NATIONAL SCIENCE CENTRE
POLAND

FACULTY OF PHYSICS
UNIVERSITY OF WARSAW

Testing Bell inequalities in $H \to \tau^+\tau^-$ @ high energy lepton colliders

Kazuki Sakurai

(University of Warsaw)

In collaboration with:

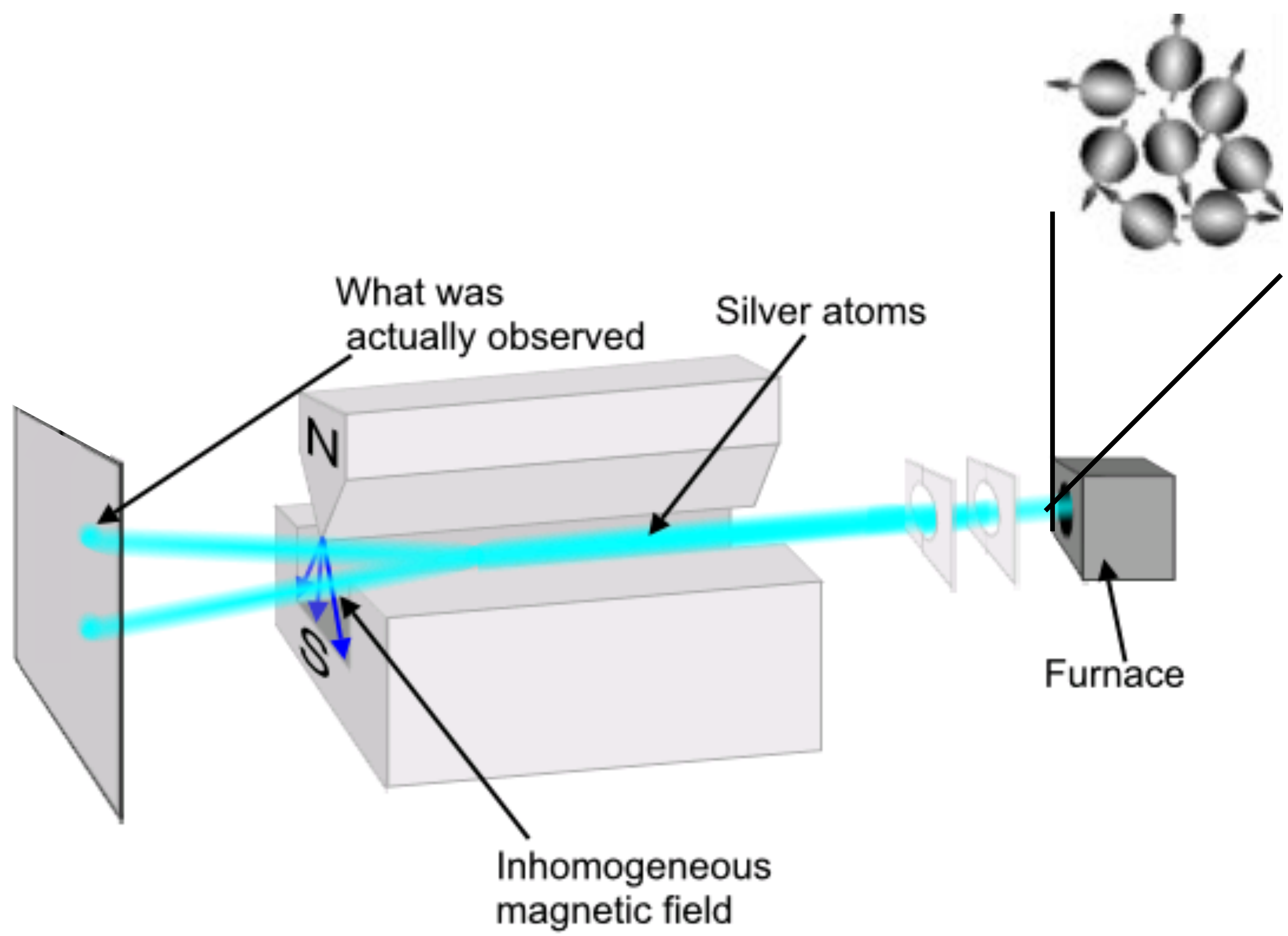
Mohammad Altakach, Fabio Maltoni, Kentarou Mawatari, Priyanka Lamba

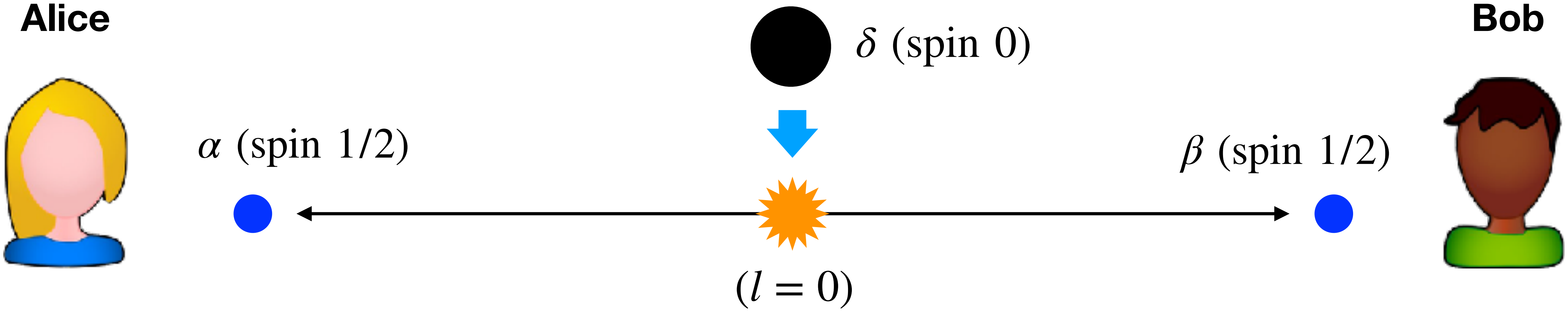
29/08/2022, Workshop on the Standard Model and Beyond @ Corfu Summer Institute

Spin

In classical mechanics, the components of angular momentum (l_x, l_y, l_z) take continuous real numbers.

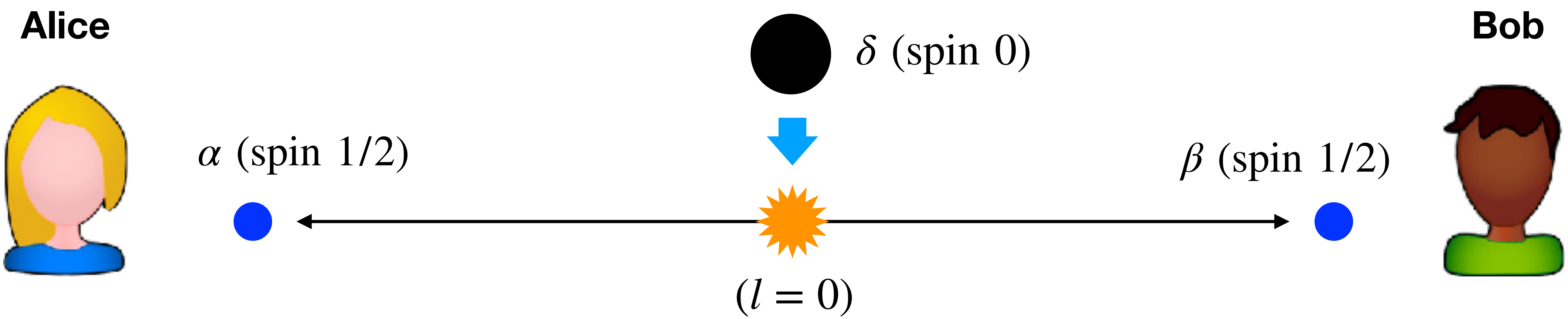
A striking fact, found in the Stern-Gerlach experiment, is that the measurement outcome of spin component is either $+1$ or -1 (in the $\hbar/2$ unit).





- Alice and Bob receive particles α and β, respectively, and measure the spin z-component of their particles. Repeat the process many times.
- Alice and Bob will find their results are completely random (+1 and -1 50-50%)
- Nevertheless, their result is 100% anti-correlated due to the angular momentum conservation. If Alice's result is +1, Bon's result is always -1 and vice versa.

Alice	+	+	-	+	-	-	+	+	+	-	+	-
Bob	-	-	+	-	+	+	-	-	-	+	-	+



- Alice and Bob receive particles α and β, respectively, and measure the spin z-component of their particles. Repeat the process many times.
- Alice and Bob will find their results are completely random (+1 and -1 50-50%)
- Nevertheless, their result is 100% anti-correlated due to the angular momentum conservation. If Alice's result is +1, Bon's result is always -1 and vice versa.

Alice	+	+	-	+	-	-	+	+	+	-	+	-
Bob	-	-	+	-	+	+	-	-	-	+	-	+
$S_z^\alpha \cdot S_z^\beta$	-	-	-	-	-	-	-	-	-	-	-	-

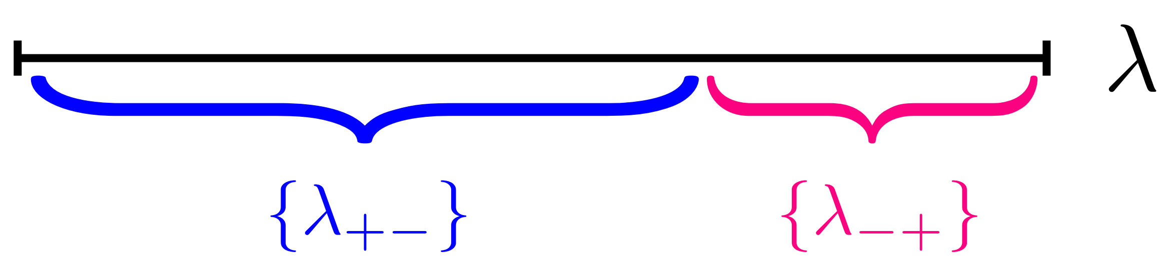
$$\langle S_z^\alpha \cdot S_z^\beta \rangle = -1$$

The most natural explanation would be as follows:

- Since their result is sometimes +1 and sometimes -1, it is natural to think that *the state of α and β are different in each decay.* The result look random, since we don't know in which sate the α and β particles are in each decay.

- This means we can parametrise the state of α and β by a set of unknown (hidden) variables, λ. For i-th decay, their states are:

$$\alpha(\lambda_i), \qquad \beta(\lambda_i)$$



If $\lambda_i \in \{\lambda_{+-}\} \implies S_z[\alpha(\lambda_i)] = +1, \;\; S_z[\beta(\lambda_i)] = -1$

If $\lambda_i \in \{\lambda_{-+}\} \implies S_z[\alpha(\lambda_i)] = -1, \;\; S_z[\beta(\lambda_i)] = +1$

$$P(\lambda \in \{\lambda_{+-}\}) = P(\lambda \in \{\lambda_{-+}\}) = \frac{1}{2}$$

The explanation in QM is very different.

