Achilleas Passias | Sorbonne University

Progress in the AdS$_3$ Landscape

arXiv: 1910.06326, 2011.00008 with Daniël Prins
arXiv: 2107.13562, 2203.09532 with Christopher Couzens & Niall Macpherson

Workshop on Holography and the Swampland | Corfu, September 2022

Anti-de Sitter

The AdS/CFT correspondence states that conformal field theories in d dimensions admit dual descriptions as theories of gravity in $d+1$ dimensional anti-de Sitter spacetimes.

Anti-de Sitter

The AdS/CFT correspondence states that conformal field theories in d dimensions admit dual descriptions as theories of gravity in $d+1$ dimensional anti-de Sitter spacetimes.

> By studying the space and properties of anti-de Sitter solutions we can gain insight into the space and properties of conformal field theories.

Anti-de Sitter

The AdS/CFT correspondence states that conformal field theories in d dimensions admit dual descriptions as theories of gravity in $d+1$ dimensional anti-de Sitter spacetimes.

- By studying the space and properties of anti-de Sitter solutions we can gain insight into the space and properties of conformal field theories.
- Vice versa, we can hope to shed light to characteristics of quantum gravity.

AdS_3/CFT_2 correspondence

AdS_3/CFT_2 dualities provide a hospitable environment for finding answers to questions on both sides of the holographic correspondence.

AdS_3/CFT_2 correspondence

AdS_3/CFT_2 dualities provide a hospitable environment for finding answers to questions on both sides of the holographic correspondence.

> Conformal field theories in two dimensions feature a highly-constraining infinite-dimensional algebra of conformal transformations that often allows for their exact solution.

AdS_3/CFT_2 correspondence

AdS_3/CFT_2 dualities provide a hospitable environment for finding answers to questions on both sides of the holographic correspondence.

> Conformal field theories in two dimensions feature a highly-constraining infinite-dimensional algebra of conformal transformations that often allows for their exact solution.

> Gravity in three-dimensional asymptotically anti-de Sitter spacetime provides a toy model for quantum gravity.

AdS_3/CFT_2 correspondence

Owing to the high dimensionality of the internal space, the problem of exploring the space of AdS_3 backgrounds is challenging.

AdS_3/CFT_2 correspondence

Owing to the high dimensionality of the internal space, the problem of exploring the space of AdS_3 backgrounds is challenging.

> A way forward is to impose a symmetry on the background, at the expense of the size of the subspace of backgrounds one can access, depending on the degree of the symmetry.

AdS_3/CFT_2 correspondence

Owing to the high dimensionality of the internal space, the problem of exploring the space of AdS_3 backgrounds is challenging.

> A way forward is to impose a symmetry on the background, at the expense of the size of the subspace of backgrounds one can access, depending on the degree of the symmetry.

> We imposed supersymmetry, as (i) a technically simplifying assumption, (ii) a computational tool and (iii) a way out of swampland.

Layout

I. Classification of minimally supersymmetric solutions.

II.

III.

Layout

I. Classification of minimally supersymmetric solutions.

II. Classification and construction of $\mathcal{N} = (2, 0)$ supersymmetric solutions.

III.

Layout

I. Classification of minimally supersymmetric solutions.

II. Classification and construction of $\mathcal{N} = (2, 0)$ supersymmetric solutions.

III. Construction of $\mathcal{N} = (2, 2)$ supersymmetric solutions from D3-branes on Riemann surfaces.

Part I

Background

$$ds^2_{10} = e^{2A} ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{M}_7)$$

&

Φ, H, F_p

preserving the symmetries of AdS_3

Supersymmetry

$$\exists\ \epsilon_{1,2} : \delta_{\epsilon_{1,2}}\psi = 0 = \delta_{\epsilon_{1,2}}\lambda$$

Supersymmetry

$$\epsilon_1 = \zeta \otimes \chi_1 \otimes \begin{pmatrix} 1 \\ -i \end{pmatrix} , \quad \epsilon_2 = \zeta \otimes \chi_2 \otimes \begin{pmatrix} 1 \\ \pm i \end{pmatrix} \quad ; \quad \nabla_\mu \zeta = \frac{1}{2} m \gamma_\mu \zeta$$

