A twistorial higher-spin theory from the IKKT-matrix model

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Based on 2203.05436 with Harold Steinacker (University of Vienna)

Why higher spin gravity (HSGRA)?

▷ **Some of the most promising approaches toward a quantum theory of gravity involve higher-spin fields (string theory, bulk reconstruction ...)**

• The main idea: the more massless fields, the more gauge symmetries. The more gauge symmetries, the fewer counter terms.

Higher-spin symmetry $\xrightarrow{?}$ Quantum Gravity

• In the context of *AdS*/*CFT*: HSGRAs in AdS should be the dual theories of (large N) free or weakly coupled Vector Model (Ising) and Chern-Simons matter theories.

HSGRAs may help us to make CFT predictions.

If higher-spin symmetry is that good, is there a free lunch?

▷ **HSGRAs are typically blocked by No-go theorems/results and plagued by pathological non-locality issues.**

• Coleman-Mandula, Weinberg, Maldacena-Zhiboedov
• ⟨Taronna, [Sleight⟩, Erdmenger-Bakaert/({Ponomarev}], Skvortsov, Boulanger); Roiban-Tseylin ...

• **3d HSGRAs.** Typically topological with no propagating dof. and can be written in Chern-Simons form
(Blencowe+(Berhshoeff-Stelle); Pope-Townsend; Fradkin-Linetsky; Kuzenko; Henneaux-Rey; Campoleoni-Fredenhagen-Pfenninger-Theisen; Gaberdiel-Gopakumar; [Mkrtchyan-(Grigoriev-Skvortsov]-Lovrekovic) ...)

$$S = \int \omega d\omega + \frac{2}{3}\omega^3 \,.$$

• **4d conformal HSGRA.** Higher-spin extension of Weyl gravity: non-unitary due to higher derivatives in the kinetic action
(Tseylin-Segal; Bakaert-Joung-Mourad; Kuzenko ...)

• **4d chiral theories.** Non-unitary theories that have complex actions. They are quasi-topological (have propagating dof but trivial S-matrix)
(Metsaev; ["Ponomarev"-({Skvortsov}]-TT-Tsulaia)||
⟨ Sharapov-Skvortsov⟩-Sukhanov-Van Dongen)

In this talk

▷ A higher-spin gauge theory induced by the IKKT-matrix model (**HS-IKKT**) on fuzzy 4-sphere S^4_N (Sperling, Steinacker).

Our main result

A twistorial description for the (HS)-IKKT on S^4_N

Other results

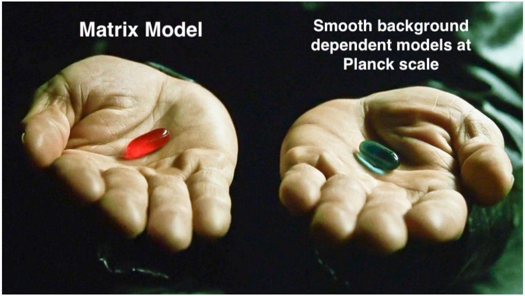
◇ Scattering amplitudes of the HS-IKKT in the flat limit.

◇ Twistor action for the self-dual YM sector of the HS-IKKT (gravHS-SDYM).

Outline

- ◇ Review of the IKKT-matrix model.
- ◇ Fuzzy S^4_N
- ◇ Twistors and higher-spin fields on fuzzy S^4_N
- ◇ A novel action of the (HS)-IKKT on fuzzy S^4_N
- ◇ Spacetime action and scattering amplitudes
- ◇ Twistor action for self-dual gauge sector of HS-IKKT

The IKKT-matrix model (1)



• The IKKT-matrix model (Ishibashi, Kawai, Kitazawa, Tsuchiya-96') is an alternative and constructive description of type IIB superstring theory.

- ▷ Obtained by dimensional reduction of 10-dim SYM theory to a point.
- ▷ Spacetime along with physical fields emerge from matrix dof.
- ▷ Similar to the Connes' approach (95') to non-commutative geometry.
- ▷ Naturally induces a HS theory on fuzzy (quantized) twistor space.

The IKKT-matrix model (2)

The IKKT has a remarkably simple action

$$S = Tr\Big([Y^I, Y^J][Y_I, Y_J] + \bar{\Psi}^{\alpha}\Gamma^I_{\alpha\beta}[Y_I, \Psi^{\beta}]\Big)\,, \quad I, J = 1, ..., 10\,.$$

Here, Y^I are $N \times N$ hermitian matrices, and Ψ are matrix-valued spinors. The action has a manifiest $SO(10)$-symmetry endowed with $\delta_{IJ} = (+, ..., +)$.

The embedding space of the IKKT is a 10-dimensional space

The action is also invariant under

$$\delta Y^I = U^{-1} Y^I U\,,$$

with U being arbitrary unitary matrix.

- Note that fields emerge as fluctuations of the background $\bar{Y}^I$

$$Y^I = \bar{Y}^I + \mathcal{A}^I\,.$$

Fuzzy S^4_N (1) - Fuzzy ambient space

Assumption: We live in 4-dimensional spacetime.

