UV/IR mixing and noncommutative gauge theories defined by using the Seiberg-Witten map

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Details can be found in

Eur.Phys.J.C 81 (2021) 10, 878 & JHEP 09 (2016) 052 by CPM, J.Trampetic and J. You

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PLAN

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My motivation

My motivation. I

- It seems that a high enough energy will shall have to make sense out of quantum (field) theories having UV/IR mixing.
- The standard hand-waving argument to make the previous statement plausible is the following

  “The more energy you pump into a black hole the larger the horizon area gets".
- A class of quantum field theories where UV/IR mixing occurs naturally, and which can be computed in a precise way, are the type of noncommutative gauge theories arising when open strings end on branes.
- Although they are very interesting theories, they have shortcomings
  - Only $U(N)$ groups ——> extra $U(1)s$; no $SU(N)$, $SO(N)$ groups.
  - Only in the fundamental, antifundamental an bifundamental representations—->No $SO(10)$ GUT.
  - No fractional hypercharges.

My motivation. II

- To formulate noncommutative gauge theories for arbitrary gauge groups in arbitrary representations, we shall need the enveloping algebra formalism which takes advantage of the Seiberg-Witten map.
- The noncommutative versions of the SM and GUTs have been formulated within this framework:
  - Eur.Phys.J.C 23 (2002) 363-376 by Calmet,Jurco,Schupp, Wess & Wohlgennant
  - Nucl.Phys.B 651 (2003) 45-70 by Aschieri, Jurco, Schupp & Wess
  - Phys.Rev.D 89 (2014) 6, 065018, CPM.
- and, also, noncommutative gravity theories
  - Class.Quant.Grav. 22 (2005) 3511-3532 by Aschieri, Blohmann, Dimitrijevic, Meyer & Schupp
  - Phys.Rev.D 79 (2009) 025004 by Marculescu & Ruiz.
  - JHEP 06 (2009) 086 by Aschieri & Castellani.
  - JHEP 11 (2014) 103 by Aschieri & Castellani.
  - Arxiv 2208.02152 by Dimitrijevic et al.

My motivation.III

- AND YET... no analysis of the existence, if any, of UV/IR mixing effects for simple gauge groups had been done.
- Our modest goal was to carry out such study in the simplest case: $SU(N)$ in the fundamental,
- for even in this case a detailed analysis is quite involved.

Generation of UV/IR mixing in standard NC field theory

- Let us assume that we are working at 1-loop.
- In the perturbative expansion of the Green functions one meets (noncommutative) Feynman integrals of the type

$$\int \frac{d^4q}{(2\pi)^4}\, e^{iq\wedge p}\, \frac{N(q,p_i)}{D(q,p_i)}, \quad q\wedge p = q_\mu \theta^{\mu\nu} p_\nu.$$

  the integral

$$\int \frac{d^4q}{(2\pi)^4}\, \frac{N(q,p)}{D(q,p)},$$

  being UV divergent.
- It so happens that the exponential $e^{iq\wedge p}$ acts as a regularization function with regulator $\theta^{\mu\nu}p_\nu$.
- This exponential renders the integral UV finite if $\tilde{p}^\mu = \theta^{\mu\nu}p_\nu \neq 0$.
- The limit $\theta^{\mu\nu}p_\nu \to 0$ is divergent: The UV divergence has turned into an IR divergence.
- Lesson: we need phases to generate UV/IR mixing, in this context.

NCQFT in the enveloping algebra framework

NCQFT formulated by using the SW map

- The enveloping algebra formalism for NCQFT, or NCQFT formulated by using the SW map, was put forward at the beginning of the current century by Aschieri, Jurco, Madore, Schupp, Wess and collaborators, in various combinations.
- The formalism for a NC Yang-Mills theory can be spelled out as follows:
    - 1) The classical action of the theory is a polynomial w.r.t the $\star$-product of the noncommutative gauge field and their derivatives, which is gauge invariant under noncommutative gauge transformations.
    - 2) The noncommutative gauge field, $A_\mu(x)$, takes values in the universal enveloping algebra of the Lie group of theory in a given arbitrary representation and it is a function –called the SW map– of the ordinary gauge field, $a_\mu(x)$, and the noncommutativity matrix, $\theta^{\mu\nu}$.
    - 3) This function is obtained by solving the (Seiberg-Witten) equations

$$s_{NC}A_\mu(x)[a_\rho;\theta] = sA_\mu[a_\rho;\theta](x), \quad s_{NC}C[c,a_\rho;\theta](x) = sC[c,a_\rho;\theta](x),$$

where

$$s_{NC}A_\mu = \partial_\mu C - i(A_\mu \star C - A_\mu \star C), \quad sa_\mu = \partial_\mu c - i[a_\mu, c]$$
$$s_{NC}C = iC\star C, \quad sc = icc$$

SOME COMMENTS:

- The NC Yang-Mills, $S_{NCYM}[A_\mu[a_\rho,\theta]]$, action reads

$$S_{NCYM}[A_\mu[a_\rho,\theta]] = -\frac{1}{4g^2}\int d^nx\,\mathrm{Tr}\,F_{\mu\nu}\star F^{\mu\nu}, \quad \text{where}$$
$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu - i(A_\mu\star A_\nu - A_\nu\star A_\mu)(x).$$

- By construction, ordinary gauge orbits are mapped into noncommutative gauge orbits.
- The Standard Model and GUTs (SU(5) and SO(10)) on noncommutative Minkowski spacetime have been formulated by utilizing this formalism.
- The noncommutative fields have no intrinsic definition, they are defined in terms of the ordinary fields by using the SW map.
- As a consequence, the physical degrees of freedom (gluons, quarks, leptons etc..) of the noncommutative theory are the same as those of the ordinary theory,
- which is nice in view of the current experimental data from the LHC.

