# CP Violating Asymmetries in the Decay of Charmed Mesons in two Pseudoscalar Mesons. 

## Franco Buccella

Istituto Nazionale di Fisica Nucleare, Sezione di Roma I, Roma, Italy

Pietro Santorelli
Dipartimento di Fisica Ettore Pancini, Napoli, I-80126, Italy Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Napoli
I-80126, Italy

## Summary

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## Summary

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## Summary

After the challenging research, which has lead the Collaboration LHCb to find more than five standard deviations for the difference between the asymmetries into charged pions or kaons $\Delta \mathrm{A}_{\mathrm{CP}}=$ $0.154 \pm 0.029 \times 10^{-2}$, the same collaboration has recently proposed for the CP violating asymmetries for the decays of the charmed neutral pseudoscalar, $D^{0}$, into two charged pions, two charged kaons and two $K_{S}$ 's the following values :
$\operatorname{Asy}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)=(0.22 \pm 0.057) \times 10^{-2}$
$\operatorname{Asy}\left(D^{0} \rightarrow K^{+} K^{-}\right)=(0.077 \pm 0.057) \times 10^{-2}$
$\operatorname{Asy}\left(D^{0} \rightarrow K_{S} K_{S}\right)=-3.1 \pm 1.2 \pm 0.4 \pm 0.2 \times 10^{-2}$
The last result differs by the measurement performed by Belle,

$$
0.0 \pm 1.5 \pm 0.2 \times 10^{-2}
$$

while the average is:
$\operatorname{Asy}\left(D^{0} \rightarrow K_{S} K_{S}\right)_{a v e}=(-1.9 \pm 1.0) \times 10^{-2}$.

## LOWER LIMITS ON THE B AMPLITUDES FROM THE MEASURED CP VIOLATING ASYMMETRIES

The CP violating asymmetries are given by:

$$
\operatorname{Asy}\left(D^{0} \rightarrow f\right)=\operatorname{Im}\left[-\frac{2 V_{c b} V_{u b}^{*}}{V_{c s} V_{u s}^{*}-V_{c d} V_{u d}^{*}}\right] \frac{\operatorname{Im}\left[2 A\left(D^{0} \rightarrow f\right) B^{*}\left(D^{0} \rightarrow f\right)\right]}{\left|A\left(D^{0} \rightarrow f\right)\right|^{2}}
$$

with:

$$
\operatorname{Im}\left[-\frac{2 V_{c b} V_{u b}^{*}}{V_{c s} V_{u s}^{*}-V_{c d} V_{u d}^{*}}\right]=5.736 \times 10^{-4}
$$

Since the moduli of the $\Delta U=1$ amplitudes are fixed by the precise measurements of the branching ratios, we can establish lower limits for the moduli of the total $\Delta U=0$ amplitudes within the standard model, since they are proportional to $V_{c b} V_{u b}^{*}$, from the values of the recent measurements of LHCb.

## LOWER LIMITS ON THE B AMPLITUDES FROM THE MEASURED CP VIOLATING ASYMMETRIES

The moduli of $B\left(D^{0} \rightarrow f\right)$ amplitudes should obey a lower limit proportional to the modulus of the CP violating asymmetry $\operatorname{Asy}\left(D^{0} \rightarrow\right.$ f) :

$$
|A s y(D \rightarrow f) A(D \rightarrow f)| \leq \operatorname{Im}\left[\frac{4 V_{c b} V_{u b}^{*}}{\left(V_{c s} V_{u s}^{*}-V_{c d} V_{u d}^{*}\right)}\right]|B(D \rightarrow f)|
$$

From the moduli of the amplitudes $A\left(D^{0} \rightarrow f\right)$ fixed by the branching ratios, the values of the asymmetries measured by LHCb and the inequality just written we deduce the following lower limits for the moduli of the $B$ amplitudes:
$2.1 \pm 1.1 \leq\left|B\left(D^{0} \rightarrow K_{S} K_{S}\right)\right|$
$0.49 \pm 0.332 \leq\left|B\left(D \rightarrow \pi^{+} \pi^{-}\right)\right|$
$0.315 \pm 0.233 \leq\left|B\left(D \rightarrow K^{+} K^{-}\right)\right|$
with a lower limit for $\left|B\left(D \rightarrow K_{S} K_{S}\right)\right|$ larger than for $\mid B\left(D \rightarrow \pi^{+} \pi^{-} \mid\right.$ and $\left|B\left(D \rightarrow K^{+} K^{-}\right)\right|$:

## LOWER LIMITS ON THE B AMPLITUDES FROM THE MEASURED CP VIOLATING ASYMMETRIES

We compare these lower limits with the moduli of the A amplitudes, which are fixed by the precisely measured branching ratios, respectively :
$\left|A\left(D^{0} \rightarrow K_{S} K_{S}\right)\right|=0.08$
$\left|A\left(D \rightarrow \pi^{+} \pi^{-}\right)\right|=0.26$
$\left|A\left(D \rightarrow K^{+} K^{-}\right)\right|=0.47$
The central value of the lower limit for $\left|B\left(D^{0} \rightarrow K_{S} K_{S}\right)\right|$ is not only larger than $\left|A\left(D^{0} \rightarrow K_{S} K_{S}\right)\right|$, which arises from the final state interaction, since the unrescattered amplitude vanishes for $S U(3)_{f}$ symmetry, but also than $\left|A\left(D \rightarrow \pi^{+} \pi^{-}\right)\right|$and $\left|A\left(D \rightarrow K^{+} K^{-}\right)\right|$

## CP VIOLATING ASYMMETRIES ARE EXPECTED

The necessity of large strong phases to account for the experimental branching ratios for the non-leptonid decays of $D$ particles into two pseudoscalar mesons has been advocated since many years with the consequence that CP violating asymmetries of order $10^{-3}$ are expected. In fact for the decays of $D^{0}$ particles into a pair of pions or kaons there are rwo contributions proportional to different combinations of the products of two elements of the Cabibbo-Kobayshi matrix elements with different phases and the presence of strong phases follows by the triangular isospin identities for the Cabibbo allowed amplitudes of $D^{0}$ and $D^{+}$into $\bar{K} \pi$ and Cabibbo forbidden into two pions. Also, as we shall see, the large $S U(3)_{f}$ violations for the branching ratios of the decays into two mesons may be explained by the final state interaction for the presence of a nonet of scalar mesons in the region of the $D$ masses. So CP violating asymmetries, smaller than the one found for the decay into two $K_{S}$, are expected.

## THE AMPLITUDES FOR NON-LEPTONIC CHARM DECAYS

The amplitudes for the Cabibbo allowed, Cabibbo first forbidden and Cabibbo double forbidden decays are proportional to the matrix elements of the $\Delta U=1$ operators:

$$
\begin{gather*}
\bar{u}_{L}(x) \gamma_{\mu} d_{L}(x) \bar{s}_{L}(x) \gamma^{\mu} c_{L}(x)  \tag{1}\\
\bar{u}_{L}(x) \gamma_{\mu} s_{L}(x) \bar{s}_{L}(x) \gamma^{\mu} c_{L}(x)-\bar{u}_{L}(x) \gamma_{\mu} d_{L}(x) \bar{d}_{L}(x) \gamma^{\mu} c_{L}(x)  \tag{2}\\
\bar{u}_{L}(x) \gamma_{\mu} s_{L}(x) \bar{d}_{L}(x) \gamma^{\mu} c_{L}(x) \tag{3}
\end{gather*}
$$

and to $V_{c s} V_{u d}^{*},\left(V_{c s} V_{u s}^{*}-V_{c d} V_{u d}^{*}\right) / 2$ and $V_{c d} V_{u s}^{*}$, respectively.
We define $A(D \rightarrow P P)$ the matrix elements of the operators defined in Eqs.(1-3).

## $S U(3)_{f}$ SYMMETRY FOR CHARM DECAYS

Asuming $S U(3)$ flavor symmetry all the amplitudes depend on three reduced matrix elements, since the $D$ particles transform as a $\overline{3}$, the operators just writen as a combination of the 15 and $\overline{6}$ and the pairs of mesons as a combination of the 8 and 27 representations and $\overline{6} \times \overline{3}=8+1 \overline{0}, 15 \times \overline{3}=8+10+27$

## FINAL STATES WITH PIONS AND (OR) KAONS

Considering only states with pions and kaons, we have several final states:

1) for the Cabibbo allowed decays:
$D^{0} \rightarrow K^{-}+\pi^{+}$
$D^{0} \rightarrow \bar{K}^{0}+\pi^{0}$
$D^{+} \rightarrow \bar{K}^{0}+\pi^{+}$
$D_{s}^{+} \rightarrow \bar{K}^{0}+K^{+}$
2) for the Cabibbo double forbidden decays:
$D^{0} \rightarrow K^{+}+\pi^{-}$
$D^{0} \rightarrow K^{0}+\pi^{0}$
$D^{+} \rightarrow K^{0}+\pi^{+}$
$D^{+} \rightarrow K^{+}+\pi^{0}$
$D_{s}^{+} \rightarrow K^{0}+K^{+}$