▷ We can split $\delta^{\boldsymbol{IJ}} = (\underline{\delta}^{ab}, \delta^{\mathcal{I}\mathcal{J}})$.

- For the case of S^4, $\delta^{ab} = (+,+,+,+,+)$ with further constraints as

$$Y_a Y^a = R^2 \,,$$
$$[M^{ab}, Y^c] = i(\delta^{bc} Y^a - \delta^{ac} Y^b) \,.$$

where M^{ab} are generators of $SO(5)$ that obey

$$[M^{ab}, M^{cd}] = i(M^{ad}\delta^{bc} + 3 \text{ more}) \,.$$

Since Y^a are matrices, they do not commute

$$\boxed{[Y^a, Y^b] = i\theta^{ab} = ir^2 M^{ab} \,, \quad r \text{ is a natural length scale}}$$

The non-commutativity of Y gives a fuzzy geometry

Fuzzy S^4_N (2) - The space of functions

Roughly speaking, the fuzzy S^4_N is described by $\mathfrak{so}(6)$ subjects to the additional constraint $Y_a Y^a = R_N^2$. In summary,

$$
\begin{aligned}
[M^{ab}, M^{cd}] &= i(M^{ad}\delta^{bc} + 3 \text{ more})\,, \\
[M^{ab}, Y^c] &= i(\delta^{bc}Y^a - \delta^{ac}Y^b)\,, \\
[Y^a, Y^b] &= i\theta^{ab} = i r^2 M^{ab}\,, \\
Y_a Y^a &= R_N^2 = r^2 N(N+4)/4\,.
\end{aligned}
$$

There are also self-duality constraints

$$
\epsilon_{abcde} M^{ab} M^{cd} = \frac{4}{r}(N+2)Y_e\,, \qquad \epsilon_{abcde} M^{cd} Y^e = r(N+2)M^{ab}\,.
$$

The space of functions are then

$$
\mathscr{C} = \sum f_{a(n)c(s)|b(n)} \theta^{ab} \ldots \theta^{ab} Y^c \ldots Y^c = \bigoplus_{n,s} \begin{array}{|c|c|}\hline \multicolumn{2}{|c|}{n+s} \\ \hline n & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}\,.
$$

Truncated higher-spin algebra as subspace of $\mathscr{C}$

$$
\mathfrak{ths}(\mathfrak{so}(5)) = \sum^{N} g_{a(n),b(n)} \theta^{ab} \ldots \theta^{ab} = \bigoplus_{n} \begin{array}{|c|}\hline n \\ \hline n \\ \hline\end{array}\,.
$$

Sperling-Steinacker

Fuzzy S_N^4 (3) - $\mathfrak{so}(6) \simeq \mathfrak{su}(4)$

By the following identification

$$Y^{AB} = -Y^{BA} = r^{-1} Y^a \gamma_a^{AB} , \qquad L^{AB} = L^{BA} = \frac{1}{2} M^{ab} \Sigma_{ab}^{AB} ,$$

we can go from $\mathfrak{so}(6)$ to $\mathfrak{su}(4)$ as

$$\begin{aligned} [L^{AB}, L^{CD}] &= i(L^{AC}C^{BD} + L^{AD}C^{BC} + L^{BD}C^{AC} + L^{BC}C^{AD}) , \\ [L^{AB}, Y^{CD}] &= i(Y^{AC}C^{BD} + Y^{BC}C^{AD} - Y^{AD}C^{BC} - Y^{BD}C^{AC}) , \\ [Y^{AB}, Y^{CD}] &= i(L^{AC}C^{BD} - L^{AD}C^{BC} - L^{BC}C^{AD} + L^{BD}C^{AC}) . \end{aligned}$$

Other relations

$$Y_{AB} Y^{AB} = 4R_N^2 = N(N+4) , \qquad \epsilon_{ABCD} Y^{AB} = -Y_{CD}$$

The space of functions

$$\mathscr{C} = \sum_{k,m} f_{A(k)B(2m)|C(k)} Y^{AC} \dots Y^{AC} L^{BB} \dots L^{BB} = \bigoplus_{k,m} \begin{array}{|c|c} \hline \quad k+2m \quad & \multicolumn{1}{|c|}{} \\ \hline k & \\ \cline{1-1} \end{array} .$$

Truncated higher-spin algebra

$$\mathfrak{ths}(\mathfrak{sp}(4)) = \sum_{n}^{N} g_{A(2n)} L^{AA} \dots L^{AA} = \bigoplus_{n} \begin{array}{|c|} \hline \quad 2n \quad \\ \hline \end{array} ,$$

Note that the symmetric coefficients $g_{A(2n)}$ are manifestly traceless wrt. C^{AB}.