The NC quantum theory

- The noncommutative quantum theory is defined by the following path integral

$$Z[j^\mu] = \int \mathscr{D}a^a_\mu \mathscr{D}ghosts\; e^{iS[A_\mu[a_\rho,\theta]] + s_{NC}X_{gauge-fixing} + i\int d^nx\,\mathrm{Tr}a_\mu(x)j^\mu(x)}, \quad a_\mu = a^a_\mu T^a$$

- Seiberg and Witten defined –for $U(1)$– the map which carries their names as a formal power series in the noncommutativity matrix $\theta^{\mu\nu}$.
- So, when it all started, for general gauge groups in arbitrary representations, the Seiberg-Witten map was defined likewise, ie, as a formal power series in $\theta^{\mu\nu}$.
- Hence, $S[A_\mu[a_\rho,\theta]]$ was ACTUALLY defined as a formal power series in $\theta^{\mu\nu}$.
- As a result the theory so formulated LACKS UV/IR mixing –unless all the terms in the series are considered at the same time: hopeless task.
- We need a solution to the SW map equations which be exact in $\theta^{\mu\nu}$ but be a formal power series in the coupling constant of the gauge theory.!
- and, thus, $S[A_\mu[a_\rho,\theta]]$ will be a formal power series in the coupling constant, each term of the series being exact in $\theta^{\mu\nu}$.

The θ-exact SW map for arbitrary gauge groups

The θ-exact SW map for arbitrary gauge groups I

- The perturbative, in the coupling constant, and θ-exact SW map was constructed in Phys.Rev.D 86 (2012) 065010 (CPM).
- The computation of such SW map would not have been possible without valuable help from the BRS cohomology results in JHEP 08 (2002) 023 (Barnich, Brandt & Grigoriev).
- For simple gauge groups, the SW map in question reads

$$A_\mu[a_\rho;h\theta] = \sum_{n=1}^{\infty} g^n A_\mu^{(n)}[a_\rho;h\theta]$$

$$\begin{aligned}
A_\mu^{(1)}[a_\rho;h\theta] &= a_\mu, \forall h,\\
A_\mu^{(2)}[a_\rho;h\theta] &= \int_0^h dt\left(\tfrac{1}{2}\,\theta^{ij}\{A_i^{(1)},\partial_j A_\mu^{(1)}\}_{\star_t} - \tfrac{1}{4}\,\theta^{ij}\{A_i^{(1)},\partial_\mu A_j^{(1)}\}_{\star_t}\right),\\
A_\mu^{(3)}[a_\rho;h\theta] &= \int_0^h dt\Big(\tfrac{1}{2}\,\theta^{ij}\{A_i^{(1)},\partial_j A_\mu^{(2)}[a_\rho;t\theta]\}_{\star_t} + \tfrac{1}{2}\,\theta^{ij}\{A_i^{(2)}[a_\rho;t\theta],\partial_j A_\mu^{(1)}\}_{\star_t}\\
&\quad - \tfrac{1}{4}\,\theta^{ij}\{A_i^{(2)}[a_\rho;t\theta],\partial_\mu A_j^{(1)}\}_{\star_t} - \tfrac{1}{4}\,\theta^{ij}\{A_i^{(1)},\partial_\mu A_j^{(2)}[a_\rho;t\theta]\}_{\star_t}\\
&\quad + \tfrac{i}{4}\,\theta^{ij}\{A_i^{(1)},[A_j^{(1)},A_\mu^{(1)}]_{\star_t}\}_{\star_t},\Big),
\end{aligned}$$

....

$$\begin{aligned}
A_\mu^{(n)}[a_\rho;h\theta] &= \int_0^h dt\Big(\tfrac{1}{2}\,\theta^{ij}\textstyle\sum_{m_1+m_2=n}\{A_i^{(m_1)},\partial_j A_\mu^{(m_2)}\}_{\star_t} - \tfrac{1}{4}\,\theta^{ij}\sum_{m_1+m_2=n}\{A_i^{(m_1)},\partial_\mu A_j^{(m_2)}\}_{\star_t}\\
&\quad + \tfrac{i}{4}\,\theta^{ij}\textstyle\sum_{m_1+m_2+m_3=n}\{A_i^{(m_1)},[A_j^{(m_2)},A_\mu^{(m_3)}]_{\star_t}\}_{\star_t}\Big),\ \forall n>3.
\end{aligned}$$

The θ-exact SW map for arbitrary gauge groups II

- The previous expression for the SW map can be Fourier transformed, eg,

$$A^{(2)}_\mu[a_\rho;h\theta](x) = \int \frac{d^np_1}{(2\pi)^n}\frac{d^np_2}{(2\pi)^n}\, e^{-i(p_1+p_2)x}\,\mathbb{A}^{(2)}_\mu[(p_1,\mu_1,a_1),(p_2,\mu_2,a_2);h\theta]\, a^{a_1}_{\mu_1}(p_1)\, a^{a_2}_{\mu_2}(p_2),$$

$$\mathbb{A}^{(2)}_\mu[(p_1,\mu_1,a_1),(p_2,\mu_2,a_2);h\theta] = \frac{1}{2}\,\theta^{ij}\,(2p_{2\,j}\,\delta^{\mu_1}_i\,\delta^{\mu_2}_\mu - p_{2\,\mu}\,\delta^{\mu_1}_i\,\delta^{\mu_2}_j)$$
$$\Big[T^{a_1}T^{a_2}\,\frac{e^{-i\frac{h}{2}p_1\wedge p_2}-1}{p_1\wedge p_2} - T^{a_2}T^{a_1}\,\frac{e^{i\frac{h}{2}p_1\wedge p_2}-1}{p_1\wedge p_2}\Big],$$

$$p\wedge q = \theta^{\mu\nu}p_\mu q_\nu,$$

- which makes it ideally suited for computations using Feynman diagrams.
- Notice that the expression above involves phases of the type

$$e^{i\frac{h}{2}p_1\wedge p_2},$$

- needed to have a chance of obtaining UV/IR mixing.
- It seems that we are on the right course!

The θ-exact SW map for arbitrary gauge groups III

- To look for UV/IR mixing we need to consider UV divergent Green functions.
- The simplest instance of which is the 2-point 1PI Green function of the gauge field at 1-loop.
- As we shall see, even the computation of this simplest Green function is awfully involved.
- A first glimpse of why it is so involved can be caught from the 3-field order contribution to the SW map:

