Emergent Space-Time and Early Universe Cosmology from Matrix Theory

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Work in collaboration with S. Brahma and S. Laliberte
`arXiv:2106.11512, arXiv:2206.12468`

Motivation

- Inflationary Scenario is the **current paradigm** of **early Universe cosmology.**
- Inflation is usually analyzed using an **effective field theory** (EFT) framework.
- **Fundamental conceptual problems** for an **EFT** description of a rapidly expanding universe.
- **Unitarity problem**, **inconsistency with the 2nd law of thermodynamics.**
- We need to look beyond an EFT description of the early universe!
- Matrix Theory Cosmology: Emergent metric space-time and early universe from the BFSS matrix model.

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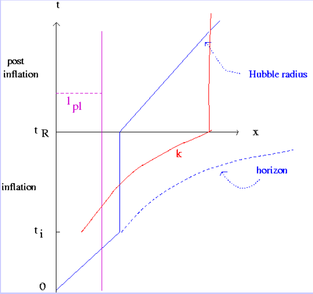
Outline

Plan

Trans-Planckian Problem

J. Martin and R.B., *Phys. Rev. D63, 123501 (2002)*



- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- $\rightarrow$ breakdown of effective field theory; new physics MUST be taken into account when computing observables from inflation.

Trans-Planckian Censorship Conjecture (TCC)

A. Bedroya and C. Vafa., arXiv:1909.11063

No trans-Planckian modes exit the Hubble horizon.

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

$$H(t) \equiv \frac{\dot{a}}{a}(t)$$

$$\boxed{\frac{a(t_R)}{a(t_i)} l_{pl} < H(t_R)^{-1}}$$

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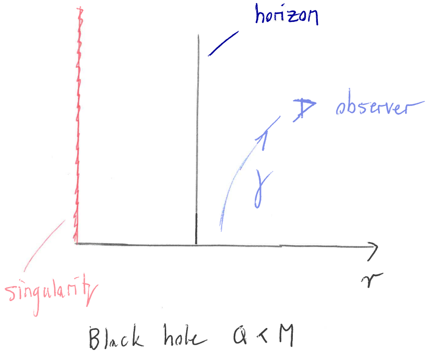
Justification

R.B. arXiv:1911.06056

Analogy with Penrose's Cosmic Censorship Hypothesis:



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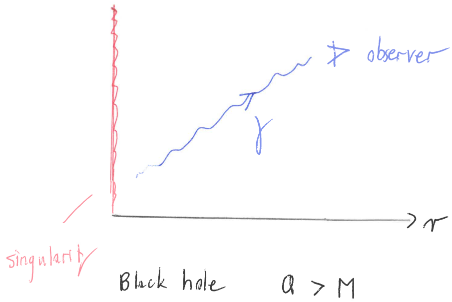

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- Effective field theory of General Relatiivty allows for solutions with timelike singularities: super-extremal black holes.
- $\rightarrow$ Cauchy problem not well defined for observer external to black holes.
- Evolution non-unitary for external observer.
- Conjecture: ultraviolet physics $\rightarrow$ **external observer** shielded from the singularity and non-unitarity by horizon.

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Cosmological Version of the Censorship Conjecture

Translation

- Position space $\rightarrow$ momentum space.
- Singularity $\rightarrow$ trans-Planckian modes.
- Black Hole horizon $\rightarrow$ Hubble horizon.

Observer measuring super-Hubble horizon modes must be shielded from trans-Planckian modes.

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Unitarity Problem

R.B. arXiv:1911.06056; A. Bedroya and C. Vafa., arXiv:1909.11063

- Recall: non-unitarity of **effective field theory** in an expanding universe (N. Weiss, Phys. Rev. D32, 3228 (1985); J. Cotler and A. Strominger, arXiv:2201.11658).
- $\mathcal{H}$ is the product Hilbert space of a harmonic oscillator Hilbert space for all **comoving** wave numbers k
- UV cutoff: time dependent k_{max} : $k_{max}(t)a(t)^{-1} = m_{pl}$
- Continuous mode creation $\rightarrow$ non-unitarity.
- Demand: **classical region be insensitive to non-unitarity**.
- $\rightarrow$ no trans-Planckian modes ever exit Hubble horizon.

