Soft Graviton Theorem in BFSS, 2208.14547

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Explicit example of flat space holographic duality

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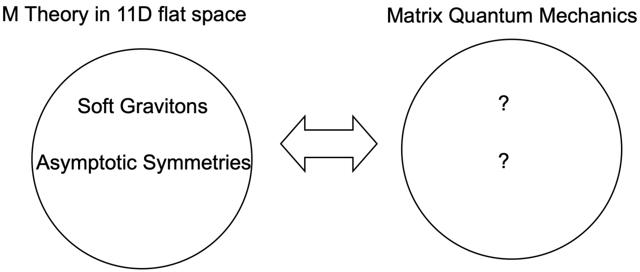

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4. When 11D supergravity is compactified on spatial circle, one gets all of the massless fields of Type 11-A string theory
5. M theory is defined to be the theory which is IIA string theory when 1 direction is compactified on circle. Low energy limit of M theory is supergravity and includes gravitons

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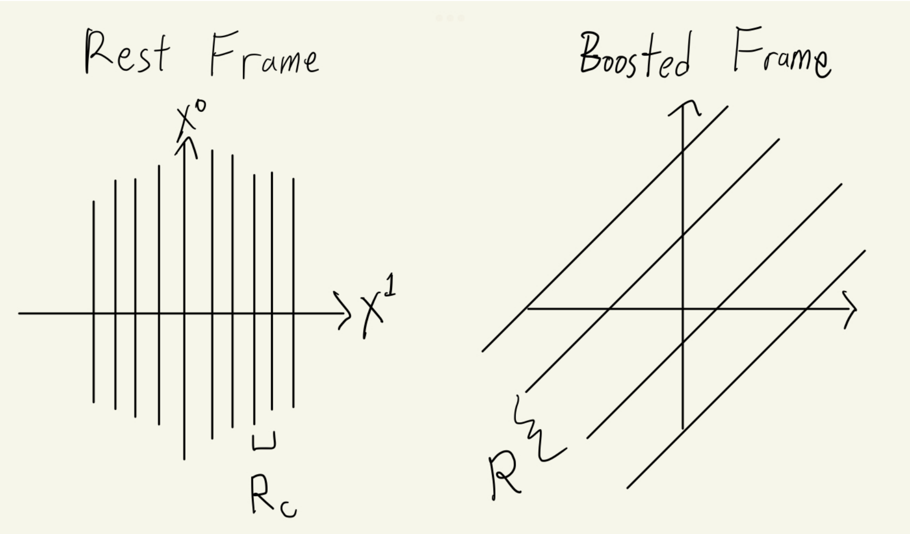
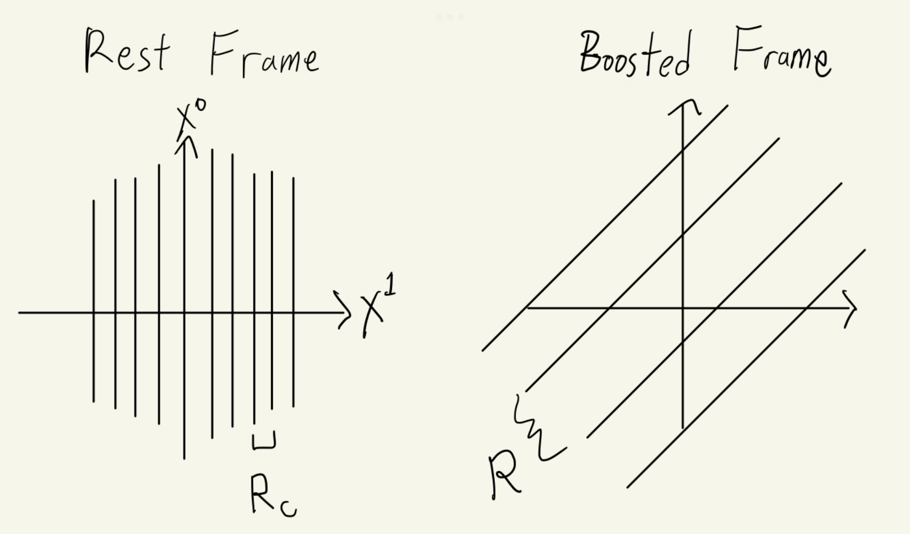
2. What are the massive charged states which are charged under this 1-form field?
3. Only possibility: D0 branes.
4. Therefore, 11D gravitons in M-theory with momentum around compact direction are bound state clumps of D0 branes in 10D.


