`Thank for the invitation`

IS THERE ADDITIONAL DARK MATTER?

Paul H. Frampton

REFERENCES

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See also
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Talk on Sept 9th.

1 Introduction

In particle theory, the concept of entropy is not regarded as fundamental. For one elementary particle entropy is neither defined nor useful.

In general relativity and cosmology, the situation is different. For black holes, entropy is a central and useful concept. For cosmology, the entropy of the universe has often been considered, although not often enough in our opinion. We shall argue that the origin and nature of cosmological dark matter can be best understood by consideration of the entropy of the universe.

We have made such an argument some years ago but that discussion was perhaps too diluted by considering simultaneously dark matter being made from elementary particles such as WIMPs and axions, as were favoured over three decades ago in *e.g.* Kolb and Turner's 1990 book, The Early Universe.

In this talk, we dispose of microscopic candidates in one paragraph. The standard model of particle theory (SM) has two examples of lack of naturalness, the Higgs boson and the strong CP problem. Our position is that to understand these we still need to understand better the SM itself. Regarding the strong CP problem, it is too *ad hoc* to posit a spontaneously broken global symmetry and consequences which include an axion.

Concerning the WIMP, the idea that dark matter experiences weak interactions arose from assuming TeV-scale supersymmetry which is now disfavoured by LHC data. To identify the dark matter, we instead look up.

Assuming dark matter is astrophysical, and that the reason for its existence lies in the Second Law of Thermodynamics, we shall be led uniquely to the dark matter constituent as the Primordial Black Hole (PBH). We must admit that there is no observational evidence for any PBH, but according to our discussion PBHs must exist. In the ensuing discussion, we shall speculate that they exist in abundance in three tiers of mass up to and including at several galactic masses.

Because PBH entropy goes like mass squared, we are mainly interested in masses satisfying $M_{PBH} > 100M_{\odot}$. Within the Milky Way, we use the acronym PIMBH for intermediate mass PBHs in the mass range $10^2M_{\odot} < M_{PIMBH} < 10^5M_{\odot}$. Outside the Milky Way we entertain all masses $10^2M_{\odot} < M_{PBH} < 10^{22}M_{\odot}$. Of these, we use PSMBH for supermassive PBHs in the mass range $10^6M_{\odot} < M_{PSMBH} < 10^{11}M_{\odot}$ and PEMBH for extra massive PBHs with $10^{12}M_{\odot} < M_{PEMBH} < 10^{22}M_{\odot}$.

Although the visible universe (VU) is not a black hole, its Schwartzschild radius is about 68% of its physical radius, 30 Gly versus 44 Gly, so it is close. This curious fact seems to have no bearing on the nature of dark matter. A few more acronyms will be useful: CMB. CIB and CXB. CMB is the familiar cosmic microwave background while I and X refer to Infra-red and X-ray respectively.

2 Entropy

We begin with the premise that the early universe be regarded in an approximate sense as a thermodynamically-isolated system for the purposes of our discussion. It certainly contains a number of particles, $\sim 10^{80}$, vastly larger than the numbers normally appearing in statistical mechanics, such as Avogadro's number, $\sim 6 \times 10^{23}$ molecules per mole.

No heat ever enters or leaves and it can be considered as though its surface were covered by a perfect thermal insulator. It is impracticable to solve all the Boltzmann transport equations so it is mandatory to use thermodynamic arguments, provided that we may argue that the system is proximate to thermal equilibrium.

Making the then-unsupported assumption in 1872 of atoms and molecules, Boltzmann discovered the quantity $S(t)$ in terms of the molecular momentum distribution function $f(\mathbf{p}, t)$

$$S(t) = -\int d\mathbf{p} f(\mathbf{p}, t) \log f(\mathbf{p}.t) \qquad (1)$$

which satisfies

$$\left(\frac{dS(t)}{dt}\right) \geq 0 \qquad (2)$$

and can be identified with the thermodynamic entropy. The crucial inequality, Eq(2), the Second Law, was derived for non-equilibrium systems assuming only the Boltzmann transport equations and the ergodic hypothesis.

