Exploring $w_{1+\infty}$ in CCFT

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Some Brief Motivation

Celestial holography allows us to use what we know about 2D conformal field theories to constrain a theory of quantum gravity in asymptotically flat spacetime

Thus far, majority of progress has been made by considering specific interactions (i.e gluons and gravitons,..) in the bulk and seeing what they imply for the boundary theory

It would be nice to have a specific duality between a bulk theory and a boundary CFT, which is the focus of some work you will hear/have already heard about (c.f talks by Casali, Strominger, Melton, Yelleshpur etc.)

What we have done here is sort of half of that story, because we considered the specific bulk theory of self-dual gravity.

OUTLINE

- Review some results about $w_{1+\infty}$ symmetry in CCFT
- Describe Self-Dual Gravity
- Show that the algebra is unchanged at all orders in perturbation theory
- Some outlook

Discovery of $w_{1+\infty}$ in CCFT

Since you will most likely hear a decent amount about this in other talks, I will try to be rather brief

The story begins with (Guevara, Himwich, Pate, Strominger, 2021) where they showed that there was a tower of soft operators

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We know that the OPE of operators in CCFT is given by the collinear limits of the scattering amplitude.

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If we take the limits of the OPE then we can get the OPE of the soft operators.

Defined by $H^k := \lim_{\varepsilon \to 0} \varepsilon G_{k+\varepsilon}$

Via the usual prescription of contour integrals, we can get the commutator of the associated currents.

In (Strominger 2021) a clever rescaling of the generators was shown to give a $w_{1+\infty}$ algebra.

$$w_n^p = \frac{1}{\kappa}(p-n-1)!(p+n-1)!H_n^{-2p+4}$$

$$\downarrow$$

$$[w_m^p, w_n^q] = [m(q-1) - n(p-1)]w_{m+n}^{p+q-2}$$

Review of OPEs from collinear limits

The dot product of null momenta is $q_1 \cdot q_2 = -2\omega_1\omega_2 |z_{12}|^2$.

A collinear limit is when $q_1 \cdot q_2 \to 0$ which means in the boundary theory either $z_{12} \to 0$ or $\bar{z}_{12} \to 0$. We will usually take the holomorphic limit. Operator product expansions (OPEs) of operators in the CCFT can be calculated from the collinear limits of bulk amplitudes: (Pate, Raclariu, Strominger, Yuan, 2019)

1. Obtain the splitting function by taking the collinear limit in momentum space

$$\lim_{q_1 || q_2} A_n(q_1, q_2, \ldots) = S(q_1, q_2) A_n(p, q_3, \ldots), \qquad p = q_1 + q_2$$

2. Upon performing a Mellin transform, the splitting functions give rise to the OPE coefficients

$$G^+_{\Delta_1}(z_1, \bar{z}_1) G^+_{\Delta_2}(z_2, \bar{z}_2) \sim -\frac{\kappa}{2} \frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - 1, \Delta_2 - 1) G^+_{\Delta_1 + \Delta_2}(z_2, \bar{z}_2)$$

Poles in the Beta function correspond to the infinite tower of soft operators

Questions to ask

There are a couple questions we could ask here:

1. We found this algebra at the classical level since the OPEs were found from tree level scattering amplitudes. One could ask whether this algebra receives quantum corrections which means if we consider loop level scattering in the bulk, does it somehow correct the algebra?

   We consider a toy model of self dual gravity to answer this!

2. We assumed minimally coupled gravity in the bulk but we could, in principle, have other interaction terms corresponding to some non-minimally coupled gravity theory. One could ask whether at tree level, this algebra gets deformed upon considering those new interaction terms.

This was explored in some detail in papers by Mago, Ren, Spradlin, Srikant and Volovich 2021,2022
(c.f Akshay's talk)

Self-Dual Gravity is nice

There are finitely many S-matrix elements! That makes our job so much easier because we can figure out the full algebra on the boundary.

Self-Dual gravity at the classical level requires

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu}^{\alpha\beta} R_{\alpha\beta\rho\sigma}$$

Requires Klein space because otherwise there are no real solutions

Solutions here are built out of positive helicity plane waves while ASDG solutions are built out of negative helicity plane waves. In order to have a theory with both helicities, you need to modify it somehow.

