The emergence of expanding space-time in a novel large-N limit of the Lorentzian type IIB matrix model

Mitsuaki Hirasawa[1]

Based on the collabolation with
Konstantinos Anagnostopoulos[2], Takehiro Azuma[3],
Kohta Hatakeyama[4], Jun Nishimura[4,5],
Stratos Papadoudis[2], Asato Tsuchiya[6]

[1]INFN Milano-Bicocca, [2]NTUA , [3]Setsunan Univ.,
[4]KEK, [5]SOKENDAI, [6]Shizuoka Univ.

Workshop on Noncommutative and generalized geometry in string theory, gauge theory and related physical models

18-25 September 2022, Corfu, Greece

Lorentzian type IIB matrix model

[N. Ishibashi, H. Kawai, Y. Kitazawa, A. Tsuchiya (1997)]

- partition function and the action

c.f.) related talks at the workshop:
Brandenberger on 19/9
Battista on 19/9
Tran on 21/9

$$Z = \int dA d\Psi d\bar{\Psi} \; e^{i(S_{\rm b}+S_{\rm f})}$$

$$S_{\rm b} = -\frac{N}{4}\mathrm{Tr}\left\{-2[A_0, A_i]^2 + [A_i, A_j]^2\right\}$$

$$S_{\rm f} = -\frac{N}{2}\mathrm{Tr}\left\{\bar{\Psi}_\alpha (C\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right\}$$

$A_\mu, \Psi_\alpha \,:\, N \times N$ Hermitian matrices $\quad (\mu = 0, \ldots, 9, \;\; \alpha = 1, 2, \ldots, 16)$

- a promising candidate for non-perturbative formulation of superstring theory
  - matrix regularization of the worldsheet action
  - The interactions of D-branes can be reproduced.
  - The string field Hamiltonian can be derived from Schwinger-Dyson equations for the Wilson loop operators.

[M. Fukuma, H. Kawai, Y. Kitazawa, A. Tsuchiya (1998)]

Lorentzian type IIB matrix model

[N. Ishibashi, H. Kawai, Y. Kitazawa, A. Tsuchiya (1997)]

- partition function and the action

$$Z = \int dAd\Psi d\bar{\Psi}\ e^{i(S_{\mathrm{b}}+S_{\mathrm{f}})}$$

$$S_{\mathrm{b}} = -\frac{N\beta}{4}\mathrm{Tr}\left\{-2[A_0,A_i]^2+[A_i,A_j]^2\right\}$$

$$S_{\mathrm{f}} = -\frac{N\beta}{2}\mathrm{Tr}\left\{\bar{\Psi}_\alpha(C\Gamma^\mu)_{\alpha\beta}[A_\mu,\Psi_\beta]\right\}$$

A_μ, Ψ_α : $N \times N$ Hermitian matrices $\quad (\mu = 0,\ldots,9,\ \ \alpha = 1,2,\ldots,16)$

- This model has $\mathcal{N}=2$ SUSY.
  evidence for the fact that this model includes gravity

- Geometry emerges from matrix degrees of freedom.
  In the SUSY algebra, translation corresponds to shift of A_μ .
  $\rightarrow$ The eigenvalues of A_μ are identified as space-time coordinates.

- This model has SO(9,1) Lorentz symmetry.

$$Z_{\mathrm{L}} = \int dA d\Psi d\bar{\Psi} e^{i(S_{\mathrm{b}}+S_{\mathrm{f}})} = \int dA e^{iS_{\mathrm{b}}} \mathrm{Pf}\mathcal{M}$$

- Wick rotation in this model

$$S_{\mathrm{b}} \to \tilde{S}_b = N\beta\, \underbrace{e^{i\frac{\pi}{2}u}}_{\text{on the worldsheet}}\, \mathrm{Tr}\left\{ \frac{1}{2} \underbrace{e^{-i\pi u}}_{\text{in the target space}} [\tilde{A}_0, \tilde{A}_i]^2 - \frac{1}{4} [\tilde{A}_i, \tilde{A}_j]^2 \right\},$$

$$u = \begin{cases} 0: & \text{Lorentzian} \\ 1: & \text{Euclidean} \end{cases}$$

This Wick rotation is equivarent to the contour deformation:

$$\begin{aligned} A_0 \quad &\to \quad \tilde{A}_0 = e^{i\frac{\pi}{2}u} e^{-i\frac{\pi}{8}u} A_0 = e^{i\frac{3}{8}\pi u} A_0 \\ A_i \quad &\to \quad \tilde{A}_i = e^{-i\frac{\pi}{8}u} A_i \end{aligned}$$