Fuzzy S^4_N (4) - Fuzzy twistor space

The above realization of $\mathfrak{su}(4)$ allows us to make connections to fuzzy twistor space $\mathbb{CP}^3_N$ that is spanned by Z^A and its dual $\hat{Z}^A$ where

$$Z^A = (Z^1, Z^2, Z^3, Z^4) \in \mathbb{C}^4 \backslash \{0\}\,, \qquad \hat{Z}^A = \bar{Z}_B C^{AB}$$

In particular,

$$\mathbb{CP}^3_N = End(\mathcal{H}_N) = (N,0,0) \otimes (0,0,N) = \sum_n^N f_{A(n)B(n)} Z^A \dots Z^A \hat{Z}^A \dots \hat{Z}^A$$

where $\mathcal{H}_N = (0,0,N) = (0,0,1)^{\otimes_{\text{sym}} N}$ is N-particle Fock space.

The relations that describe quantized twistor space are

$$\boxed{[Z^A, \bar{Z}_B] = \delta^A_B\,, \qquad [Z^A, \hat{Z}^B] = C^{AB}}$$

Take home message:

$\mathbb{CP}^3_N$ consists of "balanced" polynomials in $Z, \hat{Z}$ with cutoff at N

Fuzz S^4_N (5) - Remarks

- What we have discussed so far is fully quantum.
- There is not yet a proper notion of geometry of spacetime from the fuzzy ambient space $\mathbb{R}^5$ because "metric" contains anti-symmetric part due to non-commutativity of coordinates addressed by the symplectic structure $\theta^{ab} = r^2 M^{ab}$. However, we can have commutative geometry in the limit $r \simeq 0$.

Semi-classical (large N) limit

In the large N limit, matrices become effectively commutative since $r \sim \frac{R}{N}$

▷ Spacetime will emergence
▷ $\mathfrak{ths}$ coincides with the $\mathfrak{hs}$ of the target space

Replacement rules (Review: 1911.03162)

Quantum/fuzzy geometry	$\mapsto$	Semi-classical/dequantized geometry
(matrix) Y^a	$\mapsto$	y^a (function)
$[\,,]$	$\mapsto$	$i\{\,,\}$
Tr	$\mapsto$	$\int \Delta$

We can parametrize $y^a = (y^\mu, y^5)$ for $\mu = 1, 2, 3, 4$ as

$$y^\mu = \frac{2R^2 x^\mu}{(R^2 + x^2)}\,, \qquad y^5 = \frac{R(x^2 - R^2)}{(R^2 + x^2)}$$

which gives the 4-sphere metric

$$ds^2 = \frac{\partial y^a}{\partial x^\mu}\frac{\partial y_a}{\partial x^\nu} dx^\mu dx^\nu = g_{\mu\nu} dx^\mu dx^\nu = \frac{4R^4 dx_\mu dx^\mu}{(R^2 + x^2)^2}$$

Twistor/spinor (1) - Basic

Let $Z^A = (\lambda^\alpha, \mu^{\alpha'})$ for $\alpha = 0, 1$ and $\alpha' = 0', 1'$. The Euclidean twistor space $\mathbb{PT}$ is defined as

$$\mathbb{PT} = \{Z^A \in \mathbb{CP}^3 | \lambda^\alpha \neq 0 \;\; \& \;\; N \neq 0\}$$

where N is an $SU(4)$-invariant number operator defined as

$$N = \bar{Z}_A Z^A = -\hat{Z}^A Z_A = \langle \lambda \hat{\lambda} \rangle + [\mu \hat{\mu}]$$

Here,

$$\langle ab \rangle = a^\alpha b_\alpha \,, \qquad [ab] = a^{\alpha'} b_{\alpha'}$$

The $Sp(4)$-invariant matrix [also known as the infinity twistor]

$$C^{AB} = \begin{pmatrix} \epsilon^{\alpha\beta} & \\ & \epsilon^{\alpha'\beta'} \end{pmatrix} \,, \qquad \epsilon^{01} = 1$$

The correspondence between twistor space and spacetime is expressed via the incident relation:

$$\mu^{\alpha'} = \widetilde{\mathbf{x}}^{\alpha\alpha'} \lambda_\alpha \qquad \Rightarrow \qquad \widetilde{\mathbf{x}}^{\alpha\alpha'} = \frac{\hat{\lambda}^\alpha \mu^{\alpha'} - \lambda^\alpha \hat{\mu}^{\alpha'}}{\langle \hat{\lambda} \lambda \rangle}$$

The relation between x and $\widetilde{\mathbf{x}}$ reads

$$x_\mu = \frac{R}{2} (\hat{\sigma}_\mu)_{\alpha\alpha'} \widetilde{\mathbf{x}}^{\alpha\alpha'}$$

Twistor/spinor (2) - $\mathbb{CP}^1 \hookrightarrow \mathbb{PT} \to S^4$

Consider the symplectic form on $\mathbb{CP}^3$

$$\Omega = d\hat{Z}^A \wedge dZ_A = (1+\widetilde{\mathbf{x}}^2)\left[D\hat{\lambda}^\alpha \wedge D\lambda_\alpha + \hat{\lambda}_\alpha \frac{d\widetilde{\mathbf{x}}^{\alpha\alpha'} \wedge d\widetilde{\mathbf{x}}^{\beta}{}_{\alpha'}}{(1+\widetilde{\mathbf{x}}^2)^2}\lambda_\beta\right],$$