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Effective Field Theory (EFT) and the CC Problem

- EFT: expand **fields** in comoving Fourier space.
- Quantize each Fourier mode like a harmonic oscillator $\rightarrow$ ground state energy.
- Add up ground state energies $\rightarrow$ CC problem.
- The usual quantum view of the CC problem is an artefact of an EFT analysis!

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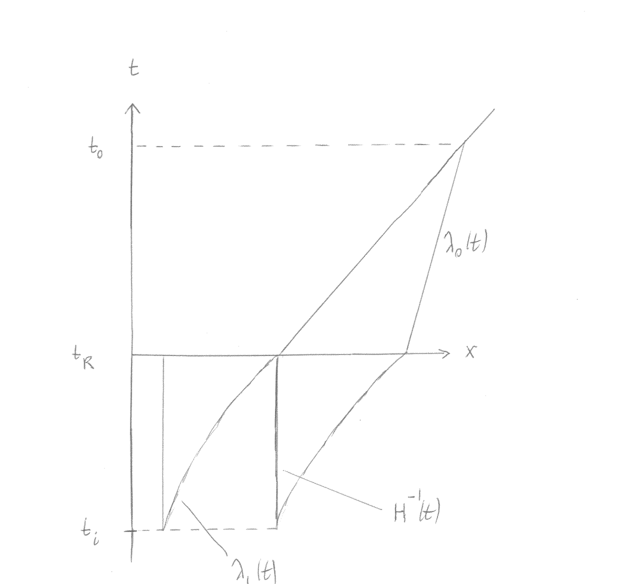
Application of the Second Law of Thermodynamics

S. Brahma, O. Alaryani and RB, arXiv:2005.09688

- Consider entanglement entropy density $s_E(t)$ between sub- and super-Hubble modes.
- Consider an **phase of inflationary expansion.**
- $s_E(t)$ increases in time since the phase space of super-Hubble modes grows.
- **Demand**: $s_E(t)$ remain smaller than the post-inflationary thermal entropy.
- $\rightarrow$ Duration of inflation is bounded from above, consistent with the TCC.

Application to EFT Description of Inflation

A. Bedroya, R.B., M. Loverde and C. Vafa., arXiv:1909.11106



Application to EFT Descriptions of Inflation

A. Bedroya, R.B., M. Loverde and C. Vafa., arXiv:1909.11106

TCC implies:

$$\frac{a(t_R)}{a(t_*)} l_{pl} < H(t_R)^{-1}$$

Demanding that inflation yields a causal mechanism for generating CMB anisotropies implies:

$$H_0^{-1} \frac{a(t_0)}{a(t_R)} \frac{a(t_R)}{a(t_*)} < H^{-1}(t_*)$$

Implications

Upper bound on the **energy scale of inflation**:

$$V^{1/4} < 3 \times 10^9 \mathrm{GeV}$$

$\rightarrow$ upper bound on the **primordial tensor to scalar ratio** r:

$$r < 10^{-30}$$

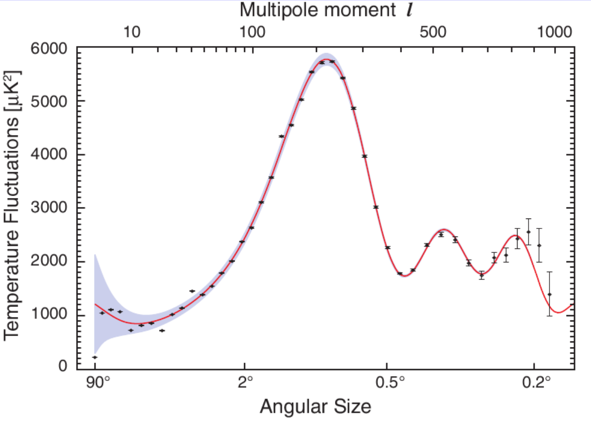
Note: Secondary tensors will be larger than the primary ones.

Implications for Dark Energy

Dark Energy cannot be a bare cosmological constant.

