Effective theories with Celestial Duals

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Based on 2111.11356 and 2206.08322 with Jorge Mago, Lecheng Ren, Marcus Spradlin and Anastasia Volovich

Introduction

- Celestial holography offers an exciting new perspective on scattering amplitudes.
- Exciting insights and developments by studying the "holomorphic sector".
- Problems in non trivially including dynamics for both helicities.
- Deformations by higher dimension operators forces us to deal with negative helicity particles.
- Associativity of Celestial OPEs in such theories imposes constraints on the theory.
- Plan for the talk:
  OPE associativity in the soft sector - Analysis using symmetry algebras
  OPE associativity in the hard sector - Analysis using scattering amplitudes.

Celestial amplitudes

- We can embed the celestial sphere in the light cone via the parametrization

$$p_i^\mu = \omega_i \, \epsilon_i \left\{1 + z_i \, \bar{z}_i, \; z_i + \bar{z}_i, \; i \left(z_i - \bar{z}_i\right), 1 - z_i \, \bar{z}_i\right\}$$

- Celestial amplitudes involving massless external particles are Mellin transforms

$$\tilde{\mathcal{A}}_n \left(\{\Delta_i, z_i, \bar{z}_i, s_i\}\right) = \int \frac{d\omega_1}{\omega_1} \dots \frac{d\omega_n}{\omega_n} \omega_1^{\Delta_1} \dots \omega_n^{\Delta_n} \mathcal{A}_n \left(\{\omega_i, z_i, \bar{z}_i, s_i\}\right)$$

- It transforms as a correlation function of primaries

$$\tilde{\mathcal{A}}_n \left(\Delta_i, \frac{a z_i + b}{c z_i + d}, \frac{\bar{a} \bar{z}_i + \bar{b}}{\bar{c} \bar{z}_i + \bar{d}}, s_i\right) = \prod_{i=1}^{n} \left[\left(c z_i + d\right)^{\Delta_i + s_i} \left(\bar{c} \bar{z}_i + \bar{d}\right)^{\Delta_i - s_i}\right] \tilde{\mathcal{A}}_n \left(\Delta_i, z_i, \bar{z}_i, s_i\right)$$

Pasterski, Shao, Strominger

- We can identify

$$\tilde{\mathcal{A}}_n \left(\{\Delta_i, z_i, \bar{z}_i, s_i\}\right) = \langle \mathcal{O}_{h_1, \bar{h}_1} \left(z_1, \bar{z}_1\right) \dots \mathcal{O}_{h_n, \bar{h}_n} \left(z_n, \bar{z}_n\right) \rangle$$

$$h_i = \frac{\Delta_i + s_i}{2}, \qquad \bar{h}_i = \frac{\Delta_i - s_i}{2}, \qquad s_i \longrightarrow \text{helicity of particle } i$$

- In this parametrization,

$$\lambda_i = \epsilon_i \sqrt{2\omega_i} \begin{pmatrix} 1 \\ z_i \end{pmatrix} \qquad \text{and} \qquad \tilde{\lambda}_i = \sqrt{2\omega_i} \begin{pmatrix} 1 \\ \bar{z}_i \end{pmatrix},$$

$$\text{where } p_i^\mu \sim \lambda_i \tilde{\lambda}_i$$

- The Lorentz invariants are

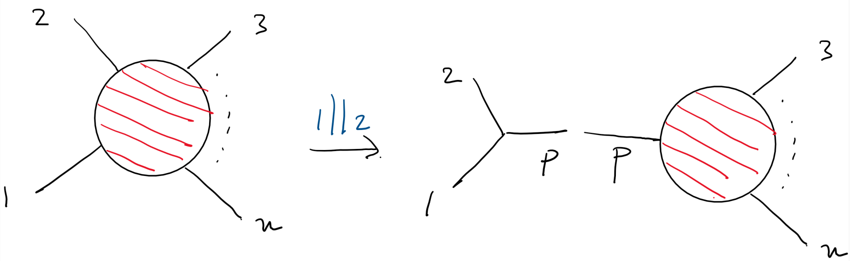
$$\langle ij \rangle = det\left(\lambda_i \lambda_j\right) \qquad [ij] = det\left(\tilde{\lambda}_i \tilde{\lambda}_j\right)$$

$$2p_i \cdot p_j = -\langle ij \rangle [ij] = 2\epsilon_i \epsilon_j \omega_i \omega_j \, z_{ij} \, \bar{z}_{ij}$$

- As $z_i \longrightarrow z_j$, the insertions $\mathcal{O}_{h_i,\bar{h}_j}\left(z_i, \bar{z}_i\right), \mathcal{O}_{h_j,\bar{h}_j}\left(z_j, \bar{z}_j\right)$ approach each other.
- OPE is controlled by the collinear limit.

