Multicritical Point Principle and Electroweak Scale

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in
the workshop on the standard model and beyond

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Based on collaborations with
Yuta Hamada, Kiyoharu Kawana, Kin-ya Oda, and Kei Yagyu:
e-Print: 2202.04221, …

Plan of the talk

1. **Possibility of Desert**
2. **Naturalness and Self-tuning Mechanisms**
3. **Generalized MPP**
4. **Emergence of Electroweak Scale**

1. Possibility of Desert

Desert

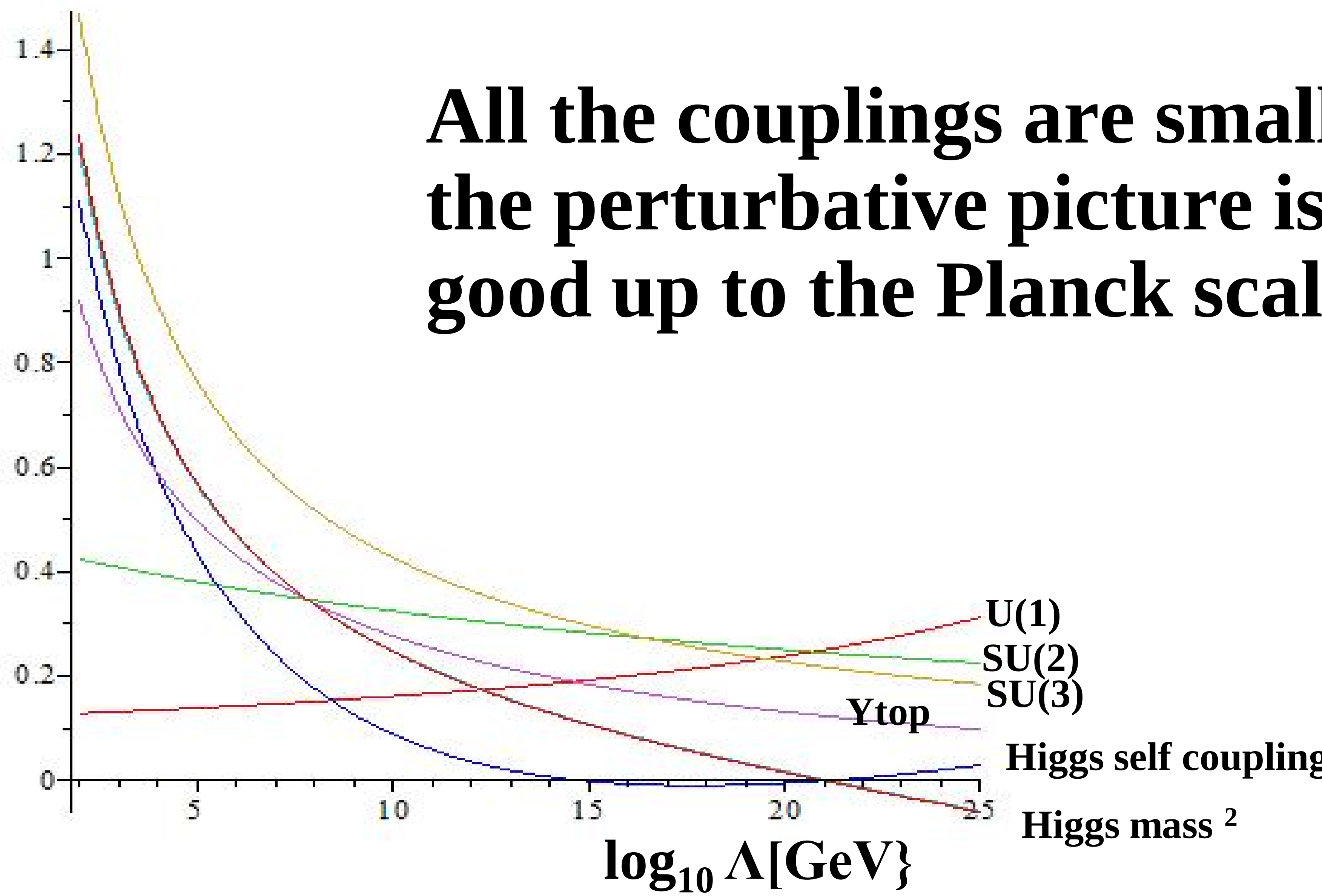
SM is good to high energy scales.

Experimentally LHC

SM is good at least below a few TeV.

Theoretically UV region of SM by RG

All the couplings are small and the perturbative picture is very good up to the Planck scale.

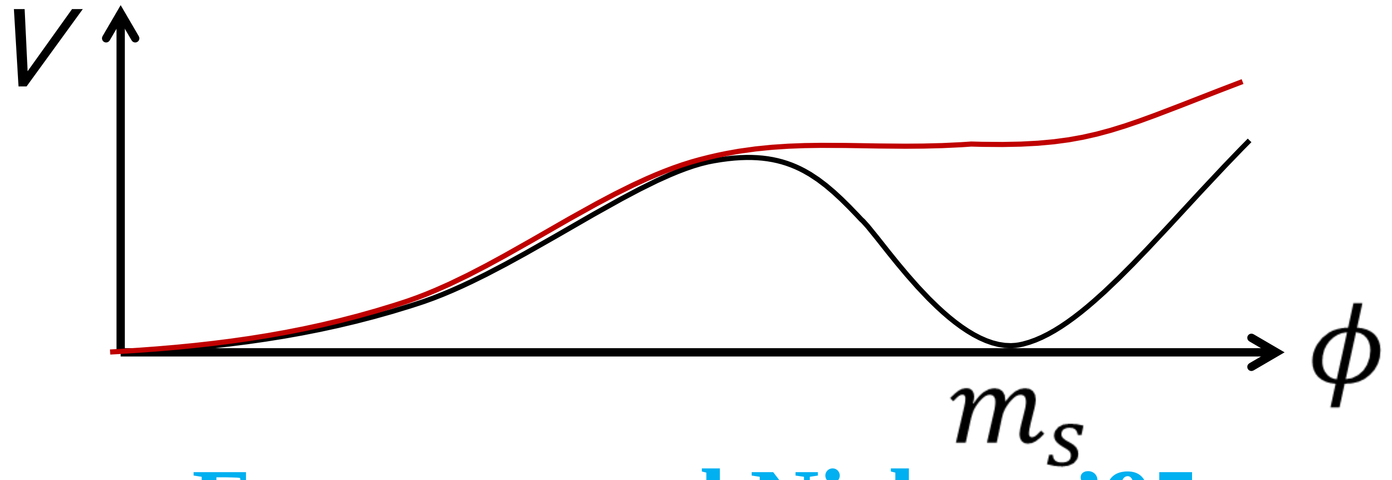


RG analyses indicate

(1) The three quantities, $\lambda_B, \beta_\lambda(\lambda_B), m_B$

become zero around the string scale.

(2) The Higgs potential becomes flat (or zero) around the string scale.



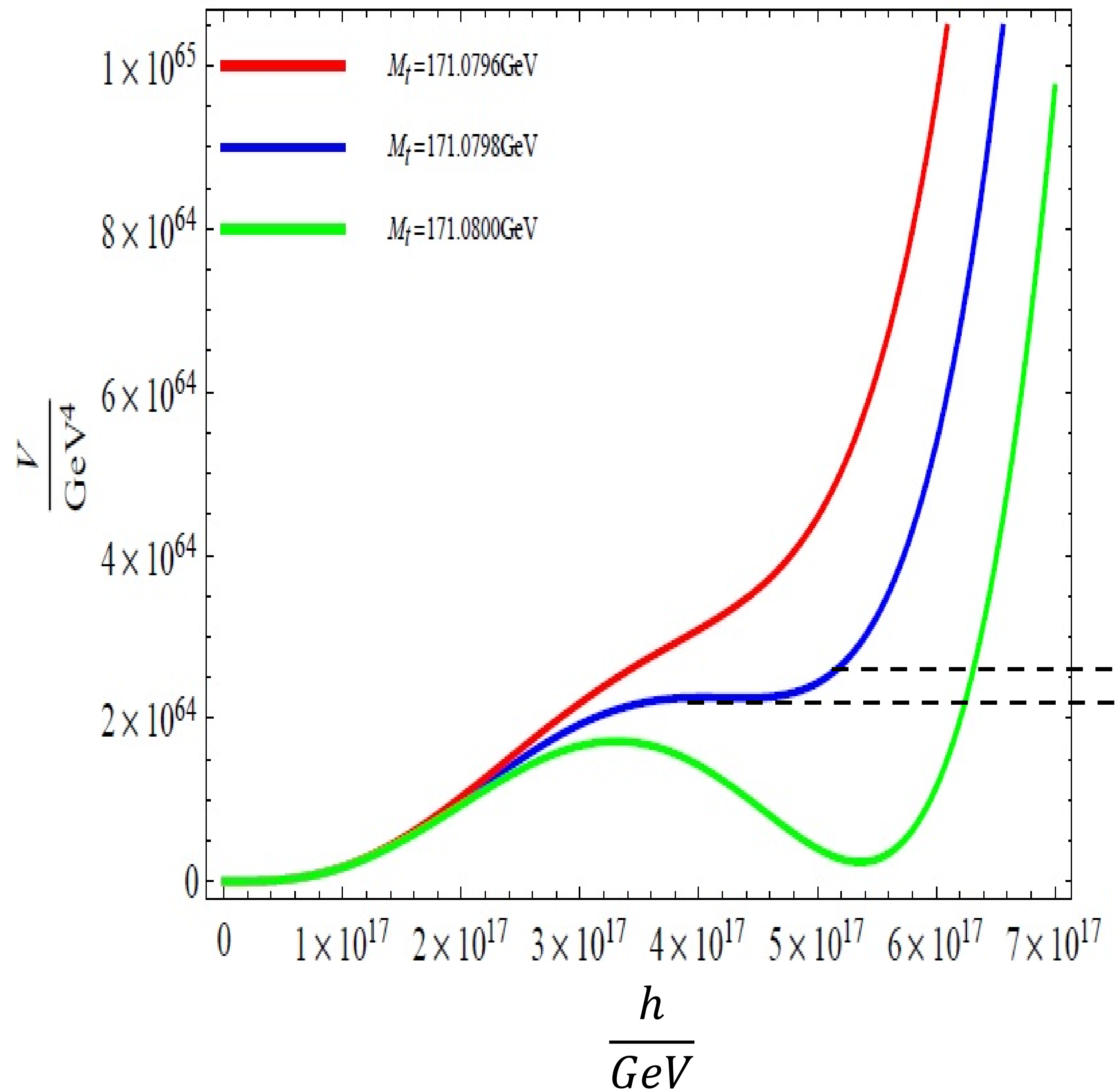
Froggatt and Nielsen ’95.