The θ-exact SW map for arbitrary gauge groups IV

$$A^{(3)}_\mu[a_\rho;h\theta](x)=\int\frac{d^np_1}{(2\pi)^n}\frac{d^np_2}{(2\pi)^n}\frac{d^np_3}{(2\pi)^n}\;e^{-i(p_1+p_2+p_3)x}\;\Big\{\mathbb{A}^{(3)}_\mu[(p_1,\mu_1,a_1),(p_2,\mu_2,a_2),(p_3,\mu_3,a_3);h\theta]\;a^{a_1}_{\mu_1}(p_1)\,a^{a_2}_{\mu_2}(p_2)\,a^{a_3}_{\mu_3}(p_3)\Big\}$$

$$\begin{aligned}\mathbb{A}^{(3)}_\mu[(p_1,\mu_1,a_1),(p_2,\mu_2,a_2),(p_3,\mu_3,a_3);h\theta]=&\\ \mathbb{P}^{(3)}_\mu[(p_1,\mu_1),(p_2,\mu_2),(p_3,\mu_3);\theta]&\Big(T^{a_1}T^{a_2}T^{a_3}\,\mathbb{I}(p_1;p_2,p_3;h,\theta)+T^{a_2}T^{a_3}T^{a_1}\,\mathbb{I}(-p_1;p_2,p_3;h,\theta)\Big)\\ +\mathbb{Q}^{(3)}_\mu[\mu_1,\mu_2,\mu_3;\theta]&\Big(T^{a_1}T^{a_2}T^{a_3}\,\mathbb{F}(p_1;p_2,p_3;h,\theta)+T^{a_2}T^{a_3}T^{a_1}\,\mathbb{F}(-p_1;p_2,p_3;h,\theta)\Big)\end{aligned}$$

$$\begin{aligned}\mathbb{P}^{(3)}_\mu[(p_1,\mu_1),(p_2,\mu_2),(p_3,\mu_3);\theta]=\tfrac{1}{4}\,\theta^{ij}\theta^{kl}\Big\{&[4(p_{3l}\,\delta^{\mu_2}_k\delta^{\mu_3}_i+p_{2l}\,\delta^{\mu_2}_i\delta^{\mu_3}_k)-2(p_3-p_2)_i\,\delta^{\mu_2}_k\delta^{\mu_3}_l]p_{1j}\,\delta^{\mu_1}_\mu\\ &+[4(p_{3l}\,\delta^{\mu_2}_k\delta^{\mu_3}_\mu+p_{2l}\,\delta^{\mu_2}_\mu\delta^{\mu_3}_k)-2(p_3-p_2)_\mu\,\delta^{\mu_2}_k\delta^{\mu_3}_l](p_2+p_3)_j\,\delta^{\mu_1}_i-[2(p_{3l}\,\delta^{\mu_2}_k\delta^{\mu_3}_i+p_{2l}\,\delta^{\mu_2}_i\delta^{\mu_3}_k)\\ &-(p_3-p_2)_i\,\delta^{\mu_2}_k\delta^{\mu_3}_l]p_{1\mu}\,\delta^{\mu_1}_j-[2(p_{3l}\,\delta^{\mu_2}_k\delta^{\mu_3}_j+p_{2l}\,\delta^{\mu_2}_j\delta^{\mu_3}_k)-(p_3-p_2)_j\,\delta^{\mu_2}_k\delta^{\mu_3}_l](p_2+p_3)_\mu\,\delta^{\mu_1}_i\Big\},\end{aligned}$$

$$\mathbb{Q}^{(3)}_\mu[\mu_1,\mu_2,\mu_3;\theta]=-\tfrac{1}{2}\,\theta^{ij}(\delta^{\mu_1}_i\delta^{\mu_2}_j\delta^{\mu_3}_\mu-\delta^{\mu_1}_i\delta^{\mu_2}_\mu\delta^{\mu_3}_j).$$

$$\mathbb{I}(p_1;p_2,p_3;h,\theta)=\frac{1}{p_2\wedge p_3}\Bigg[\frac{e^{-i\frac{h}{2}(p_1\wedge p_2+p_1\wedge p_3+p_2\wedge p_3)}-1}{p_1\wedge p_2+p_1\wedge p_3+p_2\wedge p_3}-\frac{e^{-i\frac{h}{2}p_1\wedge(p_2+p_3)}-1}{p_1\wedge(p_2+p_3)}\Bigg],$$

$$\mathbb{F}(p_1;p_2,p_3;h,\theta)=\frac{e^{-i\frac{h}{2}(p_1\wedge p_2+p_1\wedge p_3+p_2\wedge p_3)}-1}{p_1\wedge p_2+p_1\wedge p_3+p_2\wedge p_3},$$

$$\mathbb{I}(-p_1;p_2,p_3;h,\theta)=\mathbb{I}(p_1;p_2,p_3;h,\theta)|_{p_1\to-p_1},\quad\mathbb{F}(-p_1;p_2,p_3;h,\theta)=\mathbb{F}(p_1;p_2,p_3;h,\theta)|_{p_1\to-p_1}.$$

Quantisation by using the background field method

Quantisation: Background field

- To quantize the classical theory with action $S_{NCYM}[A_\mu(a_\lambda)]$, we shall use the background field method along with the BRST formalism:

$$a_\mu = b_\mu + q_\mu,$$
$$b_\mu = b^a_\mu T^a = \text{background field}, q_\mu = q^a_\mu T^a = \text{quantum field}, \ T^a = \text{Lie group generator}.$$

- This gives rise to the following splitting of the noncommutative field $A_\mu(a = b + q)$

$$A_\mu(b+q) = B_\mu(b) + Q_\mu(b,q).$$

- $B_\mu(b)$ is obtained from $A_\mu(a)$ by just replacing a_μ and A_μ with b_μ and B_μ, respectively.
- By construction, the BRST transformations of the ordinary fields

$$sb_\mu = 0, \quad sq_\mu = \partial_\mu c - ig[b_\mu + q_\mu, c], \quad sc = igcc, \quad s\bar{C} = F, \quad sF = 0,$$
$$s = \text{BRST operator}, \ c = c^a T^a = \text{ordinary ghost field}.$$

give rise to NC BRST transformations, with NC BRST operator s_{NC} acting as follows

$$sB_\mu(b) = s_{NC} B_\mu(b) = 0,$$
$$sQ_\mu(b,q) = s_{NC} Q_\mu(b,q) = D_\mu[B+Q]C(b+q,c) = \partial_\mu C(b+q,c) - ig[B_\mu + Q_\mu, C]_\star,$$
$$sC(b+q,c) = s_{NC} C(b+q,c) = igC(b+q,c) \star C(b+q,c),$$
$$s_{NC}\bar{C} = s\bar{C} = F, \quad s_{NC}F = sF = 0$$