Collinear limits of scattering amplitudes

- Amplitudes factorize on poles $\sum_i p_i \longrightarrow 0$



$$\mathcal{A}_n\left(1^{s_1}, \ldots n^{s_n}\right) \xrightarrow{1||2} \mathcal{A}_3\left(1^{s_1}, 2^{s_2}, I^{-s_I}\right) \times \frac{1}{(p_1+p_2)^2} \mathcal{A}_{n-1}\left(I^{s_I}, 3^{s_3}, 4^{s_4}, \ldots n^{s_n}\right)$$

- This determines the leading singular behavior of the OPE

OPE from collinear limits

- Three point amplitudes involving massless particles are completely fixed by Poincaré invariance.
- If $s_1 + s_2 + s_3 > 0$, "anti-holomorphic"

$$\mathcal{A}_3\left(1^{s_1}, 2^{s_2}, 3^{s_3}\right) = \kappa_{s_1,s_2,s_3}\,[12]^{s_1+s_2-s_3}[23]^{s_2+s_3-s_1}[31]^{s_3+s_1-s_2}.$$

- If $s_1 + s_2 + s_3 < 0$, "holomorphic"

$$\mathcal{A}_3\left(1^{s_1}, 2^{s_2}, 3^{s_3}\right) = \kappa_{s_1,s_2,s_3}\,\langle 12\rangle^{s_3-s_1-s_2}\langle 23\rangle^{s_1-s_2-s_3}\langle 31\rangle^{s_2-s_1-s_3}.$$

- A three point amplitude with $s_1 + s_2 - s_I = p + 1$ yields a term

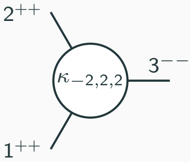
$$\mathcal{O}_{h_1,\bar{h}_1}(z_1,\bar{z}_1)\,\mathcal{O}_{h_2,\bar{h}_2}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}^{p}}{z_{12}}\,C_p\left(\bar{h}_1,\bar{h}_2\right)\mathcal{O}_{h_1+h_2-1,\bar{h}_1+\bar{h}_2+p}(z_2,\bar{z}_2) + \ldots$$

$$\begin{aligned} C_p\left(\bar{h}_1,\bar{h}_2\right) &= \kappa_{s_1,s_2,-s_I}\,B\left(\Delta_1 - s_1 + p, \Delta_2 - s_2 + p\right) \\ &= \kappa_{s_1,s_2,-s_I}\,B\left(2\bar{h}_1 + p, 2\bar{h}_2 + p\right) \end{aligned}$$

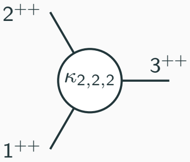
Adamo, Bu, Casali, Fan, Fotopoulos, Himwich, Jiang, Pate, Raclariu, Sharma, Singh, Strominger, Taylor, Yuan

OPE of gravitons

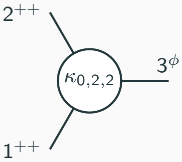
- OPE of two positive helicity gravitons



$$= \kappa_{2,2,-2} \frac{[12]^6}{[23]^2[13]^2}$$



$$= \kappa_{2,2,2} [12]^2[23]^2[13]^2$$



$$= \kappa_{0,2,2} [12]^4$$

$$\begin{aligned}\mathcal{O}^{+2}_{\Delta_1}(z_1,\bar{z}_1)\,\mathcal{O}^{+2}_{\Delta_2}(z_2,\bar{z}_2) \sim \frac{1}{z_{12}}\Big[&\kappa_{2,2,-2}B(\Delta_1-1,\Delta_2-1)\,\bar{z}_{12}\mathcal{O}^{+2}_{\Delta_1+\Delta_2}(z_2,\bar{z}_2)+\dots\\ &\kappa_{2,2,0}B(\Delta_1+1,\Delta_2+1)\,\bar{z}^3_{12}\mathcal{O}^{0}_{\Delta_1+\Delta_2+2}(z_2,\bar{z}_2)+\dots\\ &\kappa_{2,2,2}B(\Delta_1+3,\Delta_2+3)\,\bar{z}^5_{12}\mathcal{O}^{-2}_{\Delta_1+\Delta_2+4}(z_2,\bar{z}_2)+\dots\Big]\end{aligned}$$

Some features of the OPE coefficients

$$\begin{aligned}\mathcal{O}^{+2}_{\Delta_1}(z_1,\bar{z}_1)\,\mathcal{O}^{+2}_{\Delta_2}(z_2,\bar{z}_2) \sim \frac{1}{z_{12}}\Big[&\kappa_{2,2,-2}B(\Delta_1-1,\Delta_2-1)\,\bar{z}_{12}\mathcal{O}^{+2}_{\Delta_1+\Delta_2}(z_2,\bar{z}_2)+\dots\\ &\kappa_{2,2,0}B(\Delta_1+1,\Delta_2+1)\,\bar{z}_{12}^{3}\mathcal{O}^{0}_{\Delta_1+\Delta_2+2}(z_2,\bar{z}_2)+\dots\\ &\kappa_{2,2,2}B(\Delta_1+3,\Delta_2+3)\,\bar{z}_{12}^{5}\mathcal{O}^{-2}_{\Delta_1+\Delta_2+4}(z_2,\bar{z}_2)+\dots\Big]\end{aligned}$$

- Each 3 point amplitude contributes one term to the OPE
- We are assuming that there are no massless higher spins
- There are only a finite number of interactions which produce singularities in the OPE
- The OPE coefficients have poles in Δ_1, Δ_2 which has a natural interpretation in terms of soft limits.