Plan

Angular Power Spectrum of CMB Anisotropies



Credit: NASA/WMAP Science Team

Early Work

1970 p.

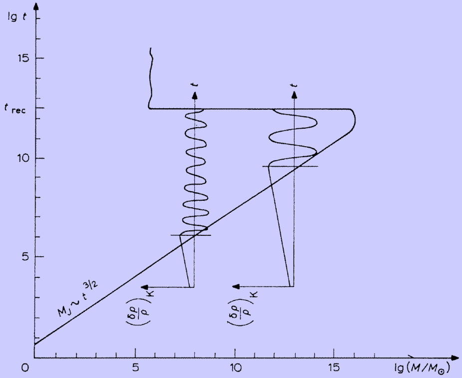


Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Predictions from 1970

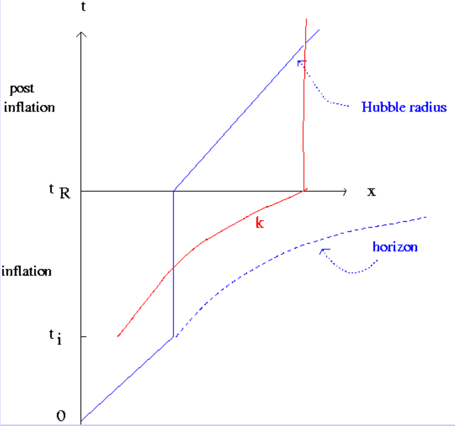
R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.
- $\rightarrow$ baryon acoustic oscillations in matter power spectrum.

Criteria for a Successful Early Universe Scenario

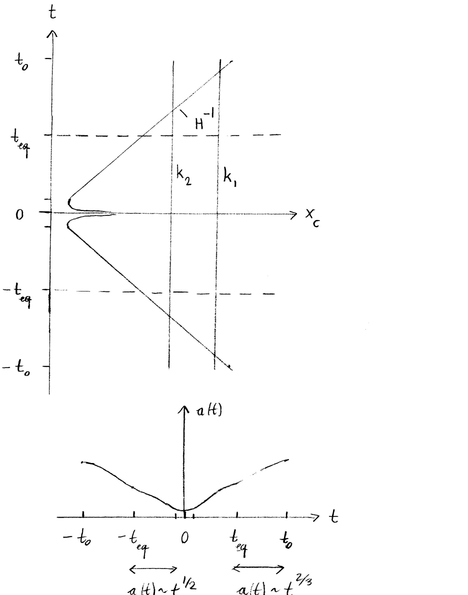
- Horizon $\gg$ Hubble radius in order for the scenario to solve the "horizon problem" of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

Inflation as a Solution



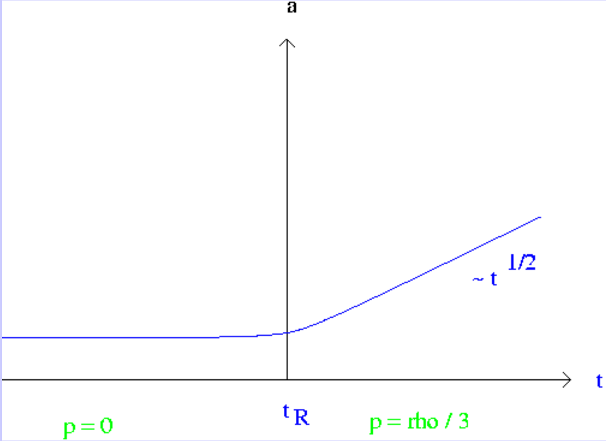
Bouncing Cosmology as a Solution

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



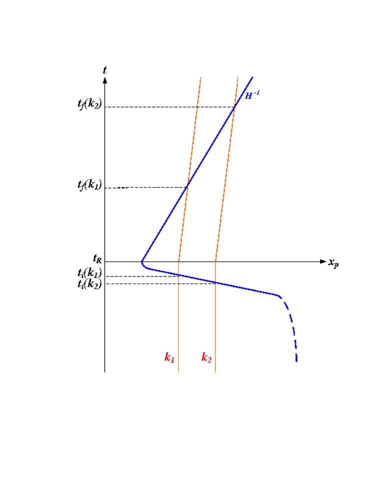
Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Trans-Planckian Censorship and Cosmological Scenarios

- **Bouncing cosmologies** are consistent with the TCC provided that the energy scale at the bounce is lower than the Planck scale.
- **Emergent cosmologies** are consistent with the TCC provided that the energy scale of the emergence phase is lower than the Planck scale.
- **Inflationary cosmologies** are inconsistent with the TCC unless the energy scale of inflation is fine tuned.