G-structure

$$\mathrm{SO}(7)$$

$$\Downarrow$$

$$G$$

stabilizer

G-structure

M_7 acquires a G-structure characterized by a set of tensors constructed as bilinears of $\{\chi_1, \chi_2\}$

$$G = \begin{cases} SU(3) : \{v,\ J,\ \Omega\ :\ v \lrcorner J = v \lrcorner \Omega = 0,\ \Omega \wedge J = 0\} \\ \\ G_2 : \varphi \end{cases}$$

G-structure

The exterior derivatives of the G-structure tensors determine its intrinsic torsion, which is parametrized by torsion classes

$$
\begin{aligned}
dv &= RJ + T_1 + \mathrm{Re}(\overline{V_1} \lrcorner \Omega) + v \wedge W_0 \\
dJ &= \tfrac{3}{2}\mathrm{Im}(\overline{W_1}\Omega) + W_3 + W_4 \wedge J + v \wedge \left(\tfrac{2}{3}\mathrm{Re}EJ + T_2 + \mathrm{Re}(\overline{V_2} \lrcorner \Omega)\right) \\
d\Omega &= W_1 J \wedge J + W_2 \wedge J + \overline{W_5} \wedge \Omega + v \wedge (E\Omega - 2V_2 \wedge J + S)
\end{aligned}
$$

Supersymmetry Equations

$$\psi_+ + i\psi_- \equiv \chi_1 \otimes \chi_2^t$$

$$+$$

$$\chi_1 \otimes \chi_2^t \propto \sum_p \chi_2^t \gamma_{m_1 \dots m_p} \chi_1 \gamma^{m_1 \dots m_p}$$

$$+$$

$$\gamma^{m_1 \dots m_k} \to dx^{m_1} \wedge \cdots \wedge dx^{m_k}$$

$$\Downarrow$$

polyforms

$$\psi_\pm(v,\ J,\ \Omega \parallel \varphi)$$

Supersymmetry Equations

$$\delta_{\epsilon_{1,2}}\psi = 0 = \delta_{\epsilon_{1,2}}\lambda$$

$$\Downarrow$$

constraints on $\{\chi_1, \chi_2\}$

$$\Downarrow$$

$$\begin{aligned}
d_H(e^{A-\Phi}\psi_\mp) &= 0 \\
d_H(e^{2A-\Phi}\psi_\pm) \mp 2me^{A-\Phi}\psi_\mp &= \frac{1}{8}e^{3A} \star_7 \lambda F \\
(\psi_\mp \wedge \lambda F)_7 &= \mp\frac{m}{2}e^{-\Phi}\mathrm{vol}_7
\end{aligned}$$

Classification

- We obtain a set of constraints on the intrinsic torsion of the G-structure and expressions for the supergravity fields in terms of the geometric data.
- This allows for charting the AdS_3 landscape and the discovery of new solutions.

Classification

A family of solutions for the strict $SU(3)$-structure case, were examined in [AP, Prins '19]: the internal manifold M_7 is a $U(1)$ fibration over a conformally Kähler base, and they feature a varying axio-dilaton, a primitive $(2,1)$-form flux $H + ie^{\Phi}F_3$, and five-form flux F_5.

$$\nabla^2(R-2|\partial\Phi|^2)-\frac{1}{2}R^2+R_{ij}R^{ij}+2|\partial\Phi|^2R-4R_{ij}\partial^i\Phi\overline{\partial}^j\Phi-\frac{8}{3}e^{-\Phi}H^{(2,1)}_{ijk}\left(H^{(1,2)}\right)^{ijk}=0$$

The solutions of [Kim '05], [Donos, Gauntlett, Kim '08], [Benini, Bobev '13], [Benini, Bobev, Crichigno '15], [Couzens, Martelli, Schafer-Nameki '17], with $\mathcal{N}=(2,0)$ supersymmetry, belong in this family.

GK geometries

[Gauntlett, Kim '07]

Y_{2n+1} consisting of a metric, a scalar function B and a closed two-form F.

The metric on Y_{2n+1} has a unit norm Killing vector ξ defining a foliation $\mathcal{F}_\xi$ of Y_{2n+1}

$$\xi = \frac{2}{n-2}\partial_z\,, \qquad \eta = \frac{n-2}{2}(dz + P)$$

The metric on Y_{2n+1} then has the form

$$ds^2_{2n+1} = \eta^2 + e^B ds^2_{2n}$$

where ds^2_{2n} is a Kähler metric transverse to $\mathcal{F}_\xi$.