Rest Frame
Boosted Frame
x^0
x^1
R_c
R



If boost is big, recover whole space!

BFSS Conjecture (Banks-Fischler-Shenker-Susskind 1996) says that the lightcone quantization of M theory is equal to a certain 0+1D quantum mechanical theory of $N \times N$ matrices, with $N \to \infty$, describing D0 branes moving in 9+1D Type IIA string theory.

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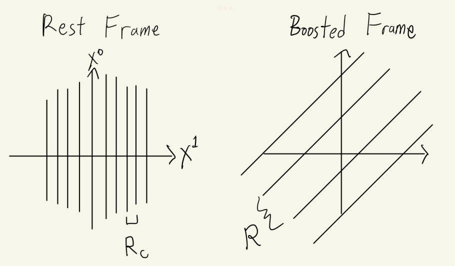
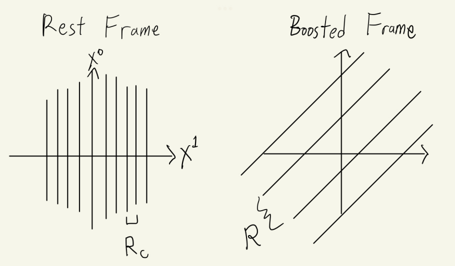
(polarization data stored in fermions)



$$X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^{10}), \qquad P^\pm = \frac{1}{\sqrt{2}}(P^0 \pm P^{10}).$$

$$(X^+, X^-) \sim (X^+, X^- + 2\pi R).$$

$$P^+ = N/R \text{ is fixed, } N, R \to \infty$$



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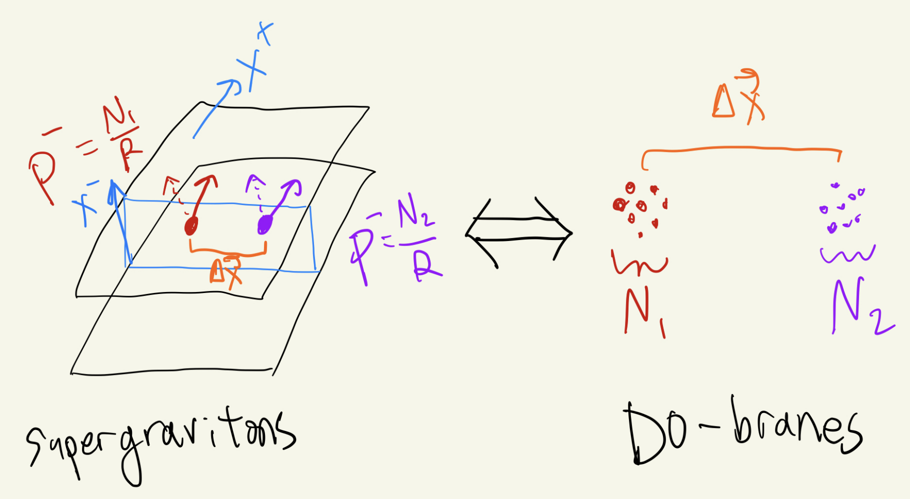
Graviton with N units of momentum is bound state of N D0 branes. Because of heavy boosting, rest mass in 10D theory is huge, D0 branes are non relativistic, $P^{\perp}/M \ll 1$.



x^+
$p^- = \frac{N_1}{R}$
x^-
$\Delta \vec{x}$
$p^- = \frac{N_2}{R}$
$\Longleftrightarrow$
$\Delta \vec{x}$
N_1
N_2
supergravitons
D0-branes

Quantum variables, $I = 1 \dots 9$, $\alpha = 1 \dots 16$:

$$X^I = X^I_A T^A, \qquad \Psi^\alpha = \Psi^\alpha_A T^A$$

T^A generate $\mathfrak{su}(N)$. States are functions of these variables.