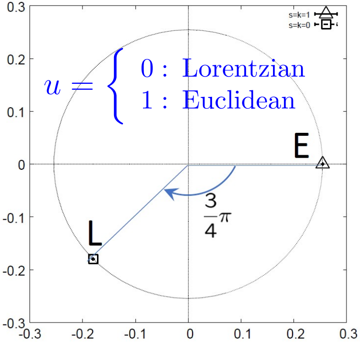
equivalence between the Euclidean and Lorentzian model

Cauchy's theorem

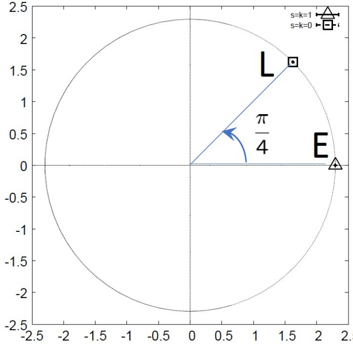
$$\left\langle \mathcal{O}(e^{-i\frac{3}{8}\pi u}\tilde{A}_0, e^{i\frac{\pi}{8}u}\tilde{A}_i) \right\rangle_u \text{ is independent of } u.$$

numerical confirmation of the equivalence using complex Langevin method

$$\left\langle \frac{1}{N}\mathrm{Tr}(A_0)^2 \right\rangle_{\mathrm{L}} = e^{-i\frac{3}{4}\pi} \left\langle \frac{1}{N}\mathrm{Tr}(\tilde{A}_0)^2 \right\rangle_{\mathrm{E}},$$



$$\left\langle \frac{1}{N}\mathrm{Tr}(A_i)^2 \right\rangle_{\mathrm{L}} = e^{i\frac{1}{4}\pi} \left\langle \frac{1}{N}\mathrm{Tr}(\tilde{A}_i)^2 \right\rangle_{\mathrm{E}}$$



- $\mathrm{Pf}\mathcal{M}$ is complex valued in the Euclidean model.
  - SSB of SO(10) does not occur in the phase quenched model.

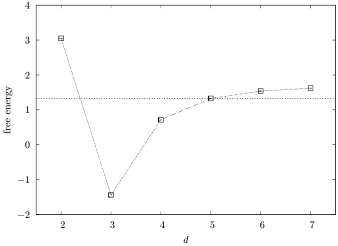
[J. Ambjørn, K. Anagnostopoulos, W. Bietenholz, T. Hotta, J. Nishimura (2000)]

$\rightarrow$ The phase of $\mathrm{Pf}\mathcal{M}$ plays an important role.

[J. Nishimura, G. Vernizzi (2000)]

- Gaussian expansion analysis [J. Nishimura, T. Okubo, and F. Sugino (2011)]

free energy for SO(d) symmetric vacuum



SO(10) $\rightarrow$ SO(3) is predicted.

non-perturbative aspects of the Euclidean model

[K. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, S. Papadoudis (2020)]

- sign problem ($\because \mathrm{Pf}\mathcal{M}$ is complex valued.)

  Conventional Monte Carlo methods are not applicable.

  $\rightarrow$ The problem was overcome by using complex Langevin method.

- SSB: SO(10) $\rightarrow$ SO(3) occurs dynamically.
  - SO(4) does not appear.

- Relation between the emergent space and our universe is not clear.

[K. Hatakeyama, A. Matsumoto, J. Nishimura, A. Tsuchiya, A. Yosprakob (2019)]

- solving the equation of motion.

$$[A^{\nu}, [A_{\nu}, A_{\mu}]] = 0.$$

  - The solution to this EOM is exhausted by diagonal matrices.
  - no strong reasons for the emergence of expanding space

- introducing an additional term

$$[A^{\nu}, [A_{\nu}, A_{\mu}]] - \gamma A_{\mu} = 0.$$

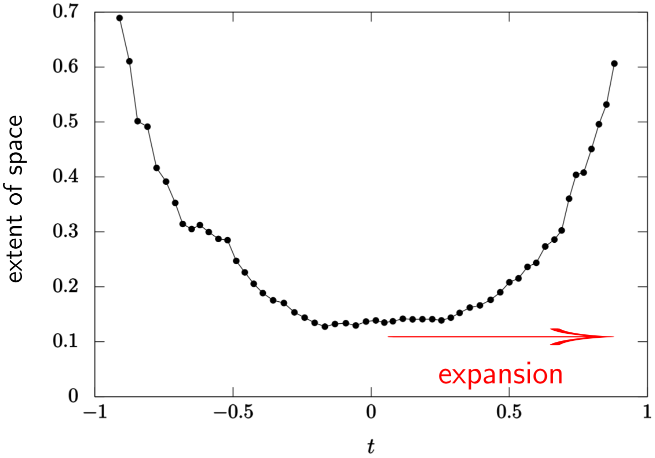
$$(\gamma > 0)$$

  - Typical solutions have expanding space although its dimensionality is not fixed.