Multiple Point Criticality Principle (MPP)

Higgs inflation

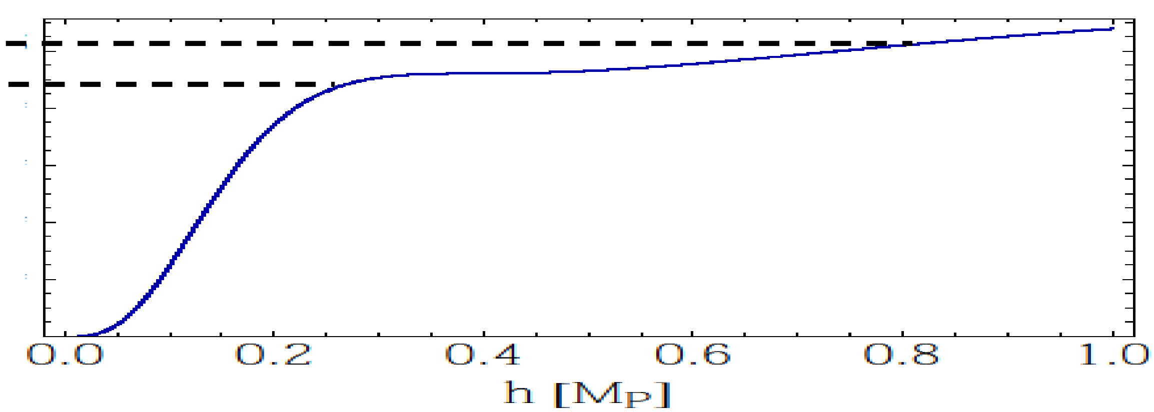
Hamada, Oda, Park and HK
Bezrukov,Shaposhnikov

We can make a realistic model of inflation by introducing non-minimal coupling $\xi R h^2$ with a rather small $\xi \sim 10$.



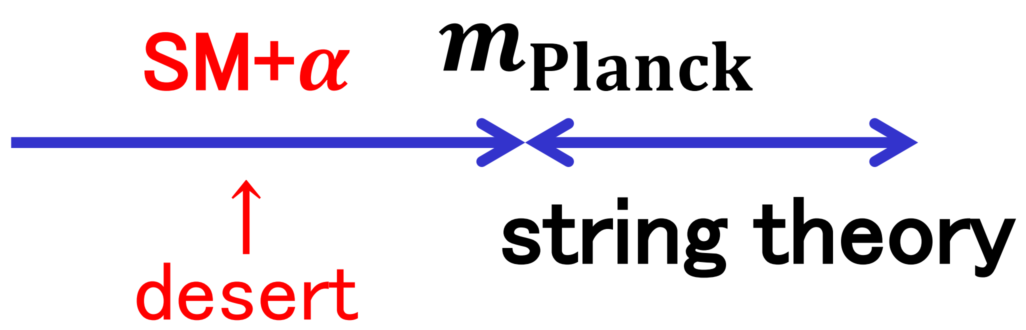
In the Einstein frame the effective potential becomes

$$V\left(\varphi_h\right), \quad \varphi_h = \frac{h}{\sqrt{1+\xi h^2 / M_P{}^2}}.$$



In short, desert is very probable.

It is natural to imagine that SM is directly connected to the string theory at the Planck scale without large modification.



Then we have to think about the naturalness problem seriously.

2. Naturalness and Self-tuning mechanisms

The naturalness problem

Suppose the underlying fundamental theory, such as string theory, has the momentum scale m_s and the coupling constant g_s .

Then, by dimensional analysis, the parameters of the low energy effective theory are given as follows:

dimension -2 (Newton constant) $G_N = \dfrac{C_{G_N}(g_S)}{m_S^2}$

dimension 0

(gauge and Higgs couplings) $g_1, g_2, g_3, \lambda_H = C_{g_1,g_2,g_3,\lambda_H}(g_S)$

dimension 2 (Higgs mass) $m_H^2 = C_{m_H}(g_s)\, m_S^2$

unnatural ! $\rightarrow$ $m_H^2 \sim (100\mathrm{GeV})^2 \leftrightarrow m_S^2 \sim (10^{17}\mathrm{GeV})^2$

dimension 4 (cosmological constant) $\Lambda = C_\Lambda(g_S)\, m_S^4$

unnatural ! ! $\rightarrow$ $\Lambda \sim (2 \sim 3\ \mathrm{meV})^4 \leftrightarrow m_S^4 \sim (10^{17}\mathrm{GeV})^4$

The real values of the cosmological constant and Higgs mass are very unnatural.

Therefore, if nature is described by a fundamental theory with a definite momentum scale such as string theory, the theory should do fine tunings by itself.

There are several attempts to extend the conventional framework of local field theory in order to solve the fine tuning problem.

- asymptotic safety
  Weinberg, Kitazawa-Ninomiya-HK, Shaposhnikov, …
- multicritcal point principle
  Bennett-Froggatt-Nielsen
- classical conformality
  Bardeen, Meissner-Nicolai, Foot-Kobakhidze-McDonald-Volkas, Iso-Okada-Orikasa, …
- baby universe and multi-local action
  Coleman,
  Okada-HK, Hamada-Kawana-HK, …

They are related.

MPP of Bennett Froggatt and Nielsen

Imagine a system that is described by the path integral of not the canonical ensemble

$$\int [d\varphi] \exp\left(-S[\varphi]\right),$$

but the micro canonical ensemble

$$\int [d\varphi] \delta\left(S[\varphi]-C\right),$$

or an even more general ensemble (next slide)

$$\int [d\varphi]\, f\left(S_1[\varphi], S_2[\varphi], \cdots\right).$$

Still the system is equivalent to the ordinary field theory in the large space-time volume limit.

But the parameters of the corresponding field theory are automatically fixed such that the vacuum is at a (multi-) critical point.

Integrating coupling constants

In fact we can show that the low energy effective theory of QG / string theory is expressed as a function of local actions:

Coleman '89
Tsuchiya-Asano-HK

$$S_{\text{eff}} = f\left(S_1, S_2, \cdots\right)$$
$$= \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$
$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

Here O_i are local scalar operators such as

$1\,, R\,, R_{\mu\nu}R^{\mu\nu}\,, F_{\mu\nu}F^{\mu\nu}, \bar{\psi}\gamma^{\mu}D_{\mu}\psi\,, \cdots.$

Because S_{eff} is a function of S_i's , we can express $\exp(iS_{\mathrm{eff}})$ by a Fourier transform as

$$\exp\left(iS_{\mathit{eff}}\left(S_1, S_2, \cdots\right)\right) = \int d\lambda\, w(\lambda_1, \lambda_2, \cdots) \exp\left(i\sum_i \lambda_i S_i\right),$$

where λ_i's are Fourier conjugate variables to S_i's, and w is a function of λ_i's .

Then the path integral for S_{eff} becomes

$$Z = \int [d\phi] \exp\left(iS_{\mathrm{eff}}\right) = \int d\lambda\, w(\lambda) \int [d\phi] \exp\left(i\sum_i \lambda_i S_i\right).$$

Because O_i are local operators, $\sum_i \lambda_i S_i$ is an ordinary local action where λ_i are regarded as the coupling constants.