Quantisation: The background-field effective action

- The quantum theory is defined by the following background-field effective action $\Gamma[b]$:

$$e^{i\Gamma[b]} = \int \mathscr{D}q^a_\mu \mathscr{D}F^a \mathscr{D}c^a \mathscr{D}\bar{C}^a \, e^{i\mathscr{S} - i\int d^4x\, q^a_\mu(x)\, \frac{\delta\Gamma[b]}{\delta b^a_\mu(x)}},$$

with

$$\mathscr{S} = S_{NCYM}[A_\mu = B_\mu(b) + Q_\mu(b,q)] + S_{BRSTexact}.$$

$$S_{BRSTexact} = s\int d^4x\,\mathrm{Tr}\left(\bar{C}\star\left(\tfrac{1}{2}F + D_\mu[B]Q^\mu(b,q)\right)\right)$$

$$= s_{NC}\int d^4x\,\mathrm{Tr}\left(\bar{C}\star\left(\tfrac{1}{2}F + D_\mu[B]Q^\mu(b,q)\right)\right)$$

$$= \int d^4x\,\mathrm{Tr}\left(\tfrac{1}{2}F\star F + F\star D_\mu[B]Q^\mu(b,q) - \bar{C}\star D_\mu[B]D^\mu[B+Q]C(b+q,c)\right).$$

Computing the UV/IR mixing for $U(N)$ in the fundamental

UV/IR mixing for $U(N)$ in the fundamental. I

- Before discussing the $SU(N)$ case, let us exhibit in terms of the ordinary fields the already known UV/IR mixing phenomenon for $U(N)$ in the fundamental.
- For $U(N)$ in the fundamental both $Q_\mu(b,q)$ and q take values in the Lie algebra of $U(N)$ in the fundamental.
- Likewise for $C(b+q,c)$ and c.
- It can be shown that the change of variables

$$q^a \to Q^a = Q^a(b,q), \quad c^a \to C^a = C^a(b+q,c).$$

  has, in dimensional regularization, a trivial determinant in perturbation theory of the coupling constant for the θ-exact SW map.
- Proof of this triviality in JHEP 09 (2016) 052 by CPM, J.Trampetic and J. You.
- Actually, in JHEP 09 (2016) 052 it is shown that NC $U(N)$ in the fundamental defined by using the SW map is the same quantum theory as NC $U(N)$ defined without using the SW map, provided the θ-exact SW map is utilised: at least in perturbation theory of the coupling constant and DIM. REG.

UV/IR mixing for $U(N)$ in the fundamental. II

- Let b_μ be on-shell.
- Then, the change of field variables in the previous slide leads to

$$e^{i\Gamma[b]} = \int \mathscr{D}q^a_\mu \mathscr{D}F^a \mathscr{D}c^a \mathscr{D}\bar{C}^a \, e^{i\mathscr{S}} = \int \mathscr{D}Q^a_\mu \mathscr{D}F^a \mathscr{D}C^a \mathscr{D}\bar{C}^a \, e^{i\hat{\mathscr{S}}} = e^{i\hat{\Gamma}[B(b)]}.$$

where

$$\hat{\mathscr{S}} = S_{NCYM}[A_\mu = B_\mu(b) + Q_\mu] + S_{BRSTexact}.$$
$$S_{BRSTexact} = \int d^4x \,\mathrm{Tr}\left(\tfrac{1}{2} F \star F + F \star D_\mu[B]Q^\mu - \bar{C} \star D_\mu[B]D^\mu[B+Q]C\right).$$

- Obviously, the path integral

$$\int \mathscr{D}Q^a_\mu \mathscr{D}F^a \mathscr{D}C^a \mathscr{D}\bar{C}^a \, e^{i\hat{\mathscr{S}}} = e^{i\hat{\Gamma}[B(b)]}.$$

yields the standard perturbation theory in terms of the noncommutative $U(N)$ quantum fields.

- We thus conclude that –>

UV/IR mixing for $U(N)$ in the fundamental. III

- the 1-loop 2-point contribution to $\hat{\Gamma}[B(b)]$ develops an IR singularity due to UV/IR mixing, namely

$$g^2 \int \frac{d^4p}{(2\pi)^4}\,[\mathrm{Tr}\,b_\mu(p)][\mathrm{Tr}\,b_\nu(-p)]\,\frac{2}{\pi^2}\,\frac{\tilde{p}^\mu \tilde{p}^\nu}{(\tilde{p}^2)^2},$$
$$\tilde{p}^\mu = \theta^{\mu\nu} p_\nu,$$

- COMMENTS
  - Only the $U(1)$ part of the $U(N)$ field b_μ carries the UV/IR mixing: $\mathrm{Tr}\,b_\mu$.
  - $\mathrm{Tr}\,\mathrm{b}_\mu$ transverse to $\tilde{p}^\mu = \theta^{\mu\nu} p_\nu$ —> No UV/IR mixing.
  - First obtained in the $U(N=1)$ case in JHEP 08 (2008) 107 by P. Schupp & J. You.

Computing the UV/IR mixing for $SU(N)$ in the fundamental

UV/IR mixing for $SU(N)$ in the fundamental. I

- We are now ready to see whether there is UV/IR mixing for SU(N) in the fundamental.
- If things were to go as in the $U(N)$ case the answer would be: NO UV/IR mixing.
  - No $U(1)$ physical degrees of freedom for $SU(N)$—>$\mathrm{Tr}\, b_\mu = 0$.
- Now, the computation of the UV/IR mixing effect is simple in the $U(N)$ case in the fundamental because both the ordinary field q_μ and the noncommutative field $Q(q,p)$ both belong to the fundamental rep of the Lie algebra of $U(N)$.
- So, for $U(N)$, one is able to fully trade q_μ for $Q_\mu(b,q)$ in the path integral—->huge simplification of the interaction vertices.
- This not so in the $SU(N)$ case, for $Q(b,q)$ does not belong to the Lie algebra of $SU(N)$.