GK geometries

This Kähler metric, with transverse Kähler two-form J, Ricci two-form $\rho = dP$ and Ricci scalar R, determines all of the remaining fields.

$$e^B = \frac{(n-2)^2}{8} R \,, \qquad F = -\frac{2}{n-2} J + d\left(e^{-B}\eta\right)$$

These off-shell geometries become solutions provided that the transverse Kähler metric satisfies the non-linear partial differential equation

$$\Box R = \frac{1}{2} R^2 - R_{ij} R^{ij} \,.$$

One can define an extremal problem which is dual to c-extremization. [Couzens, Gauntlett, Martelli, Sparks '18]

Part II

$\mathcal{N} = (2,0)$ AdS$_3 \times$ M$_7$

$$\epsilon_1 = \sum_{I=1}^{2} \zeta^I \otimes \chi_1^I \otimes \begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad \epsilon_2 = \sum_{I=1}^{2} \zeta^I \otimes \chi_2^I \otimes \begin{pmatrix} 1 \\ \pm i \end{pmatrix} \quad ; \quad \nabla_\mu \zeta^I = \frac{1}{2} m \gamma_\mu \zeta^I$$

$$d_H(e^{A-\Phi}\Psi_{\mp}^{(IJ)}) = \mp \frac{c}{16}\delta^{IJ} F_{\pm}$$

$$d_H(e^{2A-\Phi}\Psi_{\pm}^{(IJ)}) \mp 2m e^{A-\Phi}\Psi_{\mp}^{(IJ)} = \frac{1}{8} e^{3A} \star_7 \lambda f_{\pm} \delta^{IJ}$$

$$d_H(e^{-\Phi}\Psi_{\pm}^{[IJ]}) = \frac{1}{16}\epsilon^{IJ}(\tilde{\xi}\wedge + \iota_\xi) F_{\pm}$$

$$d_H(e^{3A-\Phi}\Psi_{\mp}^{[IJ]}) = \pm \frac{1}{16}\epsilon^{IJ}(\tilde{\xi}\wedge + \iota_\xi) e^{3A} \star_7 \lambda F_{\pm}$$

where

$$\Psi^{IJ} \equiv \chi_1^I \otimes \chi_2^{J\dagger}$$

$\mathcal{N} = (2, 0)$ AdS$_3 \times$ M$_7$

The ten-dimensional background admits a Killing vector which can be either time-like or null. We have explored the null class.

$\mathcal{N} = (2,0)$ AdS$_3 \times$ M$_7$

- The ten-dimensional background admits a Killing vector which can be either time-like or null. We have explored the null class.
- The internal geometry admits a Killing vector which generates a symmetry of the whole solution. This no other than the $U(1)$ R-symmetry of the $\mathcal{N} = (2,0)$ superalgebra.

$\mathcal{N} = (2, 0)$ $AdS_3 \times M_7$

- The ten-dimensional background admits a Killing vector which can be either time-like or null. We have explored the null class.
- The internal geometry admits a Killing vector which generates a symmetry of the whole solution. This no other than the $U(1)$ R-symmetry of the $\mathcal{N} = (2, 0)$ superalgebra.
- The six-dimensional space transverse to the Killing vector supports an $SU(2)$-structure characterized by (z, j_2, ω_2). The $SU(2)$-structure "lives" on a four-dimensional subspace which is complex.

$\mathcal{N} = (2,0)$ AdS$_3 \times M_7$

- The ten-dimensional background admits a Killing vector which can be either time-like or null. We have explored the null class.
- The internal geometry admits a Killing vector which generates a symmetry of the whole solution. This no other than the $U(1)$ R-symmetry of the $\mathcal{N} = (2,0)$ superalgebra.
- The six-dimensional space transverse to the Killing vector supports an $SU(2)$-structure characterized by (z, j_2, ω_2). The $SU(2)$-structure "lives" on a four-dimensional subspace which is complex.
- The problem of finding solutions to the equations of motion consists of solving PDEs coming from the Bianchi identities for the p-form fields.

$\mathcal{N} = (2,0)$ $\mathrm{AdS}_3 \times M_7$

In order to make progress we have imposed an additional isometry. The metric of the internal space reads

$$ds_7^2 = \frac{e^{2A}}{4m^2}(d\psi + \mathcal{A})^2 + e^{\Phi-3A}\left[e^{2U}D\varphi^2 + e^{\Phi-A}dy^2 + e^{\frac{1}{2}(5A-\Phi)-U}ds^2(M_4)\right]$$

with the metric on M_4 Kähler at fixed y coordinate.