Hence, we can identify $\mathbb{CP}^3$ as $\mathbb{CP}^1$-bundles over S^4, where S^4 is the base space and $\mathbb{CP}^1$ are the fibers. This can also be understood using the Hopf map following by a stereographic projection

$$\mathbb{CP}^1 \hookrightarrow \mathbb{CP}^3 \simeq S^7/_{U(1)} \to S^4\,,$$
$$Z^A \mapsto y^a := -\frac{r}{2}\hat{Z}^A(\gamma^a)_{AB}Z^B\,,$$

The above allows us to read off two important equations

$$\boxed{\langle\lambda\hat{\lambda}\rangle = \frac{NR^2}{R^2+x^2}\,, \qquad [\mu\hat{\mu}] = \frac{Nx^2}{R^2+x^2}}$$

which can be used to parametrize

$$\lambda_\alpha = \frac{R}{\sqrt{R^2+x^2}}\binom{z}{-1}\,, \qquad \hat{\lambda}_\alpha = \frac{R}{\sqrt{R^2+x^2}}\binom{+1}{\bar{z}}\,,$$

for $|z|^2+1=N$.

Twistor/spinor (3) - The measure + effective metric

On twistor space there is a natural holomorphic measure (Penrose 68')

$$D^3 Z = \epsilon_{ABCD} Z^A dZ^B dZ^C dZ^D = \frac{R^4}{(R^2+x^2)^2} \langle \lambda d\lambda \rangle \wedge [d\mu \wedge d\mu]$$

The anti-holomorphic measure is

$$D^3 \bar{Z} = \frac{R^4}{(R^2+x^2)^2} \langle \hat{\lambda} d\hat{\lambda} \rangle \wedge [d\hat{\mu} \wedge d\hat{\mu}]$$

The total measure is chosen as

$$\Delta = D^3 Z \wedge D^3 \bar{Z} = \frac{R^8}{(R^2+x^2)^4} \frac{\langle \lambda d\lambda \rangle \wedge \langle \hat{\lambda} d\hat{\lambda} \rangle}{\langle \hat{\lambda} \lambda \rangle^2} d^4 \widetilde{\mathtt{x}} = e^{2\sigma(x)} K_{\mathbb{CP}^1} d^4 \widetilde{\mathtt{x}}$$

Note that the tensorial part of the effective metric that emerges from the IKKT-matrix model can be obtained by considering

$$\{\widetilde{\mathtt{x}}^{\alpha\alpha'}, \phi\}\{\widetilde{\mathtt{x}}_{\alpha\alpha'}, \phi\} = g^{\alpha\alpha'\beta\beta'} \partial_{\alpha\alpha'}\phi \partial_{\beta\beta'}\phi$$

where at large N

$$g^{\alpha\alpha'\beta\beta'} \simeq N \epsilon^{\alpha\beta} \epsilon^{\alpha'\beta'}$$

The total metric in the large N limit is therefore

$$\boxed{G^{\alpha\alpha'\beta\beta'} = e^{2\sigma(x)} g^{\alpha\alpha'\beta\beta'} = \sqrt{g}\, g^{\alpha\alpha'\beta\beta'}}$$

Twistor/spinor (4) - Higher-spin modes

Using the incident relation, we can write $\omega(\lambda, \mu; \hat{\lambda}, \hat{\mu})$ as $\omega(\widetilde{\mathtt{x}}, \lambda, \hat{\lambda})$. Hence, the space of functions in terms of spinors $\lambda, \hat{\lambda}$ reads

$$\boxed{\mathscr{C} = \sum_n f^{\alpha(n)\beta(n)}(\widetilde{\mathtt{x}})\lambda_\alpha \ldots \lambda_\alpha \hat{\lambda}_\beta \ldots \hat{\lambda}_\beta \,.}$$

For vector modes, we have

$$\boxed{\mathscr{A} = \sum_{m=n} A^{\alpha(m)\beta(n)\gamma,\gamma'}(\widetilde{\mathtt{x}})\lambda_\alpha \ldots \lambda_\alpha \hat{\lambda}_\beta \ldots \hat{\lambda}_\beta}$$

The coefficients $f^{\alpha(n)\beta(n)}(\widetilde{\mathtt{x}})$ and $A^{\alpha(m)\beta(n)\gamma,\gamma'}$ are tensorial fields in spacetime, which for irreducible modes are totally symmetric in all $2n$ unprimed indices.

Twistor fields live in the balanced weight representation (BWR)

Spacetime fields live in the maximally unbalanced representation (MUR)

MUR: Comprises of $S(m-1,1)$ and $S(m,0)$ irrep of the Lorentz group. (Krasnov-Zhenya-TT)

Twistor/spinor (5) - Remarks

- The effective metric at the large N limit coincides with the usual one.
- Twistor/spinor formalism allows for a straightforward analysis of higher-spin fields.
- On twistor space, fields live in the BWR which constrains higher-rank tensors to increase with integers in spins.
- The Penrose transform will carry fields in the BWR on twistor space to the MUR on spacetime.