# FINAL STATES WITH PIONS AND (OR) KAONS 

2) for the Cabibbo forbidden decays:
$D^{0} \rightarrow \pi^{+}+\pi^{-}$
$D^{0} \rightarrow \pi^{0}+\pi^{0}$
$D^{0} \rightarrow K^{+}+K^{-}$
$D^{0} \rightarrow K^{0}+\bar{K}^{0}$
$D^{+} \rightarrow \pi^{0}+\pi^{+}$
$D^{+} \rightarrow K^{+}+(\bar{K})^{0}$
$D_{s}^{+} \rightarrow \pi^{0}+K^{+}$
$D_{s}^{+} \rightarrow K^{0}+\pi^{+}$

Seventeen amplitudes and three reduced $S U(3)_{f}$ matrix elements .

## LARGE VIOLATIONS OF $S U(3)_{f}$ SYMMETRY

Many $S U(3)_{f}$ relationship are badly violated . For istance, while $S U(3)_{f}$ predicts : $A\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)=-A\left(D^{0} \rightarrow K^{+} K^{-}\right)$ and $A\left(D^{0} \rightarrow K^{0} \bar{K}^{0}\right)=0$
the experimental branching ratios are $1.454 \pm 0.026 \times(10)^{-3}$ for the charged pions $4.06 \pm 0.06 \times(10)^{-3}$ for the charged kaons and $0.141 \pm 0.005 \times(10)^{-3}$ for a pair of $K_{S}$.

## FINAL STATE INTERACTION FOR THE DECAY AMPLITUDES

These large $s U(3)_{f}$ violations may be explained by the final state interaction induced by a nonet of scalar mesons with masses around the mass of the $D$ particles. Indeed an important final state interaction is implied by the isospin relationship for the Cabibbo allowed and first forbidden amplitudes, namely :
$\sqrt{2} A\left(D^{+} \rightarrow \pi^{+} \pi^{0}\right)=A\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)-A\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right)$
and
$A\left(D^{+} \rightarrow \pi^{+} \bar{K}^{0}\right)-A\left(D^{0} \rightarrow K^{-} \pi+\right)=\sqrt{2} A\left(D^{0} \rightarrow \pi^{0} \bar{K}^{0}\right)$

## CP ASYMMETRIES EXPECTED

In fact the moduli of the three quantities related by the isospin sum rules describe a triangle, which implies a large difference between the phases for the final isospin, $\frac{1}{2}$ and $\frac{3}{2}$ for the Cabibbo allowed amplitudes, 0 and 2 for the Cabibbo forbidden . The presence of these large strong phases opens the possibility of $C P$ violating asymmetries for the Cabibbo forbidden decays .

Alain Le Youanc, Luis Oliver and Claude Raynal Enrico Franco, Koshiba and Luca Silvestrini
Maurizio Lusignoli, Gennaro Miele, Alessandra Pugliese, Pietro Santorelli and F.B.

## PREDICTIONS FOR THE EXPERIMENTAL BRANCHING RATIOS

To reproduce the experimental branching ratios beyond the three reduced $S U(3)_{f}$ matrix elements, six angles have been introduced to describe the final state interacion for the different isospin channels, two parameters for the non conservation of the strangness changing vector currents in the annichilation graphs and finally two "ad hoc" $S U(3)_{f}$ violating parameters, since the precision of the experimental branching ratios is better than the one expected for $S U(3)_{f}$. In conclusion 13 parameters for 17 branching ratios.

## PREDICTIONS FOR THE EXPERIMENTAL BRANCHING RATIOS

In a previous work the branching ratios have been predicted with a reasonable agreement with the 2018 data and for each decay the experimental measurement is compared with the prediction and the 2022 data .
P. Ayan, Pietro Santorelli and F. B.

For the Cabibbo allowed data $\left(\times 10^{-3}\right)$ :

| Channel | Experiment 2018 | Previous fit | Experiment 2022* |
| :--- | :---: | :---: | :---: |
| $D^{0} \rightarrow K^{-} \pi^{+}$ | $38.9 \pm 0.4$ | $38.96 \pm 0.32$ | $39.47 \pm 0.30$ |
| $D^{0} \rightarrow K_{S} \pi^{0}$ | $11.9 \pm 0.4$ | $12.29 \pm 0.21$ | $12.40 \pm 0.22$ |
| $D^{0} \rightarrow K_{L} \pi^{0}$ | $10 \pm 0.7$ | $9.73 \pm 0.21$ | $9.76 \pm 0.32$ |
| $D^{+} \rightarrow K_{S} \pi^{+}$ | $14.7 \pm 0.8$ | $15.80 \pm 0.29$ | $15.62 \pm 0.31$ |
| $D^{+} \rightarrow K_{L} \pi^{+}$ | $14.6 \pm 0.5$ | $14.37 \pm 0.52$ | $14.6 \pm 0.5$ |
| $D_{s}^{+} \rightarrow K_{S} K^{+}$ | $15 \pm 0.5$ | $14.67 \pm 0.41$ | $14.50 \pm 0.35$ |