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$$H = \frac{R}{2} \mathrm{Tr} \left[P^I P^I - \frac{1}{2(2\pi l_p^3)^2} [X^I, X^J][X^I, X^J] - \frac{1}{2\pi l_p^3} \Psi^T \Gamma^I [X^I, \Psi] \right]$$

with constraint on physical states to be $U(N)$ invariant

$$f_{ABC}(X^I_B P^I_C - \frac{i}{2} \Psi^\alpha_B \Psi^\alpha_C) |\psi_{\text{phys}}\rangle = 0$$

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$$X^I = \begin{pmatrix} x_1^I & 0 & \dots & 0 \\ 0 & x_2^I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_N^I \end{pmatrix}$$

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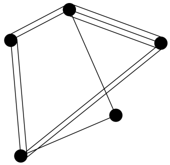
4. The off diagonal terms are (super) harmonic oscillators with frequency proportional to distance of D0 branes. They correspond to the occupation number of open strings stretched between the branes.



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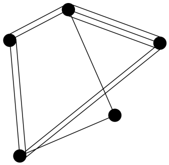

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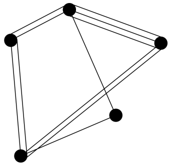

5. This can be considered a theory of emergent spacetime if you like.

Attraction of far separated clumps comes from strings being in ground states with some associated ground state energy which depends on relative distance and velocity of clumps.

The Galillei Lie algebra has the generators

p^i	spatial translations
h	time translation
l_{ij}	rotations
mx^i_{CM}	Gallilean boosts.

Note that this has the central element $[mx^i_{CM}, p^i] = im$. The Poincare lie algebra has the generators

P_i	transverse translations
$P_+ = H$	X^+translation
P_-	X^-translation
L_{ij}	transverse rotations
L_{iz}	rotation in (x^i, z) plane
K_{0z}	boost in z-direction
K_{0i}	boost in x^i-direction.

The Galilei Lie algebra is a subalgebra of the Poincare Lie Algebra if we make the identifications

$$
\begin{aligned}
P_i &\longleftrightarrow p_i \\
P_+ = H &\longleftrightarrow h \\
P_- &\longleftrightarrow m \\
L_{ij} &\longleftrightarrow l_{ij} \\
B_i = \frac{K_{0i} - L_{zi}}{\sqrt{2}} &\longleftrightarrow m x^i_{CM}
\end{aligned}
$$

Note that the Galilean boosts B_i are (real) combinations of the conformal primary operators $L_1 \pm \bar{L}_1$.

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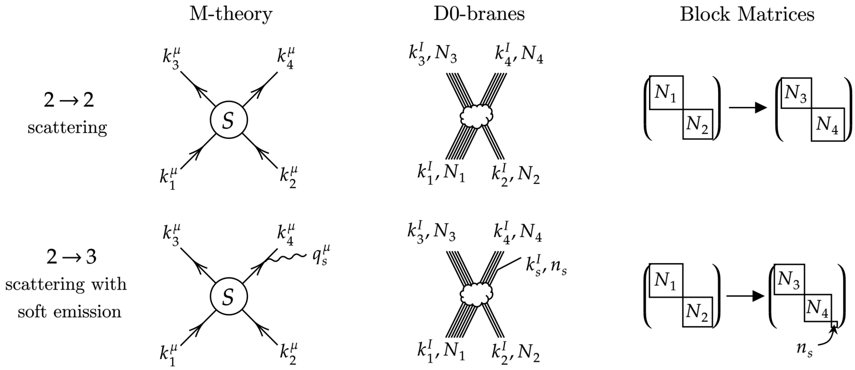
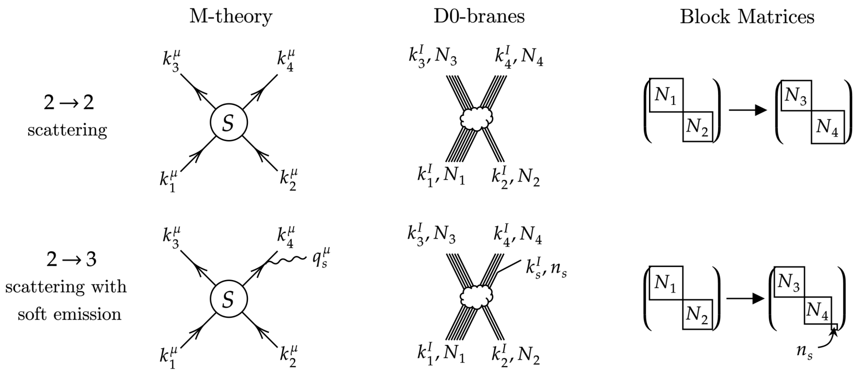