Ascertaining the nature of the dark matter can be regarded as a detective's mission and there are useful clues in the visible universe. We can made an inventory of the entropies of the known objects in the visible universe, using a venerable source, Weinberg's 1972 book.

Let us model the visible universe as containing 10^{11} galaxies each of mass $10^{12} M_{\odot}$ and each containing one central SMBH with mass $10^{7} M_{\odot}$. We recall the dimensionless entropy of a black hole $S/k(M_{BH} = \eta M_{\odot}) \sim 10^{78} \eta^{2}$. Then the inventory is

- SMBHs $\sim 10^{103}$
- Photons $\sim 10^{88}$
- Neutrinos $\sim 10^{88}$
- Baryons $\sim 10^{80}$

We regard this entropy inventory as a **first clue**. From the point of view of entropy the universe would be only infinitesimally changed if everything except the SMBHs were removed. This suggests that more generally black holes totally dominate the entropy, as we shall find in the sequel.

A second remarkable fact about the visible universe is the near-perfect black-body spectrum of the CMB which originated some 300,000 years after the beginning of the present expansion era, or after the Big Bang in a more familiar language. We are not tied to a Big Bang which could well be replaced by a bounce in a cyclic cosmology.

The precise CMB spectrum is a **second clue** about dark matter. It suggests that the plasma of electrons and protons prior to recombination is in excellent thermal equilibrium, and hence the matter sector was in thermal equilibrium for the first 300,000 years. This, combined with the thermal isolation mentioned already, underwrites the use of entropy, and the second law, during this period.

A **third clue** and final one about dark matter lies with the holographic principle 't Hooft which provides, as upper limit on the entropy of the visible universe, the area of its surface in units of the Planck length. Given it present co-moving radius 44 Gly this requires $S/k \leq 10^{123}$. The entropy of the contents which is so bounded might nevertheless tend to approach a limit which is many orders of magnitude higher than the total entropy in the limited inventory listed above.

3 Second Law

For primordial black holes (PBHs) formed at cosmic time t, their mass may be taken to be governed by the horizon size, giving

$$M_{PBH} = 10^5 M_{\odot} \left(\frac{t}{1\,\mathrm{sec}} \right) \tag{3}$$

so that PBHs with masses $10^2 M_{\odot} < M_{PBH} < 10^{22} M_{\odot}$ are produced for $10^{-3}s < t < 3Gy$. The top few orders of magnitude seem unlikely, but are possible and must be entertained.

A tendency to increase the entropy of the universe towards $S_U/k \sim 10^{123}$ can be most readily be achieved by the formation of PBHs, the more massive the better, because $S_{BH}/k \sim 10^{78}\eta^2$ for mass $M_{BH} = \eta M_{\odot}$. For example, in the case that a PEMBH existed with $M_{PEMBH} \sim 10^{22} M_{\odot}$ it would have $S/k \sim 10^{122}$ within an order of magnitude of the holographic maximum.

The PBH mass function is unknown so we must make reasonable conjectures which may approximate Nature. For a preliminary discussion we may take monochromatic distributions separately for PIMBHs, PSMBHs and PEMBHs. The real mass function is expected to be smoother but the general features in our discussion of entropy should remain valid.

In a toy model for the visible universe we include 10^{11} galaxies each with mass $10^{12} M_\odot$. As a hierarchical dark matter we shall take as illustration all PIMBHs with $100 M_\odot$; all PSMBHs with $10^7 M_\odot$; all PEMBHs at $10^{14} M_\odot$. Let the number of each type be n_I, n_S and n_E, respectively. The total dark matter mass is then

$$M = \left(10^2 n_I + 10^7 n_S + 10^{14} n_E\right) M_\odot \quad (4)$$

while the total entropy contributed by all PBHs is

$$S/k = \left(10^{82} n_I + 10^{92} n_S + 10^{106} n_E\right) \quad (5)$$

Let us begin with the middle one of the three hierarchical tiers, the supermassive black holes known to reside in galactic centres. In our toy model, n_S is equal to the number of galaxies $n_S = 10^{11}$ so their total mass and entropy are, from Eq.(4),

$$M(PSMBHs) = 10^{18} M_{\odot} \quad (6)$$

and, from Eq.(5),

$$S(PSMBHs)/k = 10^{103} \quad (7)$$

Before considering Eqs.(4) and (5) further, let us step back and ask which of the three terms in each equation is most likely to be dominant? The answer is different for Eqs.(4) and (5) because entropy S/k and mass M have the relationship $S/k \propto M^2$.