All early universe scenarios require going beyond EFT.

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Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Starting point: BFSS matrix model at high temperatures.

- BFSS model is a quantum mechanical model of 10 $N \times N$ Hermitean matrices.
- Note: no space!
- Note: no singularities!
- Note: BFSS matrix model is a proposed non-perturbative definition of M-theory: 10 dimensional superstring theory emerges in the $N \to \infty$ limit.

BFSS Model (bosonic sector)

T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997)

$$L = \frac{1}{2g^2}\left[\mathrm{Tr}\left(\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2\right)\right]$$

- $X_i, i = 1, ...9$ are $N \times N$ Hermitean matrices.
- D_t: gauge covariant derivative (contains a matrix A_0)

't Hooft limit: $N \to \infty$ with $\lambda \equiv g^2 N = g_s l_s^{-3} N$ fixed.

Thermal Initial State

N. Kawahara, J. Nishimura and S. Takeuchi, JHEP **12**, 103 (2007)

- Consider a high temperature state.
- At high temperatures, the bosonic sector of the (Euclidean) BFSS model is well approximated by the bosonic sector of the (Euclidean) IKKT matrix model.
- $S_{BFSS} = S_{IKKT} + \mathcal{O}(1/T)$
- Matsubara expansion:

$$X_i(t) = \sum_n X_i^n e^{2\pi i T t}$$

$$A_i \equiv T^{-1/4} X_i^0$$

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IKKT Matrix Model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

Proposed as a non-perturbative definition of the IIB Superstring theory.

Action:

$$S_{IKKT} = -\frac{1}{g^2}\mathrm{Tr}\Big(\frac{1}{4}[A^a, A^b][A_a, A_b] + \frac{i}{2}\bar{\psi}_\alpha(\mathcal{C}\Gamma^a)_{\alpha\beta}[A_a, \psi_\beta]\Big) ,$$

Partition function:

$$Z = \int dA d\psi e^{iS}$$

Emergent Time from Matrix Theory

Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795

- Eigenvalues of A_0 become emergent time.
- Work in the basis in which A_0 is diagonal.
- Numerical studies: $\frac{1}{N}\langle \mathrm{Tr} A_0^2 \rangle \sim \kappa N$
- $\rightarrow t_{max} \sim \sqrt{N}$
- $\rightarrow \Delta t \sim \frac{1}{\sqrt{N}}$
- $\rightarrow$ infinite continuous time.

Note: $\sum_{n=0}^{N} n^2 = \frac{1}{6} N(N+1)(2N+1)$

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Emergent Space from Matrix Theory

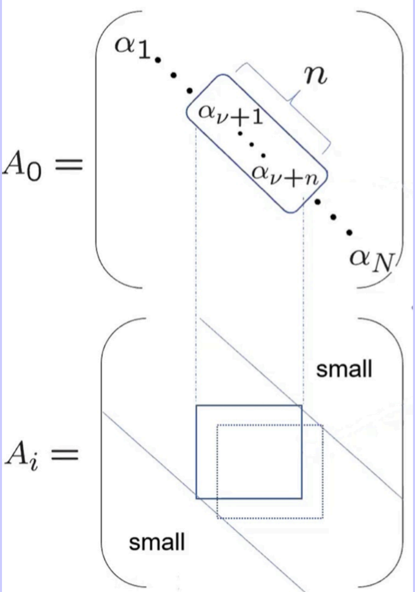
Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal: A_i matrices elements decay when going away from the diagonal.
- $\sum_i \langle |A_i|^2_{ab} \rangle$ decays when $|a - b| > n_c$
- $\sum_i \langle |A_i|^2_{ab} \rangle \sim$ constant when $|a - b| < n_c$
- $n_c \sim \sqrt{N}$