Conformally soft limits

- The criterion for the convergence of Mellin integrals is known. Nilsson, Passare
  In particular,

$$\tilde{\mathcal{A}}_n(1,2,\dots n) = \int_0^\infty \frac{d\omega_1}{\omega_1}\omega_1^{\Delta_1}\sum_{r=-1}^{\infty}\omega_1^r \mathcal{A}_n^{\prime(r)}$$

  has poles as $\Delta_1 = 1, 0, -1, -2, \dots$.

- A compact contour in ω_1 space computes the residues at these poles

$$\begin{aligned}\lim_{\Delta_1\to k}(\Delta_1 - k)\tilde{\mathcal{A}}_n &= \oint d\omega_1 \omega_1^{k-1}\mathcal{A}_n \\ &= \tilde{\mathcal{A}}_n{}^{\prime(k)} \qquad k = 1, 0, -1, -2\dots\end{aligned}$$

Donnay, Puhm, Strominger

- Celestial amplitudes have an infinite number of poles as $\Delta_i \to k$ for $k = 1, 0, -1, -2, \dots$. These are the conformally soft limits.

Comparison with energetically soft limits

$$\mathcal{O}^{+2}_{\Delta_1}(z_1,\bar{z}_1)\,\mathcal{O}^{+2}_{\Delta_2}(z_2,\bar{z}_2) \sim \frac{1}{z_{12}}\Big[\kappa_{2,2,-2}B(\Delta_1-1,\Delta_2-1)\,\bar{z}_{12}\mathcal{O}^{+2}_{\Delta_1+\Delta_2}(z_2,\bar{z}_2)+\dots$$
$$\kappa_{2,2,0}B(\Delta_1+1,\Delta_2+1)\,\bar{z}_{12}^{3}\mathcal{O}^{0}_{\Delta_1+\Delta_2+2}(z_2,\bar{z}_2)+\dots$$
$$\kappa_{2,2,2}B(\Delta_1+3,\Delta_2+3)\,\bar{z}_{12}^{5}\mathcal{O}^{-2}_{\Delta_1+\Delta_2+4}(z_2,\bar{z}_2)+\dots\Big]$$

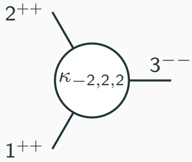
- $B(x,y)$ has poles at $x,y=0,-1,-2,\dots$
- The $\kappa_{2,2,-2}$ term starts contributing from $\Delta_1,\Delta_2=1$, $\kappa_{2,2,0}$ from $\Delta_1,\Delta_2=-1$ and $\kappa_{2,2,2}$ from $\Delta_1,\Delta_2=-3$
- The OPE of two subleading soft currents is not modified by the $\kappa_{2,2,-2}$ term.
- This hierarchy is consistent with expectations from the study of modifications to energetically soft theorems

Elvang, Jones, Naculich

- We can define single soft currents by take residues on the poles

$$H^{k,s}(z,\bar{z}) \equiv \lim_{\epsilon\to 0} \epsilon\, \mathcal{O}^{s}_{k+\epsilon}(z,\bar{z})$$

- For pure Einstein gravity



$$H^{k_1,2}(z_1,\bar{z}_1)\, H^{k_2,2}(z_2,\bar{z}_2) \sim -\frac{\kappa_{2,2,-2}}{2}\frac{\bar{z}_{12}}{z_{12}} \binom{-2\bar{h}_1 - 2\bar{h}_2 - 2}{-2\bar{h}_2 - 1} H^{k_1+k_2,2} + \dots$$

OPEs to algebras - Einstein gravity

$$H^{k_1,2}(z_1,\bar{z}_1)\,H^{k_2,2}(z_2,\bar{z}_2) \sim -\frac{\kappa_{2,2,-2}}{2}\frac{\bar{z}_{12}}{z_{12}}\begin{pmatrix}-2\bar{h}_1-2\bar{h}_2-2\\ -2\bar{h}_2-1\end{pmatrix}H^{k_1+k_2,2}+\dots$$

