - \sqrt{\frac{2}{3}} \frac{\alpha'}{96 \kappa} \mathcal{K}^\mu \right] \right) = 0, \quad (10)$$

we deduce that

$$\partial_\mu b = \sqrt{\frac{2}{3}} \frac{\alpha'}{96 \kappa} \mathcal{K}^\mu$$

for non-zero condensate value of the temporal component of the topological current $\mathcal{K}^0 \simeq \text{constant} \neq 0$ during inflation, where isotropy and homogeneity holds, implies the solution

$$\dot{b} \equiv \dot{\bar{b}} = \frac{\alpha'}{96 \kappa} \mathcal{K}^0 \simeq \text{constant} \neq 0. \quad (11)$$

Stringy vacuum [11] III

- The background violates Lorentz and CPT symmetry
- Such an axion configuration is linear in (cosmic) time and satisfies the conformal invariance conditions in non-critical strings for a consistent string background

If the anomaly condensate ceases to exist at the end of the RVM-inflationary phase EOM implies

$$\partial_\mu \left(\sqrt{-g} \partial^\mu b \right) = 0$$

- For FLRW universe the radiation era scale factor $a(t) \sim 1/T$, with T the temperature
- Implies scaling of CPTV axion background B^0 with T :

$$B^0(T) \sim B(t_{\text{exit}}) \left(\frac{T}{T_{\text{exit}}} \right)^3$$

- The suffix “exit” denotes quantities at the exit phase (end) of the RVM inflation

Stringy vacuum [11] IV

- This provides a stringy justification of the leptogenesis model
- $T_{\text{exit}} = \frac{H^I}{2\pi}$ where H^I is the inflationary scale
- $H^I \sim 10^{-5} M_{\text{Pl}}$ from Planck data
- $B(t_{\text{exit}})$ can be identified with the constant background due to the primordial-GW-induced condensate
- The model will be shown more generally to be consistent with torsion models

See the talk of N Mavromatos for a phenomenological discussion.

Generic Einstein-Cartan theory (ECT) with quantum torsion I

Emergence of restricted Einstein-Cartan like theory

The *totally antisymmetric* KR field leads to a totally antisymmetric torsion

- What effect does relaxing this requirement have on phenomenology?

$$\mathbf{e}^a \equiv e^a_{\mu}(x) dx^{\mu}, \quad \omega^a_b \equiv \omega^a_{b\mu}(x) dx^{\mu}. \quad (12)$$

$$\mathbf{T}^a \equiv d\mathbf{e}^a + \omega^a_b \wedge \mathbf{e}^b, \quad \mathbf{R}^a_b = d\omega^a_b + \omega^a_c \wedge \omega^c_b. \quad (13)$$

- The *contorsion* 1-form \mathbf{K}^{ab} is by definition

$$\mathbf{K}^{ab} \equiv \omega^{ab} - \tilde{\omega}^{ab} \quad (14)$$

where $\tilde{\omega}^{ab}$ is the spin connection in the *absence* of torsion

Generic Einstein-Cartan theory (ECT) with quantum torsion II

- $\mathbf{T}^a = \mathbf{K}_b^a \wedge \mathbf{e}^b$.
- $S_{ECT} = \frac{1}{4\kappa^2} \int \varepsilon_{abcd} \mathcal{R}^{ab} \wedge \mathbf{e}^c \wedge \mathbf{e}^d$ gravitational part
- The torsion tensor can be decomposed into irreducible representations as follows:

$$T_{\mu\nu\rho} = \frac{1}{3} (T_\nu g_{\mu\rho} - T_\rho g_{\mu\nu}) - \frac{1}{3!} \epsilon_{\mu\nu\rho\sigma} S^\sigma + q_{\mu\nu\rho} \quad (15)$$

where $T_\beta = T^\alpha_{\beta\alpha}$, $S^\nu = \epsilon^{\alpha\beta\mu\nu} T_{\alpha\beta\mu}$ and $q^\alpha_{\beta\alpha} = 0$, $\epsilon^{\alpha\beta\mu\nu} q_{\alpha\beta\mu} = 0$