Although their outcomes are different in each decay, QM says
the state of the particles are exactly the same for all decays:

$$|\,\Psi^{(0,0)}\rangle \underset{\text{up to a phase } e^{i\theta}}{\doteq} \frac{|\overset{\alpha}{+}\,,\overset{\beta}{-}\,\rangle_z - |-\,,+\,\rangle_z}{\sqrt{2}}$$

- Before Alice's measurement, Bob's outcome is undetermined

The explanation in QM is very different.

Although their outcomes are different in each decay, QM says *the state of the particles are exactly the same for all decays*:

$$|\Psi^{(0,0)}\rangle \doteq \frac{|\overset{\alpha}{+},\overset{\beta}{-}\rangle_z - |-,+\rangle_z}{\sqrt{2}}$$

up to a phase $e^{i\theta}$

- Before Alice's measurement, Bob's outcome is undetermined

- At the moment when Alice makes her measurement, the state collapses into:

$$|\Psi\rangle \xrightarrow{\text{Alice's measurement}} \begin{cases} |+,-\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = +1 \\ |-,+\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = -1 \end{cases}$$

The explanation in QM is very different.

Although their outcomes are different in each decay, QM says
the state of the particles are exactly the same for all decays:

$$|\Psi^{(0,0)}\rangle \doteq \frac{|\overset{\alpha}{+},\overset{\beta}{-}\rangle_z - |-,+\rangle_z}{\sqrt{2}}$$

up to a phase $e^{i\theta}$

- Before Alice's measurement, Bob's outcome is undetermined

- At the moment when Alice makes her measurement, the state collapses into:

$$|\Psi\rangle \xrightarrow[\text{Alice's measurement}]{} \begin{cases} |+,-\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = +1 \\ |-,+\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = -1 \end{cases}$$

Bob's outcome is completely determined (before his measurement)
and 100% anti-correlated with Alice's

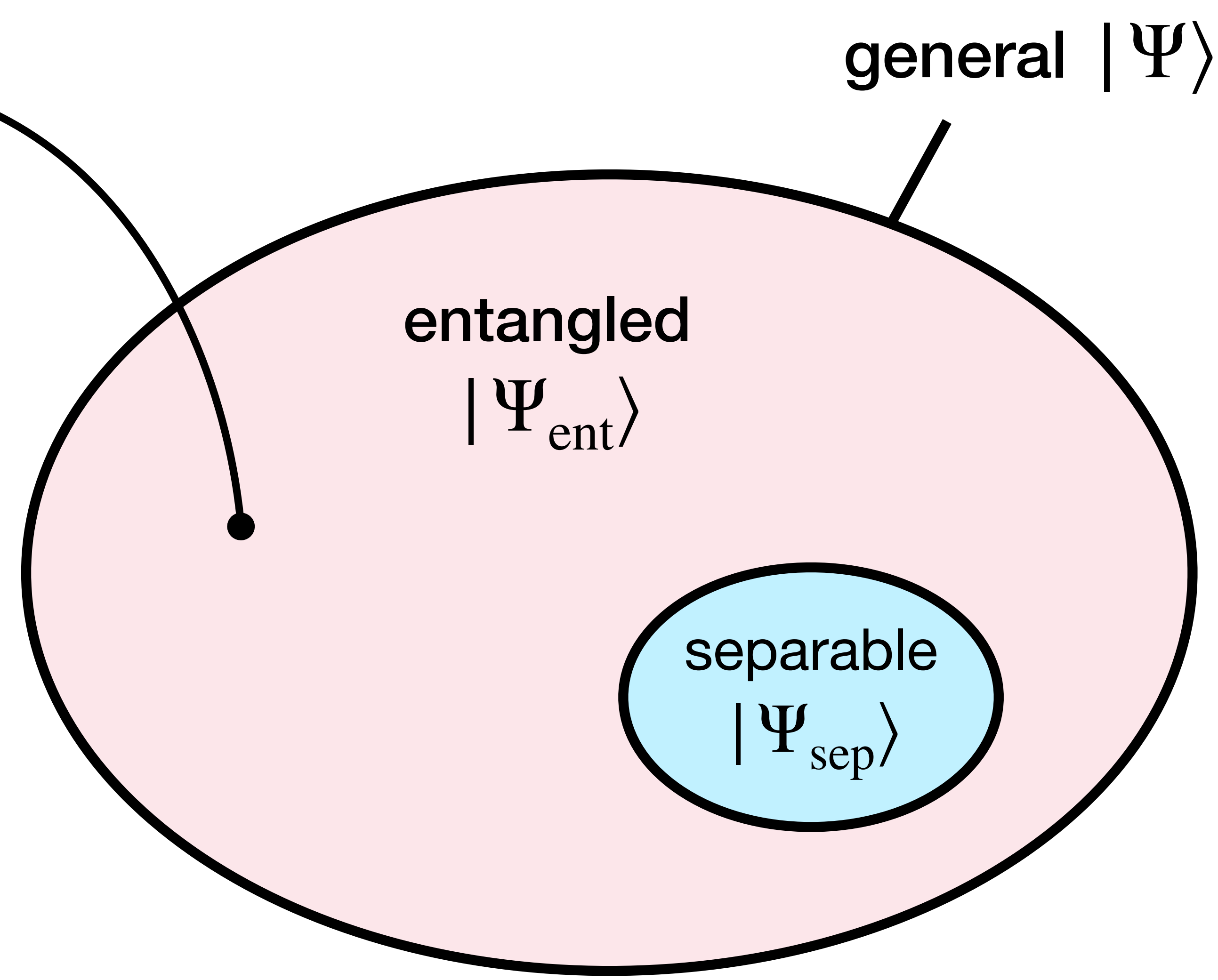
The origin of this bizarre feature is ***entanglement***.

general: $|\Psi\rangle \doteq c_{11}|+,+\rangle_z + c_{12}|+,-\rangle_z + c_{21}|-,+\rangle_z + c_{22}|-,-\rangle_z$

separable: $|\Psi_{\text{sep}}\rangle \doteq \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$

entangled: $|\Psi_{\text{ent}}\rangle \not\doteq \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$

entangled: $|\Psi^{(0,0)}\rangle \doteq \dfrac{|+,-\rangle_z - |-,+\rangle_z}{\sqrt{2}}$



The origin of this bizarre feature is ***entanglement***.

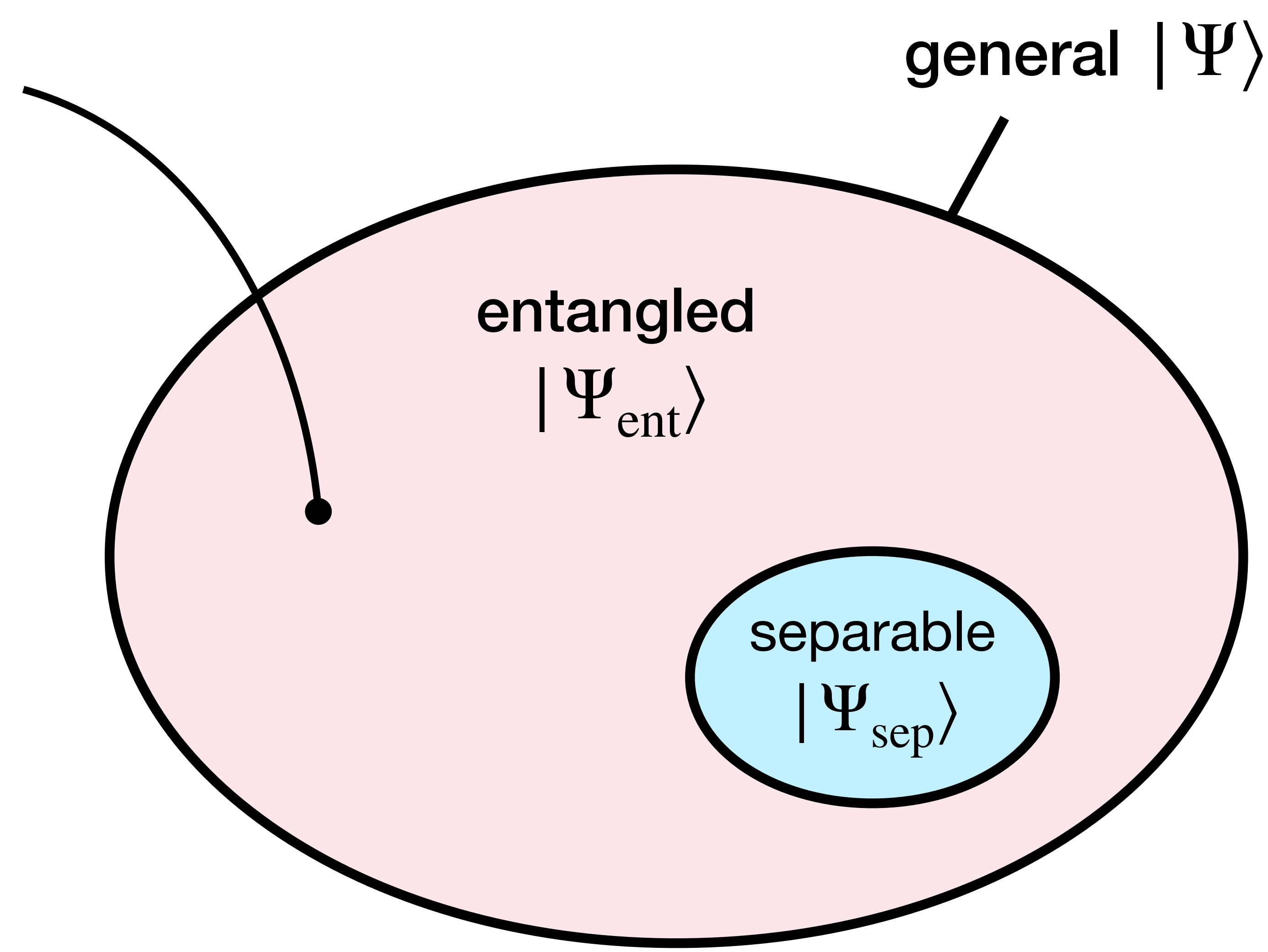
general: $|\Psi\rangle \doteq c_{11}|+,+\rangle_z + c_{12}|+,-\rangle_z + c_{21}|-,+\rangle_z + c_{22}|-,-\rangle_z$

separable: $|\Psi_{\text{sep}}\rangle \doteq \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \boxed{\left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]} \longrightarrow |+\rangle_z$

Bob's measurement collapses the state of β but doesn't influence the state of α

entangled: $|\Psi_{\text{ent}}\rangle \cancel{\doteq} \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$

entangled: $|\Psi^{(0,0)}\rangle \doteq \dfrac{|+,-\rangle_z - |-,+\rangle_z}{\sqrt{2}}$



EPR paradox

Einstein, Podolsky and Rosen (EPR) did not like the QM explanation.