The total mass in Eq.(4) is comparable to the total mass of the visible universe which is $\sim 10^{123} M_{\odot}$. Comparison with $M(PSMBHs)$ in Eq.(6) then show that the second term in the R.H.S. of Eq.(4) is sub-dominant, being several orders of magnitude less than the L.H.S.

Now let us discuss the first term on the R.H.S. In our toy model every galaxy has mass $10^{12} M_{\odot}$ which is dominated by the dark matter halo made up of $100 M_{\odot}$ PIMBHs and therefore, since there are 10^{11} galaxies, we take $n_I = (10^{11}) \times (10^{10}) = 10^{21}$ whereupon the total mass and entropy of the PIMBHs are, from Eq.(4),

$$M(PIMBHs) = 10^{23} M_{\odot} \tag{8}$$

and, from Eq.(5),

$$S(PIMBHs)/k = 10^{103} \tag{9}$$

From Eq.(8) we deduce that the first term on the R.H.S. of Eq.(4) is a dominant term. We already know that the second term on the R.H.S. is relatively small. What about the third and last term? At this stage, we can say little except that observation is consistent with it vanishing. Perhaps surprisingly, to jump ahead, after discussion of the entropy equation, Eq.(5), we shall suggest the third term on the R.H.S. of Eq.(4) is comparable to the first term on the R.H.S. of Eq.(4), thus providing a rather novel viewpoint of dark matter.

Substituting our choices $n_I = 10^{21} M_\odot$ and $n_S = 10^{11}$ into Eq.(5) we find for the total entropy

$$S/k = \left(2 \times 10^{103} + 10^{106} n_E\right) \qquad (10)$$

to be compared to the total mass

$$M = \left(10^{23} + 10^{18} + 10^{14} n_E\right) M_\odot \qquad (11)$$

In Eq.(11), for consistency we must bound the parameter n_E from above by $n_E \leq 10^9$ to avoid overclosing the universe. It is interesting to study the upper limit of n_E in the entropy equation, Eq.(10). This gives $\sim 10^{115}$ to be compared with the holographic bound on the entropy which is $\sim 10^{123}$.

In the absence of any observational evidence about either dark matter or primordial black holes, we need to look at the visible universe from the two theoretical viewpoints of mass and entropy. This suggests the most likely scenario which is $n_E \sim 10^9$. This predicts that our toy universe contains of order one billion extra-massive black hole with masses $O(10^{14} M_\odot)$ or perhaps a smaller number of even more massive PBHs. Because of their extraordinarily high masses, these PEMBHs are not expected to be associated with a specific galaxy or cluster of galaxies. (Sept 9th talk).

4 Primordial Black Holes

If black holes make up all the dark matter, they cannot be all gravity-collapse black holes because of baryon mumber conservation. The amount of dark matter is more than five times that of baryons. Therefore, most or all dark-matter black holes must instead be primordial.

PBHs are black holes formed in the early universe when there is s high density and sufficiently large fluctuations and inhomogeneities. Their existence was first conjectured in the 1960s in the Soviet Union and independently in the 1970s, in the West. Initially it was realised that only PBHs with mass greater than $10^{-18}M_{\odot}$ could survive until the present time because of Hawking evaporation. Nevertheless, it was generally assumed that PBHs were all very much lighter than the Sun.

During this early era of extremely light PBHs, the seminal idea that PBHs could form all the dark matter was proposed in 1975 by Chapline. In 2009 and 2010 the relevance of entropy in cosmological evolution emerged.

Beginning in 2010, the upper limit on PBH mass was removed by showing that in a specific model of hybrid inflation, with two stages of inflation, a parametric resonance could mathematically yield fluctuations and inhomogeneities of arbitrarily large size. We regard this as merely an existence theorem and that such formation might take place without inflation.

