Spin fields for N=1 particle in the worldline

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Motivation: want a full non-linear theory of R-R fields, on the worldsheet

For NS-NS: [Bonezzi, Meyer, Sachs, 2020]

Super-worldsheet

- Super-Virasoro algebra $\Longrightarrow$ the fermionic components of the superfield are 2d spinors and thus double-valued

$$\varphi^{NS}(e^{2i\pi}z) = +\varphi^{NS}(z)$$
$$\varphi^{R}(e^{2i\pi}z) = -\varphi^{R}(z)$$

- R states are created from the NS vacuum by a spin field $\vartheta(z)$

- $\vartheta(z)$ may be represented as endpoint of a branch cut in the Grassmann odd fields

[Friedan, Martinec, Shenker, 1986]

Super-worldline

A particle can be seen as the zero mode of a string

Could there be a ***spin field*** *operator for the particle?*

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Idea (inspired by [Sorokin, Tkach, Volkov, Zheltukhin, 1989]):
use a Grassmann degree shifting involution $\uparrow$ and six Weyl spinors $\vartheta_\alpha, \tilde{\vartheta}^{\dot{\alpha}}, \varepsilon_\alpha, \tilde{\varepsilon}^{\dot{\alpha}}, \lambda^\alpha, \tilde{\lambda}_{\dot{\alpha}}$
with a product $\circ$ inducing the *sole* commutators:

$$[\vartheta_\alpha, \lambda^\beta] = \delta_\alpha^\beta = [\varepsilon_\alpha, \lambda^\beta], \qquad [\lambda^\alpha, \uparrow] = 0 = [\lambda^\alpha, \lambda^\beta],$$

(+ antichiral counterparts)

Clifford algebra

The algebra for a particle on a superworldline

$$[x^\mu, p_\nu] = \delta^\mu_\nu, \quad \{\psi^\mu, \psi^\nu\} = 2\eta^{\mu\nu}$$

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$$[x^\mu, p_\nu] = \delta^\mu_\nu, \quad \{\psi^\mu, \psi^\nu\} = 2\eta^{\mu\nu}\mathcal{I} + \cancel{f^{(\mu\nu)}(\vartheta, \tilde{\vartheta}, \uparrow, \lambda, \tilde{\lambda})}$$

remains defined if the Gamma matrices are represented as:

$$\psi^\mu := \vartheta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}} \uparrow + \tilde{\vartheta}_{\dot{\alpha}} \tilde{\sigma}^{\mu\,\dot{\alpha}\beta} \lambda_\beta \uparrow \tag{1}$$

- The right (anti)commutators relation are satisfied up to a projector $\mathcal{I} := \vartheta \cdot \lambda + \tilde{\vartheta} \cdot \tilde{\lambda}$, and a correction $f^{(\mu\nu)}(\vartheta, \tilde{\vartheta}, \uparrow, \lambda, \tilde{\lambda})$

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- The right (anti)commutators relation are satisfied up to a projector $\mathcal{I} := \vartheta \cdot \lambda + \tilde{\vartheta} \cdot \tilde{\lambda}$, and a correction $f^{(\mu\nu)}(\vartheta, \tilde{\vartheta}, \uparrow, \lambda, \tilde{\lambda})$
- The operator $f^{\mu\nu}$ vanishes if the iterated *adjoint* action of λ a/o $\tilde{\lambda}$ on every state in the representation space is zero

States

- Supposing that there is a (NS) vacuum, the Weyl spinors ϑ, ε, $\tilde{\vartheta}$ and $\tilde{\varepsilon}$ are spin fields
- A Hilbert space annihilated by iterated action of λ a/o $\tilde{\lambda}$ and that survives $\mathcal{I}$-projection consists of:
  - $|\varphi\rangle = \varphi_\alpha(x)\vartheta^\alpha \begin{pmatrix} |0\rangle \\ \uparrow |0\rangle \end{pmatrix}, \quad +$ anti-chiral state (1-particle)

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  - $C_{\alpha\beta}(x)\,\vartheta^\alpha \uparrow (\vartheta^\beta - \varepsilon^\beta) \begin{pmatrix} |0\rangle \\ \uparrow|0\rangle \end{pmatrix}$, $\tilde{C}^{\dot{\alpha}}{}_\beta(x)\,\tilde{\vartheta}_{\dot{\alpha}} \uparrow (\vartheta^\beta - \varepsilon^\beta) \begin{pmatrix} |0\rangle \\ \uparrow|0\rangle \end{pmatrix}$, + other combinations of different chiralities (2-particles)

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  - 3-particles, 4-particles, . . .

