The Geometric νSMEFT

Jim Talbert, DAMTP, Cambridge

[2208.11139] J. Talbert

[2107.03951] J. Talbert, M. Trott

Building on work from: *Alonso, Corbett, Hays, **Helset**, Jenkins, Kim, Manohar, **Martin**, Paraskevas, **Trott**…*:

[1605.03602]		[2007.00565]		[2203.11976]
[1803.08001]	+	[2102.02819]	+	…
[1909.08470]		[2106.10284]		
[2001.01453]		[2107.07470]		

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Graham Garland Ross

The SMEFT, briefly:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{Q}_i^{(d)}$$

$$\bar{v}_T \equiv \sqrt{2\langle H^\dagger H \rangle}$$

$$\bar{v}_T/\Lambda << 1$$

- The SMEFT's operator basis can be expanded order by order in mass dimension. At **dim-5**, the 'Weinberg Operator' [PRL 43, '79] is the unique new-physics contribution (and accounts for neutrino masses!).

$$\mathcal{L}^{(5)} = \frac{c_{ij}^{(5)}}{2} \left(\ell_i^T \tilde{H}^\star\right) C \left(\tilde{H}^\dagger \ell_j\right) + \text{h.c.} \quad \xrightarrow{\text{EWSB}} \quad \mathcal{L} \supset -\frac{m_{\nu,k}}{2} \overline{\nu_L^{c,k}} \nu_L^k + \text{h.c.}$$

- The 'Warsaw Basis' of **[1008.4884]** is a non-redundant, complete set of **dim-6** operators.

$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{\varphi}\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{\varphi}\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

$\psi^2 \varphi^3$	
$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$
$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \widetilde{\varphi})$
$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$

$$+\,\mathcal{O}(1/\Lambda^{n\geq 3})$$

Expanding the SMEFT Lagrangian

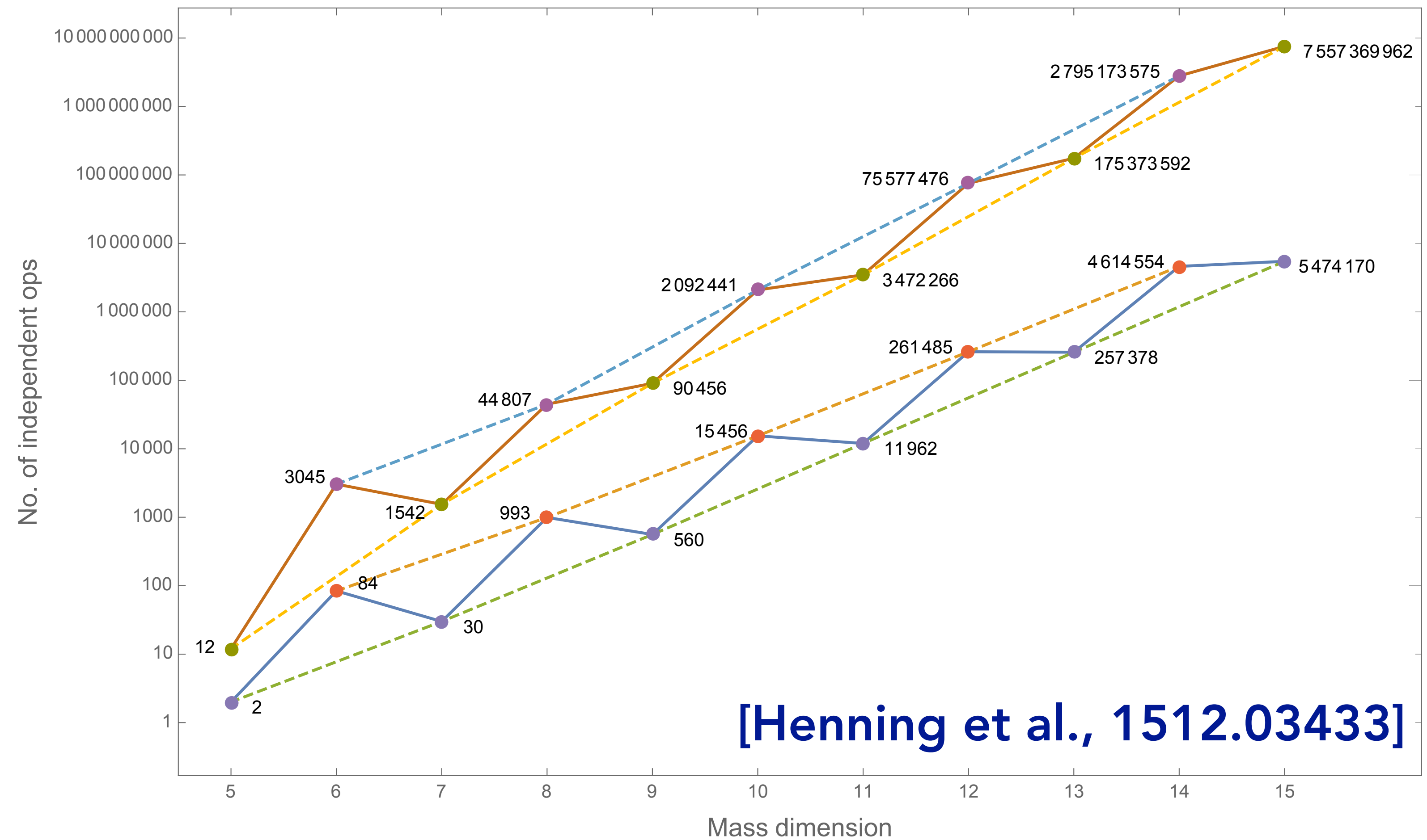
- Application of **Hilbert Series** to SMEFT shows large growth in operators order-by-order in mass dimension:

Upper lines: $n_f = 3$

Lower lines: $n_f = 1$

Higher-order corrections a priori present ⇒ mathematically **unwieldy**, and consistent fits to data **difficult** ⇒ physics conclusions can be **obscured**…

also see talks from earlier today!



$$\mathscr{L}(1/\Lambda^n) \supset \boxed{\bar{v}_T/\Lambda < 1} + \boxed{p^2/\Lambda^2 < 1} + \boxed{g_i < 1}$$

Coming from successive… **Higgs insertions…** **derivative insertions…** **loops…**

We will explore `geometric' insight that, for the first time, puts $\bar{v}_T/\Lambda$ under control at ALL ORDERS!

