A Consistent Limit of 11D Supergravity

Eric Bergshoeff

Groningen University

based upon work in progress with C. Blair, J. Lahnsteiner and J. Rosseel

Workshop on Holography and the Swampland

Corfu, September 8 2022

rijksuniversiteit groningen

Motivation

- D-branes $\leftrightarrow$ YM gauge theory
- 'Critical' limit of open strings with $\mathcal{F} = da - B \neq 0$ $\rightarrow$ Non-commutative Open Strings (NCOS)

  Seiberg, Susskind, Toumbas (2000), Gopakumar, Maldacena, Minwalla, Strominger (2000)
- NR closed superstrings with $B \neq 0$ $\leftrightarrow$ NR $10D$ $\mathcal{N} = 1$ Supergravity

  Danielsson, Güijosa, Kruczenski (2000), Gomis, Ooguri (2001); Lahnsteiner, Romano, Rosseel, Şimşek (2021)
- 'critical' limit of open membranes with $\mathcal{H} = db - C \neq 0$ $\rightarrow$ Non-commutative Open Membrane (OM-theory)

  Strominger (2000); Berman, van der Schaar, Sundell + E.B. (2000)
- NR Closed Supermembranes with $C \neq 0$ $\leftrightarrow$

  Consistent Limit of 11D Supergravity?

  García, Güijosa, Vergara (2002); Blair, Gallegos, Zinnato (2021)

Outline

A New Limit of 10D $\mathcal{N} = 1$ Supergravity

Outline

A New Limit of $10D$ $\mathcal{N} = 1$ Supergravity

A Consistent Limit of 11D Supergravity

Outline

Outline

The Bosonic Case

The basic fields are:

$$\{E_\mu{}^{\hat{A}}\,, B_{\mu\nu}\,, \Phi\}$$

with relativistic action given by

$$S_{\mathrm{rel}} = \frac{1}{2\kappa^2}\int d^{10}x\, E e^{-2\Phi}\left(\mathcal{R} - \frac{1}{12}\mathcal{H}_{\mu\nu\rho}\mathcal{H}^{\mu\nu\rho} + 4\partial_\mu\Phi\partial^\mu\Phi\right)$$

with $\mathcal{H}_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]}$. We decompose $\hat{A} = (A, a)$ and redefine

$$E_\mu{}^A = \omega\tau_\mu{}^A\,, \qquad E_\mu{}^a = e_\mu{}^a\,, \qquad B_{\mu\nu} = -\omega^2\epsilon_{AB}\tau_\mu{}^A\tau_\nu{}^B + b_{\mu\nu}$$

and find

$$S = \omega^2 \overset{(2)}{S} + \overset{(0)}{S} + \omega^{-2}\overset{(-2)}{S} + \omega^{-4}\overset{(-4)}{S}$$

Note: after taking the limit $\omega\to\infty$, $b_{\mu\nu}$ becomes a geometric field

Two miracles:

(i) the metric and 2-form contributions to $\overset{(2)}{S}$ precisely cancel

(ii) $\overset{(0)}{S}$ is invariant under local an-isotropic dilatations!

- due to the emergent local dilatation symmetry there is one 'missing e.o.m.' M
- This 'missing' e.o.m. M follows from taking the NR limit of the e.o.m. and is precisely the Poisson equation of the Newton potential
- The full set of e.o.m. $\{B; M\}$ form a reducible, but indecomposable representation under boosts:

$$\delta_B M = B \quad \text{but} \quad \delta_B B = B$$

Conventional versus Geometric Tensors

conventional tensors are curvature components of $\{\tau_\mu\,, e_\mu{}^a\,, b_{\mu\nu}\}$ that, by setting them to zero, can be used to solve for certain components of the spin-connection fields and dilatation gauge fields

geometric tensors are the remaining curvature components that do not contain any spin-connection or dilatation gauge field

We have 442 conventional tensors given by

$$\{\,T_{AB}{}^C(2)\,,\,T_a{}^{[AB]}(8)\,,\,T_a{}^A{}_A(8)\,,\,T_{\mu\nu}{}^a(360);\,h_{ABa}(8)\,,\,h_{Aab}(56)\}$$

that solve for the following gauge-field components

$$\omega_\mu{}^{AB}(10)\,,\,b_\mu \text{ except } b_A(8)\,,\omega_\mu{}^{Aa} \text{ except } W_{\{AB\}a}(144)\,,\omega_\mu{}^{ab}(280)\}$$

The remaining 128 geometric tensors are given by

$$\{\,T_{ab}{}^A(56)\,,\,T_a{}^{\{AB\}}(16)\,,\,h_{abc}(56)\}$$

Supersymmetry

Taking the naive NR limit leads to divergent terms in the susy rules

These divergences lead to emergent symmetries and geometric constraints:

- $\overset{(2)}{\delta_\epsilon}\ \overset{(0)}{S} = \omega^2\ \overset{(2)}{S} = 0$ $\rightarrow$ we find 2 'superconformal' Stueckelberg symmetries beyond dilatations

- imposing $\overset{(2)}{\delta_\epsilon}\ \overset{(-2)}{S} = 0$ we need to impose the following constraints on the geometric tensors $T_{ab}{}^{A}$, $T_a{}^{\{AB\}}$

$$T_{ab}{}^{-} = T_{a+}{}^{-} = 0 \qquad \text{or} \qquad \tau_{[\mu}{}^{-}\partial_\nu\tau_{\rho]}{}^{-} = 0$$

Simplifying feature: the geometric constraints are invariant under supersymmetry !