classical solution with (3+1)D space-time

[K. Hatakeyama, A. Matsumoto, J. Nishimura, A. Tsuchiya, A. Yosprakob (2019)]

- (3+1)D solutions (The dimensionality is chosen by hand)
  - The 3d space expands in typical classical solutions.



novel large-N limit

- In order to obtain a large-N limit inequivalent to the Euclidean model, we add a Lorentz invariant "mass" term to the action.

$$S_\gamma = -\frac{1}{2}N\gamma\mathrm{Tr}(A_\mu)^2 = \frac{1}{2}N\gamma\left\{\mathrm{Tr}(A_0)^2 - \mathrm{Tr}(A_i)^2\right\}$$

$(\gamma > 0)$

Motivation for this extra mass term
comes from the previous work on classical solutions.

$$[A^\nu,[A_\nu,A_\mu]] - \gamma A_\mu = 0$$

[K. Hatakeyama, A. Matsumoto, J. Nishimura, A. Tsuchiya, A. Yosprakob (2019)] [H. Steinacker (2017)]

We consider taking the $\gamma \to 0^+$ limit after taking the large-N limit.

We will see that γ can be also interpreted as
an "infrared regulator" for the expanding space-time.

novel large-N limit

$$Z = \int dA e^{-S(A)} \mathrm{Pf}\mathcal{M}(A), \qquad e^{-S(A)} = e^{i(S_{\mathrm{b}}(A)+S_\gamma(A))}$$

- contour deformation $\tilde{F}_{\mu\nu} \equiv -i[\tilde{A}_\mu, \tilde{A}_\nu]$

positive real part for $0 < u \leq 1$

$$S(A) \to S(\tilde{A}) \sim 2e^{i\frac{\pi}{2}(1-u)}\mathrm{Tr}(\tilde{F}_{0i})^2 + e^{-i\frac{\pi}{2}(1-u)}\mathrm{Tr}(\tilde{F}_{ij})^2$$
$$+ \gamma e^{-i\frac{\pi}{2}(1+\frac{3}{2}u)}\mathrm{Tr}(\tilde{A}_0)^2 + \gamma e^{i\frac{\pi}{2}(1+\frac{1}{2}u)}\mathrm{Tr}(\tilde{A}_i)^2$$

negative real part for $0 < u \leq 1$

The action is unbounded for $0 < u \leq 1$

One cannot define the model by contour deformation any more!
($\because$ The corresponding Euclidean model is ill-defined.)

setup

- We choose an SU(N) basis :

$$A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N)$$

$$(\alpha_1 < \alpha_2 < \cdots < \alpha_N)$$

- sign problem

$$Z_{\mathrm{L}} = \int dA \underbrace{e^{i(S_{\mathrm{b}}+S_{\gamma})}}_{\text{phase factor}} \mathrm{Pf}\mathcal{M}(A)$$

We cannot regard the Boltzmann weight as the probability.
$\rightarrow$ Conventional Monte Carlo methods are not applicable.

We use complex Langevin method to overcome the problem.

complex Langevin method

[G. Parisi (1983)] [J. Klauder (1984)]

- a way to realize the ordering : $\alpha_1 < \alpha_2 < \cdots < \alpha_N$ $(A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N))$

$$\alpha_1 = 0,\ \alpha_2 = e^{\tau_1},\ \alpha_3 = e^{\tau_1} + e^{\tau_2},\ \ldots,\ \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a}$$

[J. Nishimura, A. Tsuchiya (2019)]

- complexify the variables

$$A_i\ :\ \text{Hermitian matrices} \ \to\ \text{general matrices}$$

$$\tau_a\ :\ \text{real} \ \to\ \text{complex}$$

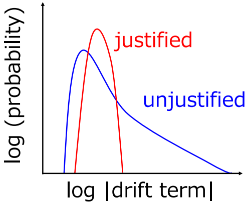
- complex Langevin equation

$$\frac{d\tau_a}{dt_{\mathrm{L}}} = -\frac{\partial S}{\partial \tau_a} + \eta_a(t_{\mathrm{L}}), \qquad \frac{d(A_i)_{ab}}{dt_{\mathrm{L}}} = -\frac{\partial S}{\partial (A_i)_{ba}} + (\eta_i)_{ab}(t_{\mathrm{L}})$$

criterion for the correct convergence

The drift histogram falls off exponentially or faster with the magnitude of the drift term.