- The modes $H^{k,2}_{\bar{m}}(z)$ form a symmetry algebra. Guevara, Himwich, Pate, Strominger

$$H^{k,2}=\sum_{\bar{m}=\bar{h}}^{-\bar{h}}\frac{1}{\bar{z}^{\bar{m}+\bar{h}}}H^{k,2}_{\bar{m}}(z) \qquad H^{k,2}_{\bar{m}}(z)=\oint_{|\bar{z}|=\epsilon}\frac{d\bar{z}}{2\pi i}\bar{z}^{\bar{m}+\bar{h}-1}H^{k,2}(z,\bar{z}).$$

- The resulting algebra is simpler in terms of the rescaled currents

$$W^{q,2}_{\bar{m}}(z)=(-\bar{m}+q-1)!\,(\bar{m}+q-1)!H^{4-2q,2}_{\bar{m}}(z)\propto \bar{\mathbf{L}}\left[H^{4-2q,2}(z,\bar{z})\right]$$

- Moreover $\bar{h}\le\bar{m}\le-\bar{h} \implies q-1\le\bar{m}\le 1-q$
- The commutator of these currents is

$$\left[W^{q_1,2}_{\bar{m}_1},\,W^{q_2,2}_{\bar{m}_2}\right]=-\frac{\kappa_{2,2,-2}}{2}\left[\bar{m}_1(q_2-1)-\bar{m}_2(q_1-1)\right]W^{q_1+q_2-2,2}_{\bar{m}_1+\bar{m}_2}$$

Penrose, Strominger

- This is the wedge subalgebra of the loop algebra of $w_{1+\infty}$. Bakas

OPEs to algebras - deformed gravity

$$
\begin{aligned}
H^{k_1,+2}(z_1,\bar{z}_1)\,H^{k_2,+2}(z_2,\bar{z}_2) \sim &-\frac{\kappa_{2,2,-2}}{2}\frac{\bar{z}_{12}}{z_{12}}\binom{-2\bar{h}_1-2\bar{h}_2-2}{-2\bar{h}_2-1}H^{k_1+k_2,2}+\dots\\
&-\frac{\kappa_{2,2,0}}{2}\frac{\bar{z}_{12}^3}{z_{12}}\binom{-2\bar{h}_1-2\bar{h}_2-6}{-2\bar{h}_2-3}H^{k_1+k_2+2,0}+\dots\\
&-\frac{\kappa_{2,2,2}}{2}\frac{\bar{z}_{12}^5}{z_{12}}\binom{-2\bar{h}_1-2\bar{h}_2-10}{-2\bar{h}_2-5}H^{k_1+k_2+4,-2}+\dots
\end{aligned}
$$

- Define the rescaled currents

$$
W^{q,s}(z,\bar{z}) \propto \bar{\mathbf{L}}\left[H^{s+2(1-q),s}(z,\bar{z})\right]
$$

- $W^{q,2}, W^{q,0}, W^{q,-2}$ are the contributions from the positive helicity graviton, scalar and negative helicity graviton respectively.

- The commutator of these currents is

$$\left[W^{q_1,2}_{\bar{m}_1},\, W^{q_2,2}_{\bar{m}_2}\right] = -\sum_{p=1,3,5} \frac{\kappa_{2,2,p-3}}{2} N\left(q_1, q_2, \bar{m}_1, \bar{m}_2, p\right) W^{q_1+q_2-p-1,3-p}_{\bar{m}_1+\bar{m}_2}$$

Mago, Ren, AY, Volovich

where

$$N\left(q_1, q_2, \bar{m}_1, \bar{m}_2, p\right) = \sum_{x=0}^{p} (-1)^{p-x} \binom{p}{x} [\bar{m}_1 + q_1 - 1]_{p-x} [-\bar{m}_1 + q_1 - 1]_x \times [\bar{m}_2 + q_2 - 1]_x [-\bar{m}_2 + q_2 - 1]_{p-x}.$$

$$[a]_n := a(a-1)\cdots(a-n+1)$$

- This is the algebra of single soft currents. Other orders might define other structures/algebras Ball talk
- This commutator is defined to be sensitive only to the holomorphic poles.
- Restricting to $p = 1$ gives the $w_{1+\infty}$ loop algebra of the previous slide. In general, this must correspond to a deformation of $w_{1+\infty}$.

A family of W algebras

- $w_{1+\infty}$ has a one parameter deformation (μ).

$$\left[W^{q_1}_{\bar{m}_1},\, W^{q_2}_{\bar{m}_2}\right] = -\sum_{p=0}^{\infty} q^{2p} f^{q_1 q_2}_{2p}\left(\bar{m}_1, \bar{m}_2, \mu\right) W^{q_1+q_2-2p-2}_{\bar{m}_1+\bar{m}_2} + c_{q_1}\left(\bar{m}_1\right) q^{2(q_1-2)} \delta^{q_1,q_2} \delta_{\bar{m}_1+\bar{m}_2}$$