UV/IR mixing for $SU(N)$ in the fundamental. II

- In the $SU(N)$ case, the best one can do to simplify the structure of the vertices is to make the following change of variables

$$q^a --- > \tilde{q}^a_\mu = \mathrm{Tr}\,(T^a Q_\mu(b,q)), c^a --- > \tilde{c}^a = \mathrm{Tr}\,(T^a C(b,q)).$$

- It can be shown that, in Dimensional Regularization, the determinant of this transformation is trivial at any order in the coupling constant for the θ-exact SW map.
- By making the previous change of field variables in the path integral

$$e^{i\Gamma[b]} = \int \mathscr{D}q^a_\mu \mathscr{D}F^a \mathscr{D}c^a \mathscr{D}\bar{C}^a\, e^{i\mathscr{S} - i\int d^4x\, q^a_\mu(x)\frac{\delta\Gamma[b]}{\delta b^a_\mu(x)}},$$

one obtains the following result giving the 2-point, $\Gamma_2[b]$, contribution to $\Gamma[b]$ at 1-loop: —>

UV/IR mixing for $SU(N)$ in the fundamental. III

$$i\Gamma_2[b] = \mathrm{Ln}\int \mathscr{D}\tilde{q}^a_\mu \mathscr{D}\tilde{c}^a \mathscr{D}\bar{C}^a\, e^{i(S_0+S_1+S_2+S_{1eom}+S_{2eom}+S_{0gh}+S_{1gh}+S_{2gh})} + O(g^3)$$

where

$$\begin{aligned}
S_0 &= -\tfrac{1}{2}\int d^4x\, \mathrm{Tr}\, \partial_\mu \tilde{q}_\nu \partial^\mu \tilde{q}^\nu,\\
S_1 &= +ig\int d^4x\, \mathrm{Tr}\, b_\mu[\tilde{q}_\nu, \partial^\mu \tilde{q}^\nu]_\star + 2ig\int d^4x\, \mathrm{Tr}\, \partial_\mu b_\nu [\tilde{q}^\mu, \tilde{q}^\nu]_\star,\\
S_2 &= +\tfrac{1}{2}g^2\int d^4x\, \mathrm{Tr}\, [b_\mu, \tilde{q}_\nu]_\star [b^\mu, \tilde{q}^\nu]_\star - \tfrac{N}{2}g^2\int d^4x\, \partial_\mu \hat{A}^{(2)\,0}_\nu(b,\tilde{q})\partial^\mu \hat{A}^{(2)\,0\,\nu}(b,\tilde{q})\\
&\quad + ig^2\int d^4x\, \mathrm{Tr}\, \hat{A}^{(2)}_\mu(b,b)[\tilde{q}_\nu, \partial^\mu \tilde{q}^\nu]_\star + ig^2\int d^4x\, \mathrm{Tr}\, \hat{A}^{(2)\,0}_\nu(b,\tilde{q})[\partial_\mu \tilde{q}^\nu, b^\mu]_\star\\
&\quad + ig^2\int d^4x\, \mathrm{Tr}\, \partial_\mu \hat{A}^{(2)\,0}_\nu(b,\tilde{q})[b^\mu, \tilde{q}^\nu]_\star + g^2\int d^4x\, \mathrm{Tr}\, \{\partial^2 - \partial_\nu \partial^\rho \hat{A}^{(2)}_\rho(b,b)\}\hat{A}^{(2)\,0\,\nu}(\tilde{q},\tilde{q})\\
&\quad + g^2\int d^4x\, \mathrm{Tr}\, [b_\mu, b_\nu]_\star [\tilde{q}^\mu, \tilde{q}^\nu]_\star + 2ig^2\int d^4x\, \mathrm{Tr}\, \partial_\mu \hat{A}^{(2)}_\nu(b,b)[\tilde{q}^\mu, \tilde{q}^\nu]_\star\\
&\quad + 2ig^2\int d^4x\, \mathrm{Tr}\, \partial_\mu \hat{A}^{(2)\,0}_\nu(b,\hat{q})[b_\nu, \tilde{q}^\mu]_\star + \tfrac{N}{2}g^2\int d^4x\, \partial_\mu \hat{A}^{(2)\,0\,\mu}(b,\tilde{q})\partial_\nu \hat{A}^{(2)\,0\,\nu}(b,\tilde{q})\\
&\quad - \tfrac{1}{2N}g^2\int d^4x\, \mathrm{Tr}\, [b_\mu, \tilde{q}^\mu]_\star [b^\nu, \tilde{q}^\nu]_\star - ig^2\int d^4x\, \mathrm{Tr}\, \partial_\mu \hat{A}^{(2)\,0\,\mu}(b,\tilde{q})[b_\nu, \tilde{q}^\nu]_\star,
\end{aligned}$$

and....(I APOLOGIZE FOR IT!)

UV/IR mixing for $SU(N)$ in the fundamental. IV

$$
\begin{aligned}
S_{1eom} &= g\int d^4x\,\mathrm{Tr}\{(\partial^2 b^\nu - \partial^\nu\partial_\rho b^\rho)\hat{A}^{(2)}_\nu(\tilde{q},\tilde{q})\},\\
S_{2eom} &= +g^2\int d^4x\,\mathrm{Tr}\{(\partial^2 b^\nu - \partial^\nu\partial_\rho b^\rho)\hat{A}^{(3)}_\nu(\tilde{q},\tilde{q},b)\}\\
&\quad -2g^2\int d^4x\,\mathrm{Tr}\{(\partial^2 b^\nu - \partial^\nu\partial_\rho b^\rho)\hat{A}^{(2)}_\nu(\hat{A}^{(2)\,a}(\tilde{q},b)T^a,\tilde{q})\}\\
&\quad -g^2\int d^4x\,\mathrm{Tr}\left(E_\nu(b,b)\hat{A}^{(2)}_\nu(\tilde{q},\tilde{q})\right)+\frac{g^2}{N}\int d^4x\,\mathrm{Tr}\left(E_\nu(b,b)\right)\mathrm{Tr}\left(\hat{A}^{(2)}_\nu(\tilde{q},\tilde{q})\right)
\end{aligned}
$$

$$
E_\nu(b,b)=\partial^2\hat{A}^{(2)}_\nu(b,b)-\partial_\nu\partial^\rho\hat{A}^{(2)}_\rho(b,b)-i\partial^\mu[b_\mu,b_\nu]_\star - i[b^\mu,\partial_\mu b_\nu-\partial_\nu b_\mu]_\star\,,
$$

