Dirac, Fierz, Pauli and charged particles with spin >1

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Based on

K.B., Nathan Berkovits, Cassiano Daniel, Matheus Lize (hep-th 2110.07623 /JHEP)
K.B., Nathan Berkovits, Cassiano Daniel, Wenqi Ke (in progress ...)

Workshop on the Standard Model and Beyond, Corfu 2022

Introduction

Massive Free fields E.O.M

Spin 0 $$\boxed{(\partial^r \partial_r - M^2)\Phi = 0}$$

Spin 1/2 $$\boxed{(i\gamma^n \partial_n + M)\Psi = 0}$$

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$$\partial_m \quad \rightarrow \quad D_m = \partial_m - ieA_m$$

$$\boxed{(D^r D_r - M^2)\Phi = 0}$$

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A theoretical challenge: Dirac (1936)

447

Relativistic Wave Equations

By P. A. M. Dirac, F.R.S., St. John's College, Cambridge

(*Received March 25, 1936*)

448 P. A. M. Dirac

The elementary particles known to present-day physics, the electron, positron, neutron, and proton, each have a spin of a half, and thus the work of the present paper will have no immediate physical application. All the same, it is desirable to have the equations ready for a possible future discovery of an elementary particle with a spin greater than a half, or for approximate application to composite particles. Further, the underlying theory is of considerable mathematical interest.

Dirac 1936:

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All the same, it is desirable to **have the equation ready for** a possible future discovery of an elementary particle with **a spin greater than a half,** or for approximate application to composite particles.

Further, the underlying theory is of considerable mathematical interest. »

A theoretical challenge in the 30's

447

Relativistic Wave Equations

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ion. This equivalence between our equations (40) and the usual electron quations persists when there is an electromagnetic field present, provided he effect of the field on equations (40) is the usual one of requiring p to be eplaced by $p + e\mathrm{A}$, A being the vector potential. Thus our equation

Equations of Motion for spin 3/2

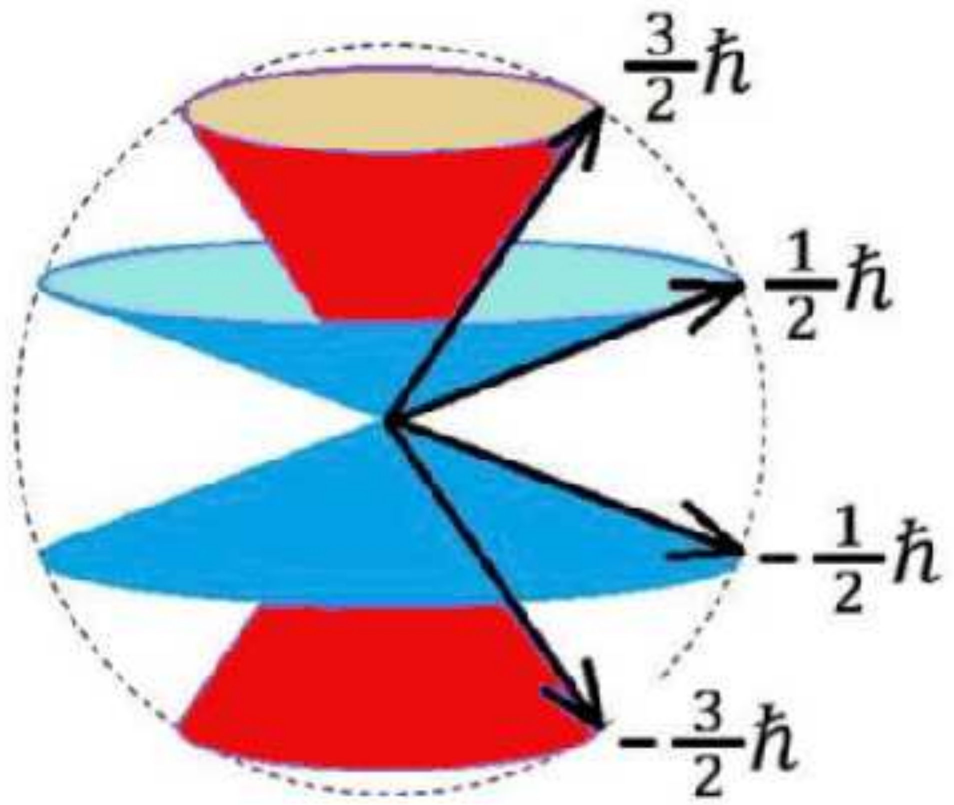
- The spinor-vector field ψ_μ has eight degrees of freedom.

- The spin 3/2 has 4 degrees of freedom; there are 4 extra ones that are not physical; one needs to impose constraints to get rid of them:

$$(i\gamma^\mu D_\mu - m)\psi_\nu = 0$$

$$\gamma^\mu \psi_\mu = 0$$

$$\partial^\mu \psi_\mu = 0$$



Massive spin 2: Fierz-Pauli (1939)

A massive spin 2 particle has 5 degrees of freedom: helicities -2, -1, 0, +1,+2
Represented by a symmetric tensor h_{mn} that satisfies

Equation of motion **10 d.o.f** → $(\partial^r \partial_r - M^2)h_{mn} = 0$

Constraints **-5 d.o.f**
- -4 d.o.f → $\partial^m h_{mn} = 0$
- -1 d.o.f → $h \equiv h^m_m = 0$

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- -4 d.o.f $\longrightarrow$ $\partial^m h_{mn} = 0$
- -1 d.o.f $\longrightarrow$ $h \equiv h^m_m = 0$

These can be obtained from

$$\mathcal{L}_{FP} = \boxed{\frac{1}{2} h^{mn} \partial^2 h_{mn} - \frac{1}{2} h \partial^2 h + h_{mn} \partial^m \partial^n h + \partial^n h_{mn} \partial_k h^{mk}} \; \boxed{- \frac{1}{2} M^2 \left(h^{mn} h_{mn} - h^2 \right)}$$

Linear expansion of Einstein-Hilbert

Mass terms:
FP combination allows to get the constraints

On relativistic wave equations for particles of arbitrary spin in an electromagnetic field

By M. Fierz and W. Pauli

Physikalisches Institut der Eidgenössischen Technischen Hochschule, Zürich

(*Communicated by P. A. M. Dirac, F.R.S.—Received* 31 *May* 1939)