Therefore the system is the ordinary field theory, but we have to integrate over the coupling constants with some weight $w(\lambda)$.

Nature does fine tunings

$$Z = \int [d\phi] \exp(iS_{\text{eff}}) = \int d\lambda w(\lambda) \underbrace{\int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right)}_{= Z(\lambda)}$$

$$= \int d\lambda w(\lambda) Z(\lambda).$$

Ordinary field theory

If a small region $\lambda \sim \lambda^{(0)}$ dominates the λ integral, it means that the coupling constants are fixed to $\lambda^{(0)}$.

3. Generalized MPP

Justification of MPP

’14 ’15 Hamada, Kawana, HK

Essence:

space−time volume

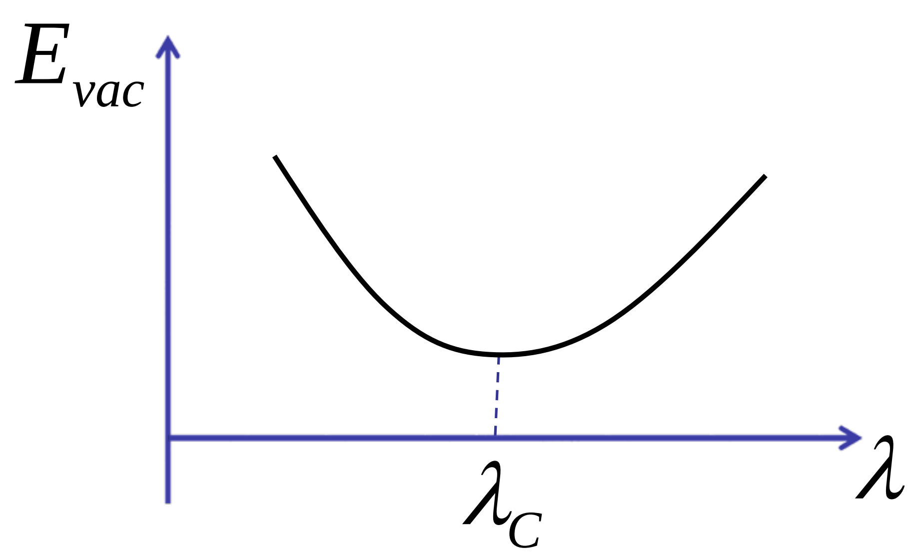
We can approximate $Z(\lambda) = \exp\left(-iVE_{vac}(\lambda)\right)$ **,**
because our universe has been cooled down for long time.

1) extremum

If $E_{vac}(\lambda)$ **is smooth and has an extremum at** λ_C **, the stationary point dominates and we have**

$$\exp\left(-iVE_{vac}(\lambda)\right) \sim \frac{\sqrt{2\pi}}{\sqrt{i\,V|E''(\lambda_c)|}}\,\delta(\lambda-\lambda_c) + O\left(\frac{1}{V}\right).$$

Thus λ **is fixed to** λ_C **in the limit** $V \to \infty$ **.**



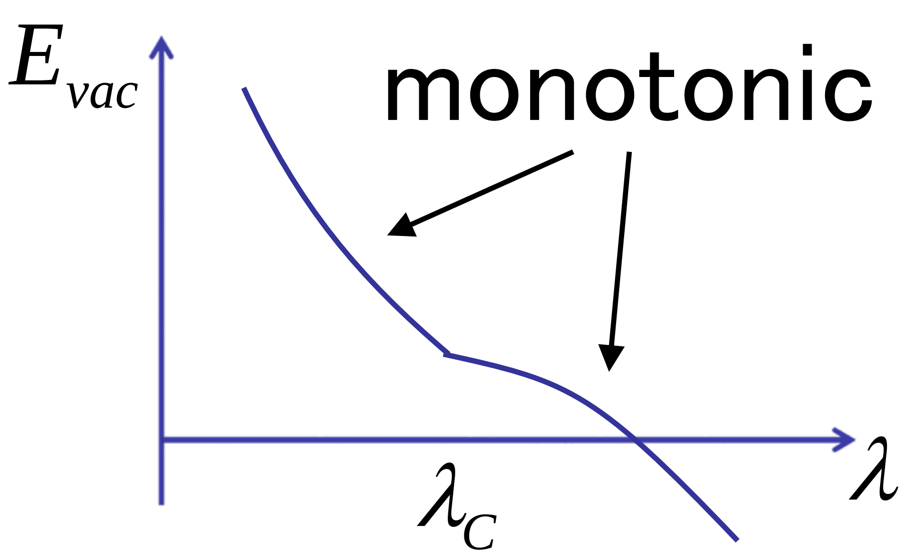
2) Kink (need not be an extremum)

If $\boldsymbol{E_{vac}(\lambda)}$ has a kink (as in the first order phase transition),

$$Z(\lambda) = \exp\left(-iVE_{vac}(\lambda)\right)$$

$$\sim \frac{i}{V}\left(\frac{1}{E_{vac}{}'(\lambda_c+0)} - \frac{1}{E_{vac}{}'(\lambda_c-0)}\right)\delta(\lambda-\lambda_c) + O\left(\frac{1}{V^2}\right)$$

Thus $\boldsymbol{\lambda}$ is fixed to $\boldsymbol{\lambda_C}$ in the limit $\boldsymbol{V \to \infty}$.



$$\int_a^b \mathrm{dx}\, \exp(iVx)\,\varphi(x)$$

$$= \left[\frac{1}{iV}\exp(iVx)\,\varphi(x)\right]_{\mathrm{a}}^{\mathrm{b}} + \mathrm{O}\left(\frac{1}{\mathrm{V}^2}\right)$$

$$\int_a^b \mathrm{dx}\, \exp(iVf(x))\,\varphi(x)$$

$$= \left[\frac{1}{iV}\exp(iVf(x))\frac{1}{f'(x)}\varphi(x)\right]_{\mathrm{a}}^{\mathrm{b}} + \mathrm{O}\left(\frac{1}{\mathrm{V}^2}\right)$$

(f is monotonic)

Generalization

On the other hand, if we consider the time evolution of universe, the definition of $Z(\lambda)$ is not a priori clear. For example, we need to specify the initial and final sates.

However, even if we do not know the precise form of $Z(\lambda)$, we expect that $Z(\lambda)$is determined by the late stage of the universe, because most of the space-time volume comes from the late stage.

From this we can make some predictions on λ's under some circumstances as follows.

(1) Symmetry example θ_{QCD}

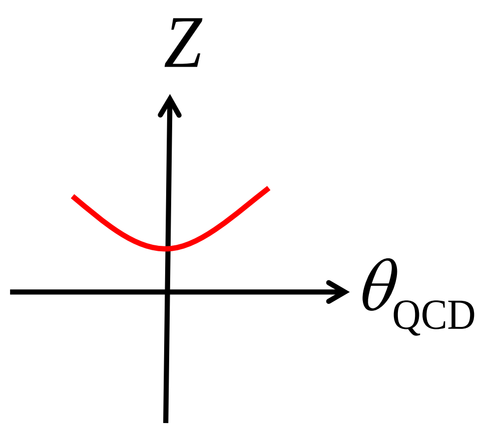
Nielsen, Ninomiya

1. It becomes important only after the QCD phase transition.
2. The masses and life-times of hadrons are invariant under

$$\theta_{QCD} \to -\theta_{QCD}\ .$$

$\Rightarrow$ We expect that Z is even in θ_{QCD}.

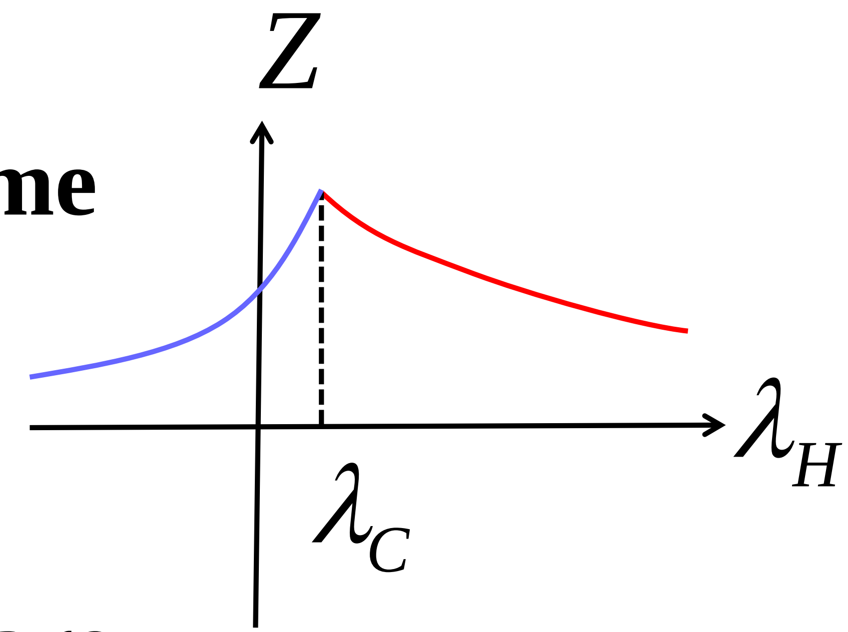
$\Rightarrow$ θ_{QCD} is tuned to 0 if Z behaves like