[^0]
## PREDICTIONS FOR THE EXPERIMENTAL BRANCHING RATIOS

Cabibbo single forbidden $\left(\times 10^{-3}\right)$ :

| Channel | Experiment 2018 | Previous fit | Experiment 2022* |
| :--- | :---: | :---: | :---: |
| $D^{0} \rightarrow K^{+} K^{-}$ | $3.97 \pm 0.07$ | $4.064 \pm 0.04$ | $4.08 \pm 0.06$ |
| $D^{0} \rightarrow K_{S} K_{S}$ | $0.170 \pm 0.07$ | $0.168 \pm 0.04$ | $0.141 \pm 0.005$ |
| $D^{0} \rightarrow \pi^{+} \pi^{-}$ | $1.407 \pm 0.025$ | $1.448 \pm 0.019$ | $1.454 \pm 0.024$ |
| $D^{0} \rightarrow \pi^{0} \pi^{0}$ | $0.822 \pm 0.025$ | $0.816 \pm 0.0299$ | $0.826 \pm 0.025$ |
| $D^{+} \rightarrow \pi^{+} \pi^{0}$ | $1.17 \pm 0.06$ | $1.235 \pm 0.033$ | $1.241 \pm 0.005$ |
| $D^{+} \rightarrow K^{+} K_{S}$ | $2.89 \pm 0.16$ | $3.164 \pm 0.056$ | $3.21 \pm 0.165$ |
| $D_{s}^{+} \rightarrow K^{+} \pi^{0}$ | $0.63 \pm 0.21$ | $1.41 \pm 0.15$ | $0.74 \pm 0.05$ |
| $D_{s}^{+} \rightarrow K_{S} \pi^{+}$ | $1.22 \pm 0.06$ | $1.24 \pm 0.06$ | $1.09 \pm 0.05$ |

* and 2023 update


## PREDICTIONS FOR THE EXPERIMENTAL BRANCHING RATIOS

Cabibbo double forbidden $\left(\times 10^{-3}\right)$ :

| Channel | Experiment 2018 | Previous fit | Experiment 2022* |
| :--- | :---: | :---: | :---: |
| $D^{0} \rightarrow K^{+} \pi^{-}$ | $0.1385 \pm 0.0027$ | $0.145 \pm 0.003$ | $0.150 \pm 0.007$ |
| $D^{+} \rightarrow K^{+} \pi^{0}$ | $0.181 \pm 0.027$ | $0.151 \pm 0.013$ | $0.208 \pm 0.021$ |

* and 2023 update


## VARIATIONS OF THE AGREEMENT OFTHE PREVIOUS FIT THE 2022 DATA

For the Cabibbo forbidden branching ratios in varius cases the agreement with the new data improves with important exception for the decay $D^{0} \rightarrow K_{S} K_{S}$ and $D_{s}^{+} \rightarrow K^{+} \pi^{0}$, where the better precision of the measurement enhances the disagreement between theory and experiment.

## VARIATIONS OF THE PARAMETERS FROM THE PREVIOUS FIT TO THE ONE TO THE 2022 DATA

It is instructive to compare the variation of the parameters in the two fits to gain confidence on the stability of our predictions:

Т : $0.424 \rightarrow 0.423$
C: $-0.211 \rightarrow-0.210$
$\phi: 0.435 \rightarrow 0.413$
$\delta_{1}:-1.085 \rightarrow-1.08$
$\delta_{0}:-2.373 \rightarrow-2.43$
$\delta_{0}^{\prime}:-0.840 \rightarrow-0.9$
$\delta_{\frac{1}{2}}:-1.632 \rightarrow-1.62$
$K: 0.100 \rightarrow 0.081$
These parameters are relevant for $D^{0}$ decays.