Outline

The geoSMEFT

The geoνSMEFT

Towards (Flavored) Phenomenology

The geoSMEFT, intuited

[1605.03602]
[1803.08001]
[1909.08470]
[geoSMEFT,2001.01453] $+ \dots$

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{Q}_i^{(d)} \quad \Rightarrow \quad \boxed{\mathcal{L}_{\rm SMEFT} = \sum_i G_i\left(I, A, \phi, \ldots\right)\, f_i}$$

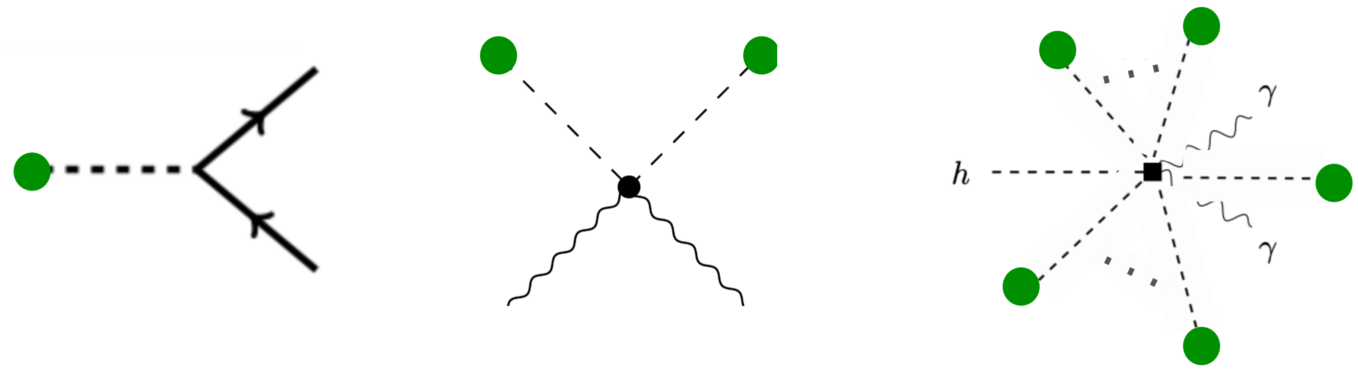
G: '*field space connections*' built from successive insertions of Higgs fields

f: operator forms composed of Lorentz-index-carrying building blocks of the Lagrangian

$$H\left(\phi_I\right) = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

$$\bar{v}_T \equiv \sqrt{2\langle H^\dagger H \rangle}$$

$D \leq 4$ $\qquad\qquad$ $D > 4$



[M. Trott KITP Talk]

vev -> fermion masses $\quad$ *-> boson masses* $\quad$ *-> geometries*

Gauge Field-Strength Terms at D=6 (e.g.)

$$\mathcal{W}^A = \{W_1, W_2, W_3, B\}$$

$$\mathcal{L}_{\rm WB} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{C_{HB}}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} + \frac{C_{HW}}{\Lambda^2} H^\dagger H W^a_{\mu\nu} W^{a,\mu\nu} + \frac{C_{HWB}}{\Lambda^2} H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu},$$

$$\boxed{\equiv -\frac{1}{4} g_{AB}(H) \mathcal{W}^A_{\mu\nu} \mathcal{W}^{B,\mu\nu}}$$

$$g_{ab} = \left(1 - 4\frac{C_{HW}}{\Lambda^2} H^\dagger H\right) \delta_{ab}$$

$$g_{a4} = g_{4a} = -2\frac{C_{HWB}}{\Lambda^2} H^\dagger \sigma_a H.$$

$$g_{44} = 1 - 4\frac{C_{HB}}{\Lambda^2} H^\dagger H$$

*Connection amounts to **metric in field space**, whose degree of curvature depends on size of $\bar{v}_T/\Lambda$. The **SM** is therefore a **FLAT** direction!*

Building up the $g_{AB}(\phi)$ metric

[2001.01453]
[2203.06771]

- Consider the higher-order operators that can connect two gauge field strengths:

Dim 6+

$$Q_{HB}^{(6+2n)} = (H^\dagger H)^{n+1} B^{\mu\nu} B_{\mu\nu},$$
$$Q_{HW}^{(6+2n)} = (H^\dagger H)^{n+1} W_a^{\mu\nu} W^a_{\mu\nu},$$
$$Q_{HWB}^{(6+2n)} = (H^\dagger H)^n (H^\dagger \sigma^a H) W_a^{\mu\nu} B_{\mu\nu}$$

Dim 8+

$$Q_{HW,2}^{(8+2n)} = (H^\dagger H)^n (H^\dagger \sigma^a H)(H^\dagger \sigma^b H) W_a^{\mu\nu} W_{b,\mu\nu}$$

That the operator forms saturate at all orders can be seen with **Hilbert Series** techniques:

	Mass Dimension				
Field space connection	6	8	10	12	14
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4

- Expanding in terms of real scalar fields, and combining into a single gauge field (A,B = 1,2,3,4), one can write

$$H(\phi_I) = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

$$\mathcal{W}^A = \{W_1, W_2, W_3, B\}$$

$$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$$

$$\begin{aligned} g_{AB}(\phi_I) = & \left[1 - 4\sum_{n=0}^{\infty}\left(C_{HW}^{(6+2n)}(1-\delta_{A4}) + C_{HB}^{(6+2n)}\delta_{A4}\right)\left(\frac{\phi^2}{2}\right)^{n+1}\right]\delta_{AB} \\ & + \sum_{n=0}^{\infty} C_{HW,2}^{(8+2n)}\left(\frac{\phi^2}{2}\right)^n \left(\phi_I \Gamma^I_{A,J}\phi^J\right)\left(\phi_L \Gamma^L_{B,K}\phi^K\right)(1-\delta_{A4})(1-\delta_{B4}) \\ & + \left[\sum_{n=0}^{\infty} C_{HWB}^{(6+2n)}\left(\frac{\phi^2}{2}\right)^n\right]\left(\phi_I \Gamma^I_{A,J}\phi^J\right)(1-\delta_{A4})\delta_{B4}, \end{aligned}$$