(2) Edge or drastic change

Conditions:

1. **Physics changes drastically at some value of the couplings.**
2. ***Z* is monotonic elsewhere.**



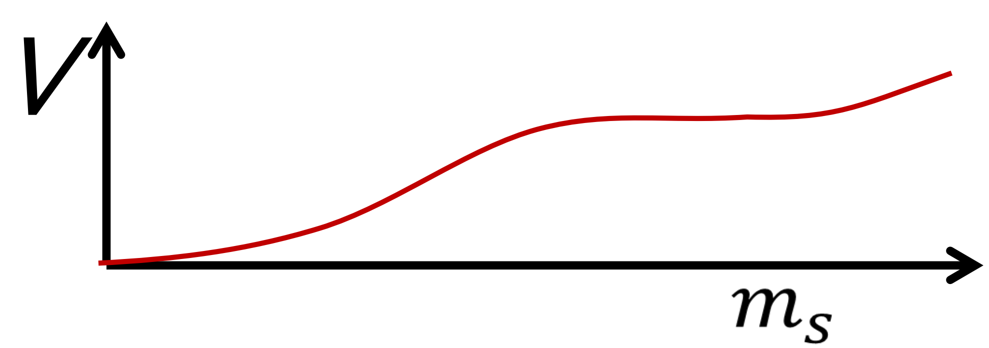
⇒ It is probable that the couplings are tuned to that value as in the case of kink.

Examples:

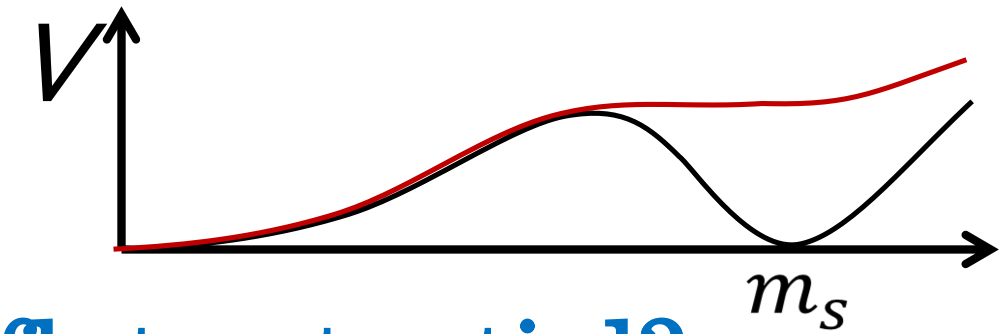
Cosmological constant,

Higgs inflation,

…



In this way we may introduce
the generalized MPP,
“ Coupling constants, which are relevant in low energy region, are tuned to values that significantly change the history of universe (or multiverse) when they are changed.”



Open questions

- Degenerate vacuum or flat potential?
- Origin of the weak scale?
- Origin of the cosmological constant?
- How many parameters are tuned?
  Too much big fix?

$\Rightarrow$ We need the precise form of $Z(\lambda)$.

$\Rightarrow$ We should investigate the wave function of multiverse.

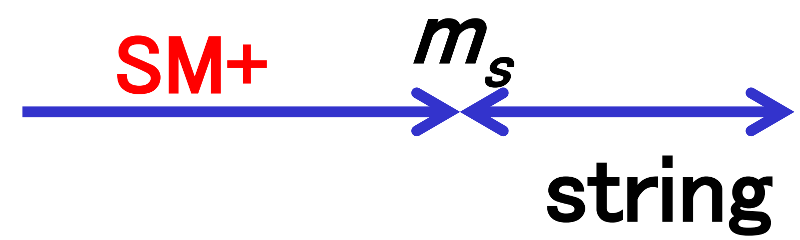
Okada-HK

4. Emergence of Electroweak Scale

Weak scale as a non-perturbative effect

Basic assumptions:

(1) SM is directly connected to the string theory without large modification.



(2) The fundamental scale is only the Planck/string scale, which appears as the cut-off of the field theory that we are considering.

(3) Relevant operators (couplings with positive mass dimensions) are tuned by nature itself through the generalized MPP.

Question:

How does the weak scale appear?

The simplest guess:

Weak scale appears as a non-perturbative effect.

Then it is related to the Planck scale as

$$m_H = M_P\, e^{-\mathrm{const.}/g_s}.$$

and the large hierarchy is naturally understood.

In order to make a phenomenologically acceptable model, we consider a Coleman-Weinberg like model in which

we first make a mass scale independently to SM sector, and then transfer it to SM through VEV.

4-1 Multi-criticalities of Two Real Scalar Model

Two real scalar model

The simplest model of the Coleman-Weinberg like mechanism is the two real scalar model:

$$\mathcal{L}_{\phi S} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial S)^2 - V$$

$$\begin{aligned} V = {} & \mu_1\phi + \mu_2\phi^2 + \mu_3\phi^3 + \frac{\lambda_\phi}{4!}\phi^4 \\ & + \mu_S S^2 + \frac{\lambda_S}{4!}S^4 \\ & + \cancelto{0}{\mu_{\phi S}}\phi S^2 + \lambda_{\phi S}\phi^2 S^2 \end{aligned}$$

Here we assume the Z_2 symmetry for S:

$$S \to -S.$$

The $\mu_{\phi S}\phi S^2$ term is eliminated by the shift of ϕ.

The basic assumptions are

(1) This action is valid up to the Planck scale where the theory is connected to string theory.
⇒ We can treat the theory as a local field theory with cutoff momentum $\Lambda \sim M_P$.

(2) The coupling constants with positive mass dimensions are tuned to one of the critical points with maximum criticality.

$$V = \mu_1 \phi + \mu_2 \phi^2 + \mu_3 \phi^3 + \frac{\lambda_\phi}{4!} \phi^4$$
$$+ \mu_S S^2 + \frac{\lambda_S}{4!} S^4 + \frac{\lambda_{\phi S}}{4} \phi^2 S^2$$

What kind of criticality?

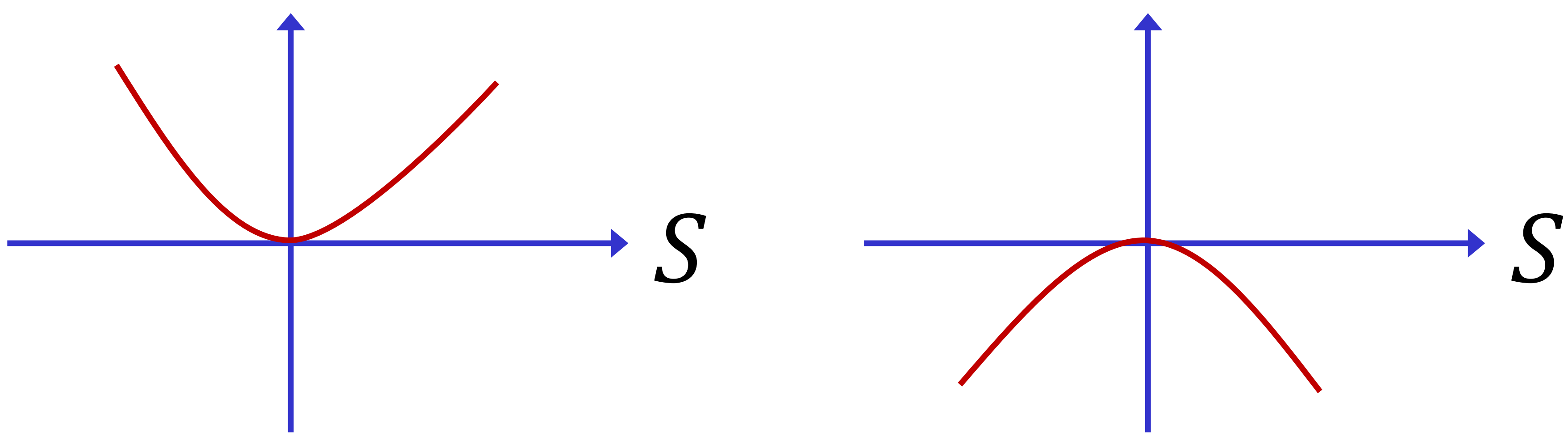
According to the generalized MPP, we consider the history of the universe.
A critical point is a point in the space of coupling constants at which the history of the universe changes significantly in its neighborhood..

For simplicity, we consider the critical point of the vacuum. That is, in the space of coupling constants, we consider the point at which the effective potential at zero temperature changes significantly.