M-theory
D0-branes
Block Matrices
$2 \to 2$ scattering
k_3^μ k_4^μ S k_1^μ k_2^μ
k_3^I, N_3 k_4^I, N_4 k_1^I, N_1 k_2^I, N_2
N_1 N_2 N_3 N_4
$2 \to 3$ scattering with soft emission
k_3^μ k_4^μ q_s^μ S k_1^μ k_2^μ
k_3^I, N_3 k_4^I, N_4 k_s^I, n_s k_1^I, N_1 k_2^I, N_2
N_1 N_2 N_3 N_4 n_s



soft graviton = D0 brane clump with few D0 branes

	M-theory	**BFSS**
Soft Limit	$\omega_s/\omega_j \to 0$	$N_s/N_j \to 0$
Leading Soft Term	$\frac{\kappa}{2}\epsilon^s_{\mu\nu}\sum_j \eta_j \frac{k_j^\mu k_j^\nu}{q_s \cdot k_j}$	$\kappa \sum_j \eta_j \frac{N_j}{N_s} e_{IJ}\mathcal{I}^{IJ}$
Subleading Terms	Expansion in $\frac{\omega_s}{\omega_j}$	Expansion in $\frac{1}{N}$
Asymptotic Symmetry	Supertranslations	Large gauge transformations of RR 1-form gauge field

Linking of infrared divergences— flat directions in the potential and soft gravitons

$$\mathcal{A}_{\mathrm{M}}(q_s^\mu, \epsilon_s^{\mu\nu}; k_1^\mu, \ldots, k_n^\mu) = \underbrace{\left(\frac{\kappa}{2}\epsilon^s_{\mu\nu}\sum_{j=1}^{n}\eta_j \frac{k_j^\mu k_j^\nu}{q_s \cdot k_j}\right)}_{\text{phase shift from supertranslation}} \mathcal{A}_{\mathrm{M}}(k_1^\mu, \ldots, k_n^\mu)$$

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soft part (adds soft graviton) hard part (shifts hard particles)

$$q_s^\mu = \omega_s \hat{q}_s^\mu = \frac{\omega_s}{2}(1 + v_s^2, 2v_s^I, 1 - v_s^2).$$

$$\epsilon^{\mu\nu}_{IJ}(v) \equiv \frac{1}{2}\left(\epsilon^\mu_I \epsilon^\nu_J + \epsilon^\nu_I \epsilon^\mu_J\right) - \frac{1}{d}\delta_{IJ}\epsilon^\mu_K \epsilon^{K\nu} \quad \text{with} \quad \epsilon^\mu_J(v) \equiv \partial_J \hat{q}_s^\mu$$