Pope, Romans, Shen

$$c_q = \frac{2^{2q-3} q!(q+2)!}{(2q+1)!!(2q+3)!!} c \qquad f^{q_1 q_2}_{2p}(\bar{m}_1, \bar{m}_2) = \frac{1}{2(2p+1)!} \phi^{q_1 q_2}_{2p}\left(\mu\right) N(q_1, q_2, \bar{m}_1, \bar{m}_2, 2p)\,,$$

$$\phi^{q_1 q_2}_{2p}\left(\mu\right) = {}_4F_3\left[\left\{-\frac{1}{2} - 2\mu, \frac{3}{2} + 2\mu, -p - \frac{1}{2}. - p\right\}, \left\{\frac{3}{2} - q_1, \frac{3}{2} - q_2, q_1 + q_2 - \frac{3}{2} - 2p\right\}, 1\right]$$

- The zeroes of $\phi^{q_1 q_2}_{2p}$ ensure that the sum in the commutator truncates.
- $\phi^{q_1 q_2}_{2p}\left(-\frac{1}{2}\right) = 1$

A family of W algebras

- Let us compare $W\left(-\frac{1}{2}\right)$ with the soft current algebra of gravitons.

$$\left[W^{q_1,2}_{\bar{m}_1},\, W^{q_2,2}_{\bar{m}_2}\right] = -\sum_{p=1,3,5} \frac{\kappa_{2,2,p-3}}{2} N\left(q_1, q_2, \bar{m}_1, \bar{m}_2, p\right) W^{q_1+q_2-p-1,3-p}_{\bar{m}_1+\bar{m}_2}$$

$$\left[W^{q_1}_{\bar{m}_1},\, W^{q_2}_{\bar{m}_2}\right] = -\sum_{p=0}^{\infty} \frac{q^{2p}}{2(2p+1)!} N(q_1, q_2, \bar{m}_1, \bar{m}_2, 2p) W^{q_1+q_2-2p-2}_{\bar{m}_1+\bar{m}_2}$$

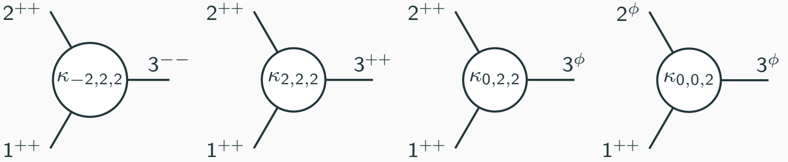
- No central charge in the soft current algebra
- There is only one free parameter q in $W\left(-\frac{1}{2}\right)$ while the soft current algebra has three $\kappa_{2,2,-2}, \kappa_{2,2,0}, \kappa_{2,2,2}$.
- The sum doesn't truncate for $W\left(-\frac{1}{2}\right)$. If we allow for massless higher even spins in the soft current algebra, this difference is eliminated.
- Remarkably, within the wedge , all algebras in the family $W(\mu)$ are equivalent.

Fairlie, Nuyts; Also see Bu, Heuveline, Skinner

- Jacobi identity fails for generic values of the κ_{s_1,s_2,s_3}.

$$\left[\left[W^{q_1,2}_{\bar{m}_1}, W^{q_2,2}_{\bar{m}_2}\right], W^{q_3,2}_{\bar{m}_3}\right] + \text{cyclic} \neq 0$$

- This implies that the OPE of soft currents derived before is not associative.
- This imposes constraints on the spectrum and the couplings.



$$\left(\kappa_{-2,2,2} - \kappa_{0,0,2}\right)\kappa_{0,2,2} = 0 \qquad 3\kappa^2_{0,2,2} = 10\kappa_{-2,2,2}\,\kappa_{2,2,2}$$

Mago, Ren, AY, Volovich

Analysis of constraints

- Graviton - Scalar sector

$$\left(\kappa_{-2,2,2}-\kappa_{0,0,2}\right)\kappa_{0,2,2}=0 \qquad \left(\kappa_{-2,2,2}-\kappa_{0,0,2}\right)\kappa_{0,0,2}=0$$

$$3\kappa_{0,2,2}^2=10\,\kappa_{-2,2,2}\,\kappa_{2,2,2}\,.$$

- Gluon-Scalar sector

$$\left(\kappa_{-1,1,1}-\kappa_{0,0,1}\right)\kappa_{0,0,1}=0 \qquad \left(\kappa_{-1,1,1}-\kappa_{0,0,1}\right)\kappa_{0,1,1}=0$$

$$\kappa_{0,1,1}^2=2\kappa_{-1,1,1}\,\kappa_{1,1,1}\,.$$

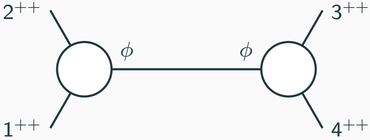
- Equations in red are consequences of gauge invariance and the equivalence principle.
- The role of the remaining constraints is less obvious.
- We can understand it better by computing amplitudes.