Criticality for μ_S

The behavior of V_{eff} around $\phi = S = 0$ changes significantly at $\mu_S = 0$:

$$V_{\text{eff}}(\phi{\sim}0, S{\sim}0) \sim \mu_1 \phi + \mu_S S^2$$



$\mu_S > 0$ $\mu_S < 0$

$\Rightarrow$ $\mu_S = 0$ is a criticality.

In the following, we concentrate on this case:

$$\mu_S = 0 \,.$$

One-loop effective potential

In the following, we also assume that the Z_2 invariance for S does not spontaneously break down.

$$V_{\text{eff}}(\phi, S=0) = \mu_1\phi + \frac{\mu_2}{2}\phi^2 + \frac{\mu_3}{3!}\phi^3 + \frac{\lambda_\phi}{4!}\phi^4 + \frac{M_\phi^4}{64\pi^2}\log\left(\frac{M_\phi^2}{\mu^2}\right) + \frac{M_S^4}{64\pi^2}\log\left(\frac{M_S^2}{\mu^2}\right)$$

$$M_\phi^2(\phi) = \mu_2 + \mu_3\phi + \frac{\lambda_\phi}{2}\phi^2,$$

$$M_S^2(\phi) = \underset{0}{\cancel{\mu_S}} + \frac{\lambda_{\phi S}}{2}\phi^2 \,.$$

V_{eff} has three parameters with positive mass dimensions μ_1, μ_2, μ_3.

$\Rightarrow$ Find triple critical points of V_{eff} .

RG analysis

$$V = \frac{\lambda_\phi}{4!}\phi^4 + \frac{\lambda_{\phi S}}{4}\phi^2 S^2 + \frac{\lambda_S}{4!}S^4 + ..$$

Beta functions:

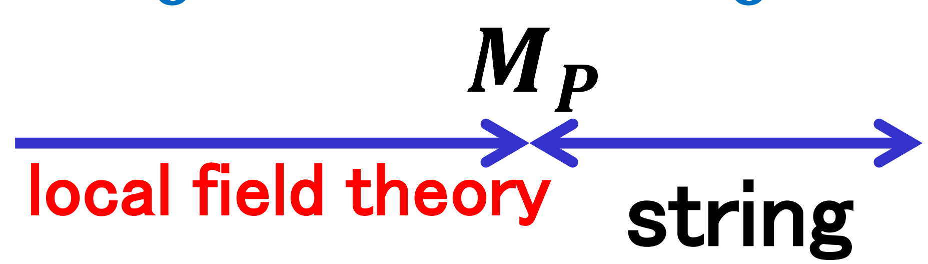
$$\beta_{\lambda_\phi} = \frac{3}{16\pi^2}\left(\lambda_\phi^2 + \lambda_{\phi S}^2\right)$$

$$\beta_{\lambda_S} = \frac{3}{16\pi^2}\left(\lambda_S^2 + \lambda_{\phi S}^2\right)$$

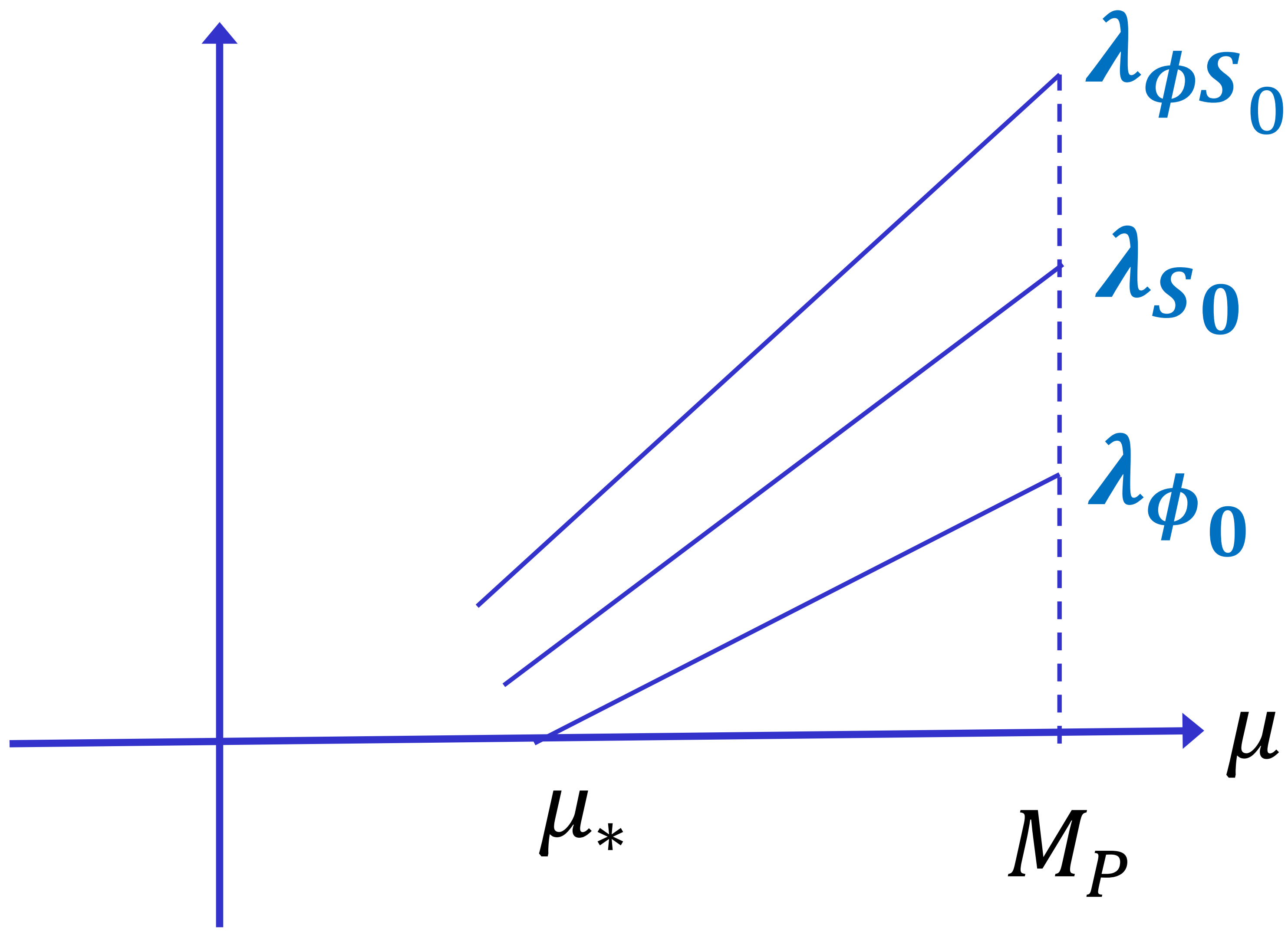
$$\beta_{\lambda_{\phi S}} = \frac{1}{16\pi^2}\left(\lambda_\phi\lambda_{\phi S} + \lambda_S\lambda_{\phi S} + 4\lambda_{\phi S}^2\right)$$

Assumption: Bare couplings at the Planck scale

$$\lambda_{\phi_0},\ \lambda_{S_0},\ \lambda_{\phi S_0} > 0.$$



When we decrease the renormalization point, one of the couplings becomes zero.



$$\mu_* \sim M_P e^{-\frac{16\pi^2 \lambda_{\phi_0}}{3\lambda_{\phi S_0}^2}}$$

We assume λ_ϕ becomes zero first at $\mu = \mu_*$.

This is possible if $\lambda_{\phi S_0} \gg \lambda_{S_0} > \lambda_{\phi_0}$.

Then it is expected

$\langle \phi \rangle \neq 0 \Rightarrow S$ **becomes massive through** $\frac{\lambda_{\phi S}}{4}\phi^2 S^2$

$\Rightarrow \langle S \rangle = 0$ **(consistent)**

Triple criticality for μ_1, μ_2, μ_3

Taking the renormalization point to μ_*, we have

$$V_{\text{eff}}(\phi, S=0) = \mu_1\phi + \frac{\mu_2}{2}\phi^2 + \frac{\mu_3}{3!}\phi^3 + \frac{M_\phi^4}{64\pi^2}\log\left(\frac{M_\phi^2}{\mu_*^2}\right) + \frac{M_S^4}{64\pi^2}\log\left(\frac{M_S^2}{\mu_*^2}\right)$$

$$M_\phi^2(\phi) = \mu_2 + \mu_3\phi,$$
$$M_S^2(\phi) = \frac{\lambda_{\phi S}}{2}\phi^2\,.$$

As we will see, at each critical point, the first three terms balance with the last term.