What is left?

- ~~Review of the IKKT-matrix model.~~
- ~~Fuzzy S^4_N~~
- ~~Twistors and higher-spin fields on fuzzy S^4_N~~
- A novel action of the (HS)-IKKT on fuzzy S^4_N
- Spacetime action and scattering amplitudes
- Twistor action for self-dual gauge sector of HS-IKKT

Twistor action for the IKKT (1) - Rewriting

Due to $\mathfrak{sp}(4) \simeq \mathfrak{so}(5)$, we have the following decomposition

$$Y^{AB} = P^{AB} + Q^{AB} = \begin{pmatrix} 0 & P^{\alpha\beta'} \\ -P^{\beta'\alpha} & 0 \end{pmatrix} + \begin{pmatrix} Q^{\alpha\beta} & 0 \\ 0 & Q^{\alpha'\beta'} \end{pmatrix}$$

where P are the 4-tangential modes and Q is the transverse mode of the $5th$ direction.

▷ **Small remarks:**

• The $SO(5)$ external symmetry breaks SUSY explicitly since it acts on Q.
• The other 5 coordinates of the IKKT model will be treated as the scalar fields of the internal group $SU(4)$.

Consider the following fluctuation

$$\begin{pmatrix} P^{\alpha\alpha'} \\ Q^{\alpha\beta} \end{pmatrix} = \begin{pmatrix} \mathtt{Y}^{\alpha\alpha'} \\ \mathtt{Y}_5\epsilon^{\alpha\beta} \end{pmatrix} + \begin{pmatrix} A^{\alpha\alpha'} \\ \hat{\phi}\epsilon^{\alpha\beta} \end{pmatrix},$$

• For large enough R in the semi-classical limit, all contributions associated to $\mathtt{Y}_5$ can be neglected. We refer to this limit as the semi-classical and flat (SCF) limit.
• In the SCF limit, the $\hat{\phi}$ scalar will rejoin with other 5 scalars and transform in the adjoint of $SU(4)$.

Twistor action for IKKT (2) - The action in SCF limit

The action of the IKKT in the SCF limit reads

$$S = \int \Big[\frac{i}{2} F_{\alpha\alpha} F^{\alpha\alpha} + i\{P^{\alpha\alpha'}, \phi^{\mathcal{I}\mathcal{J}}\}\{P_{\alpha\alpha'}, \phi_{\mathcal{I}\mathcal{J}}\} + 2\bar{\chi}^{\alpha}\{P_{\alpha\beta'}, \widetilde{\chi}^{\beta'}\}$$

$$+ \bar{\chi}^{\mathcal{I}}\{\phi_{\mathcal{I}\mathcal{J}}, \chi^{\mathcal{J}}\} + \widetilde{\bar{\chi}}^{\mathcal{I}}\{\phi_{\mathcal{I}\mathcal{J}}, \widetilde{\chi}^{\mathcal{J}}\} + \frac{i}{2}\{\phi^{\mathcal{I}\mathcal{J}}, \phi^{\mathcal{M}\mathcal{N}}\}\{\phi_{\mathcal{I}\mathcal{J}}, \phi_{\mathcal{M}\mathcal{N}}\}\Big],$$

where

$$F_{\alpha\alpha} F^{\alpha\alpha} = 4\{y^{\alpha}{}_{\gamma'}, A^{\alpha\gamma'}\}\{y_{\alpha\zeta'}, A_{\alpha}{}^{\zeta'}\} + 2\{y^{\alpha}{}_{\gamma'}, y^{\alpha\gamma'}\}\{A_{\alpha\zeta'}, A_{\alpha}{}^{\zeta'}\}$$

$$+ 4\{y^{\alpha}{}_{\gamma'}, A^{\alpha\gamma'}\}\{A_{\alpha\zeta'}, A_{\alpha}{}^{\zeta'}\} + \{A^{\alpha}{}_{\gamma'}, A^{\alpha\gamma'}\}\{A_{\alpha\zeta'}, A_{\alpha}{}^{\zeta'}\}\,.$$

The significance of spinor formalism:

- There is no gauge-fixing term $\{Y_a, A^a\}^2$ in the action like in 1704.02863.
- The strange term $\{y_{\alpha\kappa'}, y_{\alpha}{}^{\kappa'}\}\{A^{\alpha}{}_{\zeta'}, A^{\alpha\zeta'}\}$ inside F^2 can be absorbed by introducing an auxiliary field B.