Emergent Space from Matrix Theory

S. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal: A_i matrices elements decay when going away from the diagonal.
- Pick $n \times n$ blocks $\tilde{A}_i(t)$ about the diagonal ($n < n_c$)


$A_0 =$
α_1
n
$\alpha_{\nu+1}$
$\alpha_{\nu+n}$
α_N
$A_i =$
small
small

Spontaneous Symmetry Breaking in Matrix Theory

J. Nishimura, PoS CORFU **2019**, 178 (2020) [arXiv:2006.00768 [hep-lat]].

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal.
- Work in the basis in which A_0 is diagonal: A_i matrices become block diagonal.
- **Extent of space** in direction i

$$x_i(t)^2 \equiv \left\langle \frac{1}{n} \mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- In a thermal state there is spontaneous symmetry breaking: $SO(9) \to SO(6) \times SO(3)$: three dimensions of space become larger, the others are confined. [J. Nishimura and G. Vernizzi, JHEP **0004**, 015 (2000);]S.-W. Kim, J. Nishimura and A. Tsuchiya, Phys. Rev. Lett. **109**, 011601 (2012)]

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Emergent Metric from Matrix Theory

S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468

- Eigenvalues of A_0 become emergent time, continuous in $N \rightarrow \infty$ limit.
- Work in the basis in which A_0 is diagonal: pick n (comoving spatial coordinate) and consider the block matrix $\tilde{A}_i(t)$.
- Physical distance between $n = 0$ and n (emergent space):

$$l^2_{phys,i}(n,t) \equiv \left\langle \mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- $l_{phys,i}(n) \sim n$ (for $n < n_c$)
- Emergent infinite and continuous space in $N \rightarrow \infty$ limit.
- Emergent metric (S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468).

$$g_{ii}(n)^{1/2} = \frac{d}{dn} l_{phys,i}(n)$$

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- Emergent metric (S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468).

$$g_{ii}(n)^{1/2} = \frac{d}{dn} l_{phys,i}(n)$$

Emergent Metric from Matrix Theory

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No Flatness Problem in Matrix Theory Cosmology

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Emergent metric:

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$$g_{ii}(n,t) = \mathcal{A}(t)\delta_{ii} \;\; i = 1,2,3$$

$SO(3)$ symmetry $\rightarrow$

$$g_{ij}(n,t) = \mathcal{A}(t)\delta_{ij} \;\; i = 1,2,3$$

$\rightarrow$ spatially flat.

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Plan

Late Time Dynamics

$$\mathcal{A}(t) \sim t^{1/2}$$

Note: no sign of a cosmological constant.

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

- We assume that the spontaneous symmetry breaking $SO(9) \to SO(3) \times SO(6)$ observed in the IKKT model also holds in the BFSS model.
- Using the Gaussian approximation method we have shown the existence of a symmetry breaking phase transition in the IKKT model (S. Brahma, RB and S. Laliberte, arXiv:2209.01255).
- **Thermal correlation functions** in the three large spatial dimensions calculated in the high temperature state of the BFSS model (following the formalism developed in String Gas Cosmology).
- $\to$ curvature fluctuations and gravitational waves.

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Matrix Theory Cosmology: Thermal Fluctuations

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- Start with the BFSS partition function .
- Note: $\frac{1}{T}$ correction terms in the BFSS action are crucial!
- Calculate matter correlation functions in the emergent phase.
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Computation of Fluctuations I

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

$$P(k) = k^3(\delta\Phi(k))^2 = 16\pi^2 G^2 k^2 T^2 C_V(R)$$

$$C_V(R) = \frac{\partial}{\partial T} E(R)$$

$$E = -\frac{\partial}{\partial \beta} \ln Z(\beta)$$

Computation of Fluctuations II

N. Kawahara, J. Nishumura and S. Takeuchi, arXiv:0710.2188

$$E^2 \; = \; N^2 < \mathcal{E} >_{BFSS}, \;\; \mathcal{E} = -\frac{3}{4N\beta} \int_0^\beta dt \mathrm{Tr}([X_i, X_j]^2)$$