1. Introduction

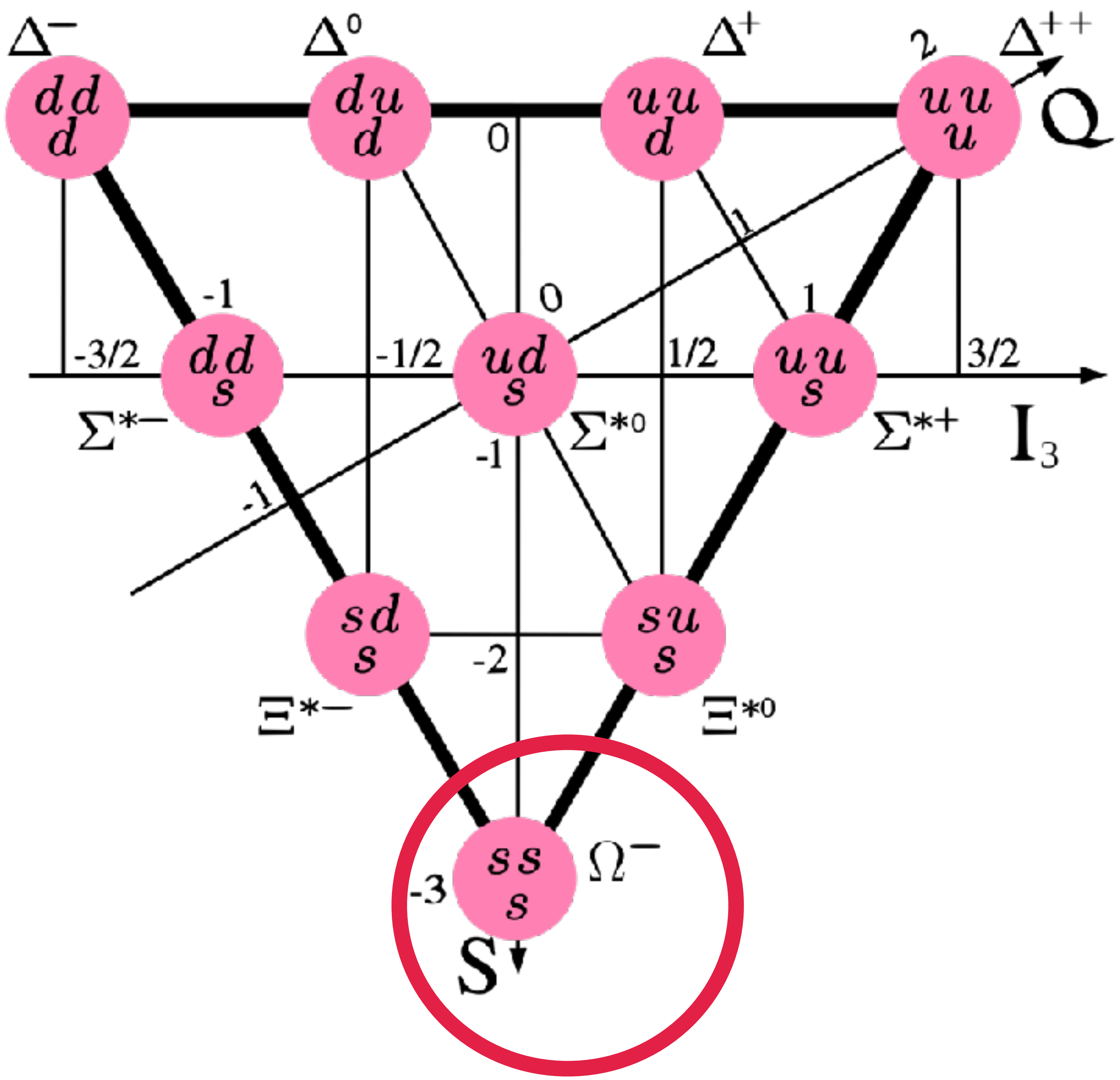
The investigations of Dirac (1936) on relativistic wave equations for particles with arbitrary spin have recently been followed up by one of us (Fierz, 1939, referred to as (A)) It was there found possible to set up a scheme of second quantization in the absence of an external field, and to derive expressions for the current vector and the energy-momentum tensor. These considerations will be extended in the present paper to the case when there is an external electromagnetic field, but we shall in the first instance disregard the second quantization and confine ourselves to a *c*-number theory.

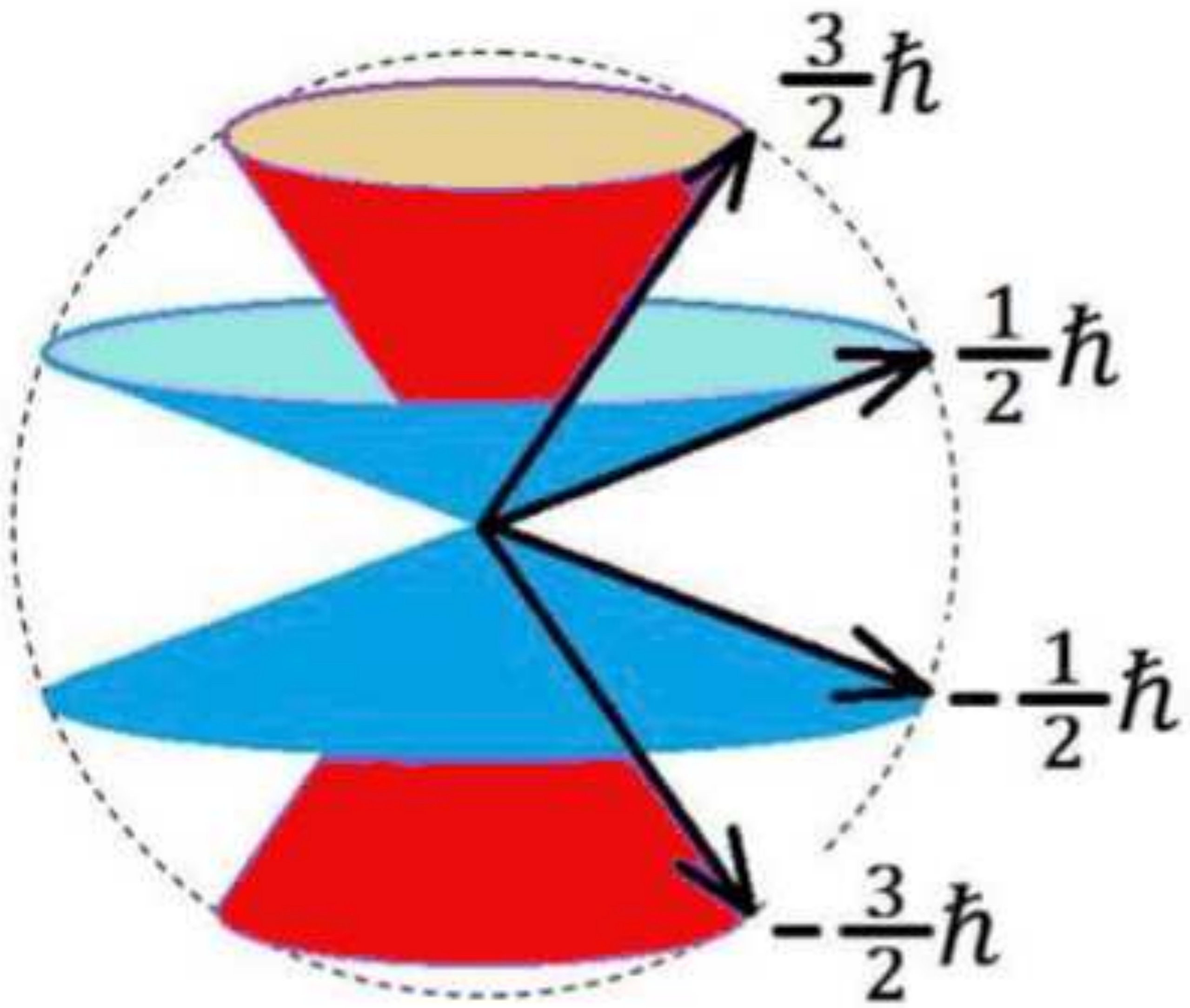
The difficulty of this problem is illustrated by the fact that the most immediate method of taking into account the effect of the electromagnetic field, proposed by Dirac (1936), leads to inconsistent equations as soon as the spin is greater than 1. To make this clear we consider Dirac's equations for a particle of spin 3/2, which in the force-free case run as follows:

For Fierz and Pauli,
the Lagrangian
in absence of external field
was a not an issue.