Twistor action for IKKT (3) - First order formalism and Self-dual sector

The first order action of the IKKT in the SCF limit reads

$$S = \int \Big[B_{\alpha\alpha} F^{\alpha\alpha} + \frac{i}{2} B_{\alpha\alpha} B^{\alpha\alpha} + i\{P^{\alpha\alpha'}, \phi^{\mathcal{I}\mathcal{J}}\}\{P_{\alpha\alpha'}, \phi_{\mathcal{I}\mathcal{J}}\} + 2\bar{\chi}^{\alpha}\{P_{\alpha\beta'}, \widetilde{\chi}^{\beta'}\}$$

$$+ \bar{\chi}^{\mathcal{I}}\{\phi_{\mathcal{I}\mathcal{J}}, \chi^{\mathcal{J}}\} + \widetilde{\bar{\chi}}^{\mathcal{I}}\{\phi_{\mathcal{I}\mathcal{J}}, \widetilde{\chi}^{\mathcal{J}}\} + \frac{i}{2}\{\phi^{\mathcal{I}\mathcal{J}}, \phi^{\mathcal{M}\mathcal{N}}\}\{\phi_{\mathcal{I}\mathcal{J}}, \phi_{\mathcal{M}\mathcal{N}}\} \Big],$$

We can drop some terms to obtain the self-dual sector as in Chalmers Siegel 96'

$$S = \int \Big[B_{\alpha\alpha} F^{\alpha\alpha} + i\{P^{\alpha\alpha'}, \phi^{\mathcal{I}\mathcal{J}}\}\{P_{\alpha\alpha'}, \phi_{\mathcal{I}\mathcal{J}}\} + 2\bar{\chi}^{\alpha}\{P_{\alpha\beta'}, \widetilde{\chi}^{\beta'}\} + \widetilde{\bar{\chi}}^{\mathcal{I}}\{\phi_{\mathcal{I}\mathcal{J}}, \widetilde{\chi}^{\mathcal{J}}\} \Big],$$

which is reminiscent of self-dual $\mathcal{N} = 4$ SYM in $4d$.

> The HS-IKKT contains higher-derivative vertices, where the interactions at the lowest order are gravitational (two-derivative) type due to the Poisson brackets.

Twistor action for IKKT (4) - Higher-spin valued eigenmodes

In terms of $\mathfrak{hs}$-modes, A has the following 2 modes

$$A^{\alpha\alpha'}_{(1)} = A^{\kappa(2s)\alpha,\alpha'}\lambda^s_\kappa\hat{\lambda}^s_\kappa\,, \qquad A^{\alpha\alpha'}_{(2)} = \epsilon^{\alpha\kappa}\omega^{\kappa(2s-1),\alpha'}\lambda^s_\kappa\hat{\lambda}^s_\kappa\,.$$

Similarly, B also has two modes

$$B^{\alpha\bullet}_{(1)} = B^{\kappa(2s)\alpha\bullet}\lambda^s_\kappa\hat{\lambda}^s_\kappa\,, \qquad B^{\alpha\bullet}_{(2)} = \epsilon^{\bullet\kappa}\psi^{\kappa(2s-1)\alpha}\lambda^s_\kappa\hat{\lambda}^s_\kappa\,.$$

The gauge transformation for $\delta A^{\kappa(2s)|\alpha,\alpha'}$ reads

$$\delta_{\xi,\vartheta}A^{\kappa(2s)|\alpha,\alpha'} = \{y^{\alpha\alpha'},\xi^{\kappa(2s)}\} + \epsilon^{\kappa\alpha}\vartheta^{\kappa(2s-1),\alpha'}\,.$$

The algebraic symmetry ϑ can be used to gauge away the (unwanted) second eigenmode $A_{(2)}$.

The second mode of B plays the role of a Lagrangian multiplier and gives us the usual generalized Lorenz gauge condition of the form

$$\int \Delta\,\psi_{\alpha(2n-1)}\{y_{\alpha\alpha'},A^{\alpha(2n),\alpha'}\}\,.$$

Only the first eigenmodes of $A^{\alpha\alpha'}$ and $B^{\alpha\alpha}$ propagate!

- A has 1 dof and describes positive helicity higher-spin fields.
- B has another one and describes negative helicity fields

(Kaparulin, Lyakhovich, Sharapov 13')

Twistor action for IKKT (5) - Solutions for free EOMs

The free equation of motion for $A^{\alpha\alpha'}$ is

$$\{y^{\alpha}{}_{\alpha'}, A^{\alpha\alpha'}\} = 0\,.$$

It has the following solution

$$A^{\alpha,\alpha'}_{(1)} = A^{\kappa(2s)\alpha,\alpha'}\lambda^s_\kappa\hat{\lambda}^s_\kappa\,,$$

$$A^{\alpha(2s+1),\alpha'} = \frac{\zeta^\alpha ... \zeta^\alpha \tilde{\upsilon}^{\alpha'}}{\langle\zeta\upsilon\rangle^{2s+1}}\, e^{i\upsilon^\alpha \widetilde{\mathtt{x}}_{\alpha\alpha'}\tilde{\upsilon}^{\alpha'}}$$