EPR's local-real requirement:

- Physical observables must be **real**: their values must be predetermined [before]/[irrespectively with] the measurement.
- Physical observables must be **local**: an action in one place cannot influence a physical observable in a space-like separated region.

QM violates both local and real requirements

EPR paradox

Einstein, Podolsky and Rosen (EPR) did not like the QM explanation.

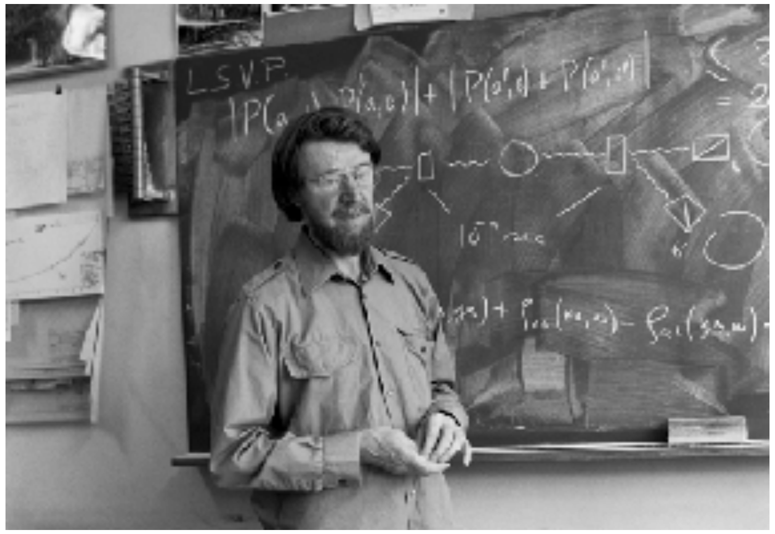
EPR's local-real requirement:

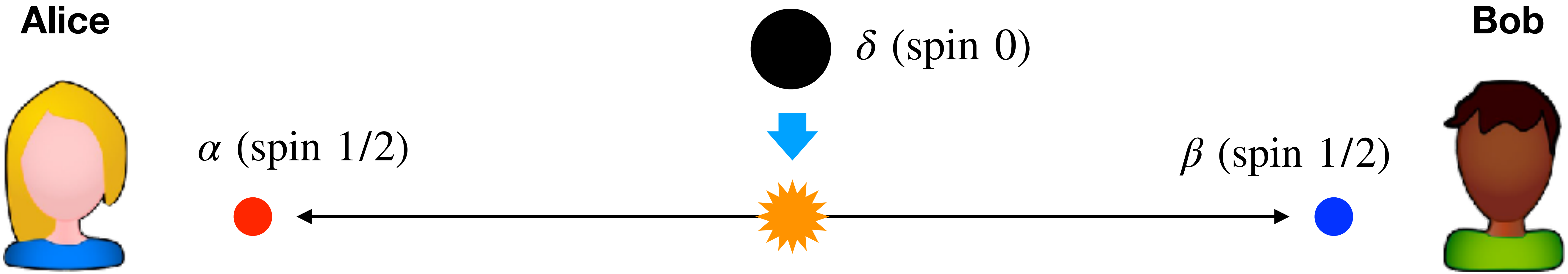
- Physical observables must be **real**: their values must be predetermined [before]/[irrespectively with] the measurement.
- Physical observables must be **local**: an action in one place cannot influence a physical observable in a space-like separated region.

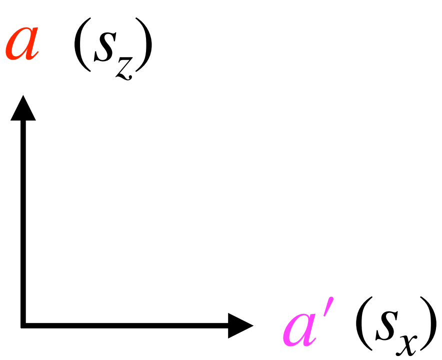
QM violates both local and real requirements

It seems difficult to experimentally discriminate QM and general hidden variable theories.

John Bell (1964) derived simple inequalities that can discriminate QM and general hidden variable theories: **Bell inequalities**

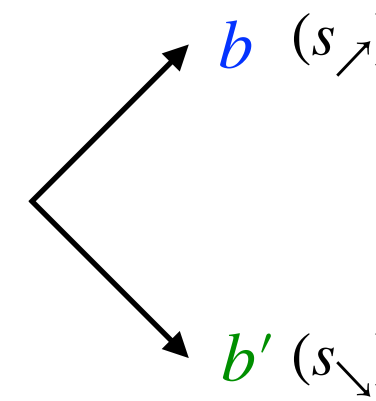






The experiment consists of 4 sessions:

1) Alice and Bob measure $s_a[\alpha]$ and $s_b[\beta]$, respectively. Repeat the measurement many times and calculate $\langle s_a \cdot s_b \rangle$.
2) Repeat (1) but for a and b'.
3) Repeat (1) but for a' and b.
4) Repeat (1) but for a' and b'.



Finally, we construct

$$R_{\mathrm{CHSH}} \equiv \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

One can show **in hidden variable theories**:

[Clauser, Horne, Shimony, Holt, 1969]

$$R_{\text{CHSH}} \equiv \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right| \leq 1$$

$$
\begin{aligned}
|\langle ab\rangle - \langle ab'\rangle| &= \left|\int d\lambda\,(ab - ab')\,P\right| \\
&= \int d\lambda\,|ab(1\pm a'b')P - ab'(1\pm a'b)P| \\
&\leq \int d\lambda\Big(|ab||1\pm a'b'|P + |ab'||1\pm a'b|P\Big) \\
&= \int d\lambda\Big[(1\pm a'b')P + (1\pm a'b)P\Big] \\
&= 2 \pm (\langle a'b'\rangle + \langle a'b\rangle)
\end{aligned}
$$

$\pm\, aba'b'P - (\pm\, aba'b'P) = 0$

$a = s_a$
$b = s_b$
$\vdots$

$|ab| = |ab'| = 1$

$|1 \pm a'b'|, |1 \pm a'b| \geq 0$

$$\boxed{\langle ab\rangle = \int a(\lambda)b(\lambda)P(\lambda)d\lambda}$$

$$\Rightarrow \quad \tilde{R}_{\text{CHSH}} = \frac{1}{2}\left(|\langle ab\rangle - \langle ab'\rangle| + |\langle a'b\rangle + \langle a'b'\rangle|\right) \leq 1$$

$$\int P(\lambda)d\lambda = 1$$

$$\max_{(\vec{a},\vec{b},\vec{a}',\vec{b}')}\left(R_{\text{CHSH}}\right) = \max_{(\vec{a},\vec{b},\vec{a}',\vec{b}')}\left(\tilde{R}_{\text{CHSH}}\right)$$

In QM, for $|\Psi^{(0,0)}\rangle \doteq \dfrac{|+,-\rangle_z - |-,+\rangle_z}{\sqrt{2}}$

one can show

$$\langle s_a s_b \rangle = \langle \Psi^{(0,0)} | s_a s_b | \Psi^{(0,0)} \rangle = (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})$$

therefore

$$R_{\text{CHSH}} = \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

$$= \frac{1}{2} \left| (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}) - (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}') + (\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}}) + (\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}}') \right|$$

In QM, for $\quad |\Psi^{(0,0)}\rangle \doteq \dfrac{|+,-\rangle_z - |-,+\rangle_z}{\sqrt{2}}$

one can show

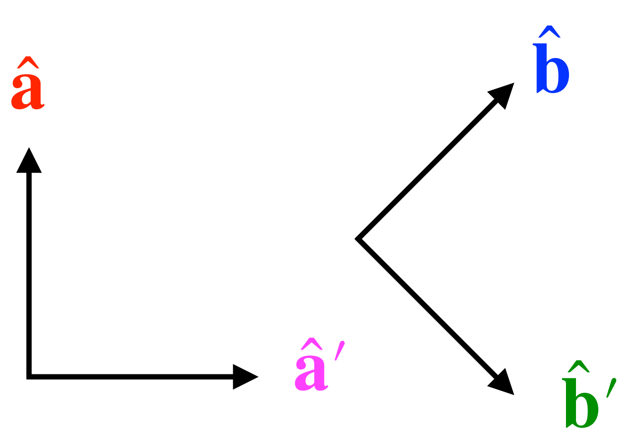
$$\langle s_a s_b \rangle = \langle \Psi^{(0,0)} | s_a s_b | \Psi^{(0,0)} \rangle = (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})$$

therefore

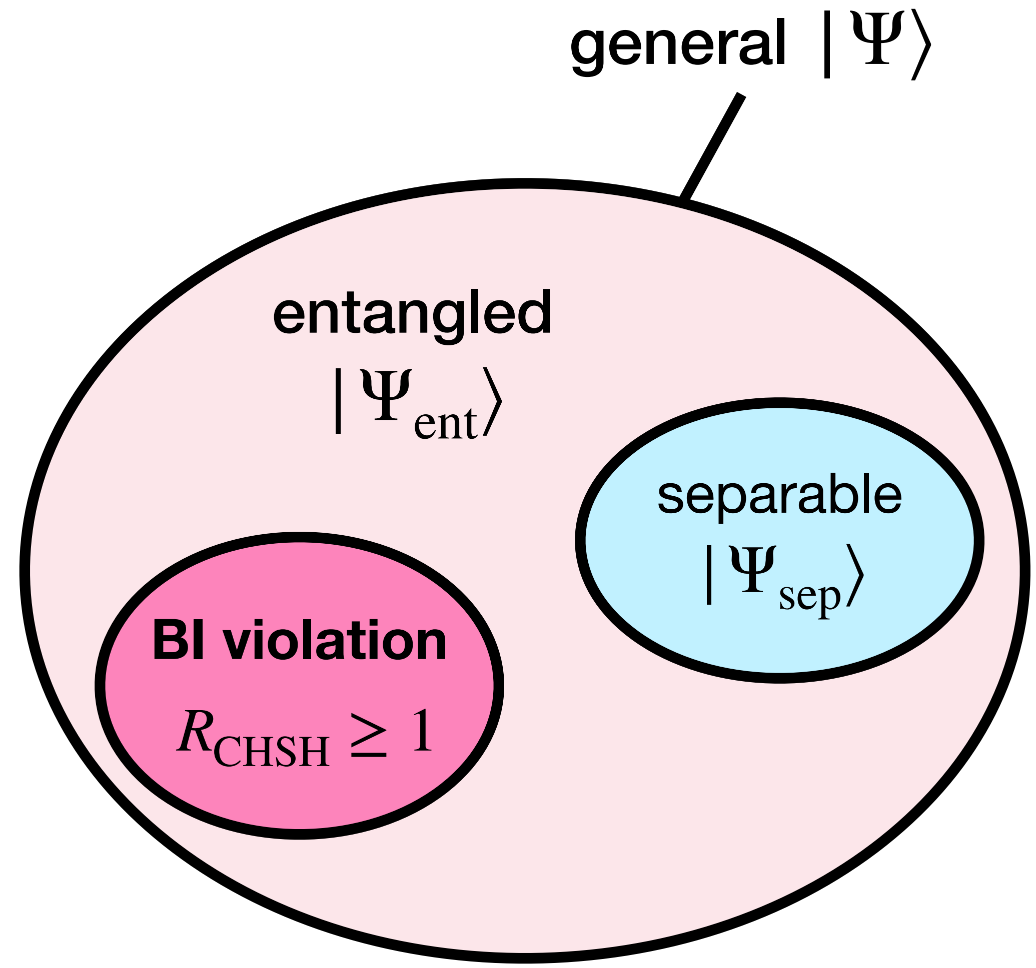
violates the upper bound of hidden variable theories!