- It is S^σ which couples to the axial fermion current
- The torsion part of the quantum gravity integral

$$\begin{aligned} Z &= \int DSDb \exp \left[i \int \frac{3}{4\kappa^2} S \wedge *S - \frac{3}{4} S \wedge *J^5 + \left(\frac{3}{2\kappa^2} \right)^{1/2} b d * S \right] \\ &= \int Db \exp \left[-i \int \frac{1}{2} db \wedge *db + \frac{1}{f_b^{\text{EC}}} db \wedge *J^5 + \frac{1}{2(f_b^{\text{EC}})^2} J^5 \wedge *J^5 \right] \end{aligned} \quad (16)$$

We have performed the Gaussian integration over S in the second line.

A model enhancement I

Apart from the presence or absence of terms in \mathcal{F}_μ and \mathcal{K}_μ , there is a close similarity between the actions in (7) and (16) to the CPTV and LV leptogenesis scenario with axion backgrounds \bar{b} satisfying $\dot{\bar{b}} \simeq \text{const}$. We note the possible role of our considerations to the strong CP problem and dark matter in addition to leptogenesis

A model for Axion cosmology and BSM

$$\begin{aligned} S_{eff} = & S_0 - \frac{1}{2(f_b^{\text{EC}})^2} \int J^5 \wedge *J^5 - \frac{\alpha_{\text{QED}}}{\pi f_b^{\text{EC}}} \int bF \wedge F \\ & - \int \frac{1}{2} db \wedge *db - \frac{1}{8\pi^2} \int \left(\Theta + \frac{N_f}{f_b^{\text{EC}}} b \right) R^{ac} \wedge R_{ac} \\ & - \frac{\alpha_s}{2\pi} \int \left(\theta + \frac{N_q}{f_b^{\text{EC}}} b \right) \text{Tr}[G \wedge G] \end{aligned} \quad (17)$$

A model enhancement II

The anomaly relation is used above

$$d^* \mathbf{J}^5 = -\frac{\alpha_{QED} Q^2}{\pi} \mathbf{F} \wedge \mathbf{F} - \frac{\alpha_s N_q}{2\pi} \text{Tr}(\mathbf{G} \wedge \mathbf{G}) - \frac{N_f}{8\pi^2} \mathbf{R}^{ab} \wedge \mathbf{R}_{ab} \quad (18)$$

α_{QED} and α_s are the fine structure constants for (electromagnetic) Abelian and (quantum chromodynamic) non-Abelian gauge fields

- $N_f(N_q)$ is the number of fermion (quark only) flavours and $Q^2 = \sum_f Q_f^2$ where Q_f is the electric charge of the fermion with flavour f ;
- \mathbf{F} , \mathbf{G} and \mathbf{R} are the field strength 2-forms of QED, QCD and gravity (with torsion)

Topological terms

Θ and θ are associated with *topological* terms: a link with axion cosmology; gravitational axion couples to the Pontryagin densities of *all* the gauge interactions in the model

The Axion model I

Consider quantum fluctuations of the axion field

- Split the b field into a quantum \tilde{b} and classical part \bar{b}
- Effective action (in tensor notation) can be written as

$$\begin{aligned} S_{eff} = & \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R + \frac{8}{3} \partial_\sigma \bar{b} \partial^\sigma \bar{b} - \Omega \right) + S_{free} - \frac{1}{\kappa} \int d^4x \sqrt{-g} \partial_\mu \bar{b} J^{5\mu} \\ & - \frac{3\kappa^2}{16} \int d^4x \sqrt{-g} J_\mu^5 J^{5\mu} + \frac{8}{3\kappa^2} \int d^4x \sqrt{-g} \partial_\sigma \bar{b} \partial^\sigma \tilde{b} \\ & + \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \frac{8}{3} \partial_\sigma \tilde{b} \partial^\sigma \tilde{b} - \frac{1}{\kappa} \int d^4x \sqrt{-g} \partial_\mu \tilde{b} J^{5\mu} \end{aligned} \quad (19)$$