[K. Nagata, J. Nishimura, S. Shimasaki (2016)]



complex Langevin method

[G. Parisi (1983)] [J. Klauder (1984)]

- singular drift problem - a cause of wrong convergence -

  If the Dirac operator has near-zero eigenvalues,
  the criterion is not satisfied.

  - adding fermionic mass term

$$S_{m_{\rm f}} = iNm_{\rm f}\mathrm{Tr}[\bar{\Psi}_\alpha(\Gamma_7\Gamma_8^\dagger\Gamma_9)_{\alpha\beta}\Psi_\beta]$$

[K. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, S. Papadoudis (2020)]

$m_{\rm f} = \infty$ corresponds to the fermion quenched model.

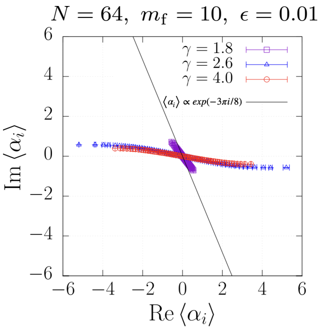
We need to make the $m_{\rm f} \to 0$ extrapolation eventually.

- We perform the following procedure at each Langevin step for stabilization. (*c.f.* dynamical stabilization for QCD [F. Attanasio, B. Jäger (2018)])

$$A_i \to \frac{1}{1+\epsilon}\left(A_i + \epsilon A_i^\dagger\right)$$

phase structure for various γ

Lorentz invariant mass term: $\frac{1}{2}N\gamma\left\{\mathrm{Tr}(A_0)^2 - \mathrm{Tr}(A_i)^2\right\}$



- $\gamma \leq 1.8$:
  qualitatively the same as the Euclidean model ($\gamma = 0$)

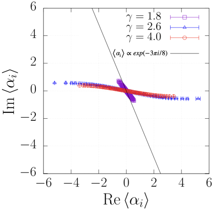
1st order phase transition (?)

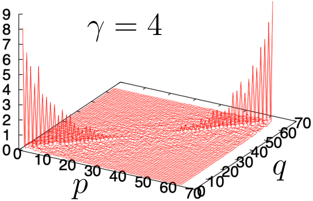
- $\gamma \geq 2.6$:
  emergence of real time at both ends
  $(\alpha_{i+1} - \alpha_i \in \mathbb{R})$

We focus on the real time phase.

how to extract time-evolution

- band diagonal structure (dynamical property)

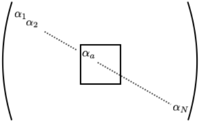




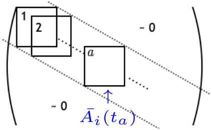
$$\mathcal{A}_{pq} \equiv \frac{1}{9}\sum_i |(A_i)_{pq}|^2$$

- how to extract time-evolution

$A_0 =$



$A_i =$



- definition of time

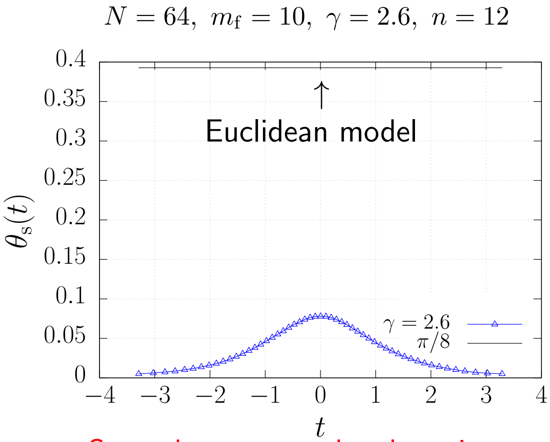
$$t_a = \sum_{i=1}^{a} |\bar{\alpha}_i - \bar{\alpha}_{i-1}|, \quad \bar{\alpha}_i = \frac{1}{n}\sum_{j=0}^{n-1} \alpha_{i+j}$$

(n: block size)

- $\bar{A}_i(t_a)$ ($n \times n$ matrix) represents the state of the universe at t_a.