$$R_{\text{CHSH}} = \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

$$= \frac{1}{2} \left| \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} - \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}')}_{-\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}}')}_{\frac{1}{\sqrt{2}}} \right| = \sqrt{2}$$



$$R_{\mathrm{CHSH}} \leq \begin{cases} 1 & \text{(hidden variable theories)} \\ \sqrt{2} & \text{(QM)} \end{cases}$$



✤ Violation of the classical bound (Bell inequality) has been observed in low energy experiments:

- **Entangled photon pairs** (from decays of Calcium atoms)

  Crauser, Horne, Shimony, Holt (1969), Freedman and Clauser (1972), A. Aspect et. al. (1981, 1982), Y. H. Shih, C. O. Alley (1988), L. K. Shalm et al. (2015) [5σ]

- **Entangled photon pairs** (from decays of ^{2}He)

  M. M. Lamehi-Rachti, W. Mitting (1972), H. Sakai (2006)

- $K^0\overline{K^0}, B^0\overline{B^0}$ **flavour oscillation** CPLEAR (1999), Belle (2004, 2007)

Bell inequality and entanglement have not been tested at high energy regime E ~ TeV

Can we test Bell inequality and entanglement at high energy colliders?

- Entanglement in $pp \to t\bar{t}$ @ LHC Y. Afik, J. R. M. de Nova (2020)

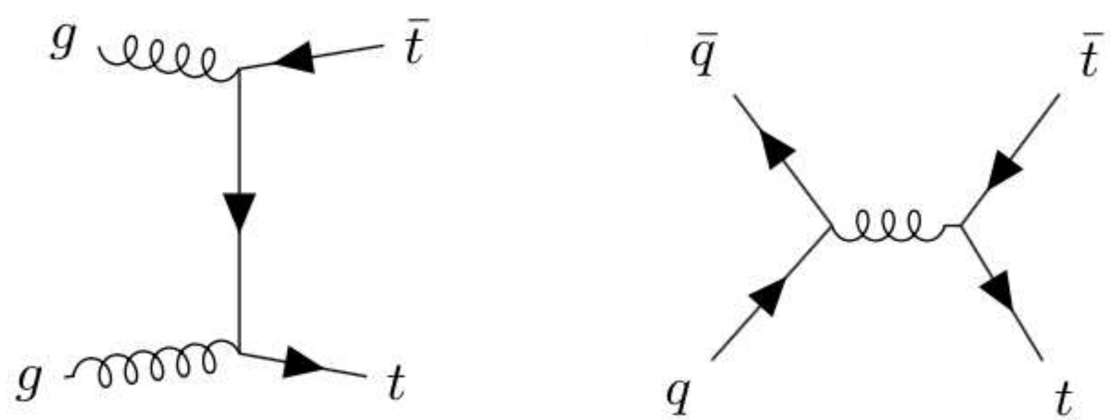
- Bell inequality test in $pp \to t\bar{t}$ @ LHC M. Fabbrichesi, R. Floreanini, G. Panizzo (2021); C. Severi, C. D. Boschi, F. Maltoni, M. Sioli (2021); J. A. Aguilar-Saavedra, J. A. Casas (2022)

- Bell inequality test in $H \to WW^*$ @ LHC A. J. Barr (2021)

At colliders,

- the spin of final state particles are correlated, but not always in $|\Psi^{(0,0)}\rangle$.

- the initial state (and therefore also for the final state) is a statistical ensemble of different pure states.



Density operator

probability of having $|\Psi_1\rangle$

- For a statistical ensemble $\left\{\{p_1 : |\Psi_1\rangle\},\ \{p_2 : |\Psi_2\rangle\},\ \{p_3 : |\Psi_3\rangle\},\ \cdots\right\}$, we define the **density operator/matrix**

$$\hat{\rho} \equiv \sum_k p_k |\Psi_k\rangle\langle\Psi_k| \qquad \rho_{ab} \equiv \langle e_a|\hat{\rho}|e_b\rangle$$

$$0 \le p_k \le 1$$

$$\sum_k p_k = 1$$

$$\langle e_a|e_b\rangle = \delta_{ab}$$

- Density matrices satisfy the conditions:
  - $\hat{\rho}^\dagger = \hat{\rho}$
  - $\mathrm{Tr}\,\hat{\rho} = 1$
  - $\hat{\rho}$ is positive definite, that is ${}^\forall|\varphi\rangle;\ \langle\varphi|\hat{\rho}|\varphi\rangle \ge 0.$

- The expectation of an observable $\hat{O}$ is calculated by

$$\langle\hat{O}\rangle = \mathrm{Tr}\left[\hat{O}\hat{\rho}\right]$$

Biparticle system

- The spin system of α and β particles has 4 independent bases:

$$\big(\ |e_1\rangle, |e_2\rangle, |e_3\rangle, |e_4\rangle\ \big) \ = \ \big(\ |+,+\rangle, |+,-\rangle, |-,+\rangle, |-,-\rangle\ \big)$$

- ==> ρ_{ab} is a 4 x 4 matrix (hermitian, Tr=1). It can be expanded as

3 x 3 matrix

$$\rho = \frac{1}{4}\left(\mathbf{1}\otimes\mathbf{1} + B_i\cdot\sigma_i\otimes\mathbf{1} + \overline{B}_i\cdot\mathbf{1}\otimes\sigma_i + C_{ij}\cdot\sigma_i\otimes\sigma_j\right) \qquad B_i, \overline{B}_i, C_{ij} \in \mathbb{R}$$

- For the spin operators $\hat{s}^\alpha$ and $\hat{s}^\beta$,

spin-spin correlation

$$\langle \hat{s}^\alpha_i \rangle = \mathrm{Tr}\left[\hat{s}^\alpha_i \hat{\rho}\right] = B_i \qquad \langle \hat{s}^\beta_i \rangle = \mathrm{Tr}\left[\hat{s}^\beta_i \hat{\rho}\right] = \overline{B}_i \qquad \boxed{\langle \hat{s}^\alpha_i \hat{s}^\beta_j \rangle = \mathrm{Tr}\left[\hat{s}^\alpha_i \hat{s}^\beta_j \hat{\rho}\right] = C_{ij}}$$

Bell inequality

$$\left\langle s_a^\alpha \cdot s_b^\beta \right\rangle = \hat{a}_i \hat{b}_j \cdot \left\langle s_i^\alpha \cdot s_j^\beta \right\rangle = \hat{a}_i C_{ij} \hat{b}_i$$

unit vectors: $\hat{a}, \hat{a}', \hat{b}, \hat{b}'$

$$R_{\rm CHSH} \equiv \frac{1}{2} \left| \left\langle s_a^\alpha \cdot s_b^\beta \right\rangle - \left\langle s_a^\alpha \cdot s_{b'}^\beta \right\rangle + \left\langle s_{a'}^\alpha \cdot s_b^\beta \right\rangle + \left\langle s_{a'}^\alpha \cdot s_{b'}^\beta \right\rangle \right|$$

$$= \frac{1}{2} \left| \hat{a}_i C_{ij} (\hat{b} - \hat{b}')_j + \hat{a}'_i C_{ij} (\hat{b} + \hat{b}')_j \right|$$

$$\max_{\hat{a}, \hat{a}', \hat{b}, \hat{b}'} \left[R_{\rm CHSH} \right] = \sqrt{\lambda_1 + \lambda_2}$$

($\lambda_1 \geq \lambda_2 \geq \lambda_3$ are 3 eigenvalues of $C^T C$)

Violation of Bell inequality implies

$$\boxed{\sqrt{\lambda_1 + \lambda_2} > 1}$$

M. Fabbrichesi, R. Floreanini, G. Panizzo (2021)

Entanglement

- If the state is separable (not entangled),

$$\rho = \sum_k p_k \rho_k^\alpha \otimes \rho_k^\beta$$

$$0 \le p_k \le 1$$

$$\sum_k p_k = 1$$

then, a modified matrix by the partial transpose

$$\rho^{T_\beta} \equiv \sum_k p_k \rho_k^\alpha \otimes [\rho_k^\beta]^T$$

is also a physical density matrix, i.e. Tr=1 and non-negative.

- For biparticle systems, entanglement $\Longleftrightarrow$ ρ^{T_β} to be non-positive.

Peres-Horodecki (1996, 1997)

- In terms of $(B_i, \overline{B}_i, C_{ij})$ expansion, entanglement implies

$$\boxed{\max_i \left(\left| \mathrm{Tr}[C] - C_{ii} \right| - C_{ii} \right) > 1}$$

Y. Afik, J. R. M. de Nova (2020)

- For $\delta\gamma \to \alpha\beta$

initial state partons

parton luminosity function

$$\rho = \frac{\widetilde{\rho}}{\mathrm{Tr}[\widetilde{\rho}]} \qquad \widetilde{\rho}_{\alpha_1\beta_1,\,\alpha_2\beta_2} = \sum_{\delta,\gamma} \boxed{P(\delta,\gamma)} \cdot \langle \alpha_2, \beta_2 | S | \delta, \gamma \rangle \langle \delta, \gamma | S | \alpha_1, \beta_1 \rangle$$

$\alpha_i\,(\beta_i) = 1, 2$ corresponds to the spin (helicity) state +, -, respectively of particle $\alpha\,(\beta)$

- For $H \to \tau^+\tau^-$, calculation is straightforward:

$$|\Psi_{\tau^+,\tau^-}\rangle = |\Psi^{(1,0)}\rangle = \frac{|+,-\rangle + |-,+\rangle}{\sqrt{2}}$$

$$\rho_{ij,\bar{i}\bar{j}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

Parity: $P = (\eta_f \eta_{\bar{f}}) \cdot (-1)^l$ with $\eta_f \eta_{\bar{f}} = -1$: $\quad P = 0 \implies l = 1$

- Let's suppose a spin 1/2 particle α is ***at rest*** and spinning in the $\mathbf{s}$ direction.
- α decays into a measurable particle l_α and the rest X $\qquad \alpha \to l_\alpha + (X)$
- The decay distribution is generally given by

$$\frac{d\Gamma}{d\Omega} \propto 1 + x_\alpha (\hat{\mathbf{l}}_\alpha \cdot \mathbf{s})$$