- S_{free} is the action of a free fermion in a gravitational background

The Axion model II

- Added a potential cosmological constant, Ω , for the sake of generality.
- This background decouples from \tilde{b} since the $\bar{b}\tilde{b}$ term vanishes (provided $\int d^4x \partial^0 \bar{b}$ vanishes).
- $\partial_\mu J^{5\mu}{}_{;\mu} = 2 i m_N \bar{N} \gamma^5 N, \quad m_N \neq 0$
- $\mathcal{S}_{\text{int}}^{\text{b-J}^5} = -\frac{1}{f_b} \int d^4x \sqrt{-g} \tilde{b}(x) J^{5\mu}{}_{;\mu},$
- $f_b^{\text{str}} = 96 \sqrt{\frac{3}{2}} \frac{\kappa}{\alpha'} \simeq 2 \times 10^2 \frac{\kappa}{\alpha'}, \quad f_b^{\text{EC}} = \sqrt{\frac{8}{3}} \frac{1}{\kappa} \approx 4 \times 10^{18} \text{ GeV}$
- The viability of the model depends on the effective potential for \tilde{b} and vacuum stability

Summary of comparative phenomenology of the string inspired and Einstein-Cartan models

- **RVM:**
 - $\langle \frac{N_f}{8\pi^2} \mathbf{R}^{ab} \wedge \mathbf{R}_{ab} \rangle \simeq \text{constant}$,
 - $\square b \simeq \frac{1}{f_b^{\text{EC}}} \langle \frac{N_f}{8\pi^2} \mathbf{R}^{ab} \wedge \mathbf{R}_{ab} \rangle \simeq \text{constant}$
 - $\ddot{b}(t) + 3H_I \dot{b} \simeq \text{constant}$,
 - $\dot{b} \simeq \frac{1}{3H_I f_b^{\text{EC}}} \langle \frac{N_f}{8\pi^2} \mathbf{R}^{ab} \wedge \mathbf{R}_{ab} \rangle \simeq \text{constant}$ up to a term proportional to $\exp(-3H_I t)$
- Background remains undiluted until the end of inflation and into the early radiation era.
- After leptogenesis in **ECT**, no primordial gravitational wave dominance and no condensate
- There is an ordinary cosmic evolution for the (massless) field b in the radiation era.

Effective axion potential and metastability in the CPTV leptogenesis model [2] I

Our theory is **UV** complete.

Effective theory

- The KR axion has a kinetic term and a Yukawa-like interaction only.
- The sterile neutrino has a heavy mass m_N
- At energies much lower than m_N , can determine effective potential for the KR axion

Effective potential

$$V_{\text{eff}}[b] = a_2 \left(W_1 \left[\tilde{b} \right] \right)^2 + a_4 \left(W_1 \left[\tilde{b} \right] \right)^4 + a_6 \left(W_1 \left[\tilde{b} \right] \right)^6$$

where $W_1 \left[\tilde{b} \right] = \frac{2m_N}{f_b} \tilde{b}$, $a_2 = 4m_N^2 \left(1 - \frac{1}{2} \ln \frac{m_N^2}{\mu^2} \right)$, $a_4 = \frac{5}{6} - \ln \frac{m_N^2}{\mu^2}$, $a_6 = -\frac{1}{3m_N^2}$

Effective axion potential and metastability in the CPTV leptogenesis model [2] II

- m_N plays the rôle of an ultraviolet cutoff
- In the regime $\mu \lesssim m_N$ run the scale μ lower to m_b
- $m_b^2 = 32 \frac{m_N^4}{f_b^2} \left(1 - \frac{1}{2} \ln\left[\frac{m_N^2}{\mu^2}\right]\right)$.
- For the string model $m_b \sim 0.14$ eV and so a possible dark matter candidate
- The potential is unstable at the sixth power of b
- Metastability analysis, similar to that used for the Higgs particle, leads to the following:
 - stability of the vacuum is determined by the ratio r of the torsion-induced-axion coupling (which depends on the string mass scale) to m_N
 - metastability for $r \gg 1$
 - highly unstable for $r \leq 1$

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