External field
was THE problem

The decuplet spin 3/2

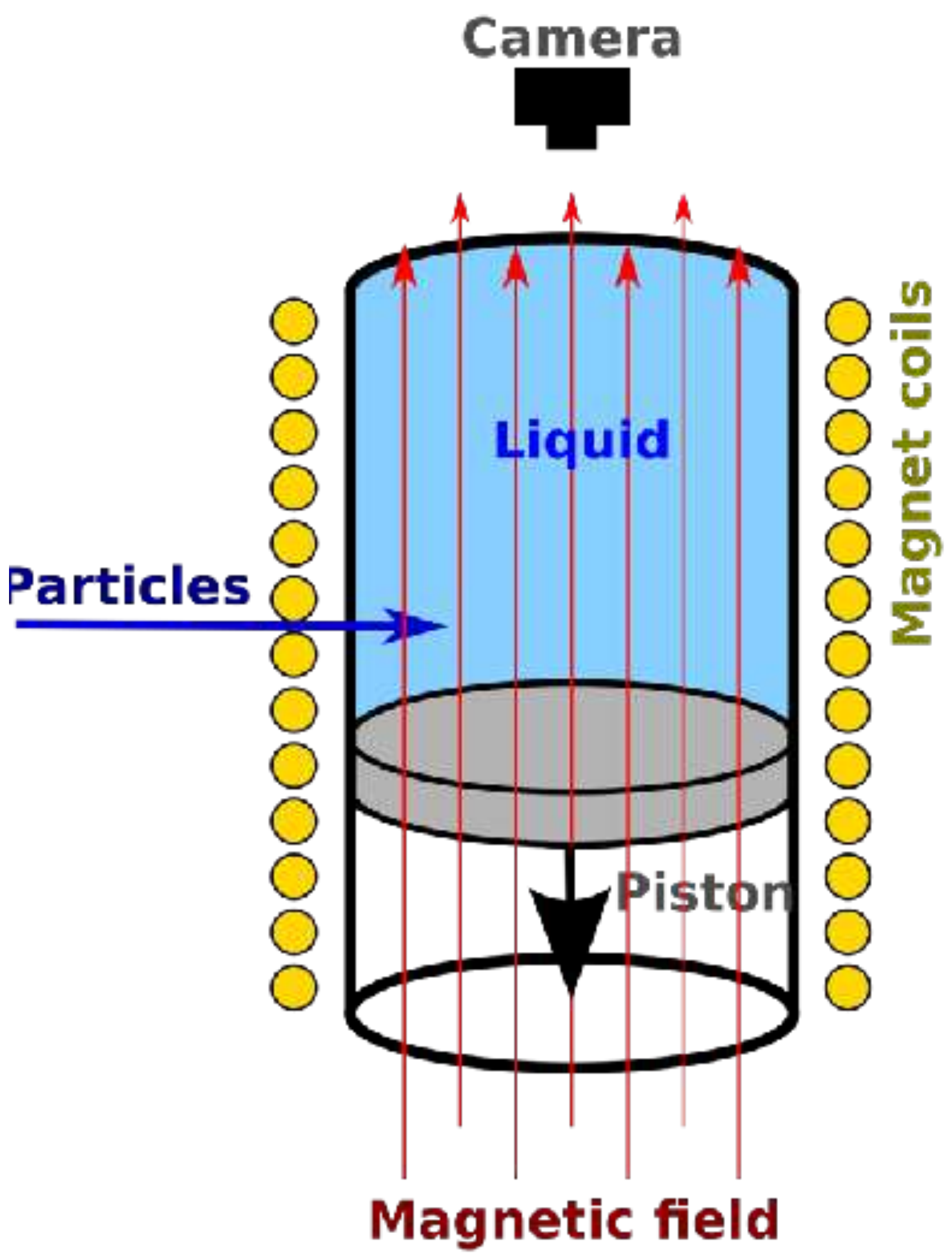


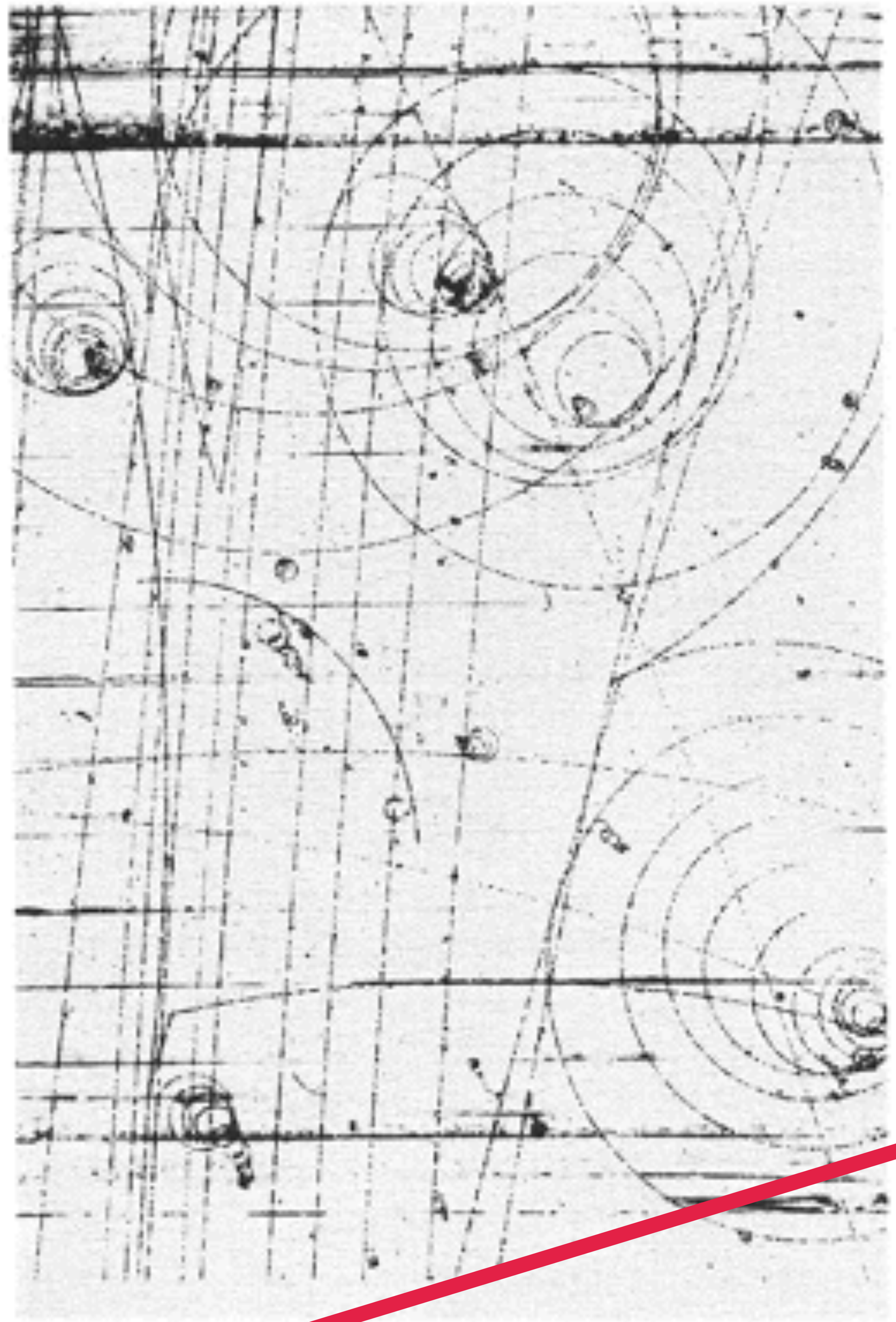


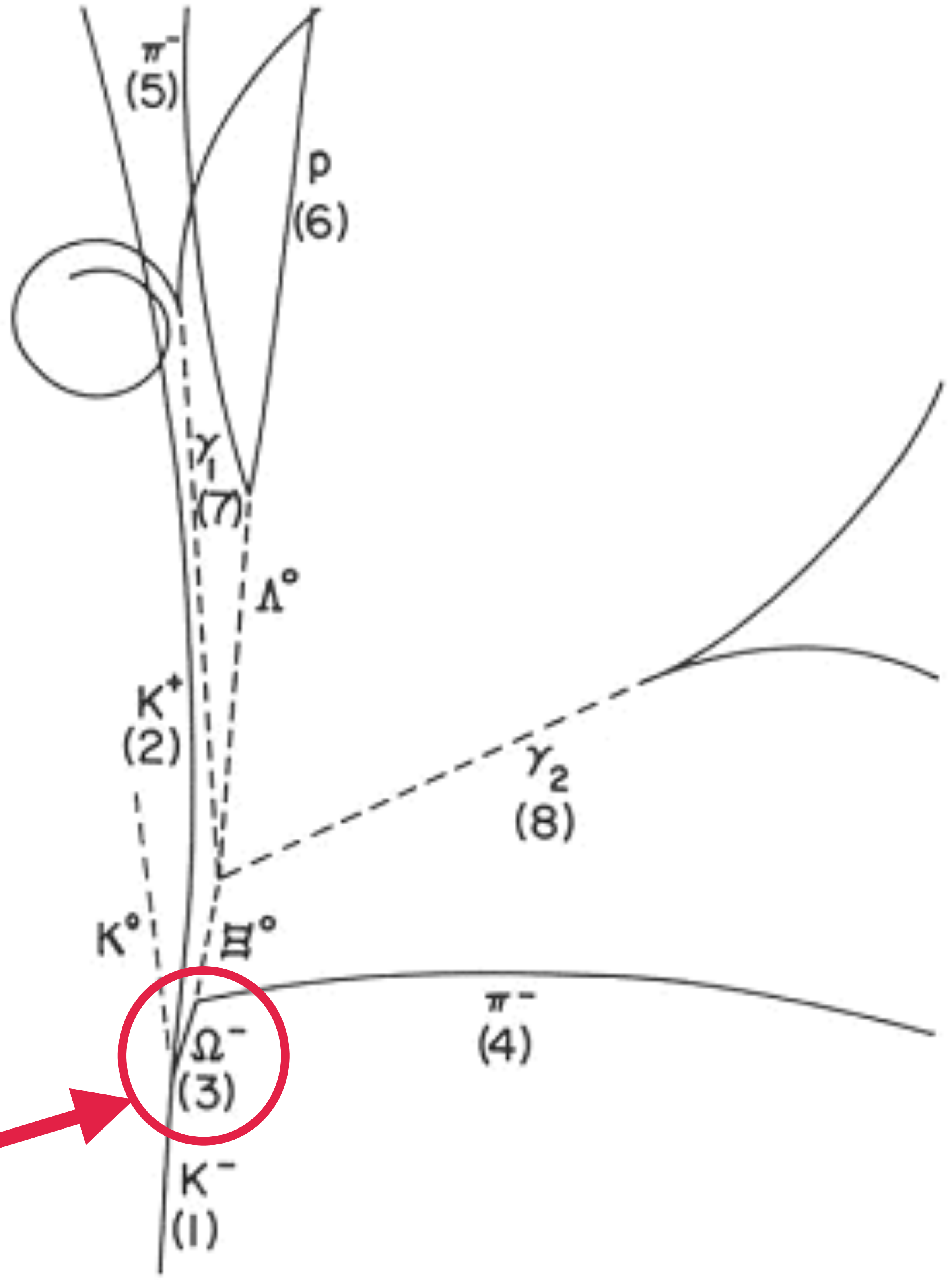
spin 3/2

In 1964, the discovery of the Ω baryon in a magnetic field

15-foot







$$K^- + p \to \Omega^- + K^- + K^0$$
$$\mapsto \Xi^0 + \pi^+$$
$$\mapsto \Lambda^0 + \pi^0$$
$$\mapsto \gamma + \gamma$$
$$\mapsto e^+ e^-$$
$$\mapsto e^+ e^-$$
$$\mapsto \pi^- p$$

In 1964, the Omega Hyperon was discovered at Brookhaven in a bubble chamber. It was predicted in 1962 by M. Gell-mann and Y. Ne'eman to have spin 3/2.

In 2006, it was shown to have spin 3/2.

Measurement of the Spin of the Omega-Minus Hyperon,
by the BaBar collaboration at SLAC (B. Aubert et al.), Phys. Rev. Lett. 97, 112001 (2006), and hep-ex/0606039.

Massive Free fields E.O.M

$$\partial_m \quad \to \quad D_m = \partial_m - ieA_m$$

Spin 0

$$\boxed{(\partial^r \partial_r - M^2)\Phi = 0}$$

$$\boxed{(D^r D_r - M^2)\Phi = 0}$$

Spin 1/2

$$\boxed{(i\gamma^n \partial_n + M)\Psi = 0}$$

$$\boxed{(i\gamma^n D_n + M)\Psi = 0}$$

Spin 1

$$\boxed{\begin{aligned} (\partial^r \partial_r - M^2)A_m &= 0 \\ \partial^m A_m &= 0 \end{aligned}}$$

$$\boxed{\begin{aligned} \left(D^2 - M^2\right) V_m + 2\mathrm{i}F_{mn}V^n &= 0 \\ D^m V_m = \frac{\mathrm{ie}}{2M^2} F_{mn}(D^m V^n - D^N V^M) & \end{aligned}}$$

Spin 3/2

$$\boxed{\begin{aligned} (i\gamma^n \partial_n + M)\Psi_m &= 0 \\ \partial^m \Psi_m &= 0 \\ \gamma^m \Psi_m &= 0 \end{aligned}}$$

Spin 2

$$\boxed{\begin{aligned} (\partial^r \partial_r - M^2)h_{mn} &= 0 \\ \partial^m h_{mn} &= 0 \\ h \equiv h_m^m &= 0 \end{aligned}}$$

Massive Free fields E.O.M

$$\partial_m \quad \to \quad D_m = \partial_m - ieA_m$$

Spin 0

$$\boxed{(\partial^r\partial_r - M^2)\Phi = 0} \qquad \boxed{(D^r D_r - M^2)\Phi = 0}$$

Spin 1/2

$$\boxed{(i\gamma^n\partial_n + M)\Psi = 0} \qquad \boxed{(i\gamma^n D_n + M)\Psi = 0}$$

Spin 1

$$\boxed{\begin{aligned}(\partial^r\partial_r - M^2)A_m &= 0\\ \partial^m A_m &= 0\end{aligned}} \qquad \boxed{\begin{aligned}\left(D^2 - M^2\right)V_m + 2\mathrm{i}F_{mn}V^n &= 0\\ D^m V_m &= \frac{\mathrm{ie}}{2M^2}F_{mn}(D^m V^n - D^N V^M)\end{aligned}}$$

Spin 3/2

$$\boxed{\begin{aligned}(i\gamma^n\partial_n + M)\Psi_m &= 0\\ \partial^m\Psi_m &= 0\\ \gamma^m\Psi_m &= 0\end{aligned}} \longrightarrow \boxed{\; ? \;}$$

Spin 2

$$\boxed{\begin{aligned}(\partial^r\partial_r - M^2)h_{mn} &= 0\\ \partial^m h_{mn} &= 0\\ h \equiv h^m_m &= 0\end{aligned}} \longrightarrow \boxed{\; ? \;}$$

Massive spin 2: minimal coupling

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Represented by a symmetric tensor h_{mn} that satisfies

Equation of motion **10 d.o.f** → $(D^r D_r - M^2)h_{mn} = 0$

Constraints **-5 d.o.f**
- -4 d.o.f → $D^m h_{mn} = 0$
- -1 d.o.f → $h \equiv h^m_m = 0$

$$[D^m, D^r D_r - M^2]h_{mn} = 0 \quad \Rightarrow \quad iQ\, F^{mr}\, D_r h_{mn} = 0$$

A new constraint !

Pathological theory: the d.o.f.'s number is different when a constant magnetic field is switched on.