$\hat{\mathbf{l}}_\alpha$ is a unit direction vector of l_α, measured at the rest frame of α

- $x \in [-1,\ 1]$ is called *spin-analysing power and* depends on the decay.

$$t \to \ell^+ + (b\nu) \quad \text{and} \quad \tau^- \to \pi^- + (\nu_\tau) \quad \Longrightarrow \quad x = 1$$

- One can show for $\alpha + \beta \ \to\ [l_\alpha + (X)] \ +\ [l_\beta + X]$ and $\xi_{ij} \equiv (\hat{\mathbf{l}}_\alpha)_i (\hat{\mathbf{l}}_\beta)_j$

$$\frac{d\sigma}{d\xi_{ij}} = \left(1 + x_\alpha x_\beta C_{ij}\right) \cdot \ln\left(\frac{1}{\xi_{ij}}\right)$$

One can measure C_{ij} by fitting $\xi_{ij} = (\hat{\mathbf{l}}_\alpha)_i (\hat{\mathbf{l}}_\beta)_j$ distribution.

$$R_{\text{CHSH}} \equiv \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right|$$

$$= \frac{9}{2\,|\,x_\alpha x_\beta\,|}\left|\left\langle (\hat{\mathbf{l}}_\alpha)_a (\hat{\mathbf{l}}_\beta)_b \right\rangle - \left\langle (\hat{\mathbf{l}}_a)(\hat{\mathbf{l}}_\beta)_{b'} \right\rangle + \left\langle (\hat{\mathbf{l}}_\alpha)_{a'} (\hat{\mathbf{l}}_\beta)_b \right\rangle + \left\langle (\hat{\mathbf{l}}_\alpha)_{a'} (\hat{\mathbf{l}}_\beta)_{b'} \right\rangle\right|$$

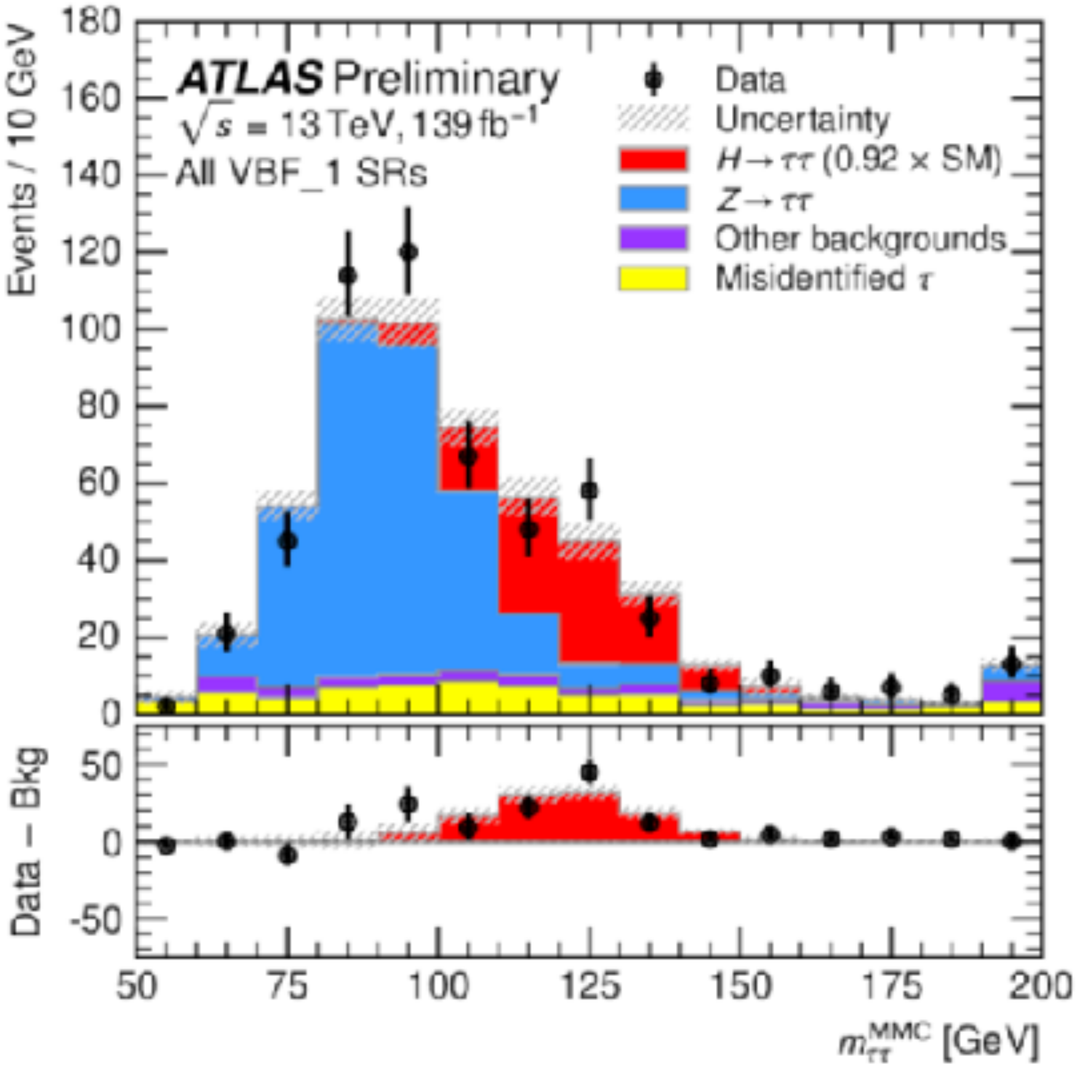
R_{CHSH} can be directly calculated

once the unit vectors $(\hat{\mathbf{a}}, \hat{\mathbf{a}}', \hat{\mathbf{b}}, \hat{\mathbf{b}}')$ are fixed.

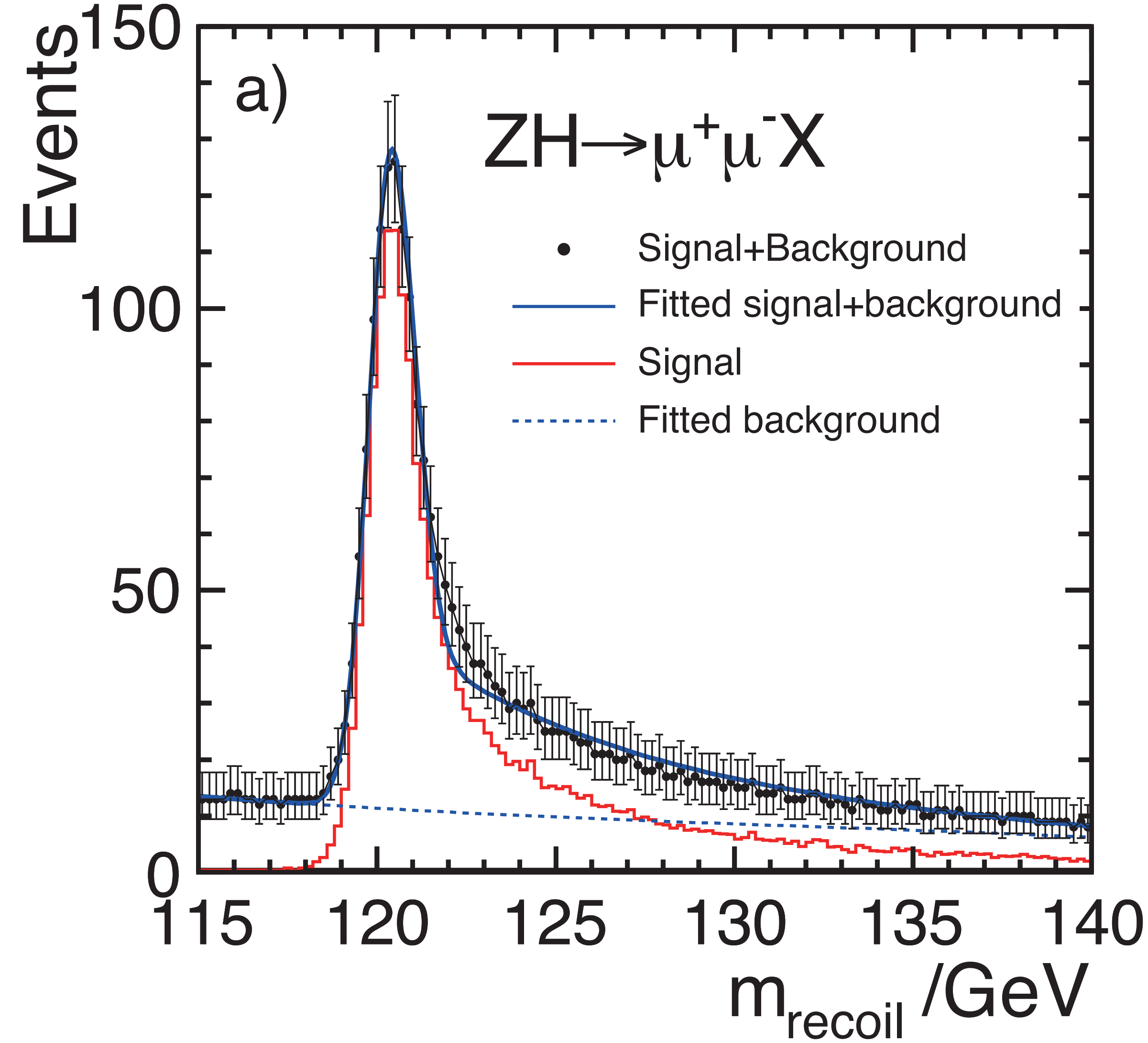
$H \to \tau^+\tau^-$ @ lepton colliders

- Background $Z/\gamma \to \tau^+\tau^-$ is much smaller for lepton colliders
- We need to reconstruct each τ rest frame to measure $\hat{\mathbf{l}}$. This is challenging at hadron colliders since partonic CoM energy is unknown for each event