$$
\begin{aligned}
S_{0gh} &= -\int d^4x\,\mathrm{Tr}\,\bar{C}\partial^2\tilde{c},\\
S_{1gh} &= ig\int d^4x\,\mathrm{Tr}\,b^\mu\{\tilde{c},\partial_\mu\bar{C}\} - ig\int d^4x\,\mathrm{Tr}\,b^\mu\{\partial_\mu\tilde{c},\bar{C}\},\\
S_{2gh} &= -g^2\int d^4x\,\mathrm{Tr}\,[b^\mu,\tilde{c}]_\star[b_\mu,\bar{C}]_\star + ig^2\int d^4x\,\mathrm{Tr}\,\bar{C}[b^\mu,\partial_\mu\hat{C}^{(2)\,0}(\tilde{c},b)]_\star\\
&\quad + ig^2\int d^4x\,\mathrm{Tr}\,\hat{A}^{(2)}_\mu(b,b)\{\tilde{c},\partial^\mu\bar{C}\}_\star - ig^2\int d^4x\,\mathrm{Tr}\,\hat{A}^{(2)}_\mu(b,b)\{\partial^\mu\tilde{c},\bar{C}\}_\star\\
&\quad - ig^2\int d^4x\,\mathrm{Tr}\,\hat{C}^{(2)\,0}(\tilde{c},b)[\partial_\mu\bar{C},b^\mu]_\star.
\end{aligned}
$$

and...... (I APOLOGIZE FOR IT, AGAIN!)

UV/IR mixing for $SU(N)$ in the fundamental. V

$$\hat{A}^{(2)0}_{\mu}(b,\tilde{q}) = \frac{1}{2N}\int e^{i(p_1+p_2)x}\big(\frac{e^{-\frac{i}{2}p_1\wedge p_2}-e^{\frac{i}{2}p_1\wedge p_2}}{p_1\wedge p_2}\big)$$
$$\times[2\tilde{p}_2^{\mu_1}\delta_{\mu}^{\mu_2}+2\tilde{p}_1^{\mu_2}\delta_{\mu}^{\mu_1}-(p_2-p_1)_{\mu}\theta^{\mu_1\mu_2}]b^a_{\mu_1}(p_1)\tilde{q}^a_{\mu_2}(p_2),$$
$$\hat{A}^{(2)}_{\nu}(b,b) = \int\frac{1}{2}e^{i(p_1+p_2)x}\big(\frac{e^{-\frac{i}{2}p_1\wedge p_2}-1}{p_1\wedge p_2}\big)$$
$$\times[2\tilde{p}_2^{\mu_1}\delta_{\mu}^{\mu_2}+2\tilde{p}_1^{\mu_2}\delta_{\mu}^{\mu_1}-(p_2-p_1)_{\mu}\theta^{\mu_1\mu_2}]b_{\mu_1}(p_1)b_{\mu_2}(p_2),$$
$$\hat{A}^{(2)}_{\mu}(\tilde{q},\tilde{q};t) = \int-\frac{i}{4}e^{i(p_1+p_2)x}\int_0^t ds\, e^{-i\frac{s}{2}p_1\wedge p_2}$$
$$\times[2\tilde{p}_2^{\mu_1}\delta_{\mu}^{\mu_2}+2\tilde{p}_1^{\mu_2}\delta_{\mu}^{\mu_1}-(p_2-p_1)_{\mu}\theta^{\mu_1\mu_2}]\tilde{q}_{\mu_1}(p_1)\tilde{q}_{\mu_2}(p_2),$$
$$\hat{A}^{(2)}_{\mu}(\tilde{q},\tilde{q}) = \hat{A}^{(2)}_{\mu}(\tilde{q},\tilde{q};t=1),$$
$$\hat{A}^{(2)}_{\mu}(\tilde{q},b;t) = \int-\frac{i}{4}e^{i(p_1+p_2)x}[2\tilde{p}_2^{\mu_1}\delta_{\mu}^{\mu_2}+2\tilde{p}_1^{\mu_2}\delta_{\mu}^{\mu_1}-(p_2-p_1)_{\mu}\theta^{\mu_1\mu_2}]$$
$$\times\int_0^t ds[e^{-i\frac{s}{2}p_1\wedge p_2}b_{\mu_1}(p_1)\tilde{q}_{\mu_2}(p_2)+e^{i\frac{s}{2}p_1\wedge p_2}\tilde{q}_{\mu_2}(p_2)b_{\mu_1}(p_1)],$$
$$\hat{A}^{(3)}_{\mu}(\tilde{q},\tilde{q},b) = -\frac{1}{4}\theta^{ij}\int_0^1 dt$$
$$\times\Big[+\{\tilde{q}_i,2\partial_j\hat{A}^{(2)}_{\mu}(\tilde{q},b;t)-\partial_{\mu}\hat{A}^{(2)}_j(\tilde{q},b;t)\}_{\star_t}+\{\hat{A}^{(2)}_i(\tilde{q},b;t),2\partial_j\tilde{q}_{\mu}-\partial_{\mu}\tilde{q}_j\}_{\star_t}$$
$$+\{b_i,2\partial_j\hat{A}^{(2)}_{\mu}(\tilde{q},\tilde{q};t)-\partial_{\mu}\hat{A}^{(2)}_j(\tilde{q},\tilde{q};t)\}_{\star_t}+\{\hat{A}^{(2)}_i(\tilde{q},\tilde{q};t),2\partial_j b_{\mu}-\partial_{\mu}b_j\}_{\star_t}$$
$$-i\{\tilde{q}_i,[\tilde{q}_j,b_{\mu}]_{\star_t}\}_{\star_t}-i\{\tilde{q}_i,[b_j,\tilde{q}_{\mu}]_{\star_t}\}_{\star_t}-i\{b_i,[\tilde{q}_j,\tilde{q}_{\mu}]_{\star_t}\}_{\star_t}\Big],$$
$$\hat{A}^{(2)}_{\nu}(\hat{A}^{(2)a}(\tilde{q},b)T^a,\tilde{q}) = -\frac{1}{16}\int\frac{d^4p_2}{(2\pi)^4}\frac{d^4p_3}{(2\pi)^4}\frac{d^4p_4}{(2\pi)^4}\quad e^{i(p_2+p_3+p_4)x}$$
$$\times[2\tilde{p}_2^{\mu_1}\delta_{\nu}^{\mu_2}+2(\tilde{p}_3+\tilde{p}_4)^{\mu_2}\delta_{\nu}^{\mu_1}+(p_3+p_4-p_2)_{\nu}\theta^{\mu_1\mu_2}][2\tilde{p}_4^{\mu_3}\delta_{\mu_1}^{\mu_4}+2\tilde{p}_3^{\mu_4}\delta_{\mu_1}^{\mu_3}+(p_3-p_4)_{\mu_1}\theta^{\mu_3\mu_4}]$$
$$\times\Big\{\int_0^1 dt\int_0^1 ds\, e^{-i\frac{t}{2}(p_3+p_4)\wedge p_2}e^{-i\frac{s}{2}p_3\wedge p_4}[\mathrm{Tr}(b_{\mu_3}(p_3)\tilde{q}_{\mu_4}(p_4)T^b)]T^b\tilde{q}_{\mu_2}(p_2)$$
$$+\int_0^1 dt\int_0^1 ds\, e^{-i\frac{t}{2}(p_3+p_4)\wedge p_2}e^{+i\frac{s}{2}p_3\wedge p_4}[\mathrm{Tr}(\tilde{q}_{\mu_4}(p_4)b_{\mu_3}(p_3)T^b)]T^b\tilde{q}_{\mu_2}(p_2)$$
$$+\int_0^1 dt\int_0^1 ds\, e^{+i\frac{t}{2}(p_3+p_4)\wedge p_2}e^{-i\frac{s}{2}p_3\wedge p_4}\tilde{q}_{\mu_2}(p_2)[\mathrm{Tr}(b_{\mu_3}(p_3)\tilde{q}_{\mu_4}(p_4)T^b)]T^b$$
$$+\int_0^1 dt\int_0^1 ds\, e^{+i\frac{t}{2}(p_3+p_4)\wedge p_2}e^{+i\frac{s}{2}p_3\wedge p_4}\tilde{q}_{\mu_2}(p_2)[\mathrm{Tr}(\tilde{q}_{\mu_4}(p_4)b_{\mu_3}(p_3)T^b)]T^b\Big\},$$
$$C^{(2)0}(\tilde{c},b) = -\frac{1}{2N}\theta^{ij}e^{i(p_1+p_2)x}p_{1i}\big(\frac{e^{-\frac{i}{2}p_1\wedge p_2}-e^{\frac{i}{2}p_1\wedge p_2}}{p_1\wedge p_2}\big)\tilde{c}^a(p_1)b_j(p_2).$$