The possibility of PBHs with many solar masses led to the 2015 dark matter proposal ithat PIMBHs provide an excellent astrophysical candidate for dark matter in the Milky Way halo, especially given the absence of a compelling elementary particle candidate either within the standard model or in any plausible extension.

This was further underscored in 2016. Both of these papers emphasised microlensing by PIMBHs of starlight from the Magellanic Clouds as a promising method for detection of PIMBHs in the Milky Way.

These PIMBHs are now to be regarded as the first of three mass tiers, the second being the supermassive PSMBHs at galactic centres and the third being extremely massive PEMBHs, more massive than galaxies.

Returning to our thermodynamic arguments about entropy, we use the entropy inventory of the known entities to observe the idea that very massive black holes already dominate entropy through the PSMBHs which we assume are primordial because there seems to be insufficient cosmic time for stellar mass black holes adequately to grow by accretion and mergers.

For the entropy of the universe to be nearer to its holographic upper limit, we are led to introduce 10^9 PEMBHs of $10^{14} M_{\odot}$ to reach $S/k \sim 10^{115}$. To achieve the maximum $S/k \sim 10^{123}$ is possible with just ten PEMBHs of $10^{22} M_{\odot}$ which, if true, would be revolutionary.

5 Microlensing

Gravitational lensing of a distant star by a nearer massive object or lens, moving across the field of view, gives rise to an enhancement of the star and to a temporal light curve whose duration is proportional to the square root of the mass of the lens, as displayed in Eq.(12).

As already mentioned, a direct way to discover PIMBHs in the Milky Way would be to use microlensing of light from the stars in the Magellanic clouds. Assuming a transit velocity 200km/s an estimate of the duration $\hat{t}$ of the light curve at half maximum is

$$\hat{t} \sim 0.2y \left(\frac{M_{lens}}{M_\odot} \right)^{\frac{1}{2}} \tag{12}$$

which means that for $10^2 M_\odot < M_{PIMBH} < 10^5 M_\odot$ the duration of the light curve is in the range $2y < \hat{t} < 60y$.

Masses below $2,500M_\odot$ with $\hat{t} < 10y$ are clearly the most practicable to measure.

A successful precursor was an experiment by the MACHO Collaboration in the 1990s. In the 2020s, microlensing searches at the Vera Rubin Observatory could repeat this success for the much higher mass ranges of the MACHOs expected for the dark matter inside the Milky Way.

The MACHO collaboration, 1992-99, used the observatory at Mount Stromlo near Canberra, Australia. it was a 1.27 m telescope with two 16-Magapixel cameras. They showed that the technique could be achieved successfully to discover MACHOs, as well as confirming this prediction by Einstein's general relativity. The highest duration of their more than a dozen light-curves was 230 days corresponding to a mass close to $10M_\odot$.

An attempt was made to use the Blanco 4m telescope at Cerro Tololo, Chile with the DECam having 570 Megapixels in order to find light-curves with durations of two years or more, and hence, by Eq.(12), lenses with $M > 100M_{\odot}$. The longer durations led, however, to crowding in the field of view such that it was impracticable to track a specific target star.

A more powerful telescope under construction at Cerro Panchon, also in Chile, is the Vera Rubin Observatory expected to start taking data in 2023. Its telescope is 8.4 metres and its camera has 3.2 Gigapixels, both significantly larger, and we can reasonably hope that it can microlens multi-year-duration light curves and possibly confirm the existence of PIMBHs in the Milky Way.

6 Cosmic Infrared Background

At large red-shifts $Z > 15$, a population of PBHs would be expected to accrete matter and emit in X-ray and UV radiation which will be redshifted into the CIB to be probed for the first time by the James Webb Space Telescope which could therefore provide support for PBH formation.

Analysis of a specific PBH formation model supports this idea that the JWST observations in the infrared could provide relevant information about whether PBHs really are formed in the early universe.

This is important because although we have plenty of evidence for the existence of black holes, whether any of them is primordial is not known.

The gravitational wave detectors LIGO, VIRGO and KAGRA have discovered mergers in black hole binaries with initial black holes in the mass range $3 - 85 M_{\odot}$. We suspect that all or most of these are not primordial but that is only conjecture.