$\Rightarrow \mu_1, \mu_2, \mu_3 \sim O(\lambda_{\phi S}^2)$

$\Rightarrow$ **the 4th term can be neglected if $\lambda_{\phi S}$ is small.**

Thus we have

$$V_{\text{eff}}(\phi) = \mu_1\phi + \frac{\mu_2}{2}\phi^2 + \frac{\mu_3}{3!}\phi^3 + \frac{c}{2\cdot 4!}\phi^4 \log\left(\frac{\phi^4}{M^2}\right),$$

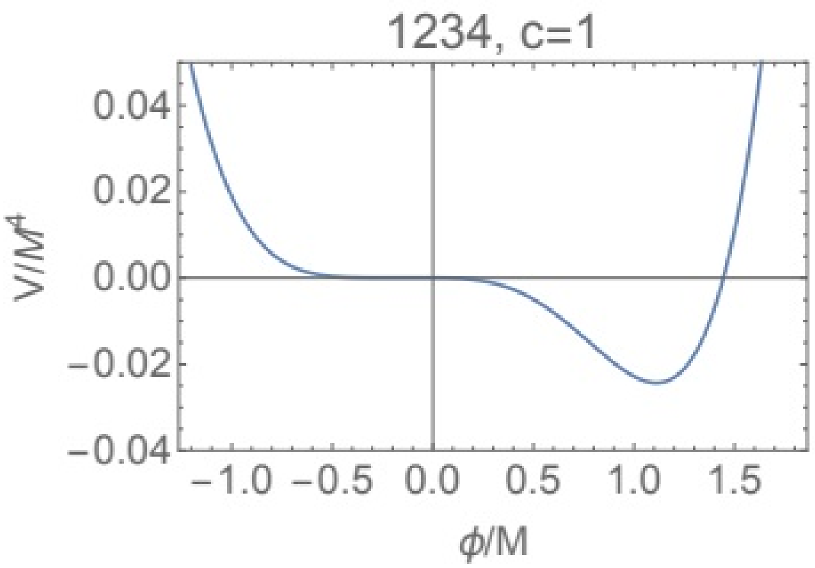
$$c = \frac{\lambda_{\phi S}^2}{6\pi^2}, \; M = \text{const.}\,\mu_* \,.$$

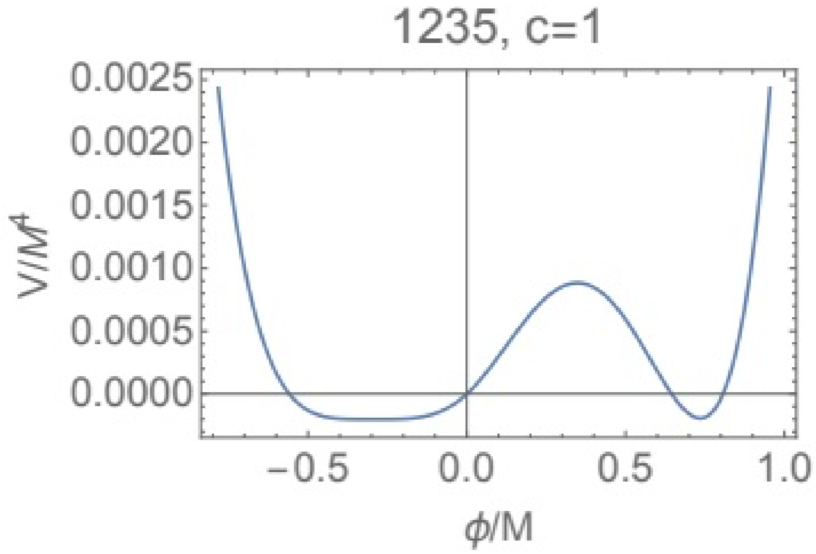
The question is reduced to classifying the triple criticalities of this function.

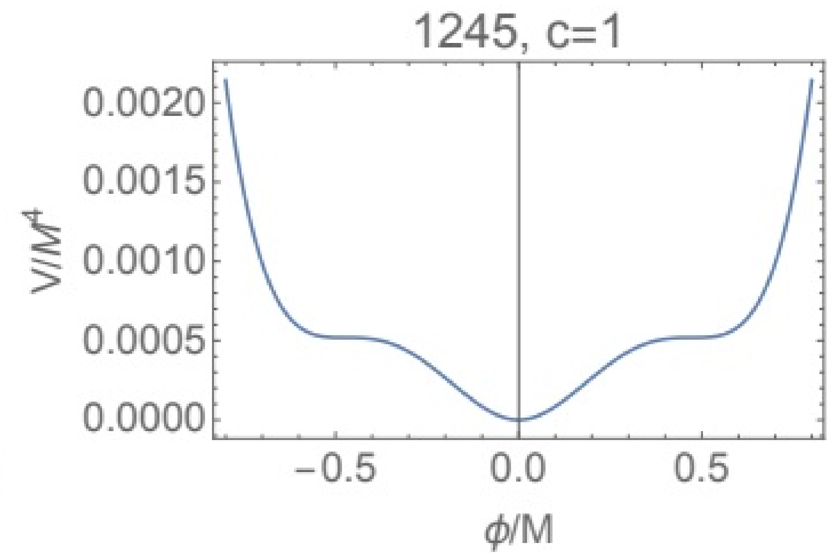
First, it is easy to show that generically $V_{\text{eff}}(\phi)$ has five extrema.

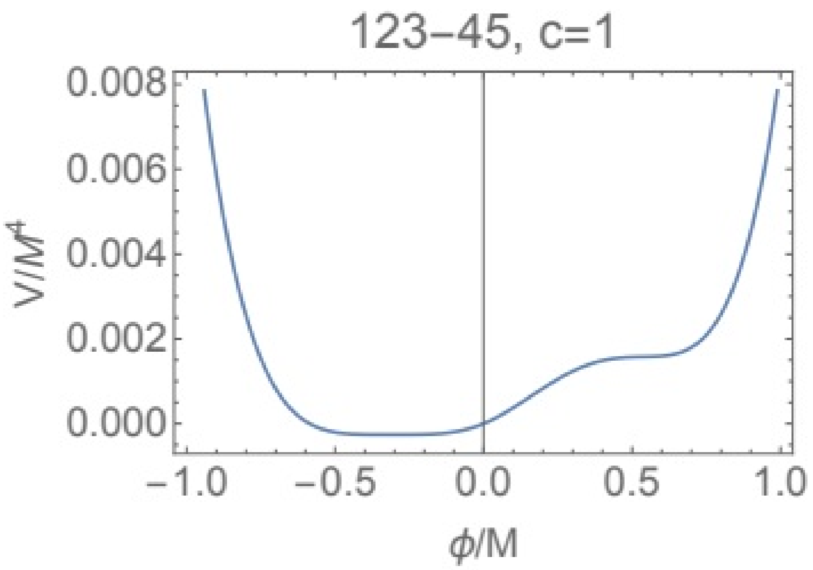
We name them 1, 2, 3, 4, and 5 from left to right on the ϕ axis. (1,3,5 local minima; 2,4 local maxima.)

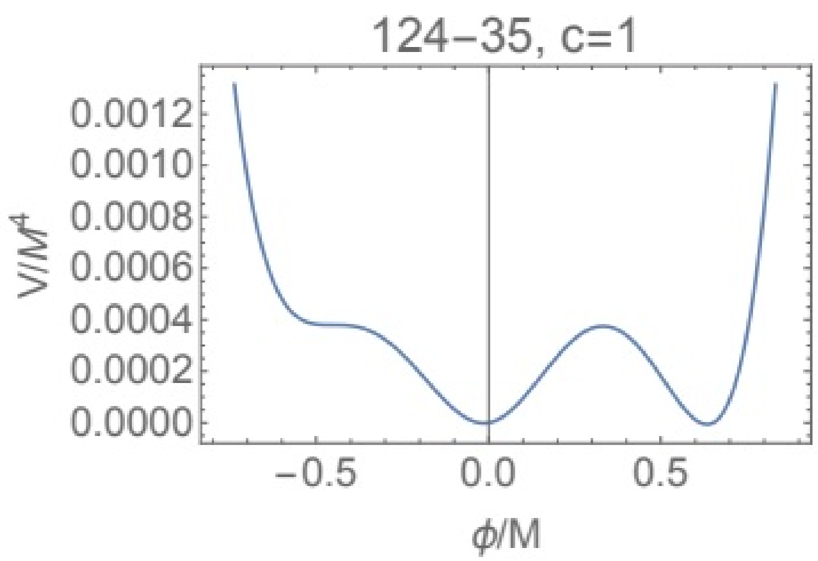
Then the triple criticality can be classified as follows.