$$\epsilon_s^{\mu\nu} = e^{IJ}\epsilon^{\mu\nu}_{IJ}$$

$$
\begin{aligned}
&\mathcal{A}_{\mathrm{BFSS}}(n_s, q_s^I, \epsilon_s, \text{out; in}) \\
&\qquad = \left(-2\kappa \sum_{j=1}^{n} \eta_j \frac{N_j}{n_s} \frac{e_{IJ}(v_s - v_j)^I (v_s - v_j)^J}{(v_s - v_j)^2} \right) \mathcal{A}_{\mathrm{BFSS}}(\text{out; in}).
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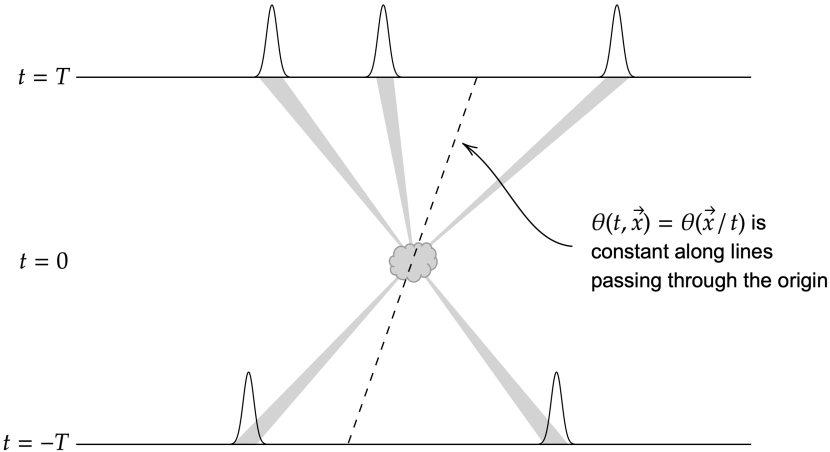
Therefore this is more like the soft *photon* theorem!

Asymptotic symmetry group of non-relativistic gauge transformations $C_\mu \mapsto C_\mu + \partial_\mu \theta$?

Asymptotic symmetry group of non-relativistic gauge transformations $C_\mu \mapsto C_\mu + \partial_\mu \theta$?

Must only depend on "rays" in asymptotic region

$$\theta(t, \vec{x}) \xrightarrow{t \to \pm\infty} \theta(t, \vec{x}) = \theta(\vec{x}/t)$$


$t = T$
$t = 0$
$t = -T$
$\theta(t, \vec{x}) = \theta(\vec{x}\,/\,t)$ is constant along lines passing through the origin

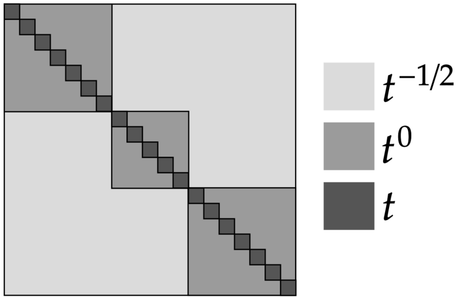
Can couple matrix model to background gauge field with the following coupling:

$$S_{RR}[C_\mu] = \int dt \sum_{n=0}^{\infty} \frac{1}{n!} \left(\partial_{I_1} \cdots \partial_{I_n} C_\mu(t, \vec{0})\right) I^{\mu(I_1 \cdots I_n)}$$

$$I^{\mu(I_1 \cdots I_n)} = \mathsf{Tr}\Big(\mathrm{Sym}(I^\mu, X^{I_1}, \cdots, X^{I_n})\Big) + I_{\mathsf{F}}^{\mu(I_1 \cdots I_n)}$$

where

$$I^\mu = (\mathbf{1}/R, \dot{X}^I/R).$$



$t^{-1/2}$
t^0
t

$$\mathcal{A}_{\mathsf{BFSS}}(\text{out; in})\Big|_{C_\mu=\partial_\mu\theta} = \exp\left(i \sum_{j=1}^{n} \eta_j \frac{N_j}{R} \theta(\vec{v}_j) \right) \mathcal{A}_{\mathsf{BFSS}}(\text{out; in})\Big|_{C_\mu=0}$$

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$$\theta_{e,v_s}(\vec{x}/t) \equiv \frac{e_{IJ}(v_s - x/t)^I (v_s - x/t)^J}{|\vec{v}_s - \vec{x}/t|^2}$$

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$$\lim_{\frac{n_s}{R} \to 0} \frac{n_s}{R} \mathcal{A}_{\text{BFSS}}(n_s, k_s^I, \epsilon_s, \text{out; in})\Big|_{C_\mu = 0} = -i \frac{d}{d\varepsilon}\Big|_{\varepsilon = 0} \mathcal{A}_{\text{BFSS}}(\text{out; in})\Big|_{C_\mu = \varepsilon \partial_\mu \theta_{e,v_s}}$$

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Very strange that supertranslations in 11D are completely captured by large *nonrelativistic* gauge transformations in 10D.... Fascinating example of duality of non-trivial two-sided asymptotic symmetry

Thank you!