Minimal Electromagnetic Coupling for Spin Two Particles (*).

P. FEDERBUSH

Department of Physics and Laboratory for Nuclear Science
Massachusetts Institute of Technology - Cambridge, Mass.

(ricevuto il 24 Ottobre 1960)

Summary. — It is noted that if the same Lagrangian that describes the linearized theory of general relativity is extended to charged massed spin 2 particles the coupling cannot be minimal electromagnetic.

It has the property of leading to a theory of massless particles like gravitons when $m = 0$. (The gauge invariance this implies probably gives a unique specification of this Lagrangian.) When $m \neq 0$ the equations describe spin 2 particles with the correct number of propagating solutions, five for particle and five for antiparticle. However, if the field is coupled to an external electromagnetic field minimally (*i.e.* by the prescription $\partial_n \to \partial_n \pm ieA_n$) in general the number of propagating fields becomes 12 rather than 10. The type of calculations necessary to reach this conclusion are described in ref. [1]. By the addition of a term

$$i\frac{e}{2}\tilde{A}^{\mu\nu}F_{\mu}{}^{\beta}A_{\beta\nu}\,,$$

to the Lagrangian the correct number of canonical variables is restored, although other inconsistencies may remain.

In conclusion we note that not only is the minimal electromagnetic coupling not unique, since it depends on the choice of the free Lagrangian, but in at least two theories, the usual spin $\frac{3}{2}$ and spin 2 theories, it leads to inconsistencies. The type of inconsistency induced in spin 2 theory is suggestive of the difficulties that one encounters in the effort to avoid subsidiary conditions; but the lack of a renormalizable theory of these spins keeps the question hypothetical.

Remarks about the Federbush Lagrangian -I

One adds to the Lagrangian gyromagnetic coupling $\quad -i\ 2\ g\ Q\ \bar{h}_{mn}\ F^{nk}\ h_k{}^m$

Gyromagnetic ratio

E.O.M's + constraints lead to

$$\boxed{\frac{3}{2}M^4 h = -i(2g\ -\ 1)iQ\,F^{mn}\,D_n D^p h_{pm} + \cdots}$$

No ghost in the Federbush Lagrangian $\quad \Rightarrow \quad g = 1/2$

Remarks about the Federbush Lagrangian -II

One adds to the minimally coupled Lagrangian the coupling:

$$-i\ 2\ g\ Q\ \bar{h}_{mn}\ F^{nk}\ h_k{}^m$$

Gyromagnetic ratio

E.O.M's + constraints lead to

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No ghost in the Federbush Lagrangian $\Rightarrow$ $g = 1/2$

Using the method of characteristics, one can show that for a magnetic field, that the vector n_μ along the normal to the characteristic hypersurfaces has components:

$$\boxed{\frac{n_0^2}{|\vec{n}|^2} = \frac{1}{1 - (\frac{3e}{2m^2})^2 \vec{B}^2}}$$

Superluminal propagation

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Summary. — It is noted that if the same Lagrangian that describes the linearized theory of general relativity is extended to charged massed spin 2 particles the coupling cannot be minimal electromagnetic.