Outline

The Bosonic Case

The basic fields are $\{E_\mu{}^{\hat{A}}, C_{\mu\nu\rho}\}$ with relativistic action given by

$$S_{\mathrm{rel}} \sim \int d^{11}x \left(E\left[\mathcal{R} - \frac{1}{48}\mathcal{F}_{\mu\nu\rho\sigma}\mathcal{F}^{\mu\nu\rho\sigma}\right] + \frac{1}{144^2}\epsilon^{\mu_1\cdots\mu_{11}}\mathcal{F}_{\mu_1\cdots\mu_4}\mathcal{F}_{\mu_5\cdots\mu_8}C_{\mu_9\mu_{10}\mu_{11}}\right)$$

with $\mathcal{F}_{\mu\nu\rho\sigma} = 4\partial_{[\mu}C_{\nu\rho\sigma]}$. We decompose $\hat{A} = (A, a)$ and redefine

$$E_\mu{}^A = \omega\tau_\mu{}^A\,, \qquad E_\mu{}^a = \omega^{-1/2}e_\mu{}^a\,, \qquad C_{\mu\nu\rho} = -\frac{1}{6}\omega^3\epsilon_{ABC}\tau_\mu{}^A\tau_\nu{}^B\tau_\rho{}^C + c_{\mu\nu\rho}$$

and find

$$S = \omega^3 \overset{(3)}{S} + \overset{(0)}{S} + \omega^{-3}\overset{(-3)}{S} + \cdots$$

Two miracles and a Trick:

(i) the metric and part of the 3-form contributions to $\overset{(3)}{S}$ precisely cancel

(ii) $\overset{(0)}{S}$ is invariant under an-isotropic local dilatations

but there are remaining divergences coming from the 3-form kinetic term and the Chern-Simons term which add up to

$$\omega^3\, f^{(+)}_{abcd}\, f^{abcd(+)}$$

This divergence can be tamed by the following 'quadratic divergence trick' introducing a Lagrange multiplier λ:

$$\omega^3 X^2 \quad \text{is equivalent to} \quad -\frac{1}{\omega^3}\lambda^2 - 2\lambda X \quad \text{for any } X$$

In our case we have $X^{(+)}_{abcd} = f^{(+)}_{abcd} \quad \rightarrow$ Lagrange multiplier $\lambda^{(+)}_{abcd}$

Geometric Constraints (G.C.)

We find the following bosonic geometric tensors:

$$\{ T_{ab}{}^{A}(84)\,,\, T_{a}{}^{\{AB\}}(40)\,,\, f^{(\pm)}_{abcd}\,(35^{\pm})\,,\, f_{Abcd}(168)\}$$

These include the e.o.m. $f^{(+)}_{abcd} = 0$

The divergences in the 'Q-supersymmetry' rules lead to

- 2 conformal 'S supersymmetries' beyond the emergent dilatations D. Note: we find that $\{Q, S\} \sim D$
- imposing $\overset{(3)}{\delta_\epsilon}\;\overset{(-3)}{S} = 0$ we need to set all geometric tensors equal to zero except for $f^{(+)}_{abcd}$ which is an equation of motion

complication: these constraints are not invariant under supersymmetry !

Fermionic Geometric Tensors

We introduce two S-supersymmetry gauge fields $\phi_{\mu-}$ and $\phi_{\mu A+}$

All components can be solved for except for $\phi_{(AB)+}$ by imposing conventional constraints

We are left with the following fermionic geometric tensors:

$$\{r_{ab}(Q_\pm)\ \ (56\times 16)\,,\ \check{r}_{Aa}(Q_-)\ \ \text{with}\ \ \Gamma^A\check{r}_{Aa}(Q_-)=0\ \ (16\times 16)\}$$

These constraints include the NR gravitino e.o.m.. Under susy we have

$$\text{bosonic G.C.}\ \to\ \underline{\text{all}}\ \text{fermionic G.C.}\ \to\ \partial\,(\text{bosonic G.C.}) + \text{constraints on } r(\omega)$$

The Poisson equation is a singlet constraint on $r(\omega)$ (boost)

String Newton-Cartan E.O.M.

Without supersymmetry we expect the following e.o.m.:

$$\tau^\mu{}_A e^\nu{}_b\, r_{\mu\nu}{}^{Ab}(G) \;=\; 0 : \qquad \mathbf{1}\ \ \text{Poisson equation}$$

$$e^\nu{}_a r_{\mu\nu}{}^{ab}(J) \;=\; 0 : \qquad \mathbf{Ab}\,,\,(\mathbf{ab})$$

with supersymmetry, we find

$$r_{\mu\nu}{}^{ab}(J) = 0$$

like in 3D NR supergravity

Andringa, Rosseel, Sezgin + E.B. (2013)

We expect that the supersymmety variation of the NR gravitino e.o.m. will give us the Poisson equation plus the NR 3-form e.o.m.

Outline

A New Limit of $10D$ $\mathcal{N} = 1$ Supergravity

A Consistent Limit of 11D Supergravity

Generalizations and Discussion

Generalizations

Our results pave the way for constructing NR IIA and IIB supergravity

Not yet done: Heterotic Supergravity

The next step is to classify the supersymmetric solutions including the NR D-brane solutions

This will pave the way for investigating NR holography using NR gravity in the bulk

Discussion

Is there a relation between the spatial double dimensional reduction of the NR 11D supergravity we constructed, i.e. NR string Type IIA supergravity and the null reduction of 11D supergravity i.e. NR particle Type IIA supergravity, if it exists ?

Does there exist besides a NR membrane 11D supergravity also a NR five-brane 11D supergravity ?