$\mathcal{N} = (2,0)$ $\mathrm{AdS}_3 \times M_7$

Taking

$$ds^2(M_4) = e^{2f_1(y)} ds^2(\Sigma_1) + e^{2f_2(y)} ds^2(\Sigma_2)$$

we have solved the differenetial equations coming from the Bianchi identities and found explicit solutions: (i) one class for non-zero Romans mass (ii) two classes for zero Romans mass

$\mathcal{N} = (2, 0)$ $\mathrm{AdS}_3 \times M_7$

$$ds^2\big(M_4(\vec{x}, y)\big) = e^{2f(y)} ds^2(\mathcal{M}_4(\vec{x}))\,,$$

$$ds^2 = e^{2A}\left[ds^2(\mathrm{AdS}_3) + \frac{1}{4m^2}(d\psi + P_K)^2 + (d\varphi + t_1\Sigma)^2 + e^{\Phi-5A}\Big(e^{\Phi-A}dy^2 + ds^2(\mathcal{M}_4)\Big)\right]$$

$$e^{-4A} = \frac{R_K}{8m^2}\sqrt{2f_0 y + c}\,, \qquad e^{-2\Phi} = \frac{(2f_0 y + c)^{5/4}\sqrt{R_K}}{2\sqrt{2}m}\,,$$

where the metric on $\mathcal{M}_4$ is Kähler and satisfies the master equation

$$\Box_K R_K - \tfrac{1}{2}R_K^2 + R^{mn}R_{mn} = 8m^2 t_1^2 |d\Sigma|^2\,.$$

For $t_1 = 0$ we have a five-dimensional GK geometry and an extremal problem can be setup for the calculation fo the central charge.

Part III

Compactifying Higher Dimensional Field Theories

- Large classes of theories in lower dimensions.
- Their properties admit a description in terms of the geometry and topology of the compact manifold.

D3-branes on a Riemann surface

- We considered D3-branes compactified on a Riemann surface with a twist, preserving $\mathcal{N} = (2, 2)$ supersymmetry and flowing to a two-dimensional SCFT.
- $SU(4) \to U(1)^3 = U(1)_L \times U(1)_R \times U(1)_F$

D3-branes on a Riemann surface

$$
\begin{tikzcd}
S^5_{\mathrm{U}(1)^3} \arrow[r] & M_7 \arrow[d] \\
& \Sigma_g
\end{tikzcd}
$$

D3-branes on a Riemann surface

[Couzens, Martelli, Schafer-Nameki '17]

$$\frac{1}{L^2}ds^2 = \frac{\sqrt{y}}{\sin\zeta}\left[ds^2(\mathrm{AdS}_3) + ds^2(X_7)\right]$$

$$ds^2(X_7) = \cos^2\zeta(d\psi_1+\sigma)^2+\sin^2\zeta d\psi_2^2+\frac{\sin^2\zeta}{4y^2\cos^2\zeta}dy^2+\frac{\sin^2\zeta}{y}g^{(4)}(y,x)_{ij}dx^i dx^j$$

- SU(2)-structure (J, Ω) on the four-dimensional base; $g^{(4)}$ is Kähler
- $R_L = \partial_{\psi_1} - \partial_{\psi_2}$, $R_R = \partial_{\psi_1} + \partial_{\psi_2}$
- The geometry is supported by F_5

D3-branes on a Riemann surface

- Assumption: $g^{(4)}$ contains an addditional flavour $U(1)$
- We have reduced the torsion conditions under this assumption. The solution is determined by a potential D satisfying a

$$(\partial_{x_1}^2 + \partial_{x_2}^2)D = 16y^2\left(\partial_y^2 D \partial_\Theta D - (\partial_y \partial_\Theta D)^2\right)e^{\partial_y D}$$

D3-branes on a Riemann surface

$$\frac{1}{L^2}ds^2 = \sqrt{\frac{yg}{h}}\left[ds^2(\mathrm{AdS}_3) + \frac{h}{g}d\psi_2^2 + \frac{he^{2A}}{yg}(dX_1^2 + dX_2^2) + \frac{h}{yg}ds^2(\mathcal{M}_4)\right] ,$$

$$ds^2(\mathcal{M}_4) = \frac{1}{4}g_{ij}du^i du^j + h^{ij}\eta_i\eta_j\,, \quad e^{2A} = 4y^2 e^{\partial_y D}g\,,$$

$$g_{ij} \equiv -\partial_i\partial_j D\,, \quad h_{ij} \equiv -\partial_i\partial_j\left(D + y(\log y - 1)\right) ,$$

$$\eta_1 \equiv d\psi_1 + \star_2 d_2(\partial_y D)\,, \quad \eta_2 \equiv \frac{1}{2}\left(d\phi + \star_2 d_2(\partial_\Theta D)\right) ,$$

with

$$u^i = \{y, \Theta\}\,, \quad g \equiv \det(g_{ij})\,, \quad h \equiv \det(h_{ij})\,.$$