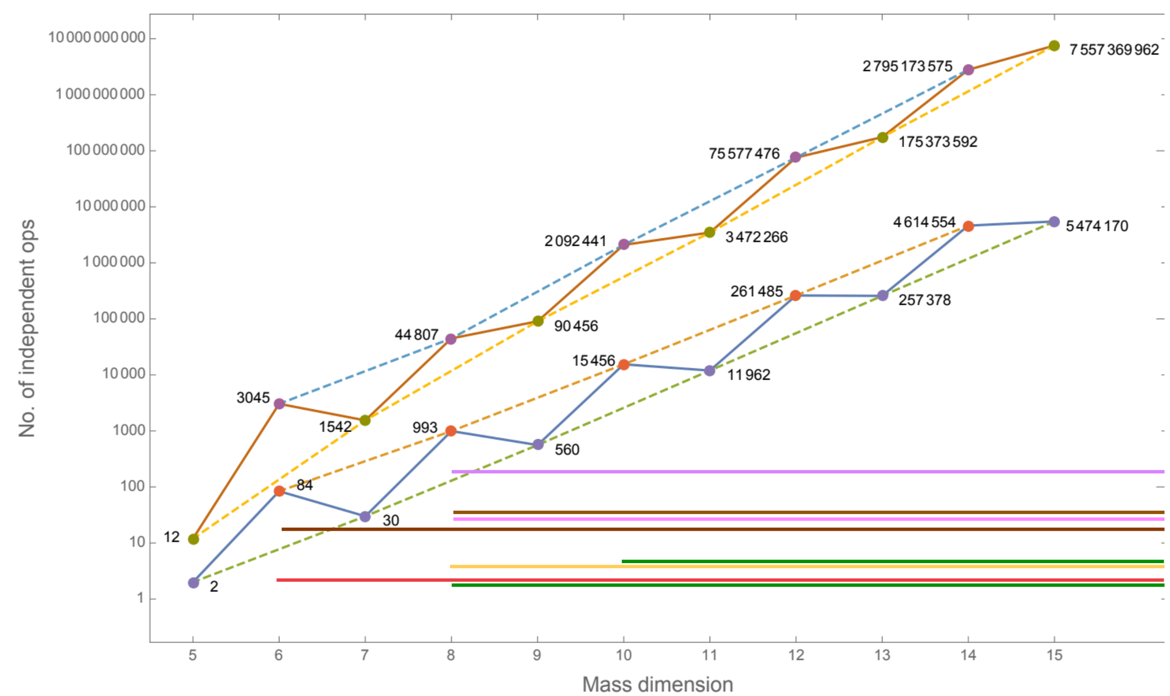
- This *field-space connection* is therefore valid at **all-orders in** $\bar{v}_T/\Lambda$! In the Higgsed phase the connection reduces to a number + emissions of *h*.

The geoSMEFT at 2 & 3 pts

[2001.01453]

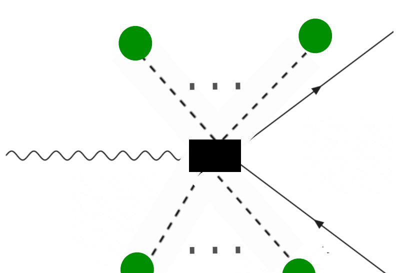
- EOM / Hilbert Series techniques allows for proof of ***all*** 2- and 3-pt field space connections!

	Mass Dimension				
Field space connection	6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$



[M. Trott KITP Talk] [1512.03433]

- Connections often *field-redefinition invariant* & yield large reduction in operators (EFT parameters)!
- Lagrangian parameters & Feynman rules obtained at all-$\bar{v}_T/\Lambda$-orders ***before*** physical amplitude calculated!
- This is a *powerful* reorganization. It allows for all-$\bar{v}_T/\Lambda$-orders amplitudes of fundamental processes:



$$\bar{\Gamma}_{Z\to\bar{\psi}\psi} = \sum_\psi \frac{N_c^\psi}{24\pi}\sqrt{\bar{m}_Z^2}|g_{\rm eff}^{Z,\psi}|^2\left(1-\frac{4\bar{M}_\psi^2}{\bar{m}_Z^2}\right)^{3/2}$$

$$g_{\rm eff}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2s^2_{\theta_Z}Q_\psi - \sigma_3)\delta_{pr} + \bar{v}_T\langle L^{\psi,pr}_{3,4}\rangle + \sigma_3\bar{v}_T\langle L^{\psi,pr}_{3,3}\rangle\right]$$

defined at all orders in $\bar{v}_T/\Lambda$!!

Consistent SMEFT Phenomenology @ dim-8: [2007.00565][2107.07470][2102.02819][2203.11976]

Towards loop calculations at all-$\bar{v}_T/\Lambda$-orders : [2106.10284]

The geoνSMEFT

Motivating (light) gauge singlets

See reviews in (e.g.)
[Kopp, **2109.00767**]
[Dasgupta & Kopp, **2106.05913**]

- Renormalizable **mass terms** for neutrinos
- Potential **dark matter** candidate (see talk from P. Di Bari later this week)
- A number of longstanding **anomalies** in neutrino oscillation physics (LSND, MiniBooNE, MicroBooNE?)

	H	q_L	ℓ_L	u_R	d_R	e_R	N
$SU(3)_c$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$	-1	0

$$\mathcal{L}_N = \overline{N}\, i\not{\partial}\, N - \frac{1}{2}\left[\overline{N}\, M\, N^c + \overline{N^c}\, M^\star\, N\right] - \overline{\ell_L}\, \tilde{H}\, Y_N\, N - \overline{N}\, Y_N^\dagger\, \tilde{H}^\dagger\, \ell_L$$

The νSMEFT

$$\mathcal{L}_{\nu\mathrm{SMEFT}} \equiv \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_N + \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{Q}_i^{(d)}$$

Complete Dim 9 basis from Li et al. **[2105.09329]**

Table calculated with **ECO! [2004.09521]**

νSMEFT Operator Counting							
Mass Dimension	5	6	7	8	9	10	11
$n_f = 1$	4	113	110	1316	1918	21540	37354
	2	29	80	323	1358	6084	25392
$n_f = 2$	14	1037	1226	14008	41720	435452	1191386
	8	343	894	4205	30102	160805	820964
$n_f = 3$	30	4659	5748	65207	334400	3513704	11347838
	18	1614	4206	20400	243944	1421263	7875572

[JT, 2208.11139]

[JT, 2208.11139]

The geoνSMEFT @ 2 and 3 pts.