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- Worldline/worldsheet description is useful for two main reasons:
  1. study of backgrounds by deformation/twist of BRST differential;
  2. calculation of tree-level and 1-loop amplitudes for given vertices.

Super-worldline BRST

- A worldline with super-reparametrization invariance:

$$\{q, q\} = H, \quad q := \psi^\mu p_\mu, \quad H := p^2 \,.$$

Corresponding BRST operator $Q = cH + \gamma q - \gamma^2 b$ with ghost-antighost pairs (c, b), (γ, β) and algebra:

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- ψ^μ taken to be (1) and the space of states as previously constructed $\Leftrightarrow$ Hamiltonian constraints/momentum mapping in BRST operator.

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- on 2-particles $|F\rangle$, which comprise 0-forms, 1-forms and 2-forms, independent equations at ghost number 0 from Fierz identities are:

$$\mathrm{d}F^{(1)} = 0 = \delta F^{(1)}$$

$$\mathrm{d}F^{(0)} + \mathrm{i}\delta F^{(2)} - \frac{1}{2}\star\mathrm{d}F^{(2)} = 0$$

For $F^{(0)} = 0$, these are compatible with **linearised R-R field equations**. Notice that $\mathrm{Im}Q \ni |\zeta\rangle_{k\geq 1}\,, |\zeta^c\rangle_{k\geq 1}$ *(ghost zero states are never Q-exact)*.

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$$\text{warped compactification with} \quad \omega_5^{\alpha\beta} \equiv F^{\alpha\beta} : \gamma\left(\vartheta_\alpha F^{\alpha\beta}\lambda_\beta + \tilde{\vartheta}^{\dot\alpha}\tilde{F}_{\dot\alpha\dot\beta}\tilde{\lambda}^{\dot\beta}\right)\uparrow$$

Chiral sector and an equivalent $H^\bullet$

Our observations concerned the chiral and antichiral sector on the same footing. Abandon reality of ψ^μ and focus on chiral supercharge:

$$\mathbf{q} := \tilde{\vartheta}^{\dot{\alpha}} \tilde{p}_{\dot{\alpha}\beta} \lambda^\beta \uparrow \, .$$

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Equivalent cohomology

In ghost degree 0, Q-cohomology $= \ker Q$ is equivalent to $\mathbf{q}$-cohomology $= \ker \mathbf{q} / \mathrm{Im}\, \mathbf{q}$ *for the chiral sector.*

In fact one retains, for the chiral sector:

- 1-particles, $\varphi^{\alpha}(x) \vartheta_{\alpha} \left|0\right\rangle$, $\varphi^{\alpha \uparrow}(x) \vartheta_{\alpha} \uparrow \left|0\right\rangle$: Weyl equation;
- 2-particles: R-R fields equations.

Chiral sector and an equivalent $H^\bullet$

The **q**-exact particles are those in the antichiral sector with some extra requirements, e.g.

$$|\tilde{\chi}\rangle, \quad \exists\, \zeta^{\beta}(x) \;\mid\; \tilde{\chi}_{\dot{\alpha}}(x) = (\tilde{p}\zeta)_{\dot{\alpha}}, \quad p\tilde{\chi} \neq 0$$

$$\left(\tilde{F}^{(2)}{}_{\dot{\alpha}\dot{\beta}}\,|e^{\dot{\alpha}\dot{\beta}}\rangle + \tilde{F}^{(1)}{}_{\dot{\alpha}}{}^{\beta}\,|e^{\dot{\alpha}}{}_{\beta}\rangle\right),$$

$$\exists\, G^{(1)}{}_{\alpha}{}^{\dot{\alpha}}(x),\, G^{(2)}_{\alpha\beta}(x) \;\mid\; F^{(2)} + F^{(1)} = \tilde{p}(G^{(1)} + G^{(2)}),\; p(F^{(2)} + F^{(1)}) \neq 0$$