- phase of space

$$\mathrm{tr}(\bar{A}_i(t))^2 = e^{2i\theta_s(t)} |\mathrm{tr}(\bar{A}_i(t))^2|$$



Space becomes real at late times.

SSB of SO(9) symmetry

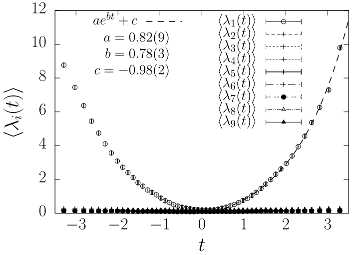
- order parameter for SSB of SO(9)

  the eigenvalues of "moment of inertia tensor"

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left(X_i(t)X_j(t)\right), \quad X_i(t) \equiv \frac{1}{2}\left(\bar{A}_i(t) + \bar{A}_i^\dagger(t)\right)$$

  - SO(9) symmetric: 9 eigenvalues are almost degenerate.
  - SO(9) broken: 9 eigenvalues are NOT degenerate.

$$N = 64,\ m_{\mathrm{f}} = 10,\ \gamma = 2.6,\ n = 12$$



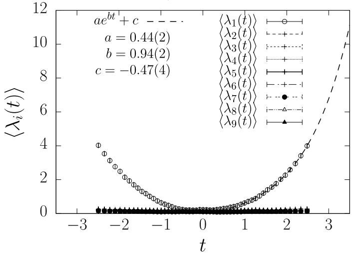
SSB of SO(9) occurs.

1d space expands exponentially.

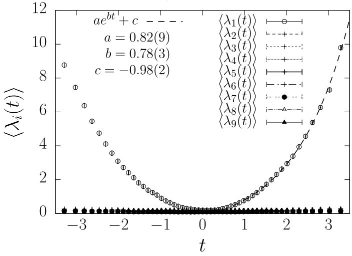
γ dependence

$$N = 64,\ m_{\rm f} = 10,\ n = 12$$

$\gamma = 4.0$



$\gamma = 2.6$



- 1d expansion occurs.
- The extent of time becomes larger at smaller γ.
- In the real time phase, the expansion of space gets more pronounced as γ decreases.

  $\rightarrow \gamma$ can be thought of as an "infrared regulator".

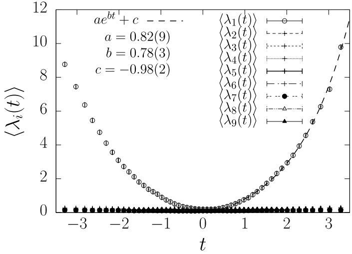
fermionic effects

$$N = 64,\ \gamma = 2.6,\ n = 12$$

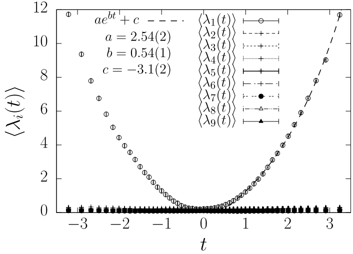
fermionic mass term: $iNm_{\mathrm{f}}\mathrm{Tr}[\bar{\Psi}_\alpha(\Gamma_7\Gamma_8^\dagger\Gamma_9)_{\alpha\beta}\Psi_\beta]$

$m_{\mathrm{f}} = \infty$ corresponds to the fermion quenched model.

$m_{\mathrm{f}} = 10$



$m_{\mathrm{f}} = 5$



- In the real time phase, the expansion of space gets more pronounced as m_{f} decreases.
  - The attractive force between space-time eigenvalues is weakened by the SUSY effects.

c.f.) Euclidean model [H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, T. Tada (1998)]

summary

- We successfully applied the complex Langevin method to the Lorentzian type IIB matrix model.
- equivalence between the Euclidean and Lorentzian model in the conventional large-N limit
  - Euclidean model exhibits SSB: SO(10) $\rightarrow$ SO(3).
    The space-time becomes complex and it has Euclidean signature.
- introducing the Lorentz invariant mass term
  - An expanding real space-time appears at late times as expected from classical solutions.
  - the dimensionality of the expanding space:
    - not fixed at the classical level
    - turned out to be 1D for $m_{\rm f} > 5$.

Does 3d expanding space appear at smaller $m_{\rm f}$?