Solves one issue: * No more ghosts

This has two issues: * Superluminal Propagation
* Gyromagnteic ratio = 1/2

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Massive Spin 3/2 Electrodynamics

S. Deser [♯], V. Pascalutsa [♭] and A. Waldron [♯]

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phvvp@flinders.edu.au

(August 6, 2018)

Introduction

Gauge interactions of massive (let alone massless) relativistic higher spin fields constitute an ancient and difficult subject. Whatever the formal problems these models encounter, *effective* higher spin theories must be constructible since approximately localised higher spin particles exist. Such models should achieve low energy consistency, and share some of the physical properties described by their lower spin hadronic physics counterparts.

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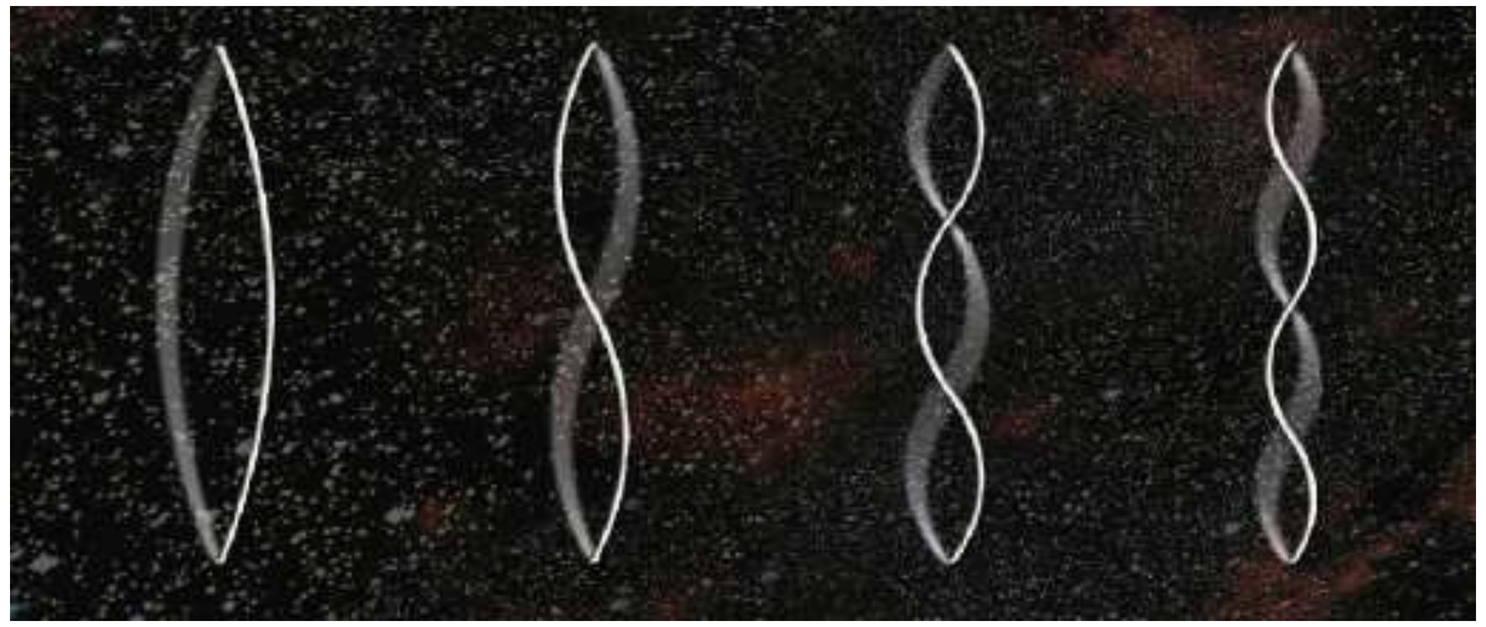
Conclusion

Our study of causality showed that no model maintaining the correct DOF avoids sharing the pathology of the minimal one. In fact this result applies to

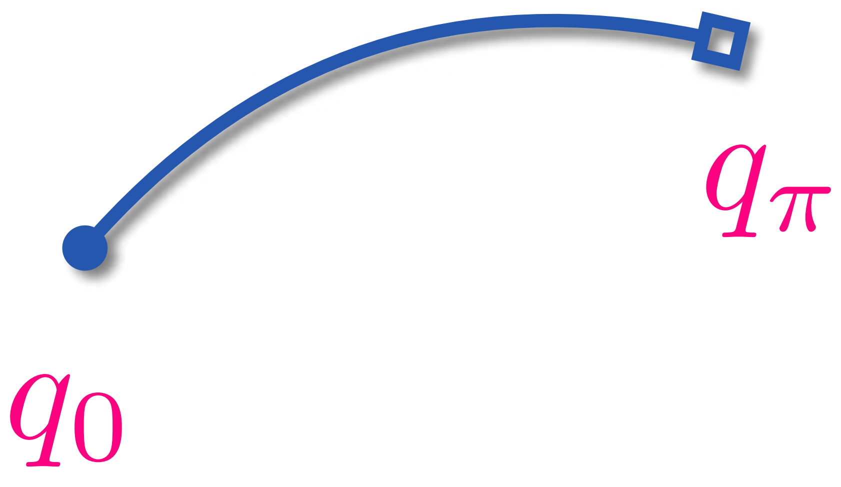
Bosonic Open Strings

Open strings massive modes

Open string



Open string



Different oscillator modes $\longrightarrow$ Different Particles

Boundary charges $\longrightarrow$ $Q = q_0 \; + \; q_\pi$

First massive string

$$|\Psi >_{(N=2)} = B_m \, a_2^{m\dagger}|0> \; + h_{mn} \, a_1^{m\dagger} a_1^{n\dagger}|0>$$

Vector *Spin 2 state*

Put in an electromagnetic background

(Abouelsaood, Callan, Nappi, Yost ’87)

Argyres-Nappi Lagrangian

B_m is a Stückelberg field for h_{mn} $\Rightarrow$ ***Only*** h_{mn} remains

Argyres-Nappi Lagrangian for charged spin-2

$$\boxed{\begin{aligned}\mathcal{L}_{AN} =&\bar{\mathcal{H}}_{mk}\mathfrak{D}^2 h_n{}^k - \bar{\mathcal{H}}\mathfrak{D}^2\mathcal{H} - \bar{\mathcal{H}}_{mn}\left\{\mathfrak{D}^m\mathfrak{D}^k\left[(1+\mathrm{i}\epsilon)\,h\right]_k{}^n - \frac{1}{2}\mathfrak{D}^m\mathfrak{D}^n\mathcal{H} + (m\leftrightarrow n)\right\} + \bar{\mathcal{H}}\mathfrak{D}^m\mathfrak{D}^n\mathcal{H}_{mn}\\ &- M^2\left(\bar{\mathcal{H}}_{mk}h_n{}^k - \bar{\mathcal{H}}\mathcal{H}\right) - 2i\bar{\mathcal{H}}_{mn}\left(\epsilon^{mk}h_k{}^n - h^m{}_k\epsilon^{kn}\right)\end{aligned}}$$

EOM's and constraints of the spin-2

$$\boxed{\begin{aligned}\left(\mathfrak{D}^2-2\right)\mathcal{H}_{mn} - 2\mathrm{i}\left(\epsilon_{km}\mathcal{H}^k{}_n + \epsilon_{kn}\mathcal{H}^k{}_m\right) &= 0\\ \mathfrak{D}^n\mathcal{H}_{mn} &= 0\\ \mathcal{H} &= 0\end{aligned}}$$

Where:

$$\epsilon = \frac{1}{\pi}(\operatorname{arctanh}(2\pi\alpha' q_0 F) + \operatorname{arctanh}(2\pi\alpha' q_\pi F)) \qquad \mathcal{M}\mathcal{M}^T = \frac{\varepsilon}{QF}, \qquad Q = q_0 + q_\pi$$

$$-i\mathcal{M}\mathcal{D} \equiv -i\mathfrak{D}, \qquad \mathcal{H}_{mn} = (\eta_{mk} - \mathrm{i}\epsilon_{mk})(\eta_{nl} - \mathrm{i}\epsilon_{nl})\,h^{kl}$$

Remarks about the Argyres-Nappi Lagrangian

- The A-N. Lagrangian $\longrightarrow$ Causal and solves the Velo-Zwanziger problem!