- Insert Matsubara expansion of the matrices: leading term in the BFSS action in the high T limit is the IKKT action.
- Express expectation values in terms of IKKT expectation values

To next to leading order in $1/T$:

$$E^2 \; = \; \frac{3}{4} N^2 \chi_2 T - \frac{3}{4} N^4 \alpha \chi_1 T^{-1/2}$$

$$\chi_1 \; = < R^2 >_{BFSS} \; T^{-1/2}$$

Matrix Theory Cosmology: Results

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Thermal fluctuations in the emergent phase →

- Scale-invariant spectrum of curvature fluctuations
- With a Poisson contribution for UV scales.
- Scale-invariant spectrum of gravitational waves.

→ BFSS matrix model yields emergent infinite space, emergent infinite time, emergent spatially flat metric and an emergent early universe phase with thermal fluctuations leading to scale-invariant curvature fluctuations and gravitational waves.

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Open Problems

- Include the effects of the fermionic sector.
- Understand **phase transition** to the expanding phase of Big Bang Cosmology.
- Spectral indices?
- What about Dark Energy?
- Emergent low energy effective field theory for localized excitations.

Plan

Conclusions

- Inflation is not the only scenario of early universe cosmology consistent with current data.
- In light of the TCC and other conceptual problems effective field theory models of inflation are not viable.
- In light of the TCC and other conceptual problems Dark Energy cannot be a cosmological constant.
- We need to go beyond point particle EFT in order to describe the very early universe.

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Conclusions

- BFSS matrix model is a proposal for a non-perturbative definition of superstring theory. Consider a high temperature state of the BFSS model.
- $\rightarrow$ emergent time, space and metric. Emergent space is spatially flat and infinite.
- **Thermal fluctuations** of the BFSS model $\rightarrow$ scale-invariant spectra of cosmological perturbations and gravitational waves.
- **Horizon problem, flatness problem** and **formation of structure problem** of Standard Big Bang Cosmology resolved without requiring inflation.
- Transition from an emergent phase to the radiation phase of expansion. No cosmological constant.

Why Hubble Horizon?

R.B. arXiv:1911.06056; A. Bedroya and C. Vafa., arXiv:1909.11063

- Recall: Fluctuations only oscillate on sub-Hubble scales.
- Recall: Fluctuations freeze out, become **squeezed states** and **classicalize** on super-Hubble scales.
- Demand: **classical region be insensitive to trans-Planckian region.**
- $\rightarrow$ no trans-Planckian modes ever exit Hubble horizon.

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Obtaining an Emergent Cosmology: String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.
Assumption: Matter is a gas of fundamental strings
Assumption: Space is compact, e.g. a torus.
Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
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T-Duality

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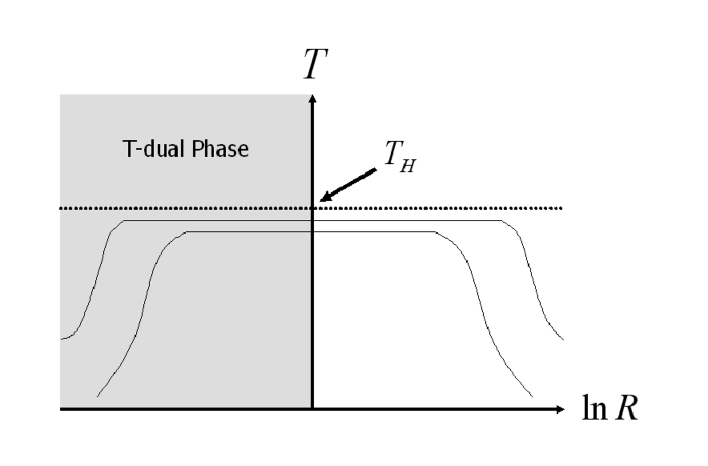
- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \to 1/R \quad (n, m) \to (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\to$ existence of D-branes