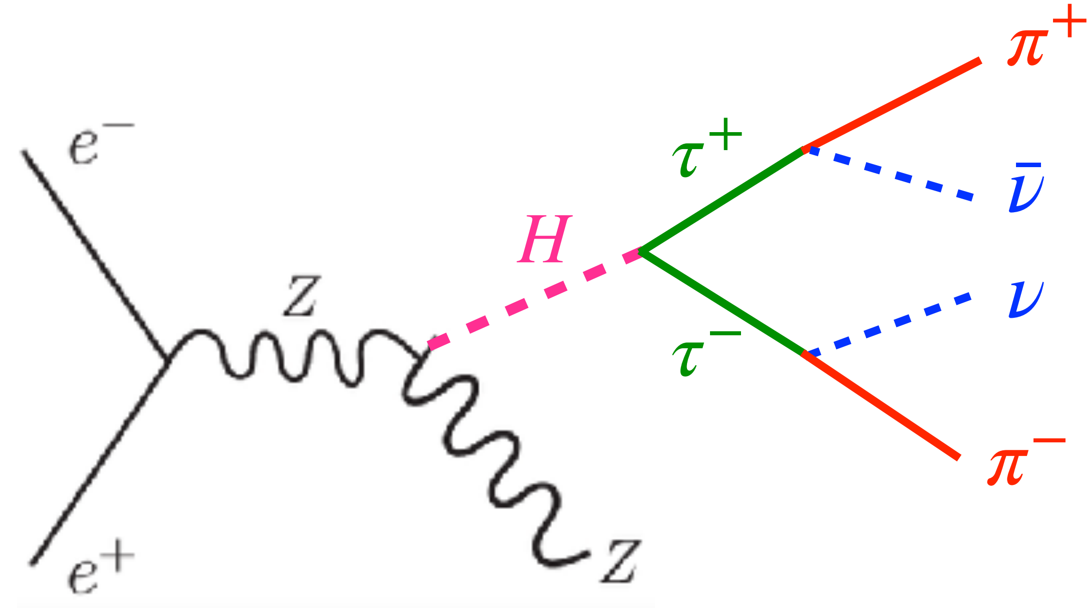
LHC



ILC

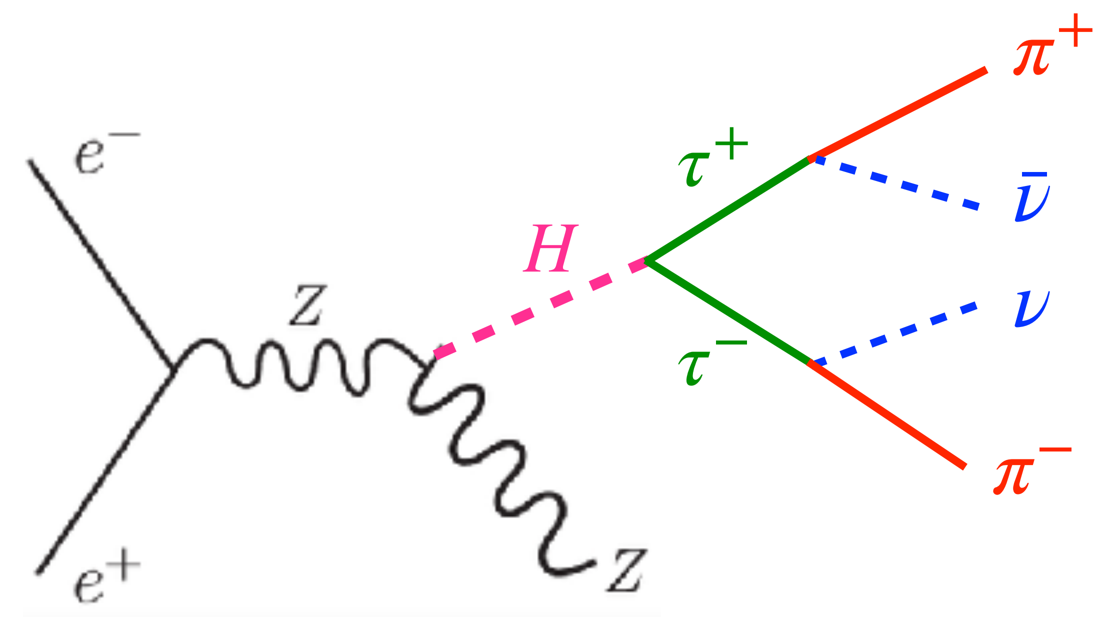


- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^{\nu}, p_y^{\nu}, p_z^{\nu})$, $(p_x^{\bar{\nu}}, p_y^{\bar{\nu}}, p_z^{\bar{\nu}})$.



- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^\nu, p_y^\nu, p_z^\nu)$, $(p_x^{\bar\nu}, p_y^{\bar\nu}, p_z^{\bar\nu})$.

- **6** unknowns can be constrained by **2** mass-shell conditions and **4** energy-momentum conservation.



$$m_\tau^2 = (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar\nu})^2$$

$$m_\tau^2 = (p_{\tau^-})^2 = (p_{\pi^-} + p_{\nu})^2$$

$$(p_{ee} - p_Z)^\mu = p_H^\mu = \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar\nu})\right]^\mu$$

- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^\nu, p_y^\nu, p_z^\nu)$, $(p_x^{\bar\nu}, p_y^{\bar\nu}, p_z^{\bar\nu})$.

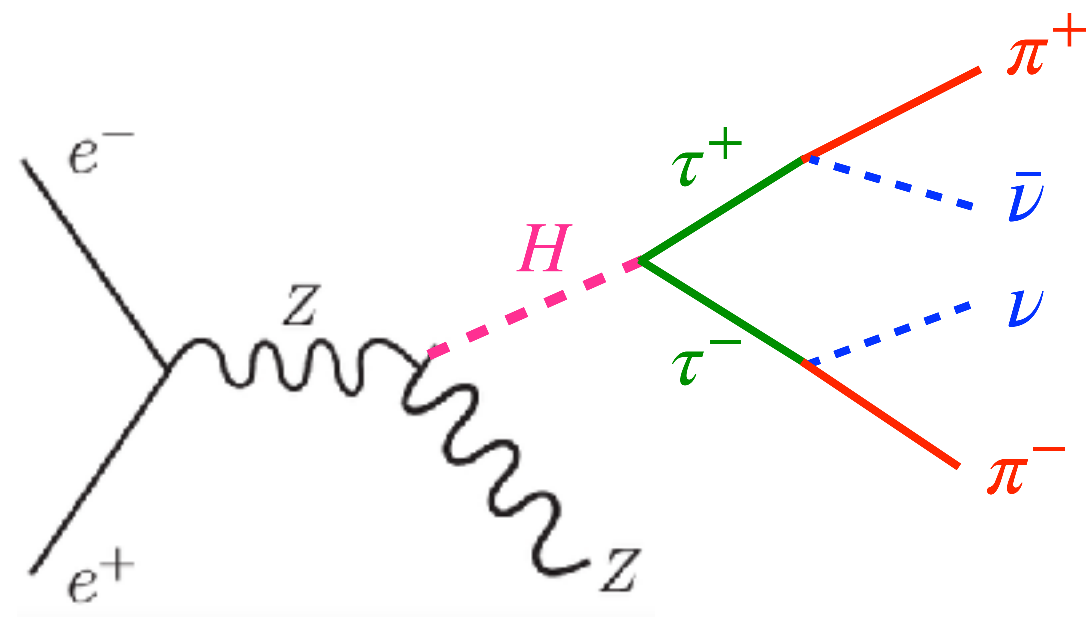
- **6** unknowns can be constrained by **2** mass-shell conditions and **4** energy-momentum conservation.

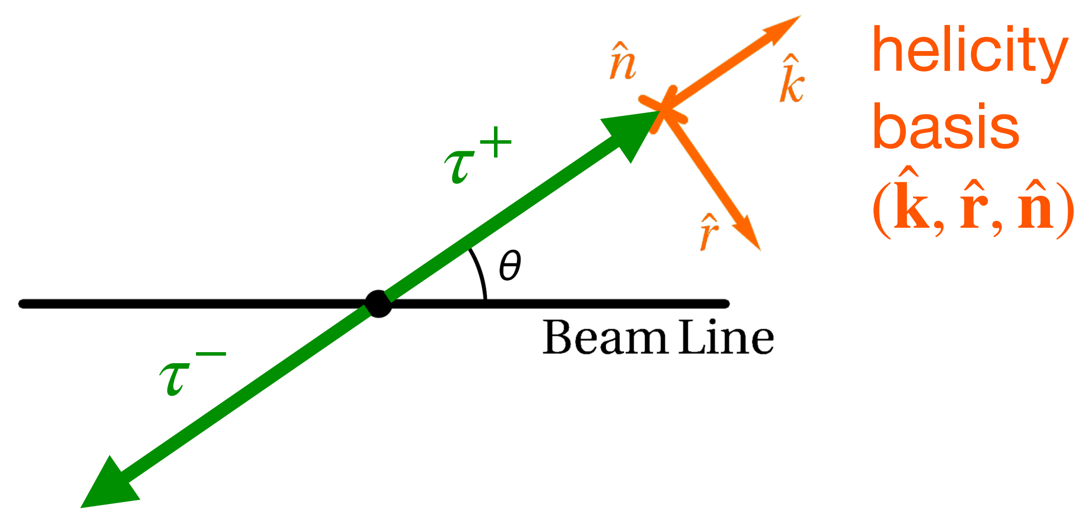
$$m_\tau^2 = (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar\nu})^2$$

$$m_\tau^2 = (p_{\tau^-})^2 = (p_{\pi^-} + p_{\nu})^2$$

$$(p_{ee} - p_Z)^\mu = p_H^\mu = \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar\nu})\right]^\mu$$

- With the reconstructed momenta, we define $(\hat{\mathbf{k}}, \hat{\mathbf{r}}, \hat{\mathbf{n}})$ basis at the Higgs rest frame.





- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^{\nu}, p_y^{\nu}, p_z^{\nu}),\ (p_x^{\bar{\nu}}, p_y^{\bar{\nu}}, p_z^{\bar{\nu}})$.

- **6** unknowns can be constrained by **2** mass-shell conditions and **4** energy-momentum conservation.

$$m_\tau^2 = (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar{\nu}})^2$$

$$m_\tau^2 = (p_{\tau^-})^2 = (p_{\pi^-} + p_{\nu})^2$$

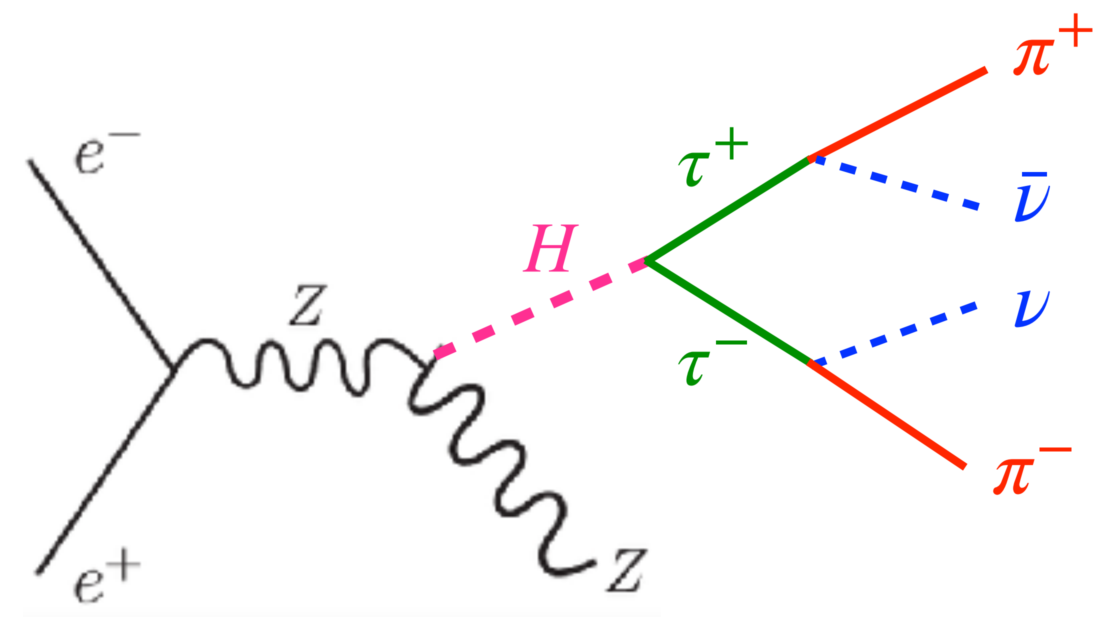
$$(p_{ee} - p_Z)^\mu = p_H^\mu = \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar{\nu}})\right]^\mu$$