D3-branes on a Riemann surface

$\star$ similar system for D4-D8/O8 [Bah, AP, Weck '18] and M5 -branes [Bah '15] on Riemann surfaces

D3-branes on a Riemann surface

Riemann surface of constant curvature:

$$e^{2A} = f(y, \Theta) e^{2A_0(X1,X2)}$$

$$\frac{1}{L^2} ds^2 = \sqrt{\Lambda}\left[ds^2(\mathrm{AdS}_3) + 4e^{4\nu} c^+ c^- e^{2A_0}(dX_1^2 + dX_2^2)\right] + \frac{1}{\sqrt{\Lambda}} ds^2(\mathcal{M}_5)\,,$$

$$ds^2(\mathcal{M}_5) = d\mu_0^2 + \frac{1}{c^+} d\mu_+^2 + \frac{1}{c^-} d\mu_-^2 + \frac{1}{c^+}\mu_+^2\eta_+^2 + \frac{1}{c^-}\mu_-^2\eta_-^2 + \mu_0^2 d\psi_2^2\,,$$

$$\Lambda \equiv \mu_0^2 + \frac{(m^+)^2}{c^+}\mu_+^2 + \frac{(m^-)^2}{c^-}\mu_-^2\,,$$

$$\eta_\pm \equiv \frac{1}{2}\left[(1 \pm \epsilon) d\psi_1 \pm d\phi + V\right]\,, \quad V = \kappa\left(\partial_{X_2}\tilde{A}_0\, dX_1 - \partial_{X_1}\tilde{A}_0\, dX_2\right)\,.$$

$g = 0$: non-compact $\qquad g = 1$: $\mathrm{AdS}_3 \times S^2 \times T^4$ $\qquad g > 1$: compact

D3-branes on a Riemann surface

$$c_{\mathsf{sugra}} = \frac{3}{2G_N^{(3)}} = \frac{c_L + c_R}{2} = 3N^2(g-1)$$

D3-branes on a Riemann surface

topological disc of non-constant curvature

$$\frac{1}{L^2}ds^2 = \sqrt{W}H(x)^{\frac{1}{3}}\left[ds^2(\mathrm{AdS}_3) + ds^2(\Sigma_2)\right] + \frac{1}{\sqrt{W}}\sum_{I=1}^{3}(X^I)^{-1}\left[d\mu_I^2 + \mu_I^2(d\phi_I + A_I)^2\right] ,$$

$$W = \sum_{I=1}^{3} X^I \mu_I^2 , \quad A_I = \frac{x - x_0}{x + 3K_I}d\varphi , \quad X^I = \frac{H(x)^{\frac{1}{3}}}{x + 3K_I} ,$$

$$ds^2(\Sigma_2) = \frac{1}{4P(x)}dx^2 + \frac{P(x)}{H(x)}d\varphi^2 ,$$

where μ_I embed a unit radius two-sphere into $\mathbb{R}^3$ and the functions of x are

$$H = (x + 3K_1)(x + 3K_2)(x + 3K_3) , \quad P = H - (x - x_0)^2 , \quad K_1 = K_2 = K, \quad K_3 = -\frac{1}{3}x_0$$

Limiting case of [Boido, Ipiña, Sparks '21]

D3-branes on a Riemann surface

$x = x_- : \mathbb{R}^2/\mathbb{Z}_k$

$x = x_0$: regular

$(x, \mu_3) = (x_0, 0)$: flavour D3-branes smeared over S^3

$$c_{\text{sugra}} = 3N^2 \frac{M^2}{4k(1+2M)}$$

Future Directions

- Study the field theory of D3-branes on a topological disc and reproduce the holographic central charge.
- Find topological disc solutions for the D4-O8/D8 and M5 -brane configurations.
- Classify & construct $\mathcal{N} = (2, 0)$ solutions in the "time-like class".
- A generalized geometry formulation of the master equation and the gravity dual of c-extremization (beyond GK geometries).

The End. Thank you!