## VARIATIONS OF THE PARAMETERS FROM THE PREVIOUS FIT TO THE ONE TO THE 2022 DATA

These parameters concern together with the others the decays of the charged $D$ 's.
$\Delta:-0.026 \rightarrow-0.026$
$K^{\prime}:-0.153 \rightarrow-0.187$
$\epsilon_{\delta}: 0.067 \rightarrow 0.062$
k: -0.036 $\rightarrow-0.036$
$k^{\prime}:-0.063 \rightarrow-0.033$

## ANALYSIS OF THE NUMBERS JUST WRITTEN

For the branching ratios of the $D^{0}$ the agreement of the predictions of the previous work with the 2022 data is better than for the 2018 data and gets even better slightly modifyng the values of the parameters, exspecially for the Cabibbo forbidden decays, for which the CP violating asymmetries have been measured. This is not surprising, since to get the four branching ratios into pions and kaons and the branching ratio for the decay $D^{+} \rightarrow(\pi)^{+}(\pi)^{0}$ six parameters, two combinations of $S U(3)_{f}$ reduced matrix elements and four angles related to the final state interaction, are involved. For the decays into $K^{+}(\pi)^{0}$ of $D_{s}^{+}$and $D^{+}$, simply and doubly Cabibbo forbidden respectively, the better precision and the change with the previous measurement bring to an important deviation, which may cast doubts on the following predictions of the $C P$ violating asymmetries.

## THE $\Delta U=0$ AMPLITUDES)

There are two sources of $\Delta U=0$ amplitudes, both proportional to $V_{c b} V_{u b}^{*}$ and to the matrix elements of the operators:
$\bar{u}_{L}(x) \gamma_{\mu} s_{L}(x) \bar{s}_{L}(x) \gamma^{\mu} c_{L}(x)+\bar{u}_{L}(x) \gamma_{\mu} d_{L}(x) \bar{d}_{L}(x) \gamma^{\mu} c_{L}(x)$
$\left[\bar{u}(x) \gamma_{\mu} \lambda_{a} u(x)+\bar{d}(x) \gamma_{\mu} \lambda_{a} d(x)+\bar{s}(x) \gamma_{\mu} \lambda_{a} s(x)\right] \bar{u}_{L}(x) \gamma^{\mu} \lambda_{a} c_{L}(x)$
The first operator transforms under $S U(3)_{f}$ as a combination of the 15 and the 3 representations, the second as a 3.

Since $15 \times \overline{3}=27+10+8$ and $\overline{3} \times 3=8+1$, the $\Delta U=0$ matrix elements depend of four parameters, two of them, the ones involving the 15 are fixed by the $\Delta U=1$ amplitudes.

## THE $\Delta U=0$ AMPLITUDES

The $\Delta U=0$ non rescattered amplitudes depend on three parameters, associated to the $U=0$ combinations of the 1,8 and 27 , but the third one is known from the $U=1$ amplitudes and gives contributions proportional to ( $T+C$ ). The other two parameters, defined in the previous work, have been called $\widetilde{P}$ and $\Delta_{4}$ and depend on the contributions of both the operators previously defined, the pseudopenguin and the penguin.

## PREDICTIONS FOR THE CP VIOLATING ASYMMETRIES

From the $A$ and $B$ amplitudes into a pair of pions or kaons of $D^{0}$ :
$A\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)=-0.079+0.244 i$
$A\left(D^{0} \rightarrow K^{+} K^{-}\right)=0.371-0.287 i$
$A\left(D^{0} \rightarrow K^{0} \bar{K}^{0}\right)=0.125+0.01 i$
$A\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right)=0.135+0.244 i$
$B\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)=(-0.683-0.308 i) \tilde{P}+(-0.3745-0.29 i) \Delta_{4}+$ $(0.438+0.2077 i)(T+C)$
$B\left(D^{0} \rightarrow K^{+} K^{-}\right)=(-0.102-0.593 i) \tilde{P}+(-0.14-0.807 i) \Delta_{4}+$ (0.069-0.1615i) $(T+C)$
$B\left(D^{0} \rightarrow K^{0} \bar{K}^{0}\right)=(-0.582+0.285 i) \tilde{P}+(0.341-0.037 i) \Delta_{4}+$ (0.561-0.19i) $(T+C)$
$B\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right)=(-0.683-0.308 i) \widetilde{P}+(-0.3745-0.029 i) \Delta_{4}+$ $(-0.562+0.203 i)((T+C)$