UV/IR mixing for $SU(N)$ in the fundamental. VI

COMMENTS

- You may not believe me, but the change

$$q^a --- > \tilde{q}^a_\mu = \mathrm{Tr}(T^a Q_\mu(b,q)), c^a --- > \tilde{c}^a = \mathrm{Tr}(T^a C(b,q)).$$

  has made the interaction vertices far simpler, for it has removed plenty of trivial contributions –those which can be encapsulated in a trivial determinant.
- $\hat{A}^{(2)\,0}_\nu(b,\tilde{q})$, $\hat{A}^{(2)}_\nu(b,b)$, $\hat{A}^{(2)}_\mu(\tilde{q},\tilde{q})$, $\hat{A}^{(3)}_\mu(\tilde{q},\tilde{q},b)$, $\hat{A}^{(2)}_\nu(\hat{A}^{(2)\,a}(\tilde{q},b)T^a,\tilde{q})$ and $C^{(2)\,0}(\tilde{c},b)$ carry the information about the Sw map.
- Our interaction vertices involve PHASES:
  - Phases from the $\star$-product, as in

$$ig\int d^4x\,\mathrm{Tr}\, b_\mu[\tilde{q}_\nu,\partial^\mu\tilde{q}^\nu]_\star$$

  - Phases from the SW map, as in

$$\hat{A}^{(2)\,0}_\mu(b,\tilde{q}) = \frac{1}{2N}\int e^{i(p_1+p_2)x}\left(\frac{e^{-\frac{i}{2}p_1\wedge p_2}-e^{\frac{i}{2}p_1\wedge p_2}}{p_1\wedge p_2}\right)$$
$$\times[2\tilde{p}_2^{\mu_1}\delta^{\mu_2}_\mu + 2\tilde{p}_1^{\mu 2}\delta^{\mu_1}_\mu - (p_2-p_1)_\mu\theta^{\mu_1\mu_2}]b^a_{\mu_1}(p_1)\tilde{q}^a_{\mu_2}(p_2),$$

UV/IR mixing for $SU(N)$ in the fundamental. VII

- We have computed the full 1-loop contribution to $\Gamma_2[b]$ and we have obtained a lengthy expression:

$$\Gamma_2[b] = \frac{g^2}{N}\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[b_\mu(p)b_\nu(-p)\right]\times$$
$$\left\{\frac{1}{16\pi^2}\frac{1}{(\tilde{p}^2)^3}\left[\frac{16}{3}p^2\theta^{\mu i}\tilde{p}_i\theta^{\nu j}\tilde{p}_j - \frac{8}{3}\tilde{p}^2(\theta^{\mu i}\tilde{p}_i p^\nu + \theta^{\nu j}\tilde{p}_j p^\mu) - \frac{32}{3}(\tilde{p}^2)^2\eta^{\mu\nu} + \frac{32}{3}\tilde{p}^2\tilde{p}^\mu\tilde{p}^\nu\right]\right.$$
$$\left. - \frac{1}{\pi^2}\frac{\tilde{p}^\mu\tilde{p}^\nu}{(\tilde{p}^2)^2}\int_0^1 dx\left[x(1-x)p^2\tilde{p}^2 K_2(\sqrt{x(1-x)p^2\tilde{p}^2})\right]\right\}$$
$$+\frac{g^2}{16\pi^2}\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[b_\mu(p)(p^2\eta^{\mu\nu}-p^\mu p^\nu)b_\nu(-p)\right]$$
$$\left\{-\frac{11}{3}(N-\frac{2}{N})\left(\frac{1}{\varepsilon}+Ln\frac{p^2}{4\pi\mu^2}+\gamma-1\right)+\frac{4}{N}\int_0^1 dx\,(3+2x)K_0(\sqrt{x(1-x)p^2\tilde{p}^2})\right\}$$
$$+\,\Gamma_2[b]^{(eom0)} + (2-\text{loop order}),$$

- The actual lengthier value of $\Gamma_2[b]^{(eom0)}$ –which we have computed– will not be relevant to our discussion here, because it fully comes from

$$\int d^4x\, q^a_\mu(x)\frac{\delta\Gamma[b]}{\delta b^a_\mu(x)},$$

  so that it vanishes on-shell.

- The actual value of $\Gamma_2[b]^{(eom0)}$ demands 2 slides to be quoted.