$$\mathcal{L}_{\nu\mathrm{SMEFT}} \overset{!}{\equiv} \sum_i G_i\,(I, A, \phi, \ldots)\; f_i$$

$$(D^\mu \phi)^I = \left(\partial^\mu \delta^I_J - \frac{1}{2}\mathcal{W}^{A,\mu}\tilde{\gamma}^I_{A,J}\right)\phi^J$$

$$D_\mu \psi = \left[\partial_\mu + i\overline{g}_3 \mathcal{G}^\mu_{\mathcal{A}} T^{\mathcal{A}} + i\frac{\overline{g}_2}{\sqrt{2}}\left(\mathcal{W}^+ T^+ + \mathcal{W}^- T^-\right) + i\overline{g}_Z\left(T_3 - s^2_{\theta_Z} Q_\psi\right)\mathcal{Z}_\mu + iQ_\psi \overline{e}\mathcal{A}^\mu\right]\psi$$

$$\mathcal{W}^A_{\mu\nu} = \partial_\mu \mathcal{W}^A_\nu - \partial_\nu \mathcal{W}^A_\mu - \tilde{\epsilon}^A_{BC}\mathcal{W}^B_\mu \mathcal{W}^C_\nu$$

2-3 pt. interactions	N-dependence	2+ ψ each

- One quickly arrives at the list of field-space connections and composite operator forms:

  - a Yukawa operator of the form $\mathcal{Y}_N(\phi)\,\overline{N}\,\ell$,
  - a Majorana mass operator of the form $\eta_N(\phi)\,\overline{N}\,N^c$,
  - dipole-type operators of the form $d_{\psi_1\psi_2}(\phi)\,\psi_1\,\sigma_{\mu\nu}\,\psi_2\,\mathcal{W}^{\mu\nu}$ with $\psi_1\psi_2 \in \{\overline{N}\ell, \overline{e}N^c, \overline{N}N^c\}$,
  - single-derivative operators of the form $L_{\psi N}(\phi)\;(D^\mu\phi)\,\psi_1\,\gamma_\mu\,\psi_2$ with $\psi_1\psi_2 \in \{\overline{e}N, \overline{N}N, \ell\mathbb{C}N\}$

Operator saturation in $\overline{v}_T/\Lambda$

[JT, 2208.11139]

It works!

geoνSMEFT Composite Operator Saturation					
Mass Dimension	d_0	d_0+2	d_0+4	d_0+6	d_0+8
$\mathcal{Y}_N(\phi)\,\overline{N}\ell$ + h.c.	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$
$d_{N\ell}(\phi)\,\overline{N}\,\sigma_{\mu\nu}\,\ell\,\mathcal{W}^{\mu\nu}$ + h.c.	$4\,n_f\cdot n_l$	$6\,n_f\cdot n_l$	$6\,n_f\cdot n_l$	$6\,n_f\cdot n_l$	$6\,n_f\cdot n_l$
$L_{eN}(\phi)\;(D^\mu\phi)\,\overline{e}\,\gamma_\mu\,N$ + h.c.	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$
$L_{NN}(\phi)\,(D^\mu\phi)\,\overline{N}\gamma_\mu N$	n_f^2	n_f^2	n_f^2	n_f^2	n_f^2
$\eta_N(\phi)\,\overline{N}N^c$ + h.c.	$(n_f+n_f^2)$	$(n_f+n_f^2)$	$(n_f+n_f^2)$	$(n_f+n_f^2)$	$(n_f+n_f^2)$
$d_{eN}(\phi)\,\overline{e}\,\sigma_{\mu\nu}\,N^c\,\mathcal{W}^{\mu\nu}$ + h.c.	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$	$2\,n_f\cdot n_l$
$d_{NN}(\phi)\,\overline{N}\sigma_{\mu\nu}N^c\,\mathcal{W}^{\mu\nu}$ + h.c.	$\frac{1}{2}(n_f+n_f^2)$	$(n_f+n_f^2)$	$(n_f+n_f^2)$	$(n_f+n_f^2)$	$(n_f+n_f^2)$
$L_{\ell N}(\phi)\;(D^\mu\phi)\,\ell\mathbb{C}\,\gamma_\mu\,N$ + h.c.	$4\,n_f\cdot n_l$	$4\,n_f\cdot n_l$	$4\,n_f\cdot n_l$	$4\,n_f\cdot n_l$	$4\,n_f\cdot n_l$

EVEN Ops (first four rows)

ODD Ops (last four rows)

*Again confirmed using **ECO**!*

d=6+

$$\left[\mathcal{Q}^{(6+2n)}_{N\ell W}\right]_{pr} = i\,(H^\dagger H)^n\,\tilde{H}^\dagger\sigma^A\,(\overline{N}_p\,\sigma_{\mu\nu}\,\ell_r\,\mathcal{W}^{\mu\nu}_A)$$

W_a: **18** operators
B: **18** operators

d=8+

$$\left[\mathcal{Q}^{(8+2n)}_{N\ell W2}\right]_{pr} = -i\,(H^\dagger H)^n\left(\tilde{H}^\dagger\sigma^A H\right)H^\dagger\,(\overline{N}_p\,\sigma_{\mu\nu}\,\ell_r\,\mathcal{W}^{\mu\nu}_A)\,(1-\delta_{A4})$$

W_a: **18** operators
B: **0** operators

$$d_{N\ell}(\phi)_{pr} = i\sum_{n=0}^{\infty}\left[\tilde{H}^\dagger(\phi)\,\sigma^A\;\tilde{C}^{(6+2n)}_{\substack{N\ell W\\ pr}} + \frac{\tilde{\phi}_I}{2}\,(\Gamma^I_{A,J} - i\,\gamma^I_{A,J})\,\phi^J\;(1-\delta_{A4})\,H^\dagger(\phi)\,\tilde{C}^{(8+2n)}_{\substack{N\ell W2\\ pr}}\right]\left(\frac{\phi^2}{2}\right)^n$$

Field-space connections

[JT, 2208.11139]

Mass-Type Connections

$$\mathcal{Y}_N(\phi)_{pr} = -\tilde{H}^\dagger(\phi_I)\,[Y_N]^\dagger_{pr} + \tilde{H}^\dagger(\phi_I)\sum_{n=0}^{\infty} \tilde{C}^{(6+2n)}_{\substack{NH\\pr}} \left(\frac{\phi^2}{2}\right)^{n+1} \qquad \eta_N(\phi)_{pr} = -\frac{1}{2}\,[M_N]_{pr} + \sum_{n=0}^{\infty} \tilde{C}^{(5+2n)}_{\substack{NN\\pr}} \left(\frac{\phi^2}{2}\right)^{n+1}$$