The supermassive black holes at galactic centres, including Sgr A* at the centre of the Milky Way, are well established and are primordial in our toy model. Whether that is the case in Nature is unknown.

Because of the no-hair theorem that black holes are completely characterised by their mass, spin and electric charge (usually taken to be zero), there is no way to tell directly whether a given black hole is primordial or the result of gravitational collapse of a star.

The distinction between a primordial and a non-primordial black hole can be made only from knowledge of its history. For example, if it existed before star formation, it must be primordial. The infra-red data from JWST might be able to provide useful insight into this central question.

It is familiar to study a mass-energy pie-chart of the universe with approximately 5% baryonic normal matter, 25% dark matter and 70% dark energy. The entropy pie-chart is very different if the toy model considered in this papers resembles Nature. The slices corresponding to normal matter and dark energy are extremely thin and the pie is essentially all dark matter.

In this talk we have attempted to justify better the discussion of our previous 2018 paper which argued that entropy and the second law applied to the early universe provide a *raison d'être* for the dark matter. We proposed that the dark matter constituents in the Milky Way are PIMBHs.

Here we have included the supermassive PSMBHs at the galactic centres as a second tier of dark matter with a similar primordial origin to replace the conventional wisdom that SMBHs arise from accretion and merging of black holes which arise from gravity collapse of stars.

We have gone one step further and discussed a third tier of the extremely massive PEMBHs, more massive than galaxies, whose entropy far exceeds that of the PIMBHs and PSMBHs. If this is correct then although normal matter contributes as much as 5% of the mass-energy pie-chart of the universe, its contribution to an entropy pie-chart is truly infinitesimal.

Since it has never been observed except by its gravity, it does seem most likely that dark matter has no direct or even indirect connection to the standard model of strong and electroweak interactions in particle theory, including extensions thereof aimed to ameliorate problems with naturalness existing therein with respect to the Higgs boson and the strong CP problem.

The three clues we have mentioned in the Introduction, the dominance of black holes in the entropy inventory, the CMB spectrum and the holographic entropy maximum all hint toward PBHs as the dark matter constituent.

One ambiguity is whether the maximum entropy limit suggested by holography should be saturated in which case the mass function for the PEMBHs must be extended to high values.

7 Generalizations

In a recent paper, we investigated the possibility that PBHs might saturate the holographic entropy bound and entertained the possibility of PEMBHs (= Primordial Extremely Massive Black Holes) with masses up to $10^{22}M_{\odot}$. Just ten of these would saturate the bound but here we discuss what is a realistic upper mass bound since such extreme masses could impinge on the cosmological principle concerning the large-scale homogeneity and isotropy of the universe.

It is appropriate to refer to PEMBHs as intergalactic dark matter because it is not associated with galaxies or clusters of galaxies but is located in intergalactic space. The total mass of intergalactic dark is comparable to that of interstellar dark matter inside clusters but its total entropy is incomparably greater.

If we take PBH production at $t < 300,000y$ to precede recombination, then

$$M_{PBH} < M_{cmb} \sim 10^{18} M_{\odot}. \qquad (13)$$

However, it is possible that more massive PBHs may be formed later provided they do not produce photons which disturb the CMB spectrum.

8 The Great Attractor

It was pointed out by Dressler that the peculiar velocities of certain galaxies point to the existence of a specific mass overdensity which corresponds to what he called the Great Attractor with mass $M_{GA} \sim M_{cmb} \sim 10^{18} M_{\odot}$. This the only such overdensity in a volume $\sim (1Gpc)^3$ so assuming a uniform density within the visible universe there could be a few thousand of them. We shall assume the approximate equality of M_{GA} snd M_{cmb} to be accidental.

For our present purposes, we shall assume that the Great Attractor is an extremely massive PBH, and use it as a jumping off point to posit the existence in the visible universe of truly cosmological size PBHs.

Table 1: Values of Schwarzschild radius and Dimensionless entropy.