Amplitudes

- The simplest amplitude is the all-plus amplitude which vanishes in Einstein gravity.

$$\mathcal{A}_4\left(1^{++},2^{++},3^{++},4^{++}\right) =$$

$$+$$



$$+ \ \text{cyclic}$$

$$\begin{aligned}\mathcal{A}_4\left(1^{++},2^{++},3^{++},4^{++}\right) &= \left(10\kappa_{-2,2,2}\kappa_{2,2,2} - 3\kappa_{0,2,2}^2\right)\, s_{12}s_{13}s_{23}\,\frac{[12][23][34][41]}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \\ &= 0\end{aligned}$$

Ren, Spradlin, AY, Volovich

More amplitudes

$$\mathcal{A}\left(1^{++},2^{++},3^{++},4^{--}\right)=\kappa_{2,2,2}\kappa_{-2,-2,2}(\langle 14\rangle[13]\langle 34\rangle)^2\frac{[12][23][31]}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

$$\mathcal{A}\left(1^{++},2^{--},3^{--},4^{++}\right)=\frac{(\langle 23\rangle\,[14])^4}{s_{14}}\Big(\kappa_{2,2,2}\,\kappa_{-2,-2,-2}s_{12}\,s_{13}-\kappa_{2,2,0}\,\kappa_{-2,-2,0}+\kappa_{2,2,-2}\,\kappa_{-2,-2,2}\frac{1}{s_{12}\,s_{13}}\Big)$$

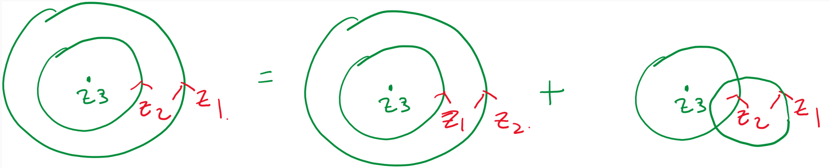
$$\mathcal{A}_4\left(1^{++},2^{++},3^{++},4^{\phi}\right)=\;0$$

- All non vanishing amplitudes involve both holomorphic ($\sum_i s_i>0$) and anti-holomorphic couplings.
- What is the connection between vanishing amplitudes and OPE associativity?

Associativity of the hard OPE

- We begin with a statement that is guaranteed to be true by associativity

$$\tilde{\mathcal{A}}_n\left(z_1,\ldots z_n\right) := \langle \mathcal{O}_{\Delta_1}(z_1,\bar{z}_1)\ldots\mathcal{O}_{\Delta_n}(z_n,\bar{z}_n)\rangle$$



$$\left[\oint_{|z_{13}|=2} \mathrm{d}z_1 \oint_{|z_{23}|=1} \mathrm{d}z_2 - \oint_{|z_{23}|=2} \mathrm{d}z_2 \oint_{|z_{13}|=1} \mathrm{d}z_1 - \oint_{|z_{23}|=2} \mathrm{d}z_2 \oint_{|z_{12}|=1} \mathrm{d}z_1\right] \tilde{\mathcal{A}}_n\left(z_1,\ldots z_n\right) = 0$$

$$\Longrightarrow \left[\operatorname*{Res}_{z_2\to z_3}\operatorname*{Res}_{z_1\to z_2} - \operatorname*{Res}_{z_1\to z_3}\operatorname*{Res}_{z_2\to z_3} + \operatorname*{Res}_{z_2\to z_3}\operatorname*{Res}_{z_1\to z_3}\right]\tilde{\mathcal{A}}_n\left(z_1,\ldots z_n\right) = 0\,.$$

- Residues in z_i are independent of ω_i

$$\left[\operatorname*{Res}_{z_2\to z_3}\operatorname*{Res}_{z_1\to z_2} - \operatorname*{Res}_{z_1\to z_3}\operatorname*{Res}_{z_2\to z_3} + \operatorname*{Res}_{z_2\to z_3}\operatorname*{Res}_{z_1\to z_3}\right]\mathcal{A}_n = 0$$

Associativity of the hard OPE

- Let us unpack the expression

$$\left[\operatorname*{Res}_{z_2\to z_3}\operatorname*{Res}_{z_1\to z_2} - \operatorname*{Res}_{z_1\to z_3}\operatorname*{Res}_{z_2\to z_3} + \operatorname*{Res}_{z_2\to z_3}\operatorname*{Res}_{z_1\to z_3}\right]\mathcal{A}_n\left(\left\{\lambda_1,\tilde{\lambda}_1\right\}^{s_1},\left\{\lambda_2,\tilde{\lambda}_2\right\}^{s_2},\dots,\left\{\lambda_n,\tilde{\lambda}_n\right\}^{s_n}\right)$$

by starting with

$$\operatorname*{Res}_{z_1\to z_2}\mathcal{A}_n = \sum_{s_I}\mathcal{A}_3\left(\tilde{\lambda}_1^{s_1},\tilde{\lambda}_2^{s_2},\tilde{\lambda}_I^{-s_I}\right)\frac{1}{2\sqrt{\omega_1\omega_2}\,[12]}\mathcal{A}_{n-1}\left(\left\{\lambda_I,\tilde{\lambda}_I\right\}^{s_I},\dots,\left\{\lambda_n,\tilde{\lambda}_n\right\}^{s_n}\right)$$