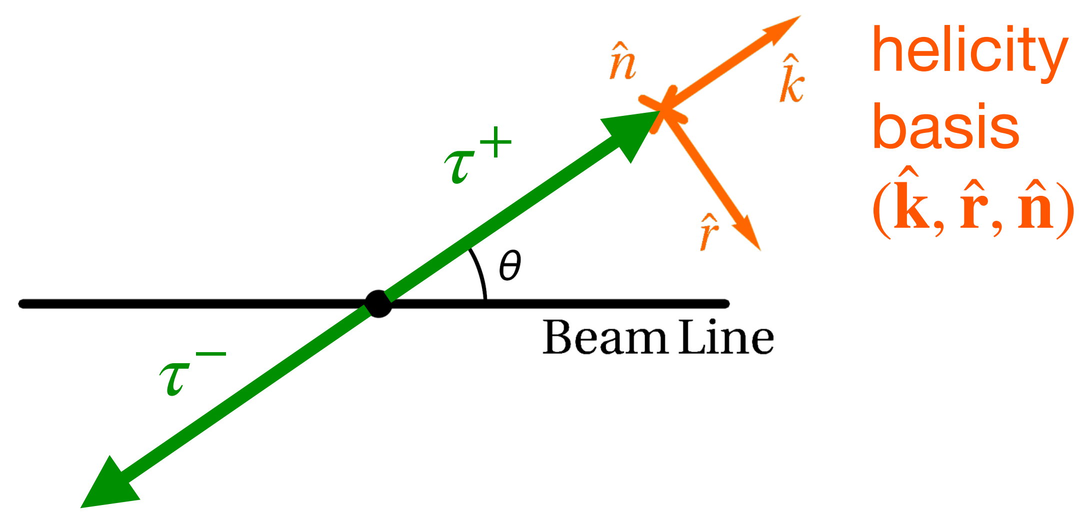
- With the reconstructed momenta, we define $(\hat{\mathbf{k}}, \hat{\mathbf{r}}, \hat{\mathbf{n}})$ basis at the Higgs rest frame.

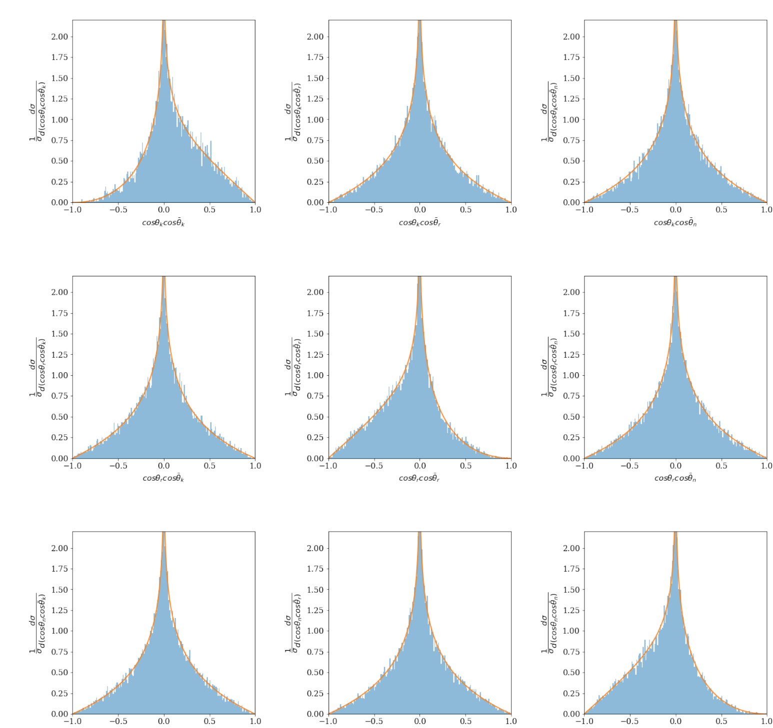
- In the $\tau^{+(-)}$ rest frame, we measure the direction of $\pi^{+(-)}$, $\hat{\mathbf{l}}^+$ and $\hat{\mathbf{l}}^-$, and calculate R_{CHSH} directly with

$$(\hat{\mathbf{a}}, \hat{\mathbf{a}}', \hat{\mathbf{b}}, \hat{\mathbf{b}}') = \left(\hat{\mathbf{k}}, \hat{\mathbf{r}}, \frac{1}{\sqrt{2}}(\hat{\mathbf{k}} + \hat{\mathbf{r}}), \frac{1}{\sqrt{2}}(\hat{\mathbf{k}} - \hat{\mathbf{r}})\right)$$

and extract C_{ij} by fitting $\hat{\mathbf{l}}_i^+ \hat{\mathbf{l}}_j^-$ distributions.







Results (preliminary)

- We assume an e^+e^- collider with $\sqrt{s} = 240\,\mathrm{GeV}$ and $L = 5\,\mathrm{ab}^{-1}$
- Generate events with MadGraph5 and perform 100 pseudo-experiments to estimate the uncertainties.

Results (preliminary)

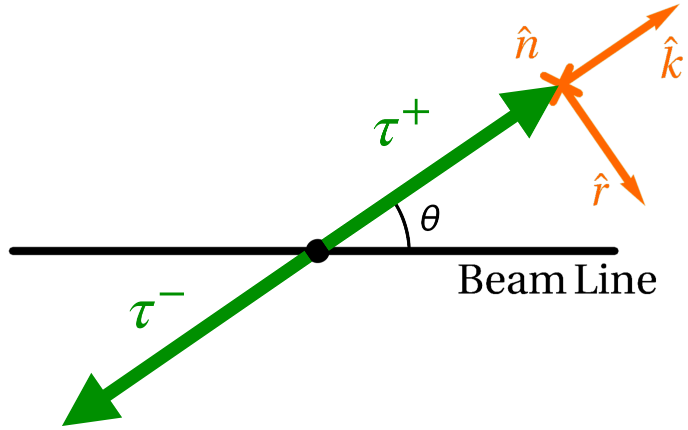
- We assume an e^+e^- collider with $\sqrt{s} = 240\,\mathrm{GeV}$ and $L = 5\,\mathrm{ab}^{-1}$
- Generate events with MadGraph5 and perform 100 pseudo-experiments to estimate the uncertainties.

Parton level analysis:

$$R_{\mathrm{CHSH}} = 1.410 \pm 0.084 \quad \Longrightarrow \quad R_{CHSH} > 1$$ **BI violation** $\sim 5\sigma$

$$C_{ij} = \begin{array}{c c} & \begin{array}{ccc} \hat{\mathbf{k}} & \hat{\mathbf{r}} & \hat{\mathbf{n}} \end{array} \\ \begin{array}{c} \hat{\mathbf{k}} \\ \hat{\mathbf{r}} \\ \hat{\mathbf{n}} \end{array} & \begin{pmatrix} -1.008_{\pm 0.123} & 0.002_{\pm 0.103} & 0.003_{\pm 0.096} \\ 0.024_{\pm 0.090} & 0.988_{\pm 0.106} & 0.001_{\pm 0.071} \\ -0.006_{\pm 0.098} & 0.004_{\pm 0.074} & 0.997_{\pm 0.108} \end{pmatrix} \end{array}$$

$$\max_i \left(\left| \mathrm{Tr}[C] - C_{ii} \right| - C_{ii} \right) = | C_{rr} + C_{nn} | - C_{kk} = 2.99 \pm 0.19 > 1$$ **Entanglement** $\gg 5\sigma$



Results (preliminary)

- We assume an e^+e^- collider with $\sqrt{s} = 240\,\mathrm{GeV}$ and $L = 5\,\mathrm{ab}^{-1}$
- Generate events with MadGraph5 and perform 100 pseudo-experiments to estimate the uncertainties.

Parton level analysis:

$$R_{\mathrm{CHSH}} = 1.410 \pm 0.084 \quad \Longrightarrow \quad R_{CHSH} > 1 \qquad \text{BI violation} \ \sim 5\sigma$$

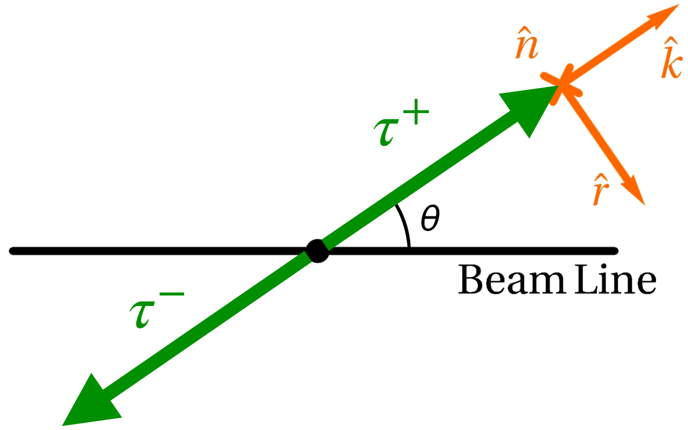
$$C_{ij} = \begin{array}{c c} & \begin{array}{ccc} \hat{\mathbf{k}} & \hat{\mathbf{r}} & \hat{\mathbf{n}} \end{array} \\ \begin{array}{c} \hat{\mathbf{k}} \\ \hat{\mathbf{r}} \\ \hat{\mathbf{n}} \end{array} & \begin{pmatrix} -1.008_{\pm 0.123} & 0.002_{\pm 0.103} & 0.003_{\pm 0.096} \\ 0.024_{\pm 0.090} & 0.988_{\pm 0.106} & 0.001_{\pm 0.071} \\ -0.006_{\pm 0.098} & 0.004_{\pm 0.074} & 0.997_{\pm 0.108} \end{pmatrix} \end{array}$$