The free equation of motion for the B field reads

$$2\{y^{\gamma}{}_{\alpha'}, B^{(1)}_{\gamma\alpha}\} = 0\,,$$

It is solved by

$$B^{\alpha\alpha}_{(1)} = B^{\alpha\alpha\kappa(2s)}\lambda^s_\kappa\hat{\lambda}^s_\kappa\,,$$

$$B^{\alpha(2s)} = \upsilon^\alpha ... \upsilon^\alpha e^{i\upsilon^\kappa \widetilde{\mathtt{x}}_{\kappa\kappa'}\tilde{\upsilon}^{\kappa'}}\,.$$

(Krasnov-Zhenya-TT)

Twistor action for IKKT (6) - Remarks

- The spinor formalism allows us to organize the action of the IKKT in full non-linearity without ambiguity of extra terms which usually appear in matrix-model.
- Our analysis shows that spacetime fields are massless higher-spin fields that carry 2 propagating degrees of freedom even though the original system lives on 5-dimensional ambient space.
- The solutions of free EOMs in the SCF limit coincide with the usual ones of twistor theory in flat space after integrating out the fibres.

Spacetime action (1) - The Penrose transform

The spacetime action can be obtained by integrating out all fibre coordinates

$$\int_{\mathbb{CP}^1} K \, \frac{\hat{\lambda}^{\alpha} \dots \hat{\lambda}^{\alpha} \, \lambda_{\beta} \dots \lambda_{\beta}}{\langle \hat{\lambda} \lambda \rangle^{m}} \, \delta_{m,n} = - \frac{2\pi i}{(m+1)} \epsilon_{\beta}{}^{\alpha} \dots \epsilon_{\beta}{}^{\alpha} \, .$$

The final result is

$$S = \int d^4 x \Big\langle B_{\alpha\alpha} F^{\alpha\alpha} + \frac{i}{2} B_{\alpha\alpha} B^{\alpha\alpha} + i \{ P^{\alpha\alpha'}, \phi^{\mathcal{I}\mathcal{J}} \} \{ P_{\alpha\alpha'}, \phi_{\mathcal{I}\mathcal{J}} \} + 2 \bar{\chi}^{\alpha} \{ P_{\alpha\beta'}, \widetilde{\chi}^{\beta'} \}$$
$$+ \bar{\chi}^{\mathcal{I}} \{ \phi_{\mathcal{I}\mathcal{J}}, \chi^{\mathcal{J}} \} + \widetilde{\bar{\chi}}^{\mathcal{I}} \{ \phi_{\mathcal{I}\mathcal{J}}, \widetilde{\chi}^{\mathcal{J}} \} + \frac{i}{2} \{ \phi^{\mathcal{I}\mathcal{J}}, \phi^{\mathcal{M}\mathcal{N}} \} \{ \phi_{\mathcal{I}\mathcal{J}}, \phi_{\mathcal{M}\mathcal{N}} \} \Big\rangle \, ,$$

where $\langle \rangle$ means all possible contractions between unprimed indices.

Remarks on the fuzzy twistor construction

- We do not need to refer to twistor cohomology.
- Everything is naturally higher-spin extensible.
- Interactions on twistor space are local thanks to BWR.

Spacetime action (2) - The 3-pt amplitudes

Consider the lowest order in derivatives of the Poisson bracket, we obtain the following vertex in the gauge sector after integrating out fibre coordinates

$$V_3 = \sum_{m+n=2s-2} B_{\alpha(2s)} \partial_{\alpha \bullet'} A^{\alpha(m),}{}_{\alpha'} \partial_{\alpha}{}^{\bullet'} A^{\alpha(n),\alpha'} + V^{\text{irrelevant}} .$$

▷ **The above vertex is of gravitational type.**

- In light-cone gauge, it matches with the vertex of HS-SDGRA in 2105.12782

$$V_3 = \sum_{s_2,s_3} \overline{\mathbb{P}}^2 (\Phi_{-(s_2+s_3-2)} \Phi_{+s_2} \Phi_{+s_3})$$

where $\boldsymbol{p} := (\beta, p^-, p, \bar{p})$ and $\overline{\mathbb{P}}_{ij} = \bar{p}_i \beta_j - \bar{p}_j \beta_i$ for $\boldsymbol{p}_i$ being the 4-momenta of the external field Φ_{s_i}.

- Using the plane wave solutions for A and B, we obtain

$$\mathcal{M}_{-s_1|s_2,s_3} = \delta(2 - (s_2 + s_3 - s_1)) \frac{[23]^{2s_2+2s_3-2}}{[31]^{2s_2-2}[12]^{2s_3-2}} .$$

Spacetime action (3) - Remarks

- The fuzzy twistor construction is suitable for finding higher-order vertices (quartic, quintic, ...) of higher-spin fields.
- Gauge invariance on twistor space is easier to control compared to spacetime. Hence, it makes sense to explore higher-spin theories on twistor space.

Question: Can we obtain the same V_3 from the usual twistor construction ??? (Mason et al.)