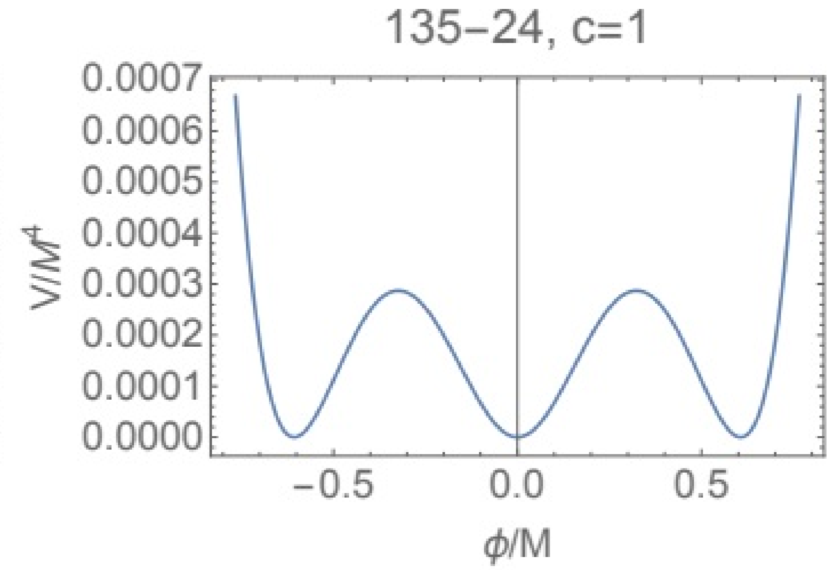


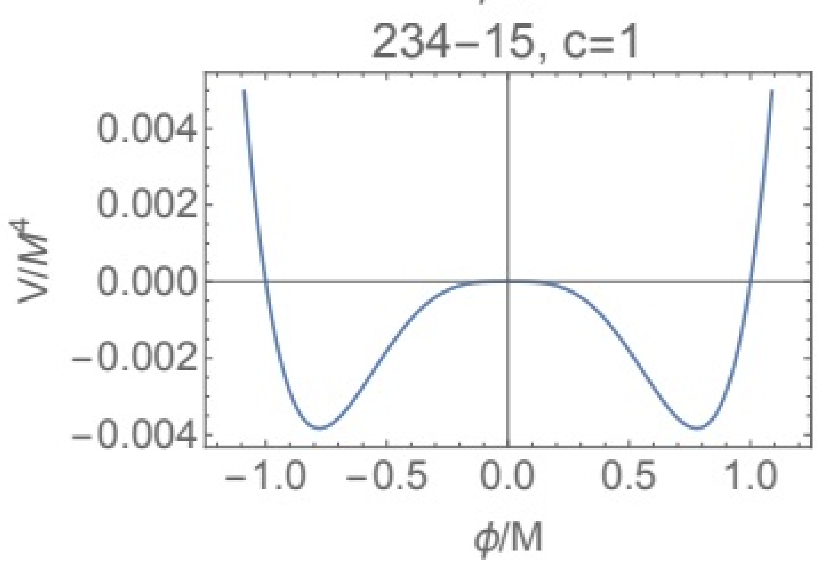












← Coleman-Weinberg

$$|v_\phi| = \text{const.}\, M$$
$$m_\phi^2 = \text{const.}\, cM^2$$

$$c = \frac{\lambda_{\phi S}^2}{6\pi^2},\ M = \text{const.}\, \mu_*$$

4-2 Coupling to SM

Coupling the two real scalar model to SM

Total action:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu S\partial^\mu S - V$$

$\mathcal{L}_{SM}$**: SM action without Higgs potential**

$$\begin{aligned} V = &\lambda_H\left(H^\dagger H\right)^2 + \lambda_\phi\phi^4 + \lambda_S S^4 \\ &+\lambda_{\phi S}\phi^2 S^2 - \lambda_{\phi H}\phi^2\left(H^\dagger H\right) + \lambda_{SH}S^2\left(H^\dagger H\right) \\ &+\mu_H\left(H^\dagger H\right) + \mu_S S^2 + \mu_{\phi H}\phi\left(H^\dagger H\right) + \mu_{\phi S}\phi S^2 \\ &+\mu_1\phi + \mu_2\phi^2 + \mu_3\phi^3 \end{aligned}$$

0 (by the shift of ϕ)

We have 6 parameters with positive mass dimensions. According to MPP, they should be tuned to a sextuple critical point.

Criticality for $\mu_H, \mu_S, \mu_{\phi S}$

In the following, we concentrate on the special case:

$$\mu_H = \mu_S = \mu_{\phi S} = 0.$$

In fact, the behavior of V around $H = S = \phi = 0$ changes significantly, when each of $\mu_H, \mu_S, \mu_{\phi S}$ changes its sign:

$$V \sim \mu_H \left(H^\dagger H\right) + \mu_S S^2 + \mu_{\phi S} \phi S^2 + \mu_1 \phi + \mu_2 \phi^2.$$

Then the problem is reduced to the two scalar case:

$$V = \lambda_H (H^\dagger H)^2 + \lambda_S S^4$$
$$-\lambda_{\phi H}\phi^2 (H^\dagger H) + \lambda_{\phi S}\phi^2 S^2 + \lambda_{SH} S^2 (H^\dagger H)$$
$$+\mu_1 \phi + \mu_2 \phi^2 + \mu_3 \phi^3 + \lambda_\phi \phi^4$$

Electroweak scale is generated non-perturbatively from the Planck scale, as

$$M_{\mathrm{P}} \to \langle\phi\rangle \sim \mu_* \to m_H^2 \sim \lambda_{\phi H} \langle\phi\rangle^2 \, .$$