- Only the first massive level can be decoupled from the rest of the Regge trajectory. (Porrati, Rahman, Sagnotti ’11)

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- Expanding in « powers of the electromagnetic field strength »: (Porrati, Rahman, Sagnotti '11)

No ghost $\longrightarrow$ the Federbush Lagrangian $\longrightarrow$ $-ie\ \bar{h}_{mn}\ F^{nk}\ h_k{}^m \Rightarrow g = 1/2$ ☹️

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ghost ⛔

Expansion with infinite series of terms !

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ghost

Expansion with infinite series of terms !

- **BUT:** The A-N Lagrangian is consistent **ONLY in D=26** dimensions. (Argyres - Nappi '89)

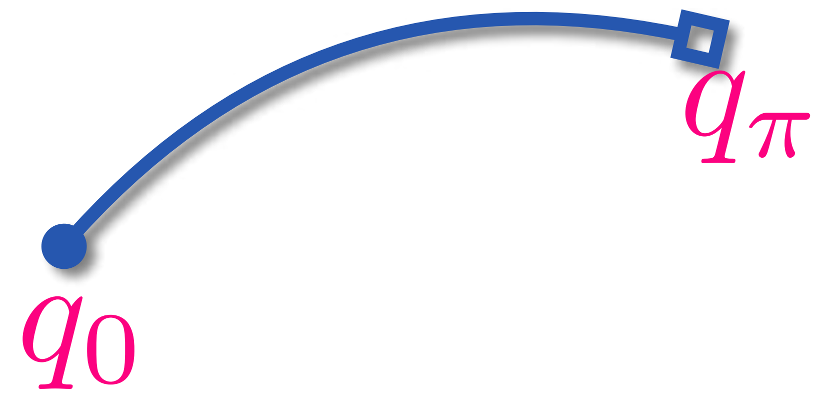
4D Open Supertrings

Strategy

The world-sheet action:

$$S_{w-s} = S_0 + q_0 S_{int}\big|_{\sigma=0} + q_\pi S_{int}\big|_{\sigma=\pi} + S_b.$$

Coupling of electromagnetic field to the charges on the boundaries



$$S_0 = \frac{1}{2\pi}\int d^2z\left\{\frac{1}{\alpha'}\partial x^m\bar{\partial}x_m + p^\alpha\bar{\partial}\theta_\alpha + \bar{p}_{\dot{\alpha}}\bar{\partial}\bar{\theta}^{\dot{\alpha}} + \hat{p}^\alpha\partial\hat{\theta}_\alpha + \hat{\bar{p}}_{\dot{\alpha}}\partial\hat{\bar{\theta}}^{\dot{\alpha}} + \frac{1}{2}\partial\rho\bar{\partial}\rho + \frac{1}{2}\partial\hat{\rho}\bar{\partial}\hat{\rho}\right\} + S_6,$$

CY

In the hybrid formalism **[Berkovits '95]**, the action is manifestly supersymmetric:

→ **Action for the spin 2 + spin 3/2 states + the rest of the supersymmetric multiplets**

The String Field Theory action (SFT): $S = \langle \Phi \mathbb{Q} \Phi \rangle$

We need the expressions of:

The BRST operator: $\mathbb{Q}$

The string field: Φ

Physical states

The string field Φ will describe on-shell:

12 bosonic d.o.f.

5 polarisations of massive spin-2

3 polarisations of massive spin-1

2 x 2 spin-0

12 fermionic d.o.f.

2 x Dirac massive spin-3/2

2 x Dirac spin-1/2

Superspace Lagrangian

[KB, N. Berkovits, C. Daniel, M. Lize '21]

- The action of the BRST operator leads to (compact form):

$$
\begin{aligned}
S = -\frac{1}{16}\int d^4x p_0^2\bar{p}_0^2 \{ & V_n^\dagger(\eta^{nm}-i\epsilon^{nm})\left[-\{d_0^2,\bar{d}_0^2\}V_m + 16\Pi_0^n\Pi_{n0}V_m - 32\left(\eta_{mp}-i\epsilon_{mp}\right)V^p\right.\\
& - 32\left(\left(\partial\bar{\theta}_0\bar{d}_0\right)V_m + \left(\partial\theta_0 d_0\right)V_m\right) + 8\bar{\sigma}_m^{\dot{\alpha}\alpha}\left(d_{\alpha 0}\bar{U}_{2\dot{\alpha}} - \bar{d}_{\dot{\alpha}0}U_{1\alpha}\right) + 32\Pi_{m0}B\\
& \left.+24\bar{\sigma}_m^{\dot{\alpha}\alpha}\left[\bar{d}_{\dot{\alpha}0},d_{\alpha 0}\right]C\right] + U_2^\alpha\left[-8\sigma^n_{\alpha\dot{\alpha}}\left(\eta_{nm}-i\epsilon_{nm}\right)\bar{d}_0^{\dot{\alpha}}V^m + 4\bar{d}_{\dot{\alpha}0}d_{\alpha 0}\bar{U}_2^{\dot{\alpha}} - 4\bar{d}_0^2U_{1\alpha}\right.\\
& \left.+d_{\alpha 0}\bar{d}_0^2(-2iB+18C) + \partial\theta_{\alpha 0}(-32iB-96C) - 48i\Pi_{\alpha\dot{\alpha}0}\bar{d}_0^{\dot{\alpha}}C\right]\\
& - \bar{U}_{1\dot{\alpha}}\left[-8\bar{\sigma}^{n\dot{\alpha}\alpha}\left(\eta_{nm}-i\epsilon_{nm}\right)d_{\alpha 0}V^m + 4d_0^2\bar{U}_2^{\dot{\alpha}} - 4d_0^\alpha\bar{d}_0^{\dot{\alpha}}U_{1\alpha} - \bar{d}_0^{\dot{\alpha}}d_0^2(2iB+18C)\right.\\
& \left.+\partial\bar{\theta}_0^{\dot{\alpha}}(-32iB+96C) + 48i\Pi_0^{\dot{\alpha}\alpha}d_{\alpha 0}C\right] + B^\dagger\left[-32\Pi_0^n\left(\eta_{nm}-i\epsilon_{nm}\right)V^m\right.\\
& \left.+\left(\{d_0^2,\bar{d}_0^2\}-64\right)B + 3i\left[d_0^2,\bar{d}_0^2\right]C - i\left(2d_0^2\bar{d}_{\dot{\alpha}0} + 32\partial\bar{\theta}_{\dot{\alpha}0}\right)\bar{U}_2^{\dot{\alpha}} + i\left(2\bar{d}_0^2d_0^\alpha + 32\partial\theta_0^\alpha\right)U_{1\alpha}\right]\\
& + 3C^\dagger\left[-8\bar{\sigma}^{n\dot{\alpha}\alpha}\left[d_{\alpha 0},\bar{d}_{\dot{\alpha}0}\right]\left(\eta_{nm}-i\epsilon_{nm}\right)V^m - \left(6d_0^\alpha\bar{d}_0^2 + 8i\Pi_0^{\dot{\alpha}\alpha}\bar{d}_{\dot{\alpha}0}\right)U_{1\alpha}\right.\\
& - \left(6\bar{d}_{\dot{\alpha}0}d_0^2 + 8i\Pi_{\alpha\dot{\alpha}0}d_0^\alpha\right)\bar{U}_2^{\dot{\alpha}} - \left[d_0^2,\bar{d}_0^2\right]iB\\
& \left.- \left(-11\{d_0^2,\bar{d}_0^2\} + 128\Pi_0^n\Pi_{n0} - 256\partial\bar{\theta}_{\dot{\alpha}0}\bar{d}_0^{\dot{\alpha}} - 256\partial\theta_0^\alpha d_{\alpha 0} - 64\right)C\right]\}
\end{aligned}
$$

No E.M $\longrightarrow$ [N. Berkovits, M. Leite '98]

Superspace Lagrangian

[KB, N. Berkovits, C. Daniel, M. Lize ’21]