UV/IR mixing for $SU(N)$ in the fundamental: removing the gauge dependence

- ....BUT, $\Gamma_2[b_\mu]$ is gauge dependent.
- To remove this gauge dependence, we shall set b_μ on shell, ie, we shall compute the 1-loop 2-point DeWitt effective action, which is gauge invariant by construction.
- At one loop we have to replace b_μ with $b_\mu^{(0)}(x)$

$$
\begin{array}{l}
b_\mu^{(0)}(x) = b_\mu^{\perp}(x) + \partial_\mu \alpha(x), \\
\partial^2 b_\mu^{\perp}(x) = 0, \quad \partial^\mu b_\mu^{\perp}(x) = 0, \\
\alpha(x) \quad \text{an arbitrary function with values in the SU(N) Lie algebra}
\end{array}
$$

- $\Gamma_2[b_\mu^{(0)}]$ develops the following IR singularity when $\tilde{p}^\mu = \theta^{\mu\nu} p_\nu \to 0$

$$
\frac{g^2}{N} \frac{-2}{3\pi^2 \tilde{p}^2} \left\{ \eta^{\mu\nu} + 2 \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2} \right\} \mathrm{Tr}\left[b_\mu^{\perp}(p) b_\nu^{\perp}(-p) \right],
$$

- THIS IS A GAUGE INVARIANT STATEMENT!

UV/IR mixing for $SU(N)$ in the fundamental: is this so?

- ....But someone may rightly say:
  - Wait a minute!
  - you have indeed obtained an IR singularity at $\tilde{p}^{\mu} = 0$,
  - but why do you say that it is due to UV/IR mixing?
  - Have you rule out the possibility that the vanishing denominators in the SW map give rise to such an IR divergence?
  - For instance, recall the following bit of the SW map

$$\hat{A}_{\mu}^{(2)\,0}(b,\tilde{q}) = \frac{1}{2N}\int e^{i(p_1+p_2)x}\left(\frac{e^{-\frac{i}{2}p_1\wedge p_2} - e^{\frac{i}{2}p_1\wedge p_2}}{p_1\wedge p_2}\right)$$

$$\times\,[2\tilde{p}_2^{\mu_1}\,\delta_{\mu}^{\mu_2} + 2\tilde{p}_1^{\mu 2}\,\delta_{\mu}^{\mu_1} - (p_2 - p_1)_{\mu}\,\theta^{\mu_1\mu_2}]\,b^a_{\mu_1}(p_1)\,\tilde{q}^a_{\mu_2}(p_2),$$

- The answer to this question is that we have rule out such possibility by analysing carefully the contributions which give rise to the IR divergence in question
- Let me give you one example ———>

UV/IR mixing for $SU(N)$ in the fundamental: Yes, this is so.

- Let $\langle .....\rangle_0$ denote average w.r.t to the free action. Then

$$-ig^2\frac{N}{2}\Big\langle\,\partial^\mu\hat{A}_\mu^{(2)\,0}(b^{(0)},\tilde{q})(x)\partial^\nu\hat{A}_\nu^{(2)\,0}(b^{(0)},\tilde{q})(x)\Big\rangle_0,$$

yields the following IR divergent contribution

$$\begin{aligned}
- & \quad \frac{g^2}{2N}\mathrm{Tr}\,(b_\mu^{(0)}(p)b_\nu^{(0)}(-p))\tilde{p}^2\int\frac{d^4k}{(2\pi)^4}\frac{k^\mu k^\nu}{k^2(k\tilde{p})^2}(e^{ik\tilde{p}}+e^{-ik\tilde{p}}-2)\\
= & \quad i\frac{g^2}{N}\frac{1}{16\pi^2}\mathrm{Tr}(b_\mu^{(0)}(p)b_\nu^{(0)}(-p))\Big\{\frac{4}{3}\Big(\frac{\eta^{\mu\nu}}{\tilde{p}^2}-4\frac{\tilde{p}^\nu\tilde{p}^\nu}{(\tilde{p}^2)^2}\Big)\Big\}+O(\varepsilon),
\end{aligned}$$

- Now,

$$\frac{(e^{ik\tilde{p}}+e^{-ik\tilde{p}}-2)}{(k\tilde{p})^2}\to\text{CONSTANT as}\quad k\tilde{p}\to 0.$$

- Hence no IR divergence comes from that integration region.
- But $e^{\pm ik\tilde{p}}$ cuts off the UV divergences if $\tilde{p}^\mu\neq 0$: UV/IR mixing phenomenon!

UV/IR mixing for $SU(N)$ in the fundamental: Comparison.

- It is time to compare our $SU(N)$ result

$$\frac{g^2}{N}\frac{-2}{3\pi^2\tilde{p}^2}\left\{\eta^{\mu\nu}+2\frac{\tilde{p}^\mu\tilde{p}^\nu}{\tilde{p}^2}\right\}\mathrm{Tr}\left[b^{\perp}_{\mu}(p)b^{\perp}_{\nu}(-p)\right],$$

with corresponding $U(N)$ expression

$$g^2\,\tfrac{2}{\pi^2}\,\tfrac{\tilde{p}^\mu\tilde{p}^\nu}{(\tilde{p}^2)^2}[\mathrm{Tr}\,b^{\perp}_{\mu}(p)][\mathrm{Tr}\,b^{\perp}_{\nu}(-p)],$$

- Both are $1/N$ corrections.
- The group structure is quite different: $\mathrm{Tr}(bb)$ versus $(\mathrm{Tr}(b))^2$.
- “Gluons" feel UV/IR mixing for $SU(N)$ and they don’t for $U(N)$.
- It is “Photons" which feel UV/IR mixing for $U(N)$.
- The Lorentz tensor structure is also quite different. In particular, if $\tilde{p}^\mu b_\mu(p)=0$, then,
  - No contribution due to UV/IR mixing for $U(N)$
  - UV/IR mixing gives rise to a contribution for $SU(N)$

Conclusions

CONCLUSIONS

- In general, there is UV/IR mixing in NC gauge theories defined by using the θ-exact SW map.
- Perhaps, the right way to define NC gauge theories for arbitrary gauge groups in arbitrary reps. is by utilizing the θ-exact SW map; of course, if one assumes that UV/IR mixing is a key feature of noncommutative gauge theories.
- $SU(N)$ and $U(N)$ Yang-Mills theories are quite different as noncommutative quantum field theories.
- Open problem: Ken Wilson showed us how to make sense of theories without UV/IR mixing: The renormalization group; but we do not know how to make sense out of NC theories with UV/IR mixing.