Dim 4 Yukawa

Dim 4 LNV mass

Dipole-Type Connections

$$d_{eN}(\phi)_{pr} = i\sum_{n=0}^{\infty}\left[\frac{\tilde{\phi}_I}{2}\,\left(\Gamma^I_{A,J} + i\gamma^I_{A,J}\right)\,\phi^J\,\left(1-\delta_{A4}\right)\,\tilde{C}^{(7+2n)}_{\substack{eNW\\pr}}\right]\left(\frac{\phi^2}{2}\right)^n,$$

$$d_{NN}(\phi)_{pr} = i\sum_{n=0}^{\infty}\left[\sigma^A\,\delta_{A4}\,\tilde{C}^{(5+2n)}_{\substack{NNB\\pr}} - \frac{\phi_I}{2}\,\Gamma^I_{A,J}\,\phi^J\,\left(1-\delta_{A4}\right)\,\tilde{C}^{(7+2n)}_{\substack{NNW\\pr}}\right]\left(\frac{\phi^2}{2}\right)^n$$

+ $d_{NI}(\phi)$!

Derivative-Type Connections

$$L_{eN}(\phi)_{pr} = \sum_{n=0}^{\infty}\left[\frac{\phi_I}{2}\,\left(i\,\Gamma^I_{4,J} + \gamma^I_{4,J}\right)\,\tilde{C}^{(6+2n)}_{\substack{DeN\\pr}}\right]\left(\frac{\phi^2}{2}\right)^n, \qquad L_{\ell N1}(\phi)_{pr} = \sum_{n=0}^{\infty}\tilde{C}^{(7+2n)}_{\substack{D\ell N1\\pr}}\left(\frac{\phi^2}{2}\right)^{n+1}$$

$$L_{NN}(\phi)_{pr} = -\sum_{n=0}^{\infty}\left[\frac{\phi_I}{2}\,\left(i\,\Gamma^I_{4,J} + \gamma^I_{4,J}\right)\,\tilde{C}^{(6+2n)}_{\substack{DNN\\pr}}\right]\left(\frac{\phi^2}{2}\right)^n, \quad L_{\ell N2}(\phi)_{pr} = -\sum_{n=0}^{\infty}\left[\frac{\phi_I}{2}\,\left(i\,\Gamma^I_{4,J} + \gamma^I_{4,J}\right)\,\tilde{H}^\dagger\,\tilde{C}^{(7+2n)}_{\substack{D\ell N2\\pr}}\right]\left(\frac{\phi^2}{2}\right)^n$$

What I want to know…

These composite operators and connections ***define*** *the tree-level* ***geoνSMEFT****, with all of the same benefits as the geoSMEFT,* ***BUT****…*

- what do these connections look like under **Renormalization Group** evolution? Is all $\bar{v}_T/\Lambda$-orders behavior preserved?
- what kind of **geometries** do these connections describe, besides the Higgs and gauge connections, which are **metrics**? Furthermore, what more can we learn about EFTs / calculating with them as a result?
- what do (e.g.) **unitarity constraints** look like and mean in a geometric context?
- what kinds of **higher-order calculations** & **fits** are most motivated by this formalism, and the ambiguities it helps resolve?

All represent interesting points of research in this highly novel class of geometric EFTs!

Towards Phenomenology

All-$\overline{v}_T/\Lambda$-orders Feynman rules

[JT, 2208.11139]

- Geometric EFTs permit the derivation of all-$\overline{v}_T/\Lambda$-orders Feynman Rules ab initio:

***G_i**: all-orders scalar vertices*

***f_i**: momentum dependence*

$$\mathbb{F}(\mathcal{O}(G_i\, f_i)) \propto \left\langle \frac{\delta G_i}{\delta \hat{h}} \right\rangle$$

- The simplest are operators with trivial momentum dependence, e.g. mass-type operators:

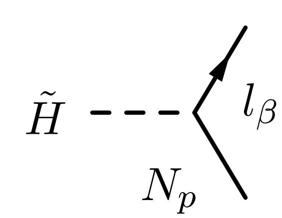
$$\{\hat{h}, \overline{N}_p, N_r^c\} = -i \left\langle \frac{\delta \eta_N(\phi)_{pr}}{\delta \hat{h}} \right\rangle = -i\sqrt{h}^{44} \sum_{n=0}^{\infty} \frac{(2n+2)}{2^{n+1}} \tilde{C}^{(5+2n)}_{\substack{NN\\pr}}\, \overline{v}_T^{2n+1}$$

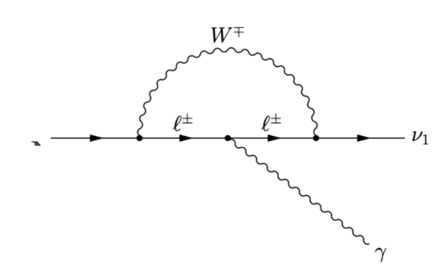
$$\{\hat{h}, \overline{N}_p, \ell_r\} = -i \left\langle \frac{\delta \mathcal{Y}_N(\phi)_{pr}}{\delta \hat{h}} \right\rangle = i\frac{\sqrt{h}^{44}}{\sqrt{2}} Y^\dagger_{N,pr} - i\frac{\sqrt{h}^{44}}{\sqrt{2}} \sum_{n=0}^{\infty} \frac{(2n+3)}{2^{n+1}} \tilde{C}^{(6+2n)}_{\substack{NH\\pr}}\, \overline{v}_T^{2n+2},$$

$$= -i\sqrt{h}^{44} \frac{\overline{M}^D_{N,pr}}{\overline{v}_T} - i\frac{\sqrt{h}^{44}}{\sqrt{2}} \sum_{n=0}^{\infty} \frac{(2n+2)}{2^{n+1}} \tilde{C}^{(6+2n)}_{\substack{NH\\pr}}\, \overline{v}_T^{2n+2}$$

- Calculate (e.g.) all-$\overline{v}_T/\Lambda$-orders, tree-level Higgs decays, $h \to \ell N$.

- More N-dependent processes can and should be pursued in this formalism!