- The action of the BRST operator leads to (compact form):

$$
\begin{aligned}
S = -\frac{1}{16}\int d^4x\, p_0^2\bar{p}_0^2 \Big\{ & V_n^\dagger(\eta^{nm} - i\epsilon^{nm})\Big[-\{d_0^2,\bar{d}_0^2\} V_m + 16\Pi_0^n\Pi_{n0} V_m - 32\left(\eta_{mp} - i\epsilon_{mp}\right) V^p \\
& - 32\left((\partial\bar{\theta}_0\bar{d}_0) V_m + (\partial\theta_0 d_0) V_m\right) + 8\bar{\sigma}_m^{\dot\alpha\alpha}\left(d_{\alpha0}\bar{U}_{2\dot\alpha} - \bar{d}_{\dot\alpha0} U_{1\alpha}\right) + 32\Pi_{m0}B \\
& +24\bar{\sigma}_m^{\dot\alpha\alpha}\left[\bar{d}_{\dot\alpha0}, d_{\alpha0}\right] C\Big] + U_2^{\alpha}\Big[-8\sigma^n_{\alpha\dot\alpha}\left(\eta_{nm} - i\epsilon_{nm}\right)\bar{d}_0^{\dot\alpha} V^m + 4\bar{d}_{\dot\alpha0}d_{\alpha0}\bar{U}_2^{\dot\alpha} - 4\bar{d}_0^2 U_{1\alpha} \\
& + d_{\alpha0}\bar{d}_0^2(-2iB + 18C) + \partial\theta_{\alpha0}(-32iB - 96C) - 48i\Pi_{\alpha\dot\alpha0}\bar{d}_0^{\dot\alpha} C\Big] \\
& - \bar{U}_{1\dot\alpha}\Big[-8\bar{\sigma}^{n\dot\alpha\alpha}\left(\eta_{nm} - i\epsilon_{nm}\right) d_{\alpha0} V^m + 4d_0^2\bar{U}_2^{\dot\alpha} - 4d_0^\alpha\bar{d}_0^{\dot\alpha} U_{1\alpha} - \bar{d}_0^{\dot\alpha} d_0^2(2iB + 18C) \\
& + \partial\bar{\theta}_0^{\dot\alpha}(-32iB + 96C) + 48i\Pi_0^{\dot\alpha\alpha} d_{\alpha0} C\Big] + B^\dagger\Big[-32\Pi_0^n\left(\eta_{nm} - i\epsilon_{nm}\right) V^m \\
& + \left(\{d_0^2,\bar{d}_0^2\} - 64\right) B + 3i\left[d_0^2,\bar{d}_0^2\right] C - i\left(2d_0^2\bar{d}_{\dot\alpha0} + 32\partial\bar{\theta}_{\dot\alpha0}\right)\bar{U}_2^{\dot\alpha} + i\left(2\bar{d}_0^2 d_0^\alpha + 32\partial\theta_0^\alpha\right) U_{1\alpha}\Big] \\
& + 3C^\dagger\Big[-8\bar{\sigma}^{n\dot\alpha\alpha}\left[d_{\alpha0},\bar{d}_{\dot\alpha0}\right]\left(\eta_{nm} - i\epsilon_{nm}\right) V^m - \left(6d_0^\alpha\bar{d}_0^2 + 8i\Pi_0^{\dot\alpha\alpha}\bar{d}_{\dot\alpha0}\right) U_{1\alpha} \\
& - \left(6\bar{d}_{\dot\alpha0} d_0^2 + 8i\Pi_{\alpha\dot\alpha0} d_0^\alpha\right)\bar{U}_2^{\dot\alpha} - \left[d_0^2,\bar{d}_0^2\right] iB \\
& - \left(-11\left\{d_0^2,\bar{d}_0^2\right\} + 128\Pi_0^n\Pi_{n0} - 256\partial\bar{\theta}_{\dot\alpha0}\bar{d}_0^{\dot\alpha} - 256\partial\theta_0^\alpha d_{\alpha0} - 64\right) C\Big]\Big\}
\end{aligned}
$$

In components and in unitary gauge

[KB, N. Berkovits, C. Daniel, W. Ke (in progress …)]

$$
\begin{aligned}
\mathcal{L}_2 =& \bar{\mathcal{C}}^m \mathfrak{D}^2 \mathcal{C}_m + \mathfrak{D}^m \bar{\mathcal{C}}_m \mathfrak{D}^n \mathcal{C}_n - 2\bar{\mathcal{C}}^m \left(\eta_{mn} - \mathrm{i}\epsilon_{mn}\right) \mathcal{C}^n \\
& + 2\bar{a}^m a_m - \mathrm{i}\epsilon_{mn}\bar{a}^m a^n + \mathfrak{D}^m \bar{a}_m \mathfrak{D}^n a_n - \left[\frac{\mathrm{i}}{\sqrt{2}}\bar{a}^m \left(\epsilon^{nk}\varepsilon_{mkpq}\mathfrak{D}^q h_n{}^p\right) + \mathrm{h.c.}\right] \\
& - 2\bar{c}^m c_m + \left[\bar{c}^m \left(\sqrt{2}\mathrm{i}\tilde{\epsilon}_{mn}a^n - \frac{2}{5}\mathfrak{D}_m h + \left(\eta^{nk} - \mathrm{i}\epsilon^{nk}\right)\mathfrak{D}_n h_{mk}\right) + \mathrm{h.c.}\right] \\
& - \frac{2}{5}\mathfrak{D}^m \bar{c}_m \mathfrak{D}^n c_n + \frac{1}{10}\bar{h}h - \bar{h}^{mn}h_{mn} + \frac{1}{2}\bar{h}_{mk}\left(\eta^{mn} - \mathrm{i}\epsilon^{mn}\right)\mathfrak{D}^2 h_n{}^k \\
& - \frac{1}{2}\bar{h}_{km}\left(\eta^{kl} - \mathrm{i}\epsilon^{kl}\right)\mathfrak{D}^m \mathfrak{D}^n h_{ln} - 3\mathrm{i}\epsilon^{kl}\bar{h}_{lm}h_k{}^m + \frac{3}{2}\epsilon^{kl}\epsilon^{mn}\bar{h}_{ln}h_{km} - \frac{1}{2}\epsilon_k{}^m\epsilon^{kl}\bar{h}_{mn}h_l{}^n \\
& + \bar{\mathcal{M}}_1\left(-2 + \mathfrak{D}^2\right)\mathcal{M}_1 + \bar{\mathcal{N}}_1\left(-2 + \mathfrak{D}^2\right)\mathcal{N}_1
\end{aligned}
$$

Equations of motion **and constraints**

$$
\left(\mathfrak{D}^2 - 2\right)\mathcal{H}_{mn} + 2\mathrm{i}\left[(\epsilon \cdot \mathcal{H})_{mn} - (\mathcal{H} \cdot \epsilon)_{mn}\right] = 0,
$$

$$
\mathfrak{D}^n \mathcal{H}_{mn} = -\frac{\mathrm{i}}{\sqrt{2}}\tilde{\epsilon}_{mn}a^n + \mathrm{i}\epsilon_{mn}c^n
$$

$$
\mathcal{H} = 0
$$

$$
\mathcal{H}_{[mn]} = \frac{1}{\sqrt{2}}\tilde{F}_{mn}(a) + F_{mn}(c)
$$

Equations of Motion for a spin 3/2 in a electromagnetic background

- The equations of motion in electromagnetic background: $R_\mu = \gamma_{\mu\nu\rho}\mathcal{D}^\nu\psi^\rho + \frac{ie}{m}\mathcal{F}_{\mu\nu}\psi^\nu = 0$
- The primary constraint can be seen as from the absence of time derivative on the zeroth component: $$\gamma^0 R_0 = \gamma_{ij}\,\mathcal{D}^i\psi^j + \frac{ie}{m}\gamma_0\,\mathcal{F}^{0i}\,\psi_i = 0$$
- The secondary constraint can be written as: $\gamma^0 R\,\psi_0 + \cdots$ Deser, Pascalutsa, Waldron

$$R = -\frac{3}{2}m^2 + \frac{ie}{2}\gamma_i F^{ij}\gamma_j - \frac{ie}{m}\gamma_0\,[\mathcal{D}_i, \mathcal{F}^{0i}] - \frac{e^2}{2m^2}\gamma_0\mathcal{F}^{0i}\gamma_0\,\gamma_j\gamma_i\mathcal{F}^{0j}$$

And when the determinant $det R = 0$ then ψ_0 is no more completely determined.