$\delta\mathbf{q}$ in chiral theory

A deformation can generate a R or R-R field by acting on reference state $|\Omega\rangle$:

$$\delta\mathbf{q}\,|\Omega\rangle$$

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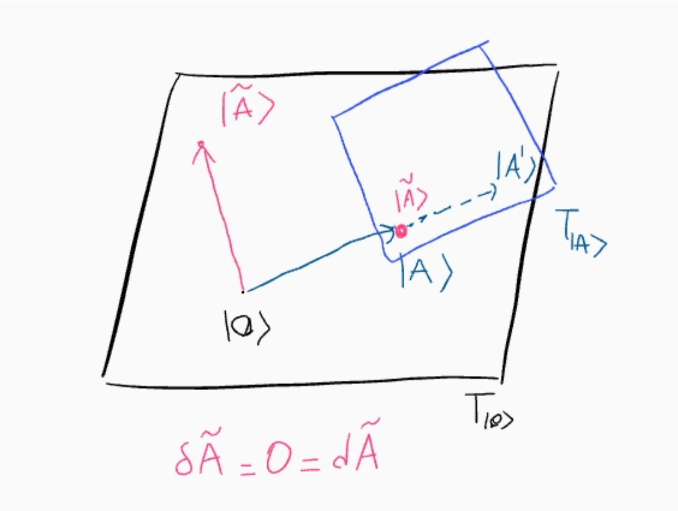
Which R-R fields can be consistent backgrounds?

$$\delta\mathbf{q} = s\,\tilde{\vartheta}^{\dot{\alpha}}\tilde{A}^{(1)}_{\dot{\alpha}\beta}\lambda^{\beta}\uparrow, \qquad s << 1$$

$$\Downarrow$$

genuine deformation: for 1-particles it's $U(1)$ or $SU(N)$ gauge potential, for 2-particles is a 1-form background

δq in chiral theory



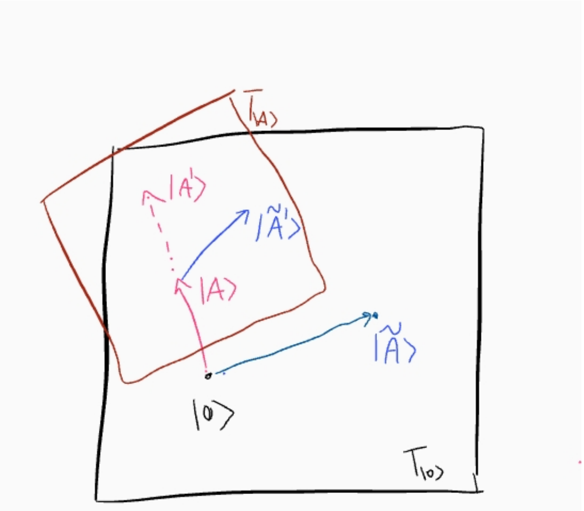
$\delta\mathbf{q}$ in chiral theory

Non-nilpotent deformations, $s << 1$:

$\delta\mathbf{q}_1 := s\,\vartheta^\beta A^{(1)}_{\beta\dot\beta}\tilde\lambda^{\dot\beta}\uparrow$

$\ker\mathbf{q}$ is in $\mathrm{Im}(\delta\mathbf{q}_1)$.
Both for chiral spinors and chiral RR fields: $\delta\mathbf{q}_1$ sets the antichiral sector to zero.

δq in chiral theory



$\delta\mathbf{q}$ in chiral theory

Another non-nilpotent deformation:

$\delta\mathbf{q}_2 := s\,\tilde{\vartheta}^{\dot\beta}\tilde{F}^{(2)}_{\dot\beta\dot\gamma}\tilde{\lambda}^{\dot\gamma}\uparrow$

Now $\mathbf{q}+\delta\mathbf{q}_2$ lands in the antichiral sector. Perturb fields in power of s:

- for spinors:

$$\tilde{\not{p}}\varphi_{(0)} = 0, \quad \tilde{\not{p}}\varphi_{(1)} = \tilde{F}^{(2)}\tilde{\chi}_{(0)}$$

- for RR-fields: $A_\alpha{}^{\dot\beta}(x)$ is deformed by Chern-Simons interaction.

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- for RR-fields: ${A_\alpha}^{\dot\beta}(x)$ is deformed by Chern-Simons interaction.

Conclusion: can have 1-form field background $\tilde{A}^{(1)}_{\dot\alpha\beta}(x)$ and, perturbatively in s, 2-form field $\tilde{F}^{(2)}_{\dot\beta\dot\gamma}(x)$!

Summary:

- We constructed the spin fields for N=1 worldline in 4d
- We moved towards non-linear theory of R-R backgrounds

To do:

- Extend to 10d and worldsheet (ambitwistor string, pure spinor [Berkovits, Howe, 2001]
- full explicit target space supersymmetry (NS-NS and R-R)?

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