Neutrino masses @ tree level

[JT, 2208.11139]

- As with the νSMEFT, the neutrino mass sector can be reorganized as follows:

$$\mathcal{L}_{\rm mass} = -\frac{1}{2}\left(\overline{\nu_L^c}\;\;\overline{N}\right)\cdot\begin{pmatrix}\langle\eta_\ell(\phi)\rangle & \langle\mathcal{Y}_N^T(\phi)\rangle\\ \langle\mathcal{Y}_N(\phi)\rangle & \langle\eta_N(\phi)\rangle\end{pmatrix}\cdot\begin{pmatrix}\nu_L\\ N^c\end{pmatrix} + \text{h.c.} \equiv -\frac{1}{2}\overline{n}\,\mathcal{M}_\nu\, n + \text{h.c.}$$

- $\eta_\ell(\phi)$ is simply the `Weinberg connection' associated to LH neutrino masses in the (geo)SMEFT:

$$\frac{1}{2\Lambda}\left[C_{\underset{pr}{5}}\left(\tilde{H}^\dagger\ell_p\right)^T\mathbb{C}\left(\tilde{H}^\dagger\ell_r\right)+\text{h.c.}\right] \qquad \eta_\ell(\phi)_{pr} \equiv \frac{\delta\mathcal{L}_{\rm SMEFT}}{\delta(\overline{\ell_p^c}\ell_r)}\Bigg|_{\mathcal{L}(\alpha,\beta,..)\to 0} = \sum_{n=0}^{\infty}\left[\tilde{H}^\dagger(\phi_I)\tilde{H}^\star(\phi_J)\,\tilde{C}^{(5+2n)}_{\underset{pr}{\ell\ell}}\right]\left(\frac{\phi^2}{2}\right)^n$$

- $\mathscr{L}_{mass}$ can readily be diagonalized with a unitary transformation on *n* vectors:

$$\boxed{\mathcal{U}_n^\dagger\,\mathcal{M}_\nu\,U_n \equiv m_\nu = \text{diag}\left(m_{\nu_1},\ldots,m_{\nu_{n_l}},m_{N_1},\ldots,m_{N_{n_f}}\right)}$$

How can these mass eigenvalues, mixing angles and phases be written at all $\bar{v}_T/\Lambda$ orders?

Flavor Invariants! [2107.03951] JT, M. Trott

All-orders formulae: quark masses

[0907.4763]
[1507.00328]
[2107.03951]

- The **Hilbert Series** associated to Yukawa couplings transforming under $U(3)_{Q_L}$ transformations can be utilized to enumerate a basis of **11 flavor invariants for (geo)SM(EFT)**.

$$\underset{U(3)_{Q_L}}{Y^{\psi} Y^{\psi\,\dagger} \to U^{\dagger}\, Y^{\psi} Y^{\psi\,\dagger}\, U} \qquad H(q) \;=\; h(q,q) = \frac{1+q^{12}}{(1-q^2)^2(1-q^4)^3(1-q^6)^4(1-q^8)}$$

(e.g.) $$I_1 \equiv \mathrm{tr}\left(\mathbb{Y}_u\right)\,, \qquad \hat{I}_3 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\right)\,, \qquad \hat{I}_6 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_u\right) = 3\det \mathbb{Y}_u$$

- Unmixed invariants can be solved to obtain exact formulae for Yukawa couplings / masses:

$$y_i^2 = \frac{(-2)^{1/3}}{3\,\psi_u}\left(I_1^2 - 3\,\hat{I}_3 + (-2)^{-1/3}\; I_1\,\psi_u + (-2)^{-2/3}\;\psi_u^2\right)\,,$$

$$y_{j,k}^2 = \frac{1}{12\psi_u}((-2)^{4/3}\; I_1^2 - 3\cdot(-2)^{4/3}\;\hat{I}_3 + 4\,I_1\,\psi_u$$

$$\mp\,\psi_u\sqrt{24\left(I_1^2 - 3\,\hat{I}_3\right) + \frac{6\cdot(-2)^{5/3}\left(I_1^2-3\,\hat{I}_3\right)^2}{\psi_u^2} - 3\cdot(-2)^{4/3}\,\psi_u^2 + (-2)^{2/3}\,\psi_u^2})$$

Valid for up-quark masses. Send $I_{1,3,6}$ to $I_{2,4,8}$ for down quark masses.

$$\psi_u = \left(-2\,I_1^3 + 9\,I_1\hat{I}_3 - 9\,\hat{I}_6 + 3\sqrt{-3\,I_1^2\hat{I}_3^2 + 12\,\hat{I}_3^3 + 4\,I_1^3\hat{I}_6 - 18\,I_1\hat{I}_3\hat{I}_6 + 9\,\hat{I}_6^2}\right)^{1/3}$$

All-orders formulae: CKM parameters

- Similarly, the mixed invariants (not shown) give predictions for (CKM) mixing angles:

$$s_{13} = \left[\frac{-\hat{I}_{10} - y_b^2\left(\hat{I}_7 - \Delta^+_{ds}\Delta^+_{uc}\Delta^+_{ut}\right) - y_u^2\left(\hat{I}_9 + y_b^2\left(\hat{I}_5 - y_b^2\Delta^+_{ct}\right) - y_d^2\, y_s^2\, \Delta^+_{ct}\right)}{\Delta^-_{bd}\Delta^-_{bs}\Delta^-_{cu}\Delta^-_{ut}}\right]^{1/2} \qquad \Delta^\pm_{ij} \equiv y_i^2 \pm y_j^2$$

$$s_{23} = \left[\frac{\Delta^-_{tu}\left(-\hat{I}_{10} + y_c^2\left(-\hat{I}_9 + (y_b^4 + y_d^2 y_s^2)\,\Delta^+_{ut}\right) + y_b^2\left(-\hat{I}_7 + y_c^2\left(-\hat{I}_5 + \Delta^+_{ct}\Delta^+_{ds}\right) + y_u^2\Delta^+_{ct}\Delta^+_{ds}\right)\right)}{\Delta^-_{ct}\left(\hat{I}_{10} + y_u^2\,\hat{I}_9 + y_b^2\left(\hat{I}_7 + y_u^2\left(\hat{I}_5 - 2\Delta^+_{ct}\Delta^+_{ds}\right)\right) - (y_u^4 + y_c^2 y_t^2)\,(y_b^4 + y_d^2 y_s^2)\right)}\right]^{1/2}$$

$$s_{12} = \left[\frac{\Delta^-_{db}\left(\hat{I}_{10} + y_s^2\left(\hat{I}_7 - y_c^2 y_t^2 \Delta^+_{db}\right)\right) + y_u^2\,\Delta^-_{bd}\left(-\hat{I}_9 - y_s^2\,\hat{I}_5 + \Delta^+_{sb}\Delta^+_{ct}\Delta^+_{ds}\right) + y_u^4 y_s^2\,(y_b^4 - y_d^4)}{\Delta^-_{ds}\left(\hat{I}_{10} + y_u^2\,\hat{I}_9 + y_b^2\left(\hat{I}_7 + y_u^2\left(\hat{I}_5 - 2\Delta^+_{ct}\Delta^+_{ds}\right)\right) - (y_b^4 + y_d^2 y_s^2)\,(y_u^4 + y_c^2 y_t^2)\right)}\right]^{1/2}$$