where

$$\lambda_1 = \sqrt{\frac{\omega_1}{\omega_1+\omega_2}}\lambda_I\,, \qquad \lambda_2 = \sqrt{\frac{\omega_2}{\omega_1+\omega_2}}\lambda_I\,,$$
$$\tilde{\lambda}_I = \sqrt{\frac{\omega_1}{\omega_1+\omega_2}}\tilde{\lambda}_1 + \sqrt{\frac{\omega_2}{\omega_1+\omega_2}}\tilde{\lambda}_2\,.$$

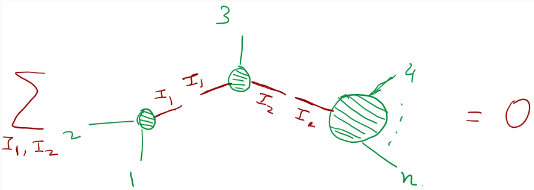
- Only anti-holomorphic three point amplitudes are non vanishing since $\lambda_1 \propto \lambda_2 \propto \lambda_I$

Associativity of the hard OPE

- The double residue

$$\operatorname*{Res}_{z_2\to z_3}\operatorname*{Res}_{z_1\to z_2}\mathcal{A}_n = \sum_{s_I}\mathcal{A}_3\left(\tilde\lambda_1^{s_1},\tilde\lambda_2^{s_2},\tilde\lambda_I^{-s_I}\right)\frac{1}{2\sqrt{\omega_1\omega_2}\,[12]}$$
$$\times\operatorname*{Res}_{z_2\to z_3}\mathcal{A}_{n-1}\left(\left\{\lambda_I,\tilde\lambda_I\right\}^{s_I},\ldots,\left\{\lambda_n,\tilde\lambda_n\right\}^{s_n}\right)$$
$$=\sum_{s_I}\mathcal{A}_3\left(\tilde\lambda_1^{s_1},\tilde\lambda_2^{s_2},\tilde\lambda_I^{-s_I}\right)\frac{1}{2\sqrt{\omega_1\omega_2}\,[12]}$$
$$\times\sum_{s_{I_2}}\frac{1}{2\sqrt{\omega_{I_1}\omega_3}\,[I_1 3]}\mathcal{A}_3\left(\tilde\lambda_{I_1}^{s_{I_1}},\tilde\lambda_3^{s_3},\tilde\lambda_{I_2}^{-s_{I_2}}\right)$$
$$\times\mathcal{A}_{n-2}\left(\left\{\lambda_{I_2},\tilde\lambda_{I_2}\right\}^{s_{I_2}},\ldots,\left\{\lambda_n,\tilde\lambda_n\right\}^{s_n}\right)$$

- The other two terms give cyclic permutations of this.

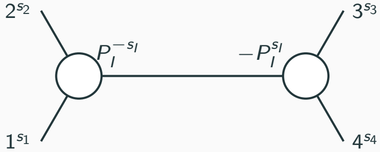


Vanishing amplitudes

- The correct statement is the part of amplitudes that can be constructed *solely* out of anti-holomorphic vertices vanish
- This can be proven rigorously using all line shift recursion relations
- When applied to four point amplitudes,

$$\mathcal{A}_4\left(\left\{\lambda_1,\tilde{\lambda}_1\right\}^{s_1},\left\{\lambda_2,\tilde{\lambda}_2\right\}^{s_2},\left\{\lambda_3,\tilde{\lambda}_3\right\}^{s_3},\left\{\lambda_4,\tilde{\lambda}_4\right\}^{s_4}\right)$$

$$=$$



$$+ \quad \text{cyclic}$$

$$= \frac{[34]}{\langle 12\rangle}\sum_{s_I}\Bigg[\mathcal{A}_3\left(\tilde{\lambda}_1^{s_1},\tilde{\lambda}_2^{s_2},\hat{\tilde{\lambda}}_I^{-s_I}\right)\frac{1}{[12]\,[34]}\mathcal{A}_3\left(\hat{\tilde{\lambda}}_I^{s_I},\tilde{\lambda}_3^{s_3},\tilde{\lambda}_4^{s_4}\right)$$