$$\mu_* \sim M_P e^{-\frac{16\pi^2 \lambda_{\phi_0}}{3\lambda_{\phi S_0}^2}}$$

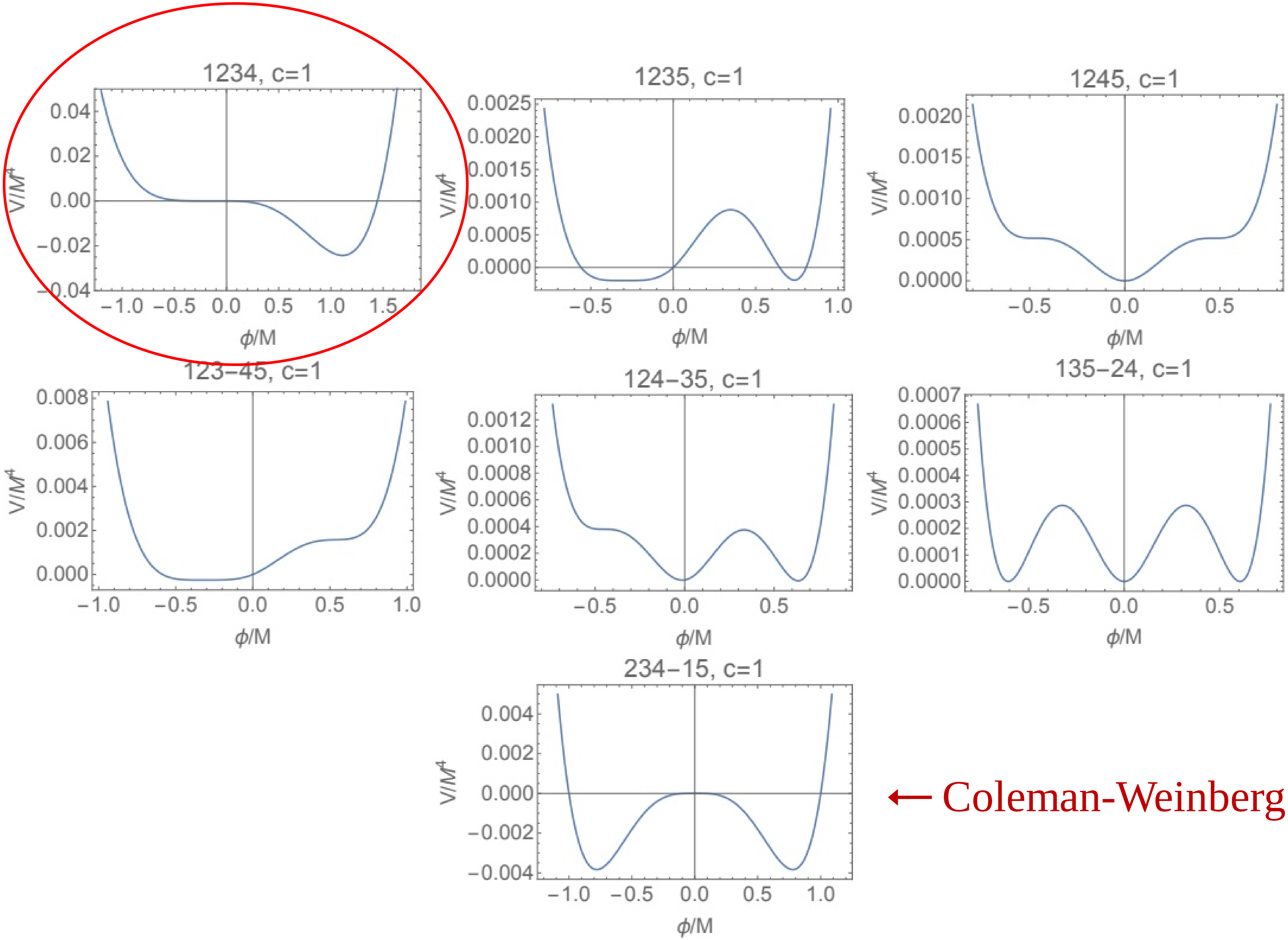
S as dark matter

no vev: $\langle S \rangle = 0$

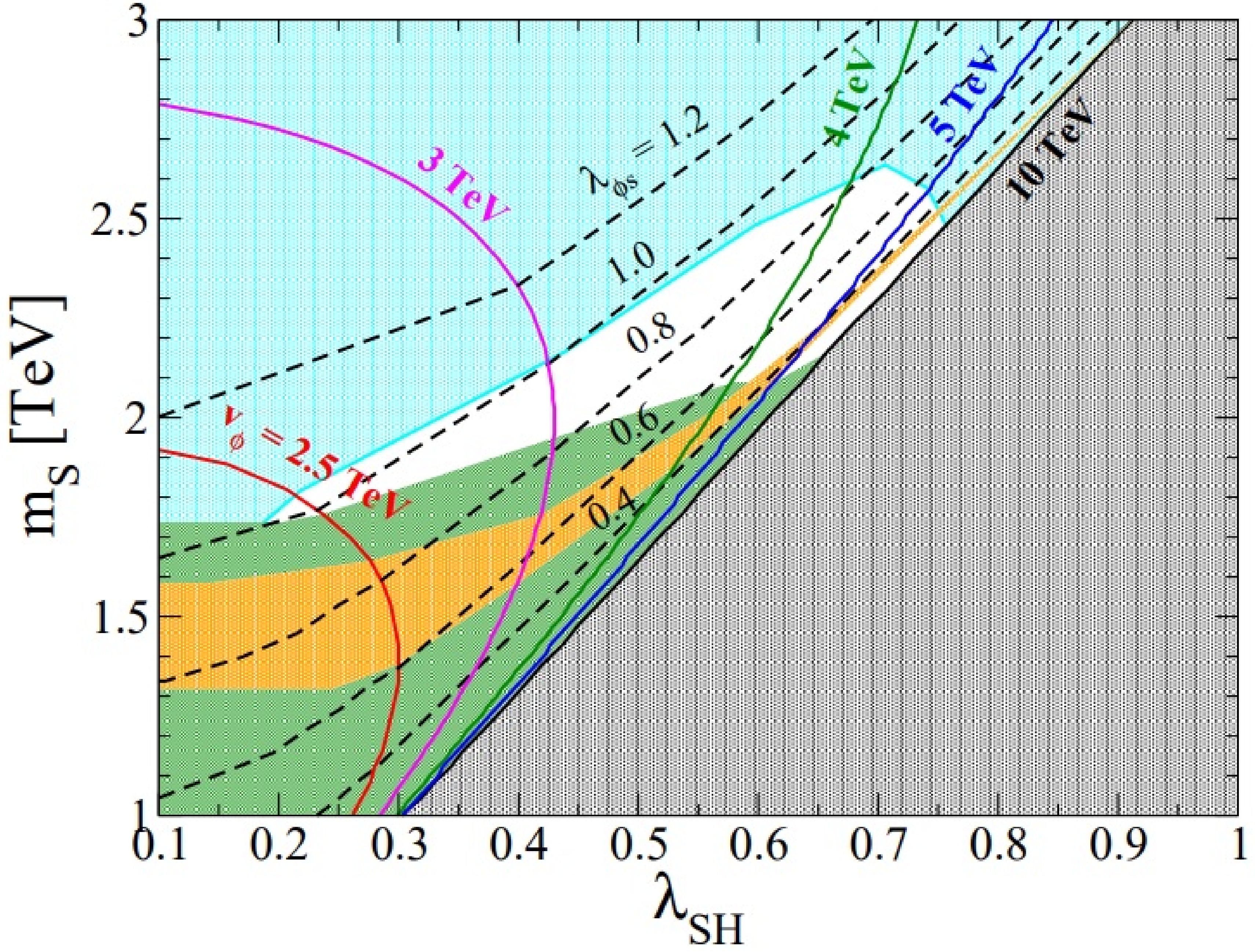
heavy but not too heavy: $m_S^2 \sim \lambda_{\phi S} \langle\phi\rangle^2$

couples to Higgs: $\lambda_{SH} S^2 (H^\dagger H)$

4-3 Phenomenological analyses for 1234 criticality model

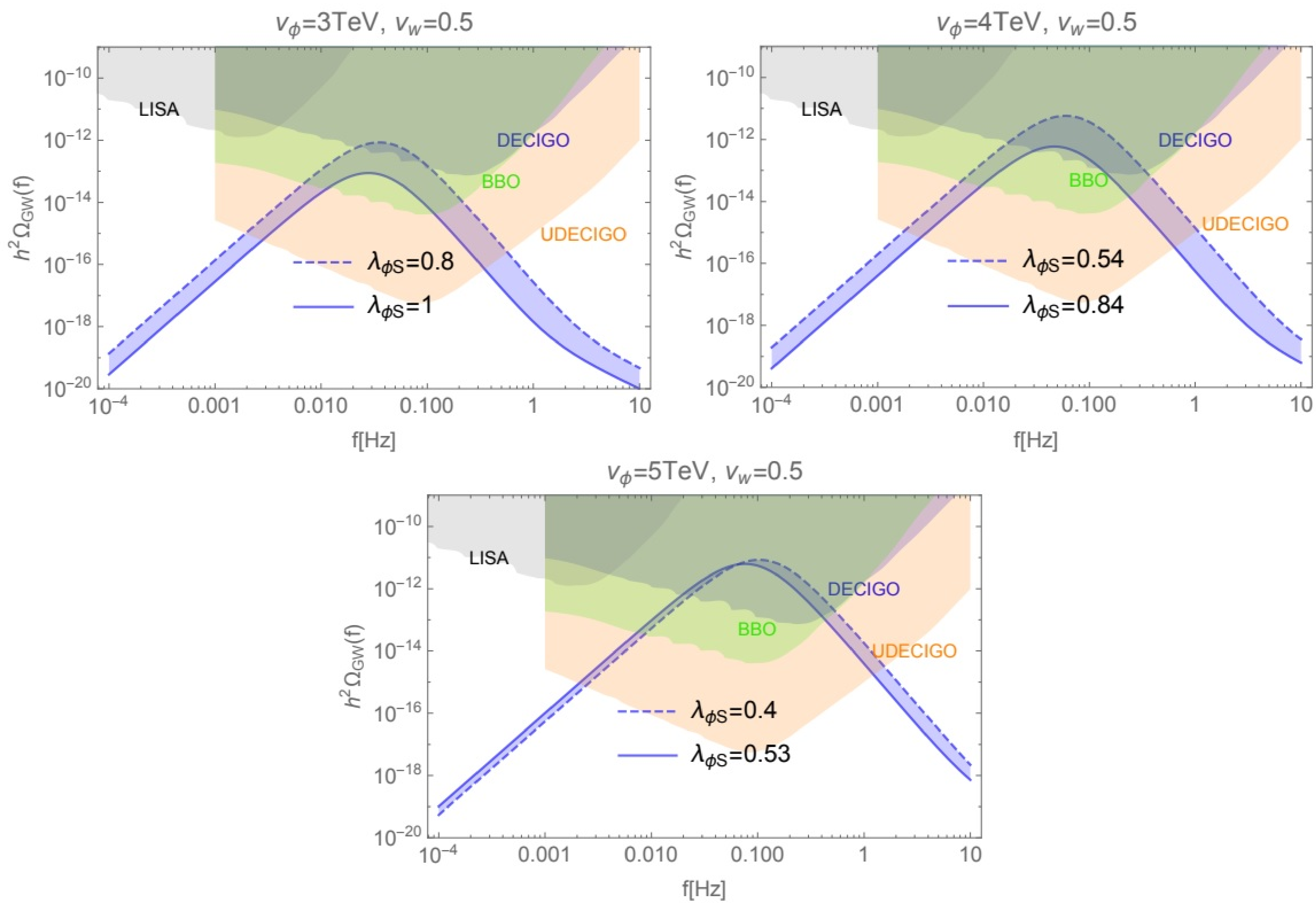

1234, c=1
1235, c=1
1245, c=1
123−45, c=1
124−35, c=1
135−24, c=1
234−15, c=1
V/M⁴
ϕ/M
← Coleman-Weinberg

1234 criticality model



Excluded by XENON1T (green), the LHC data (orange), dark matter abundance (gray), no Landau pole below 10^{17}GeV (cyan).

Gravitational wave from the 1st order phase transition



Summary

Desert is probable from experiments and observations.

It is natural to expect that SM with a small modification is directly connected to string theory at the Planck scale.

It is meaningful to investigate the possible modifications under the assumption that theory stays perturbative up to the Planck scale.

Naturalness may serve a good clue to such attempts.

QG/string theory seems to have a self-tuning mechanism. Although our understanding is not complete, we may use MPP as an ad hoc principle to reach the correct low energy theory that is valid up to the Planck scale.

Thank you very much.