- When combined with a CP-odd 11th invariant, one also can derive the Dirac CP-violating phase (and its sign!)

$$s_\delta = \frac{4}{3}\, I^-_{11}\left[\Delta^-_{tc}\Delta^-_{tu}\Delta^-_{cu}\Delta^-_{bs}\Delta^-_{bd}\Delta^-_{sd}\, s_{12}s_{13}s_{23}\left(1 - s_{23}^2\right)^{1/2}\left(1 - s_{12}^2\right)^{1/2}\left(1 - s_{13}^2\right)\right]^{-1}$$

Here one notices the proportionality to the Jarlskog as well!

Applications

[1710.01741]
[2107.03951]

- Formulae can (e.g.) be used in **higher-order fits**, or to derive **RGE for fermionic mass and mixing** in novel ways:

$$\dot{s}_\delta = s_\delta \left[\frac{\dot{I}^-_{11}}{I^-_{11}} - \sum_{(ij)\in s_2} \frac{\dot{\Delta}^-_{ij}}{\Delta^-_{ij}} - \dot{s}_{12}\frac{(1-2\,s^2_{12})}{s_{12}c^2_{12}} - \dot{s}_{23}\frac{(1-2\,s^2_{23})}{s_{23}c^2_{23}} - \dot{s}_{13}\frac{(1-3\,s^2_{13})}{s_{13}c^2_{13}} \right]$$

[J.Talbert & M. Trott: 2107.03951]

$$\mu\frac{dI^-_{11}}{d\mu} \simeq (6a_0 + 6b_0 + 2a_1 I_1 + 2b_1 I_2)\, I^-_{11}$$

$$a_0 = \frac{3}{8\pi^2}\left(I_1 + I_2 + \frac{I_1 - I_2}{2n_g}\right) - 2\frac{\alpha_s}{\pi}, \qquad a_1 = \frac{3}{16\pi^2}$$

$$b_0 = \frac{3}{8\pi^2}\left(I_1 + I_2 + \frac{I_2 - I_1}{2n_g}\right) - 2\frac{\alpha_s}{\pi}, \qquad b_1 = \frac{3}{16\pi^2}$$

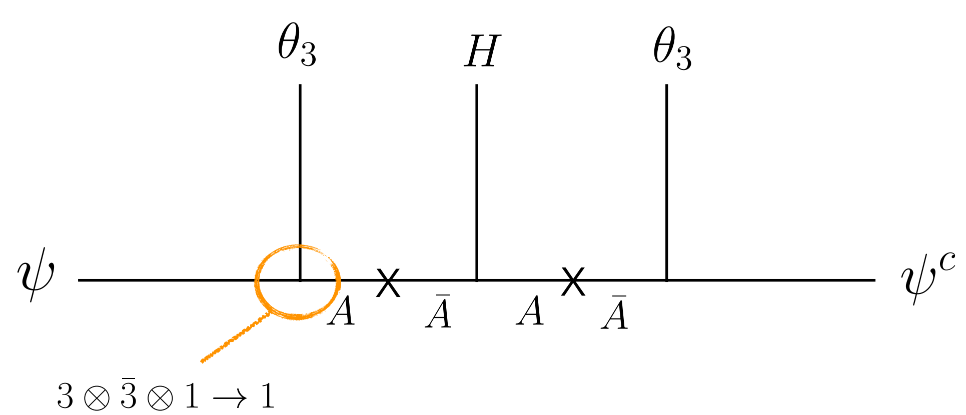
[1507.00328]

- Can also be used in **flavored model building**, including objects with explicit BSM states (e.g. scalar flavons or leptoquarks). A nice example is the UTZ model:

The Universal Texture Zero Model

Fields	$\psi_{q,e,\nu}$	$\psi^c_{q,e,\nu}$	H_5	Σ	S	θ_3	θ_{23}	θ_{123}	θ	θ_X
$\Delta(27)$	3	3	1_{00}	1_{00}	1_{00}	$\bar{3}$	$\bar{3}$	$\bar{3}$	$\bar{3}$	3
Z_N	0	0	0	2	-1	0	-1	2	0	x

[I de Medeiros Varzielas, G. Ross, J.Talbert: 1710.01741]



$$\mathcal{L}_{\rm UTZ} \supset \psi_p \left(\frac{1}{M^2_{3,f}}\theta^p_3\theta^r_3 + \frac{1}{M^3_{23,f}}\theta^p_{23}\theta^r_{23}\Sigma + \frac{1}{M^3_{123,f}}(\theta^p_{123}\theta^r_{23} + \theta^p_{23}\theta^r_{123})S \right) \psi^c_r H + \mathcal{O}(1/M^4) + \ldots$$

Proof-in-principle fits yield good agreement with global flavor data.

$$\mathcal{M}^D_f = \begin{pmatrix} 0 & a\,e^{i\gamma} & a\,e^{i\gamma} \\ a\,e^{i\gamma} & (b\,e^{-i\gamma} + 2a\,e^{-i\delta})\,e^{i(\gamma+\delta)} & b\,e^{i\delta} \\ a\,e^{i\gamma} & b\,e^{i\delta} & 1 - 2a\,e^{i\gamma} + b\,e^{i\delta} \end{pmatrix}_f$$

currently working on an MCMC fit to the UTZ!

Invariants in the neutrino sector

$$\begin{pmatrix} \langle \eta_\ell(\phi) \rangle & \langle \mathcal{Y}_N^T(\phi) \rangle \\ \langle \mathcal{Y}_N(\phi) \rangle & \langle \eta_N(\phi) \rangle \end{pmatrix}$$

No LNV: $\eta_\ell(\phi) = \eta_N(\phi) = 0$ ✅

- Treatable with (geo)SM(EFT) technologies presented in **[2107.03951]** (analogous to quarks).