There are many prescriptions for this and we went with one that was used by Chalmers and Siegel which uses a Lagrange multiplier to enforce self duality and is itself interpreted as the negative helicity graviton.

SDG S-matrix elements

The S-matrix elements have $1 - l$ negative helicity gravitons where l is the number of loops. At tree level we only have amplitudes with one negative helicity graviton which means the only non-trivial one is the three point

$$M_3^{\text{tree}}(1^+,2^+,3^-) = -\frac{i\kappa}{2}\left(\frac{[12]^3}{[23][31]}\right)^2$$

The 1-loop amplitude is given by

$$M_n^{1-\text{loop}}(1^+,2^+,\ldots,n^+) = -\frac{i}{(4\pi)^2\cdot 960}\left(-\frac{\kappa}{2}\right)^n \sum_{1\leq a<n\leq n,M,N} h(a,M,b)h(b,N,a)\text{tr}^3[aMbN]$$

where $h(a,\{1,2,\ldots,n\},b) = \frac{[12]}{\langle 12\rangle}\frac{\langle a|K_{1,2}|3]\langle a|K_{1,3}|4]\cdots\langle a|K_{1,n-1}|n]}{\langle 23\rangle\langle 34\rangle\cdots\langle n-1,n\rangle\langle 1\rangle\langle a2\rangle\cdots\langle an\rangle\langle 1b\rangle\langle nb\rangle} + \mathscr{P}(2,3,\ldots,n)$ is called a half-soft function.

Calculating the Algebra

In the literature the splitting function is shown to be the same except they take the true collinear limit where both $z_{12}, \bar{z}_{12}$ are taken to 0 together. We needed to figure out whether the splitting function was changed if we took just the holomorphic collinear limit.

We defined $P = p_1 + p_2$ and $t = \dfrac{\omega_1}{\omega_1 + \omega_2}$. The splitting function at tree level is given by $\dfrac{-\kappa}{2t(1-t)} \dfrac{\bar{z}_{12}}{z_{12}}$. After some tedious algebra we were able to show that the half soft functions obey the limit

$$h(a, \{1,2,3,\ldots,n\}, b) \to \frac{1}{t(1-t)} \frac{[12]}{\langle 12 \rangle} h(a, \{P,3,\ldots,n\}, b),$$

$$h(1,\{2,3,\ldots,n\}, b) \to \frac{\sqrt{t}\langle b | K_{3,n} | 2]}{(1-t)\langle 12 \rangle \langle Pb \rangle} h(1,\{3,\ldots,n\}, b)$$

from which you can show that for one loop

$$M_n^{1-\text{loop}}(1^+,2^+,\ldots,n^+) \to \frac{-\kappa}{2t(1-t)} \frac{[12]}{\langle 12 \rangle} M_{n-1}^{1-\text{loop}}(P^+,\ldots,n^+)$$

which is exactly the same as tree level!

A fun requirement!

It turns out that when we did the calculation we found an additional term that is proportional to $\frac{[12]^3}{\langle 12\rangle}$ which does not appear when performing the true collinear limit. But it is proportional to

$$\sum_{3\leq n\leq b,M,N} \langle Pb\rangle^2 h(P,M,b)h(b,N,P)\langle b|K_M|P]\langle P|K_N|b]^3$$

which was shown to be identically 0! (Rao and Feng 2016)

It would be nice to understand why this is zero and what its implications are.

Conclusions

There exists an infinite tower of soft operators/soft symmetries

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OPEs of these operators are found from collinear limits of bulk scattering amplitudes

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Their modes form a $w_{1+\infty}$ algebra found by contour integrating the OPE

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In self-dual gravity there is one tree level $(++-)$ amplitude at loop level there are only the all-plus one-loop amplitudes

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Loop corrections to the algebra in self-dual gravity will be found from corrections to the OPE which come from corrections to the momentum space splitting functions

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The splitting functions are the same, therefore in quantum self-dual gravity, the algebra remains uncorrected