## PREDICTIONS FOR THE CP VIOLATING ASYMMETRIES

We predict the asymmetries as a function of $\widetilde{P}$ and $\Delta_{4}$ :

$$
\operatorname{Asy}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)=\left(1.686 \tilde{P}+0.820 \Delta_{4}-0.23\right) \times 10^{-3}
$$

$$
\operatorname{Asy}\left(D^{0} \rightarrow K^{+} K^{-}\right)=\left(-0.648 \tilde{P}-0.883 \Delta_{4}-0.02\right) \times 10^{-3}
$$

$$
\operatorname{Asy}\left(D^{0} \rightarrow K_{S} K_{S}\right)=\left(1.486 \tilde{P}-0.043 \Delta_{4}-0.21\right) \times 10^{-3}
$$

$$
\operatorname{Asy}\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right)=\left(2.48 \tilde{P}+1.21 \Delta_{4}+0.45\right) \times 10^{-3}
$$

## PREDICTIONS FOR THE CP VIOLATING ASYMMETRIES

One has the following constraints for the four asymmetries:
$\operatorname{Asy}\left(D^{0} \rightarrow K_{S} K_{S}\right)=1.338\left[\operatorname{Asy}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)+\operatorname{Asy}\left(D^{0} \rightarrow K^{+} K^{-}\right)\right]+$ $0.015\left[\operatorname{Asy}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)-\operatorname{Asy}\left(D^{0} \rightarrow K^{+} K^{-}\right)\right]-4 \times 10^{-4}$
$\operatorname{Asy}\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right)=0.415\left[\operatorname{Asy}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)-\operatorname{Asy}\left(D^{0} \rightarrow K^{+} K^{-}\right)\right]+$ $0.305 A \operatorname{sy}\left(D^{0} \rightarrow K_{S} K_{S}\right)+0.41 \times 10^{-3}$

Eq.(31) implies for $\operatorname{Asy}\left(D^{0} \rightarrow K_{S} K_{S}\right)=[(1.338+0.015)(22 \pm ?)+$ $1.323(7.7 \pm 5.7)] \times 10^{-4}=3.6 \times 10^{-3}$
consistent with the Belle result, but about two and half standard deviations from LHCb1. To agree with the central values for the asymmetries for the decay of $D^{0}$ into two charged mesons one needs high values for $\tilde{P}$ and $\Delta_{4}$, respectively 2.79 and -3.03 .

## CONSEQUENCES OF THE ZWEIG SELECTION RULE

If we assume the Zweig selection rule, which implies that also the $\Delta U=0$ non rescattered amplitude $B\left(D^{0} \rightarrow K^{0} \bar{K}^{0}\right)$ vanishes as well as the $\Delta U=1$, which violates $S U(3)_{f}$ symmetry, $\Delta_{4}$ vanishes and the four CP violating asymmetries depend only on $\widetilde{P}$ and the combination $T+C$, which has been fixed by the fit to the branching ratios. The Zweig selection rule implies that the non rescattered matrix elements of the operators:
$\bar{u}_{L}(x) \gamma_{\mu} s_{L}(x) \bar{s}_{L}(x) \gamma^{\mu} c_{L}(x)-\bar{u}_{L}(x) \gamma_{\mu} d_{L}(x) \bar{d}_{L}(x) \gamma^{\mu} c_{L}(x)$
and
$\bar{u}_{L}(x) \gamma_{\mu} s_{L}(x) \bar{s}_{L}(x) \gamma^{\mu} c_{L}(x)+\bar{u}_{L}(x) \gamma_{\mu} d_{L}(x) \bar{d}_{L}(x) \gamma^{\mu} c_{L}(x)$
are equal for the processes, where a $s \bar{s}$ pair is created, and opposite when a $d \bar{d}$ pair is created.

## CONSEQUENCES OF THE ZWEIG SELECTION RULE

The Zweig rule also implies that the non rescattered amplitude $D^{0} \rightarrow K^{0} \bar{K}^{0}$ vanishes at the lowest QCD order. Indeed the Zweig rule accounted for the narrow width of the $J / \psi$ particle. This property can be imposed by relating the matrix elements of the 3's obtained by the products $6 \times \overline{3}$ or $\overline{3} \times \overline{3}$ to the 8 or the 1 , respectively, to the reduced matrix elements of the operator.

$$
\bar{u}_{L}(x) \gamma_{\mu} s_{L}(x) \bar{s}_{L}(x) \gamma^{\mu} c_{L}(x)-\bar{u}_{L}(x) \gamma_{\mu} d_{L}(x) \bar{d}_{L}(x) \gamma^{\mu} c_{L}(x)
$$

## CONSEQUENCES OF THE ZWEIG SELECTION RULE

As an example it implies for the non rescattered amplitudes that:
$B_{n r}\left(D^{0} \rightarrow K \bar{K}\right)=A_{n r}\left(D^{0} \rightarrow K \bar{K}\right)$
$B_{n r}\left(D^{0} \rightarrow \pi \pi\right)=-A_{n r}\left(D^{0} \rightarrow \pi \pi\right)$
This allows to deduce the matrix elements of the pseudopenguin $\Delta U=0$ amplitudes from the $\Delta U=1$ ones. The Zweig selection rule implies for the parameters introduced in the previous work $\Delta_{3}=\Delta_{4}=0$.