Porrati-Rahman propose a parametrized Lagrangian and to impose $\gamma^m\Psi_m = 0$

then determine the parameters of the Lagrangian order by order

The secondary constraint

$$\mathcal{D}.R = -\frac{3}{2}\,m^2\,\gamma.\psi + \frac{ie}{2}\gamma_{\mu\nu\rho}\,F^{\mu\nu}\psi^\rho + \frac{ie}{m}\,(\mathcal{D}^i\mathcal{F}_{i\nu} + \frac{1}{2}\,m\gamma^0\mathcal{F}_{0\nu})\,\psi^\nu + \frac{ie}{m}\,(\dot{\mathcal{F}}^{0i}\,\psi_i + \mathcal{F}^{0i}\,\dot{\psi}_i) = 0$$

can be written as: $\gamma^0 R\,\psi_0 + \cdots$ where

$$R = -\frac{3}{2}\,m^2 + \frac{ie}{2}\,\gamma_i F^{ij}\gamma_j - \frac{ie}{m}\,\gamma_0\,[\mathcal{D}_i, \mathcal{F}^{0i}] - \frac{e^2}{2m^2}\,\gamma_0\mathcal{F}^{0i}\gamma_0\,\gamma_j\gamma_i\mathcal{F}^{0j}$$

but when the determinant $detR = 0$ then ψ_0 is no more completely determined.

Using the method of characteristics, one can show that $detR = 0$ leads to components of the vector n_μ along the normal to the characteristic hypersurfaces:

$$\frac{n_0^2}{|\vec{n}|^2} = \frac{1}{1 - (\frac{3e}{2m^2})^2\,\vec{B}^2}$$

Thus, with a velocity bigger than 1 (in light speed units).

Massive charged spin3/2 open string oscillator

[K.B., N. Berkovits, C. Daniel, M. Lize '21]

Spin 3/2 equations of motion:

$$i\mathfrak{D}_n(\bar\sigma^n\chi_{1m})^{\dot\alpha}+\sqrt{2}\bar\lambda^{\dot\alpha}_{1m}=0$$
$$i\mathfrak{D}_n(\sigma^n\bar\chi_{2m})_\alpha+\sqrt{2}\lambda_{2\alpha m}=0$$
$$i\mathfrak{D}_n(\sigma^n\bar\lambda_{1m})_\alpha+\sqrt{2}\chi_{1\alpha m}-i\sqrt{2}\varepsilon_{mn}\chi^n_{1\alpha}=0$$
$$i\mathfrak{D}_n(\bar\sigma^n\lambda_{2m})^{\dot\alpha}+\sqrt{2}\bar\chi^{\dot\alpha}_{2m}-i\sqrt{2}\varepsilon_{mn}\bar\chi_2^{\dot\alpha n}=0.$$

Free case:

$$(i\gamma^\mu\partial_\mu-m)\psi_\nu=0$$
$$\gamma^\mu\psi_\mu=0$$
$$\partial^\mu\psi_\mu=0$$

The constraints:

$$\mathfrak{D}^m\chi_{1\alpha m}-2\varepsilon_{rs}(\sigma^{rs}\xi_1)_\alpha=0$$
$$\mathfrak{D}^m\bar\chi^{\dot\alpha}_{2m}+2\varepsilon_{rs}(\bar\sigma^{rs}\bar\xi_2)^{\dot\alpha}=0$$
$$\mathfrak{D}^m\bar\lambda^{\dot\alpha}_{1m}-i\frac{\sqrt{2}}{2}\tilde\varepsilon^{mn}(\bar\sigma_n\chi_{1m})^{\dot\alpha}+2\varepsilon_{rs}(\bar\sigma^{rs}\bar\psi_1)^{\dot\alpha}-2\sqrt{2}i\varepsilon^{mn}\mathfrak{D}_m(\bar\sigma_n\xi_1)^{\dot\alpha}=0$$
$$\mathfrak{D}^m\lambda_{2\alpha m}+i\frac{\sqrt{2}}{2}\tilde\varepsilon^{mn}(\sigma_n\bar\chi_{2m})_\alpha-2\varepsilon_{rs}(\sigma^{rs}\psi_2)_\alpha+2\sqrt{2}i\varepsilon^{mn}\mathfrak{D}_m(\sigma_n\bar\xi_2)_\alpha=0$$

Massive charged spin3/2 open string oscillator

[K.B., N. Berkovits, C. Daniel, W.Ke '22]

Spin 1/2 and 3/2 equations of motion and constraints:

Charged (NEW!)

$$\boxed{\begin{aligned}&\left(\mathrm{i}\not{\mathfrak{D}}+\sqrt{2}\right)\Phi_2=0\\&\left[\mathfrak{D}^m-\frac{\sqrt{2}}{4}\left(\epsilon^{mn}-\mathrm{i}\tilde{\epsilon}^{mn}\right)\gamma_n\right]\boldsymbol{\Psi}_{2m}=0,\quad\gamma^m\boldsymbol{\Psi}_{2m}=0\\&\left(\mathrm{i}\not{\mathfrak{D}}+\sqrt{2}\right)\boldsymbol{\Psi}_{2m}=\sqrt{2}\mathrm{i}\epsilon_{mn}\boldsymbol{\Psi}^n_{2R}\end{aligned}}$$

Neutral

Spin 1/2 $\quad (i\gamma^n\partial_n+\sqrt{2})\Phi_2=0$

Spin 3/2 $\quad \partial^m\Psi_m=0 \qquad \gamma^m\Psi_{2m}=0$

$$(i\gamma^n\partial_n+\sqrt{2})\Psi_{2m}=0$$

Massive Free fields E.O.M

$$\partial_m \quad \rightarrow \quad D_m = \partial_m - ieA_m$$

Spin 0

$$(\partial^r \partial_r - M^2)\Phi = 0$$

$$(D^r D_r - M^2)\Phi = 0$$

Spin 1/2

$$(i\gamma^n \partial_n + M)\Psi = 0$$

$$(i\gamma^n D_n + M)\Psi = 0$$

Spin 1

$$(\partial^r \partial_r - M^2)A_m = 0$$
$$\partial^m A_m = 0$$

$$\left(D^2 - M^2\right) V_m + 2\mathrm{i}F_{mn}V^n = 0$$
$$D^m V_m = \frac{\mathrm{i}e}{2M^2} F_{mn}(D^m V^n - D^N V^M)$$

Spin 3/2

$$(i\gamma^n \partial_n + M)\Psi_m = 0$$
$$\partial^m \Psi_m = 0$$
$$\gamma^m \Psi_m = 0$$

$$\longrightarrow$$

$$\left(\mathfrak{D}^2 - 2\right)\mathcal{H}_{mn} - 2\mathrm{i}\left(\epsilon_{km}\mathcal{H}^k{}_n + \epsilon_{kn}\mathcal{H}^k{}_m\right) = 0$$
$$\mathfrak{D}^n \mathcal{H}_{mn} = 0$$
$$\mathcal{H} = 0$$

Spin 2

$$(\partial^r \partial_r - M^2)h_{mn} = 0$$
$$\partial^m h_{mn} = 0$$
$$h \equiv h^m_m = 0$$

$$\longrightarrow$$

$$\left(\mathrm{i}\not{\mathfrak{D}} + \sqrt{2}\right)\Phi_2 = 0$$
$$\left[\mathfrak{D}^m - \frac{\sqrt{2}}{4}\left(\epsilon^{mn} - \mathrm{i}\tilde{\epsilon}^{mn}\right)\gamma_n\right]\mathbf{\Psi}_{2m} = 0, \quad \gamma^m \mathbf{\Psi}_{2m} = 0$$
$$\left(\mathrm{i}\not{\mathfrak{D}} + \sqrt{2}\right)\mathbf{\Psi}_{2m} = \sqrt{2}\mathrm{i}\epsilon_{mn}\mathbf{\Psi}^n_{2R}$$

Conclusions

Dirac 1936:

« The elementary particles known to present-day physics, the electron, positron, neutron, and proton, each have a spin of a half, and thus the work of the present paper will have no immediate physical application. All the same, it is desirable to have the equation ready for a possible future discovery of an elementary particle with a spin greater than a half, or for approximate application to composite particles. Further, the underlying theory is of considerable mathematical interest. »

Where do we stand now?

Higher spin (composite) states exist in Nature.

Minimal coupling to constant electromagnetic background leads to pathologies.

SFT helps construct E.O.M's and Lagrangian of the massive charged spin 3/2, 2, … fields.

Equations of motion found!

The fields in 4D appear to be coupled to lower spin ones.

Equations of motion found! Work in progress …