Twistor construction (1)

- Higher-spin extending the non-linear graviton construction (Penrose 72').
- The curved twistor space $\mathcal{PT} \sim_{\text{diff}} \mathbb{CP}^1 \times \mathbb{R}^4 \equiv \mathbb{PS}$.
- Assuming all perturbations to be sufficiently small
  ▷ The incident relations remain the same!
- The gravHS-SDYM action reads

$$S = \int D^3 Z \mathcal{B}(\bar{\partial}\omega + \frac{1}{2}\{\omega, \omega\}_h)\,,$$

where

$$\{\omega, \omega\}_h = \frac{\hat{\lambda}^{\alpha}\hat{\lambda}^{\alpha}}{\langle \lambda\hat{\lambda}\rangle^2}\partial_{\alpha\alpha'}\omega\partial_{\alpha}{}^{\alpha'}\omega\,.$$

- Higher-spin diffeomorphism is subtle due to the Poisson bracket on $\mathcal{PT}$

$$\delta Z = \sum_{n\in\mathbb{Z}}\{Z, \xi_n\}_h\,, \qquad \xi \in \mathcal{O}(2n-2)$$

▷ Non-gauge-invariant measure $D^3 Z$.

Twistor construction (2) - gravHS-(SD)YM

- The spacetime action for gravHS-SDYM

$$S = \int \sum_s B^{\alpha(2s)} \left[\partial_{\alpha\bullet'} A_{\alpha(2s-1),}{}^{\bullet'} + \sum_{m+n=2s-2} \{ A_{\alpha(m),\bullet'}, A_{\alpha(n),}{}^{\bullet'} \}_h \right]$$

where

$$\{a, b\}_h = \partial^{\alpha}{}_{\alpha'} a \, \partial^{\alpha\alpha'} b$$

Consider a deformation away from the chiral sector

$$S = \sum_s \int d^4x \, B_{\alpha(2s)} G^{\alpha(2s)} - \frac{1}{2} \sum_s \int d^4x \, B_{\alpha(2s)} B^{\alpha(2s)} .$$

By integrating out the B fields, we end up with a gravitational-type HS-YM (gravHS-YM) action

$$S = \frac{1}{2} \sum_s \int d^4x \, G_{\alpha(2s)} G^{\alpha(2s)} .$$

It is a gravitational extension of HS-YM (TT-21')

What all of these are about?

• Results from the light-cone gauge showed that <u>local</u> higher-spin theories with propagating degrees of freedom can exist. Moreover, they can avoid No-go theorems by having trivial/simple (holographic) S-matrix.
Brink, Bengtsson^2, Linden; Metsaev; Ponomarev-Zhenya; Zhenya-T-Tsulaia ...

BUT ...

" Light-cone is the second best ... " - Anders Bengtsson

• <u>Folklore:</u> Twistorial or world-sheet formalism for higher-spin theories are needed to construct consistent covariant higher-spin theories that can avoid No-go theorems.

Options ...

▷ twistor theory (Adamo-Hahnel-Mcloughlin,T, Steinacker-T)
▷ Free differential algebra (⟨ Sharapov-Skvortsov⟩-Sukhanov-Van Dongen)

On construction of viable higher-spin theories

- The assumption of higher-spin symmetry is crucial to avoid No-go theorems

BUT NOT ENOUGH ...

- Fronsdal approach faces No-go results for locality (Maldacena-Zhiboedov, Bekaert-Sleight-Ponomarev-Erdmenger, Taronna-Sleight, Ponomarev ...)
- Light-cone approach predicts there is no parity-invariant theory (Metsaev, Ponomarev-Zhenya)

NO FREE LUNCH CONJECTURE

- Unitary higher-spin theories are non-local.
- Local higher-spin theories are non-unitary.

Over the years, what have we learned?

- **Old Beliefs:**

▷ HSGRAs prefer (A)dS.

▷ Flat space HSGRA can only be written in light-cone gauge.

▷ The interactions can be very non-local!

▷ Flat limit is rather hard to achieve.

▷ Can there be a non-trivial scattering amplitudes for HS theories ?! ...

- **New Studies:**

▷ HS can also live on flat, self-dual and fuzzy backgrounds.

▷ Covariant actions for (grav) HS-(SD)YM and HS-SDGRA in flat space are found.

▷ Local interactions exist in both (A)dS and flat space.

▷ Recent developments show it is not the case.

▷ Twistor theory ?!
Adamo-TT to appear

Summary

Main Results:

• Twistorial action for the HS-IKKT.

• Fuzzy twistor construction and the usual twistor construction can complement each other in finding consistent higher-spin theories using BWR/MUR.

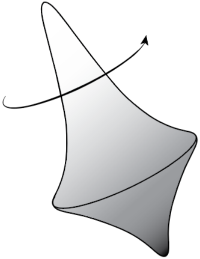
Outlook:

⋄ Study HS-IKKT on fuzzy 4-hyperboloid H^4_N using twistor formalism.

⋄ Go for higher orders in deformation!

⋄ Find more higher-spin theories from twistor space and compute their amplitudes.

⋄ And much more to come ...



More twistorial higher-spin theories are coming,
Brace yourself!