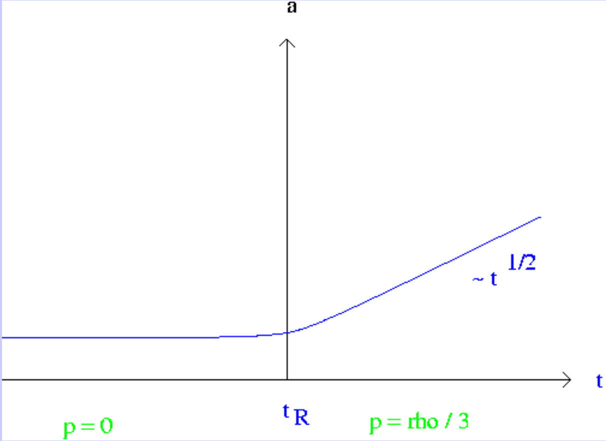
Adiabatic Considerations

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology

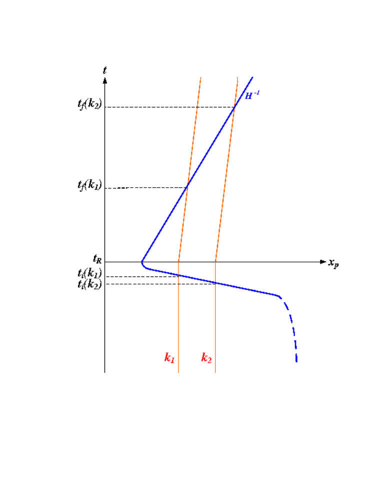


Background for string gas cosmology



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
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Power Spectrum of Cosmological Perturbations

Emergent

R. Brandenberger

TCC

Scenarios

Matrix Theory

Matrix Theory Cosmology

Conclusions

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} .$$

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Power spectrum of cosmological fluctuations

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\begin{aligned}
P_\Phi(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\
&= 8G^2 k^2 < (\delta M)^2 >_R \\
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Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

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Relationship between IKKT Model and Type IIB String Theory

Consider action of the Type IIB string theory in Schild gauge

$$S_{\text{Schild}} = \int d^2\sigma\alpha\left[\sqrt{g}\left(\frac{1}{4}\{X^\mu, X^\nu\} - \frac{i}{2}\bar{\psi}\Gamma^\mu\{X^\mu, \psi\}\right) + \beta\sqrt{g}\right].$$

$$\text{Partition function}: Z = \int \mathcal{D}\sqrt{g}\mathcal{D}X\mathcal{D}\psi e^{-S_{\text{Schild}}}.$$

$$\begin{aligned} \text{Correspondence}: \{,\} &\rightarrow -i[,] \\ \int d^2\sigma\sqrt{g} &\rightarrow \text{Tr} \end{aligned}$$

Obtain grand canonical partition function of IKKT model.

Some Details

Starting point: finite temperature partition function:

$$Z(\beta) = \int \mathcal{D}A\mathcal{D}X_i e^{-S(\beta)}$$

Internal energy

$$E = -\frac{d}{d\beta}\ln Z(\beta)$$

$$E = -\frac{3}{4}\lambda^{-1}\frac{N}{\beta}\int_0^\beta dt \mathrm{Tr}[X_i, X_j]^2$$

Matsubara expansion:

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Matsubara expansion of the action:

$$S_{BFSS} = S_0 + S_{kin} + S_{int}$$

At high temperature: S_{kin} and S_{int} suppressed compared to S_0.

To next to leading order:

$$E \simeq \lambda^{-1}\frac{3N^2}{4}\chi_2 T$$
$$-\lambda^{-1}\frac{3N^2}{4}\mathcal{O}(1)\chi_2\chi_1 T^{-1/2}$$

where $\chi_1 \simeq R^2\lambda^{4/3}T^{-1/2}$.

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- Derivative w.r.t. $T \rightarrow$ density fluctuations: both terms contribute.
- Derivative w.r.t. $R \rightarrow$ pressure fluctuations: only second term contributes.

Power spectrum $P(k)$ of density fluctuations: $(k = R^{-1})$

- First term dominates in the UV: Poisson spectrum.
- Second term dominated in the IR: Scale-invariant spectrum.

$$P(k) = 16\pi^2 G^2 \lambda^{4/3} N^2 \mathcal{O}(1) \sim (l_s m_{pl})^{-4}$$

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