(geo)SMEFT: $\eta_N(\phi) = \mathscr{Y}_N(\phi) = 0$ ✅

$$H(q) = \frac{1 + q^6 + 2q^8 + 4q^{10} + 8q^{12} + 7q^{14} + 9q^{16} + 10q^{18} + 9q^{20} + 7q^{22} + 8q^{24} + 4q^{26} + 2q^{28} + q^{30} + q^{36}}{(1-q^2)^2\,(1-q^4)^3\,(1-q^6)^4\,(1-q^8)^2\,(1-q^{10})}$$

- Invariants from [0907.4763] and parameters from [2107.06274]. Extension to all-$\bar{v}_T/\Lambda$-orders trivial.

Dynamical seesaw model: $\eta_\ell(\phi) = 0$ ⚠️

$$H(q) = \frac{N(q)}{D(q)}$$

- Hilbert Series from [1010.3161], but highly non-trivial!
- Progress on invariants from Yu et al., see e.g. [2107.11928] for $n_f = 2$ scenario.
- Parameters unknown (to my knowledge), as is….

$$\begin{aligned} N(t) = {} & 1 + t^4 + 5t^6 + 9t^8 + 22t^{10} + 61t^{12} + 126t^{14} + 273t^{16} + 552t^{18} + 1038t^{20} \\ & + 1880t^{22} + 3293t^{24} + 5441t^{26} + 8712t^{28} + 13417t^{30} + 19867t^{32} + 28414t^{34} + 39351t^{36} \\ & + 52604t^{38} + 68220t^{40} + 85783t^{42} + 104588t^{44} + 123852t^{46} + 142559t^{48} + 159328t^{50} \\ & + 173201t^{52} + 183138t^{54} + 188232t^{56} + 188232t^{58} + 183138t^{60} + 173201t^{62} + 159328t^{64} \\ & + 142559t^{66} + 123852t^{68} + 104588t^{70} + 85783t^{72} + 68220t^{74} + 52604t^{76} + 39351t^{78} \\ & + 28414t^{80} + 19867t^{82} + 13417t^{84} + 8712t^{86} + 5441t^{88} + 3293t^{90} + 1880t^{92} + 1038t^{94} \\ & + 552t^{96} + 273t^{98} + 126t^{100} + 61t^{102} + 22t^{104} + 9t^{106} + 5t^{108} + t^{110} + t^{114}, \end{aligned} \tag{2.3}$$

$$D(t) = (1-t^2)^3\,(1-t^4)^4\,(1-t^6)^4\,(1-t^8)^2\,(1-t^{10})^2\,(1-t^{12})^3\,(1-t^{14})^2\,(1-t^{16})$$

A minimal basis of invariants for the $n_f = 3$ seesaw is not known! WIP

Summary and outlook

- We have presented the first-ever **geometric νSMEFT** by identifying and deriving its composite **operators** & **field-space connections** under geometric refactorization.
- These objects describe geometries in the field spaces of the νSMEFT. Amongst many benefits, they permit the derivation of **all-$\overline{v}_T/\Lambda$-orders Feynman Rules** at the outset of an amplitude calculation. We have shown two such rules.
- Our goal is to explore **geoνSMEFT formalism & phenomenology**. One desirable ingredient is an **all-$\overline{v}_T/\Lambda$-orders** neutrino flavor formalism. WIP
- To that end, one can construct basis-independent, all-orders flavor formalisms by combining **invariant theory** with geometric EFT technologies. WIP
- Phenomenological applications to (e.g.) fits of neutrino mass and mixing can and should be explored, as should the **deeper theoretical questions** exposed by geometric factorization. WIP

THANK YOU!

Backup Slides

Basis of invariants for quarks

[0907.4763]
[1507.00328]

Do you know how to write $y^2(Y)$, $\theta(Y)$, $\delta(Y)$?

*Calculate **invariants** under U(3)!*

$$Y^\psi Y^{\psi\,\dagger} \xrightarrow[U(3)_{Q_L}]{} U^\dagger\, Y^\psi Y^{\psi\,\dagger}\, U$$

*Structure given by (known) **Hilbert Series**!*

$$H(q) = h(q,q) = \frac{1+q^{12}}{(1-q^2)^2(1-q^4)^3(1-q^6)^4(1-q^8)}$$

- A set of 11 invariants can be found to fully parameterize the theory, including six 'unmixed' *I*

$$YY^\dagger \equiv \mathbb{Y}$$

$$I_1 \equiv \mathrm{tr}\left(\mathbb{Y}_u\right), \qquad \hat{I}_3 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\right), \qquad \hat{I}_6 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_u\right) = 3\det\mathbb{Y}_u$$

$$I_2 \equiv \mathrm{tr}\left(\mathbb{Y}_d\right), \qquad \hat{I}_4 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_d\right), \qquad \hat{I}_8 \equiv \mathrm{tr}\left(\mathbb{Y}_d\,\mathrm{adj}\,\mathbb{Y}_d\right) = 3\det\mathbb{Y}_d$$

- as well as four 'mixed' *I*, relevant for extracting information about the CKM (overlap) matrix

$$\hat{I}_5 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathbb{Y}_d\right), \qquad \hat{I}_7 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\,\mathbb{Y}_d\right), \qquad \hat{I}_9 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_d\right), \qquad \hat{I}_{10} \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_d\right)$$

- and finally one mixed, CP-odd invariant relevant to pinning down the overall sign of CP violation:

$$I_{11}^- = -\frac{3i}{8}\det\left[\mathbb{Y}_u, \mathbb{Y}_d\right]$$

proportional to to the Jarlskog Invariant J!

- The fundamental geoSMEFT object we can construct at all-orders is then given by

$$\mathbb{Y}_{rp} = \frac{\hbar}{2}\left(Y_{ri}Y^\star_{pi} - \sum_{n'}^{\infty} f(n')Y_{ri}\tilde{C}^{(2n')}_{ip} - \sum_{n}^{\infty} f(n)\tilde{C}^{(2n),\star}_{ir}Y^\star_{pi} + \sum_{n,n'}^{\infty} f(n)f(n')\tilde{C}^{(2n),\star}_{ir}\tilde{C}^{(2n')}_{ip}\right)$$