Mass	Schwarzschild radius	Entropy S/k
$10^{18} M_\odot$	$100\ kpc$	10^{114}
$10^{22} M_\odot$	$1\ Gpc$	10^{122}

The size of the GA is comparable to that of galaxies and clusters $\sim 100kpc$. For a PBH with mass M_{cp} the Schwarzschild radius is comparable to that of the visible universe and too large to be detectable on a plot like that in Dressler's paper.

In Table 1, we summarise the entropy properties for two examples of intergalactic dark matter and see that they suggest an opportunity to saturate the holographic upper bound because if we take the maximum allowed number of GAs their entropy adds to $S/k \sim 10^{117}$ just a million times less than the ultimate limit.

To put this in perspective, we recall the interstellar dark matter which could be the correct explanation for the dark matter inside of galaxies such as the Milky Way. If we take those intermediate-mass black holes to be $100M_{\odot}$ their entropy adds to only $S/k \sim 10^{103}$ which is approximately the same as the entropy of the supermassive black holes (SMBHs) known to reside at galactic centres. Of the known objects in the universe, SMBHs overwhelmingly dominate the entropy. Nevertheless, the entropy of the SMBHs falls short of the holographic limit by 20 orders of magnitude.

In the present talk, we are exploring the possibility that the content of the universe possesses entropy adding to the holographic limit. Initially our expectation was to find this possibility excluded but all we would say now is that it must involve a lot of intergalactic dark matter.

Looking at the universe from the viewpoint of entropy is extremely different from the viewpoint of mass-energy. For example, normal matter is 5% in an energy pie chart, but 10^{-25} of entropy and this becomes only 10^{-45} if the total entropy is at the holographic limit.

9 Maximal Intergalactic Dark Matter

From the entropy viewpoint, it is interesting that the visible universe is so close to being itself a black hole in the sense that its Schwarzschild radius $9Gpc$ is about 2/3 of the comoving radius $13.5Gly$,

In terms of mass-energy, this is merely a restatement of the fact that the present density is close to the critical density. But for entropy it is extremely puzzling, because the content has only an infinitesimally tiny fraction of its maximum possible value. This suggests that there is something dominating the cosmological entropy which is being overlooked.

The only candidate to fill this rôle is, to our knowledge, extremely massive black holes, as in Table 1, which may be regarded as a straightforward extension of the dark matter known to be in galaxies and in clusters of galaxies.

We have no good idea for the mass function of extremely massive PBHs. It may be a smooth function or a series of almost monochromatic steps as suggested by some numerical work. Here we shall discuss the latter possibility.

First, we reconsider the Great Attractor mass, M_{GA}, and the viable possibility of one thousand PBHs of this size. As we have seen, these can contribute $S/k \sim 10^{117}$ to the entropy of the universe, very much more than the supermassive black holes, $\sim 10^{103}$.

The other, higher, mass scale mentioned *ut supra* was M_{cp} which was taken to be the largest mass which is consistent with the cosmological principle. Each such black hole provides a contribution to dimensionless entropy which is only one order of magnitude below the holographic limit. Therefore, no more than ten are allowed.

10 Testability

So far, our discussion has been totally speculative and has populated the visible universe with objects which must be the most massive ever contemplated. The nearest may be Carr, who considered almost as massive black holes. From the point of view of entropy, all these very massive objects are a natural extension of the dark matter inside galaxies and clusters.

Thus, dark matter in this generalised sense permeates all of intergalactic space not as condensed clumps of mass but spread out on all scales up to cosmological ones. This occurrence of extremely massive black holes seems inevitable if we adopt the hypothesis that the bulk contents of the universe possess an entropy which saturates the holographic limit.

The obvious question is how to test this novel view of the universe.

Additional great attractors, if they exist, require better technology to observe galaxy distributions at the largest distances. As for the most extreme black holes comparable to the size of the universe itself, we are unaware of any available observational test although it is possible that purely theoretical arguments may be able to rule them out.

There is a question on how the largest PBHs could be formed. According to Eq.(3), masses $10^{18} M_\odot$ and $10^{22} M_\odot$ would be formed at, respectively, $t = 300ky$ and $3Gy$ respectively, so there can be a concern about distorting the CMB and of affecting large-scale structure, but for the moment we have simply postulated that such PBHs exist.

`Thank you for your attention`