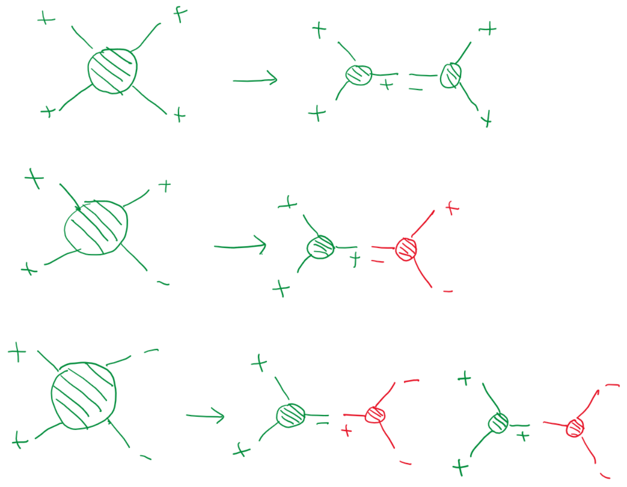
$$+\mathcal{A}_3\left(\tilde{\lambda}_1^{s_1},\tilde{\lambda}_3^{s_3},\hat{\tilde{\lambda}}_I^{-s_I}\right)\frac{1}{[31]\,[24]}\mathcal{A}_3\left(\hat{\tilde{\lambda}}_I^{s_I},\tilde{\lambda}_2^{s_2},\tilde{\lambda}_4^{s_4}\right)$$

$$+\mathcal{A}_3\left(\tilde{\lambda}_1^{s_1},\tilde{\lambda}_2^{s_4},\hat{\tilde{\lambda}}_I^{-s_I}\right)\frac{1}{[14]\,[23]}\mathcal{A}_3\left(\hat{\tilde{\lambda}}_I^{s_I},\tilde{\lambda}_3^{s_3},\tilde{\lambda}_4^{s_2}\right)\Bigg] = 0$$

Ren, Spradlin, AY, Volovich

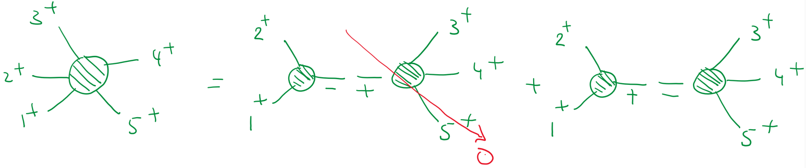
Four point amplitudes

- We can better understand the non zero amplitudes by looking at the contributing factorization channels



Higher point amplitudes

- Beyond four points, there are no amplitudes that can be constructed only from anti-holomorphic three point amplitudes
- Thus, they are generically non vanishing



- OPE associativity can be easily checked at tree-level by computing the four point amplitude of all positive helicity gravitons
- If we set $\kappa_{s_1,s_2,s_3} = 0, \quad \forall s_1 + s_2 + s_3 < 0$, then all tree-level amplitudes vanish.

Some speculations and conclusions

- The soft current algebra obtained by deforming Einstein gravity with higher derivative operators is remarkably similar to W_∞ and is identical to it (within the wedge) if we include contributions from massless higher spins
- In this case, the constraints on the couplings are

$$\kappa_{s_1,s_2,s_3} \sim \frac{(l_P)^{s_1+s_2+s_3-1}}{\Gamma(s_1+s_2+s_3)} \qquad s_1+s_2+s_3>0$$

- These constraints are known in the higher spin theory literature. They arise as consistency conditions on higher spin vertices Metsaev, Ponomarev, Skvortosov
- This algebra is also identical to the algebra obtained for Moyal deformed self dual gravity Monteiro; Bu, Heuveline, Skinner

Some speculations and conclusions

- The lack of associativity of the deformed OPE can also be stated as the failure of structure constants of the Kinematic algebra of the deformed theory to satisfy the Jacobi identity. Monteiro; Guevara
- We showed that on decoupling the holomorphic sector, all tree-level amplitudes vanish in theories with an associative celestial OPE. Is this a generalization of self-dual gravity/YM? How is it related to Moyal deformed SD gravity?
- One-loop corrections involving negative helicity gluons make the soft OPE non associative Costello, Paquette
- What is the effect of loop corrections in these theories?

Thank you for your attention!

All line shift recursion relations

- BCFW recursion fails in the presence of R^3.
- All line holomorphic shift works.

$$\hat{\lambda}_i = \lambda_i + \alpha\, w_i X \qquad i = 1, \ldots, n \qquad \text{with} \qquad \sum_{i=1}^{n} w_i \tilde{\lambda}_i = 0\,.$$

- X is an arbitrary reference spinor which drops out in the end.
- The large α behavior is

$$\hat{\mathcal{A}}_n(\alpha) \to \alpha^a \qquad 2a = 4 - n - c - \sum_{i=1}^{n} s_i\,,$$

- c is the mass dimension of the product of couplings in the amplitude.

$$\mathcal{A}_n(\alpha = 0) = \oint_{\alpha=0} \frac{\mathrm{d}\alpha}{\alpha}\, \mathcal{A}_n(\alpha) = -\sum_j \operatorname*{Res}_{\alpha=\alpha_j} \left[\frac{1}{\alpha}\mathcal{A}_n\right].$$

- No residue at infinity as long as $4 - n - c - \sum_{i=1}^{n} s_i < 0$