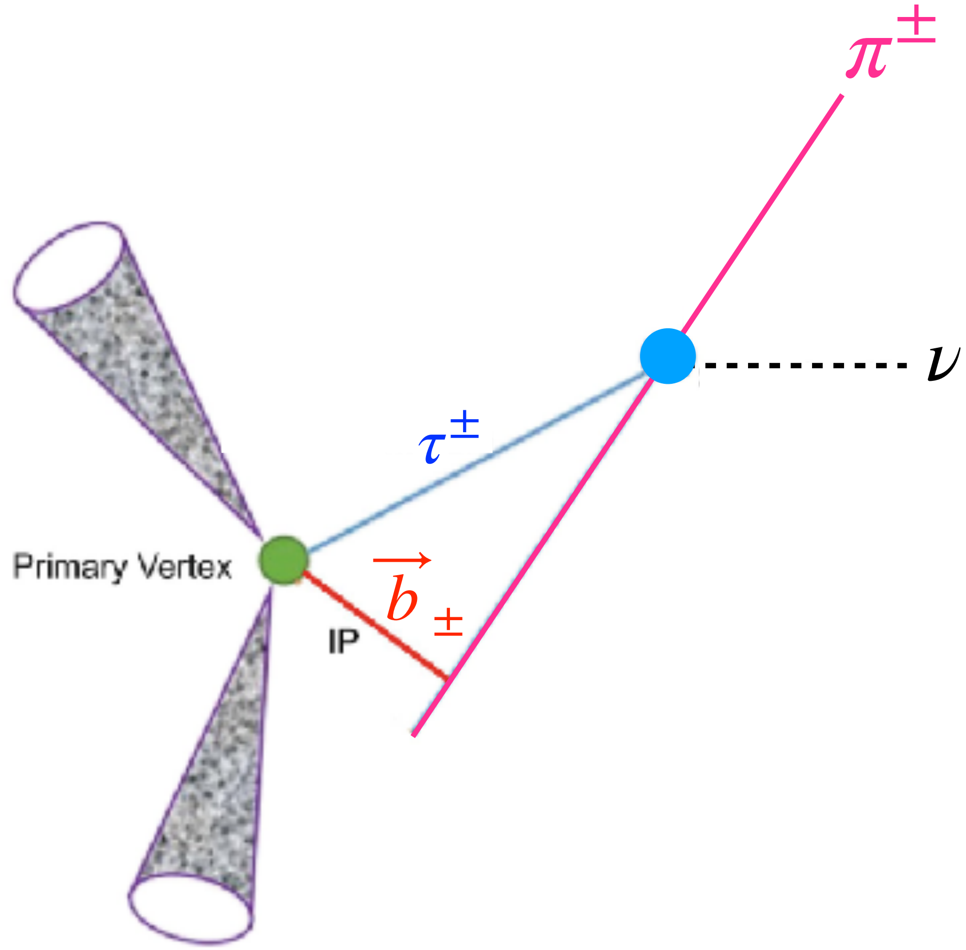
$$\max_i \left(\left| \mathrm{Tr}[C] - C_{ii} \right| - C_{ii} \right) = |C_{rr} + C_{nn}| - C_{kk} = 2.99 \pm 0.19 > 1 \qquad \text{Entanglement} \ \gg 5\sigma$$

With detector resolution:

$$R_{\mathrm{CHSH}} = 0.716 \pm 0.121$$

$$|C_{rr} + C_{nn}| - C_{kk} = 0.505 \pm 0.202$$





Use impact parameter information

- We use the information of impact parameter $\overrightarrow{b}_{\pm}$ measurement of $\pi^{\pm}$ to "correct" the observed energies of $\tau^{\pm}$ and Z decay products
- We check whether the reconstructed τ momenta are consistent with the measured impact parameters.
- We construct the likelihood function and search for the most likely τ momenta.

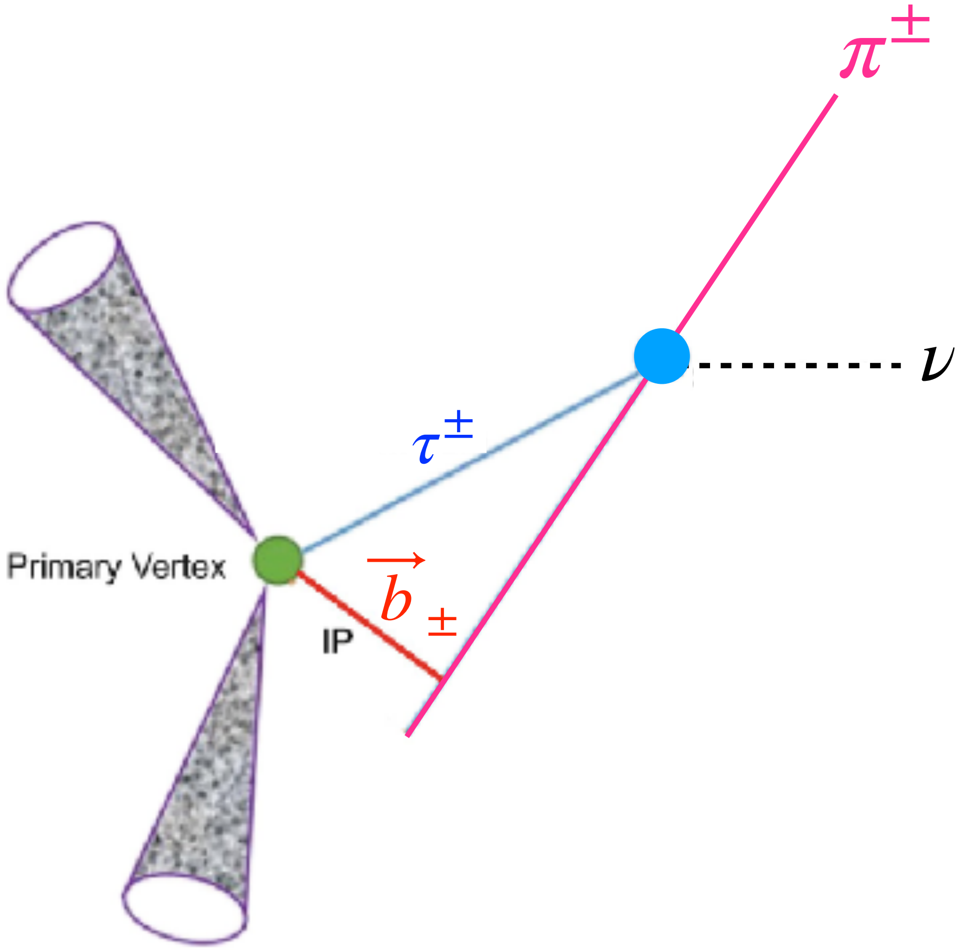
$$\boxed{E_\alpha(\delta_\alpha) = (1 + \sigma_\alpha^E \cdot \delta_\alpha) \cdot E_\alpha^{\text{obs}}}$$

$$\vec{b}_+ \ = \ |\vec{b}_+| \left(\sin^{-1}\Theta_+ \cdot \vec{e}_{\tau^+} \ - \ \tan^{-1}\Theta_+ \cdot \vec{e}_{\pi^+}\right)$$

$$\vec{\Delta}^i_{b_+}(\{\delta\}) \equiv \vec{b}_+ - |\vec{b}_+| \left(\sin^{-1}\Theta^i_+(\{\delta\}) \cdot \vec{e}^{\,i}_{\tau^+}(\{\delta\}) - \tan^{-1}\Theta^i_+(\{\delta\}) \cdot \vec{e}_{\pi^+}\right)$$

$$L^i_{\pm}(\{\delta\}) \ = \ \frac{[\Delta^i_{b_\pm}(\{\delta\})]^2_x + [\Delta^i_{b_\pm}(\{\delta\})]^2_y}{\sigma^2_{b_T}} + \frac{[\Delta^i_{b_\pm}(\{\delta\})]^2_z}{\sigma^2_{b_z}}$$

$$\boxed{L^i(\{\delta\}) \ = \ L^i_+(\{\delta\}) + L^i_-(\{\delta\})}$$



Use impact parameter information

- We use the information of impact parameter $\vec{b}_{\pm}$ measurement of $\pi^{\pm}$ to "correct" the observed energies of $\tau^{\pm}$ and Z decay products
- We check whether the reconstructed τ momenta are consistent with the measured impact parameters.
- We construct the likelihood function and search for the most likely τ momenta.

With detector resolution + IP information:

$$R_{\mathrm{CHSH}} = 1.256 \pm 0.104 \quad \Longrightarrow \quad R_{CHSH} > 1$$

BI violation $\sim 2.5\sigma$

$$C_{ij} = \begin{array}{c c} & \begin{array}{ccc} \hat{\mathbf{k}} & \hat{\mathbf{r}} & \hat{\mathbf{n}} \end{array} \\ \begin{array}{c} \hat{\mathbf{k}} \\ \hat{\mathbf{r}} \\ \hat{\mathbf{n}} \end{array} & \begin{pmatrix} -0.918_{\pm 0.120} & -0.006_{\pm 0.142} & 0.001_{\pm 0.132} \\ 0.021_{\pm 0.125} & 0.9130_{\pm 0.141} & 0.007_{\pm 0.115} \\ -0.009_{\pm 0.116} & 0.0004_{\pm 0.125} & 0.939_{\pm 0.124} \end{pmatrix} \end{array}$$

$$\max_i \left(\left| \mathrm{Tr}[C] - C_{ii} \right| - C_{ii} \right) = |\, C_{rr} + C_{nn} \,| - C_{kk} = 2.77 \pm 0.22 > 1$$

Entanglement $> 5\sigma$

Summary

- High energy tests of entanglement and Bell inequality has recently attracted an attention.
- $\tau^+\tau^-$ pairs from $H \to \tau^+\tau^-$ form the EPR triplet state $|\Psi^{(1,0)}\rangle = \dfrac{|+,-\rangle + |-,+\rangle}{\sqrt{2}}$, and maximally entangled.
- We investigated a test at a future high energy lepton collider, since the background is small and the τ momentum reconstruction is possible.
- Assuming an e^+e^- collider with $\sqrt{s} = 240\,\mathrm{GeV}$ and $L = 5\,\mathrm{ab}^{-1}$, and using IP information, we obtained the following results:

$$R_{\mathrm{CHSH}} = 1.256 \pm 0.104 \quad \Longrightarrow \quad R_{CHSH} > 1$$

BI violation $\sim 2.5\sigma$

$$\max_i \left(\left| \mathrm{Tr}[C] - C_{ii} \right| - C_{ii} \right) = |C_{rr} + C_{nn}| - C_{kk} = 2.77 \pm 0.22 > 1$$

Entanglement $> 5\sigma$

$$
\begin{aligned}
\sigma(e^+e^- \to HZ)\Big|_{\sqrt{s}=240\mathrm{GeV}} &= 240.3\,\mathrm{fb} \\
BR(H \to \tau^+\tau^-) &= 0.0632 \\
BR(\tau^- \to \pi^-\nu_\tau) &= 0.109 \\
BR(Z \to jj, \mu\mu, ee) &= 0.766 \\
\sigma(e^+e^- \to HZ)_{240}^{\mathrm{unpol}} \cdot BR_{H\to\tau\tau} \cdot [BR_{\tau\to\pi\nu}]^2 \cdot BR_{Z\to jj,\mu\mu,ee} &= 0.1382\,\mathrm{fb}
\end{aligned}
$$