(SUSY : $m_{\rm f} = 0$)

discussion

- a possible mechanism for the emergence of the 3d expanding space
  - a mechanism for collapsing space

    Quantum fluctuation is suppressed most when $\mathrm{Tr}[A_i, A_j]^2 \sim 0$.

    $\rightarrow$ 1d expanding space is favored.

  - property of the Pfaffian (at $m_\mathrm{f} = 0$)

    Pfaffian becomes zero if there are only two large matrices:

    $A_1, A_2 \neq 0, A_3, \ldots, A_9 = 0.$ [W. Krauth, H. Nicolai, M. Staudacher (1998)] [J. Nishimura, G. Vernizzi (2000)]

    (Due to the exponential expansion of space, A_0 cannot play any role here.)

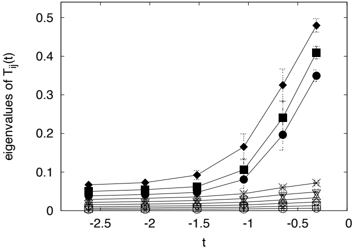
    $\rightarrow$ 3d expanding space may be favored by the Pfaffian.

- We are now trying to see whether SO(9) $\rightarrow$ SO(3) occurs by decreasing m_f further. (*c.f.* results for the Euclidean model)
- We expect that 3d expanding space appears for sufficiently small m_f.

Thank you for listening!

Lorentzian type IIB matrix model

- One of the candidates for non-perturbative definition of superstring theory
  - Monte Carlo method is applicable.
- Previous works about Monte Carlo simulation of the model
  - SSB: SO(9,1) -> SO(3,1)   Kim, Nishimura, Tsuchiya ('12)



Lorentzian type IIB matrix model

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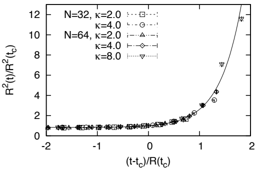
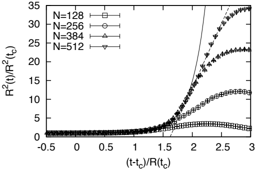
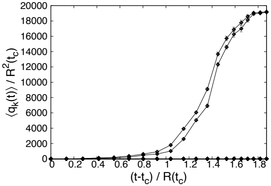
- Previous works about Monte Carlo simulation of the model
  - SSB: SO(9,1) -> SO(3,1) Kim, Nishimura, Tsuchiya ('12)
  - expansion of the 3d space
    - exponential expansion in the early time Ito, Kim, Nishimura, Tsuchiya ('13)
    - power law expansion in the late time Ito, Nishimura, Tsuchiya ('15)





Lorentzian type IIB matrix model

- Previous works about Monte Carlo simulation of the model (cont'd)
  - structure of the 3d space
    - Pauli-matrix structure



Space is not continuous.

Aoki, Hirasawa, Ito, Nishimura, Tsuchiya ('19)

So far, we had used an approximation for the partition function to avoid the sign problem.

$$e^{iS_{\mathrm{b}}} \to e^{\beta S_{\mathrm{b}}} \qquad (\beta > 0)$$

Lorentzian type IIB matrix model

- Previous works about Monte Carlo simulation of the model (cont'd)
  - structure of the 3d space
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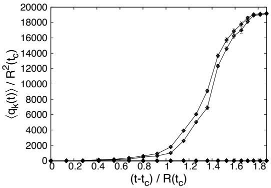


Aoki, Hirasawa, Ito, Nishimura, Tsuchiya ('19)

Space is not continuous.

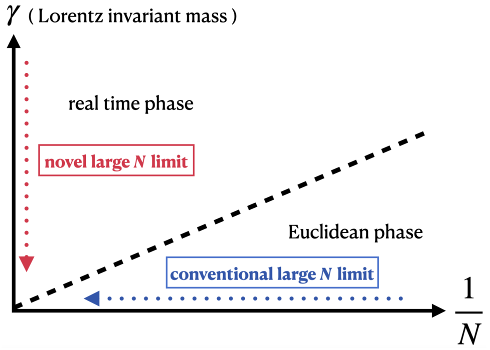
due to the approximation?

So far, we had used an approximation for the partition function to avoid the sign problem.

$$e^{iS_b} \to e^{\beta S_b} \qquad (\beta > 0)$$

Novel large-N limit

our prediction for a phase diagram $(\gamma,\ 1/N)$



We expect the phase appearing in the novel large-N limit is inequivalent to that in the conventional one.