## CONSEQUENCES OF THE ZWEIG SELECTION RULE

So $\tilde{P}$ is fixed to be $0.8 \pm 0.15$ by the measurement of $\Delta \mathrm{A}_{\mathrm{CP}}=$ $1.54 \pm 0.29 \times 10^{-3}$ and one predicts:
$A_{\text {Asy }}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)=1.09 \pm 0.2 \times 10^{-3}$
$A_{A s y}\left(D^{0} \rightarrow K^{+} K^{-}\right)=(-0.45 \pm 0.09) \times 10^{-3}$
$A_{A s y}\left(D^{0} \rightarrow K_{S} K_{S}\right)=(1 \pm 0.2) \times 10^{-3}$
$A_{\text {Asy }}\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right)=1.35 \pm 0.17 \times 10^{-3}$
all at order $10^{-3}$

## CONSEQUENCES OF THE ZWEIG SELECTION RULE

So the assumption of the Zweig selection rule allows us to deduce the three asymmetries measured in terms of the measurement by LHCb of the difference between the asymmetries in charged mesons in agreement with the result by Belle on $A_{C P}\left(D^{0} \rightarrow K_{S} K_{S}\right)$, but about two standard deviations with the three recent measurements of the asymmetries by LHCb. So with different values for $\Delta_{4}$ we get different predictions for the three measured asymmetries with deviations in all the cases. In particular, if the high negative value of $A_{a s y}\left(D^{0} \rightarrow K_{S} K_{S}\right)$ should be confirmed with a better precision, our parametrization our parametrization of the weak amplitudes should be revised.

## COMPARISONS OF THE TWO SCENARIOS

Taking the $\Delta U=1$ amplitudes, which descrive the experimental branching ratios and assuming $S U(3)_{f}$ symmetry for the unrescattered $\Delta U=0$ amplitudes and the isospin invariant final state interactions, we have assumed for the $\Delta U=1$ amplitudes, the CP violating asymmetries are given in terms of two parameters, implying a relationship between the three measured asymmetries by LHCb, which is violated by about three standard deviations from the measurement found in LHCb, but is consistent with Belle.

## COMPARISONS OF THE TWO SCENARIOS

Assuming Zweig selection rules for the unrescattered $\Delta U=0$ amplitudes, one obtains the CP violating asymmetries in terms of only one parameter, which is fixed by $\Delta \mathrm{A}_{\mathrm{CP}}=1.54 \pm 0.29 \times 10^{-3}$ and implies for $\sum \mathrm{A}_{\mathrm{CP}}=\operatorname{Asy}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)+\operatorname{Asy}\left(D^{0} \rightarrow K^{+} K^{-}\right)$
and $\operatorname{Asy}\left(D^{0} \rightarrow K_{S} K_{S}\right)$ values positive and small, two standard deviations from the ones measured by LHCb and from the combined result by LHCb and Belle, but consistent with the result by Belle for $\operatorname{Asy}\left(D^{0} \rightarrow K_{S} K_{S}\right)$. It is interesting also to see the values of the parameters in the two cases : in the first case we find $\widetilde{P}=2.97$ and $\Delta_{4}=-3.03$, while in the second one $\tilde{P}=0.8$ and $\Delta_{4}=0$, and the pseudopenguin and the penguin contributions to the $\Delta A_{C P}$ are similar and the prediction for $\operatorname{Asy}\left(D^{0} \rightarrow K_{S} K_{S}\right)$ is consistent with Belle. Smaller values for the parameters are obtained for the parameters in the second case at the price of the two standard deviation from $\Sigma A_{C P}$ the sum of the asymmetries for the charged pions and kaons measured by LHCb.

## CHANGES OF THE AMPLITUDES INDUCED BY THE FINAL STATE INTERACTIONS

It is instructive to see how the final state interaction modifies the non rescattered amplitudes to report how the $A$ amplitudes for the four channels considered are modified and to make a similar work for the $B$ amplitudes in the two scenario described.

$$
\begin{aligned}
& A\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right):-0.424 \rightarrow-0.079+0.24 i \\
& A\left(D^{0} \rightarrow K^{+} K^{-}\right): 0.424 \rightarrow 0.371-0.287 i \\
& A\left(D^{0} \rightarrow K^{0} \bar{K}^{0}\right): 0 \rightarrow 0.125-+0.01 i \\
& A\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right):-0.213 \rightarrow 0.134+0.24 i
\end{aligned}
$$

## CHANGES OF THE AMPLITUDES INDUCED BY

 THE FINAL STATE INTERACTIONS$$
\begin{aligned}
& B_{2}\left(D^{0} \rightarrow \pi^{+} p i\right): 2.97 \rightarrow-0.8+0.008 i \\
& B_{2}\left(D^{0} \rightarrow K^{+} K^{\prime}\right): 2.97 \rightarrow-0.136+0.65 i \\
& B_{2}\left(D^{0} \rightarrow K^{0} \bar{K}^{0}\right):-3.03 \rightarrow-2.642-+0.118 i \\
& B_{2}\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right): 2.657 \rightarrow-1.013+0.008 i \\
& B_{1}\left(D^{0} \rightarrow \pi^{+} p i\right): 0.8 \rightarrow--0.453+0.202 i \\
& B_{1}\left(D^{0} \rightarrow K^{+} K^{2}\right): 0.8 \rightarrow 0.065-0.508 i \\
& B_{1}\left(D^{0} \rightarrow K^{0} \bar{K}^{0}\right): 0 \rightarrow-0.355+0.182 i \\
& B_{1}\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right): 0.587 \rightarrow 0.666+0.202 i
\end{aligned}
$$

## COMPARISONS OF THE TWO SCENARIOS

If we fix the two parameters to reproduce the measurements by LHCb of the asymmetries for the decay of $D^{0}$ into a pair of charged mesons, we predict :
$A_{\text {Asy }}\left(D^{0} \rightarrow K_{S} K_{S}\right)=(3.6) \times 10^{-3}$
small and positive, while the measurement by LHCb gives a value large and negative. By assuming the Zweig selection rule, we fix the only parameter, $\tilde{P}$ to the value found by LHCb for $\Delta \mathrm{A}_{\mathrm{CP}}=$ $1.54 \pm 0.29 \times 10^{-3}$,
and predict :
$A_{\text {Asy }}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)=1.09 \pm 0.2 \times 10^{-3}$
$A_{A s y}\left(D^{0} \rightarrow K^{+} K^{-}\right)=(-0.45 \pm 0.09) \times 10^{-3}$
$A_{C P}\left(D^{0} \rightarrow K_{S} K_{S}\right)=(1 \pm 0.2) \times 10^{-3}$
$A_{C P}\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right)=1.35 \pm 0.17 \times 10^{-3}$
About tho standard deviations from the three measurement by LHCb and consistent with the measurement by Belle of $A_{A s y}\left(D^{0} \rightarrow\right.$ $\left.K_{S} K_{S}\right)=(0 \pm 1.5 \pm 0.2) \times 10^{-2}$

## COMPARISONS OF THE TWO SCENARIOS

It is istructive to compare the moduli of the $B$ amplitudes in the two scenarios with the moduli of the $A$ amplitudes for the four decays of $D^{0}$.

Channel -A- $\left|B_{2}\right| \mid B_{1}$
$(\pi)^{+}(\pi)^{-} 0.260 .80 .49$
$K^{+} K^{-} 0.470 .980 .51$
$K^{0} \bar{K}^{0} 0.132 .80 .39$
$(\pi)^{0}(\pi)^{o} 0.281 .30 .7$
which shows that to reproduce the scenario in agreement with $\sum A_{C P}$ measured by LHCb, one needs high values for the moduli of the $B$ amplitudes, while by imposing the Zweig selection rule the values of the moduli of the $A$ and $B$ amplitues are not so different.

The large values for $B_{2}$, exspecially 2.8 disfavor the scenario with two free parameters if the parametrization proposed is right.

## CONCLUSION

It is difficult with the parametrization proposed to reproduce the large negative value found by LHCb for the CP violating asymmetry for the decay of $D^{0}$ into two $K_{S}$ 's and the sum of the asymmetries into two charged mesons, Should the experimental results confirmed with a better precision, the parametrization proposed for the $\Delta U=$ ! and 0 amplitudes should be revised. Indeed Shaktar and Sony are able to predict a large negative asymmetry for $D^{0} \rightarrow K_{S} K_{S}$ induced by the presence of two isoscalar resonances with masses around $D^{0}$ mass.


[^0]:    * and 2023 update

