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Phenomenology of Supersymmetric Trinification from Dimensional Reduction of a $N = 1$, $10D$ E_8 Theory

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Motivation

Unified description of Nature

- Extra Gauge Symmetry (i.e. GUTs)
- Supersymmetry
- Extra Dimensions
  - Unify gauge and Higgs sectors
  - Also unify fermion interactions with the above sectors
  - SUSY can unify all the above in one vector supermultiplet
  - Less free parameters

Coset Space Dimensional Reduction

1. Compactification

B - compact space

$\dim B = D - 4 = d$

D dims $\rightarrow$ 4 dims

$$
\begin{array}{ccccc}
M^D & \rightarrow & M^4 & \times & B \\
| & & | & & | \\
x^M & & x^\mu & & y^a
\end{array}
$$

2. Dimensional Reduction

$\mathcal{L}$ independent of the extra coordinates y^a:

- "Naive" way: Discard the field dependence on y^a coordinates
- Elegant way: Allow field dependence on y^a
  $\rightarrow$ compensated by a symmetry of the Lagrangian $\rightarrow$ Gauge Symmetry

3. Coset Space Dimensional Reduction

Witten (1977); Forgacs, Manton (1980); Chapline, Slansky (1982); Kapetanakis, Zoupanos - Phys.Rept. (1992) Kubyshin, Mourao, Rudolph, Volobujev - Book (1989)

– $B = S/R$

– Allow a non-trivial dependence on y^a

– impose the condition that a symmetry transformation by an element of the isometry group S of B is compensated by a gauge transformation

$\rightarrow$ Gauge invariant $\mathcal{L}$ $\rightarrow$ $\mathcal{L}$ independent of y^a !

Reduction of a D-dimensional Yang-Mills Lagrangian

Consider a Yang-Mills-Dirac theory in D dims based on group G defined on $M^D \to M^4 \times S/R$, $D = 4 + d$

$$S = \int d^4x d^d y \sqrt{-g} \left[-\frac{1}{4} Tr(F_{MN} F_{K\Lambda}) g^{MK} g^{N\Lambda} + \frac{i}{2} \overline{\psi} \Gamma^M D_M \psi \right]$$

Demand: any transformation by an element of S acting on S/R is compensated by gauge transformations.

$\to$ Constraints on the fields of the theory A_α and ψ

Solution of constraints:

- Remaining gauge invariance
- 4-dimensional fields
- Potential

1) The 4D gauge group

$$H = C_G(R_G)$$

$$\text{i.e.} \quad G \supset R_G \times H$$

2) Scalar fields

$$S \supset R$$

$$\text{adj}S = \text{adj}R + \sum s_i$$

$$G \supset R_G \times H$$

$$\text{adj}G \supset (\text{adj}R, 1) + (1, \text{adj}H) + \sum (r_i, h_i)$$

for each $s_i = r_i \Rightarrow h_i$ survives in 4D.

(The 4D gauge fields are independent of the extra dimensions)

3) Fermions

$$SO(d) \supset R$$

$$\sigma_d = \sum \sigma_j$$

$$G \supset R_G \times H$$

$$F = \sum (t_i, h_i)$$

for each $t_i = \sigma_i \Rightarrow h_i$ survives in 4D.

$D = 4n + 2$ Weyl + Majorana fermions in vector-like rep $\longrightarrow$ 4D chiral theory.

The 4D Theory

Integrate out the extra coordinates (+ take into account constraints):

$$S = C \int d^4x \,\mathrm{tr}\left[-\frac{1}{8}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(D_\mu\phi_a)(D^\mu\phi^a)\right] + V(\phi) + \frac{i}{2}\bar{\psi}\Gamma^\mu D_\mu\psi - \frac{i}{2}\bar{\psi}\Gamma^a D_a\psi$$

- $D_\mu = \partial_\mu - igA_\mu$
- $D_a = \partial_a - \theta_a - ig\phi_a$
- C: coset space volume
- $\phi_a \equiv A_a$

$$V(\phi) = -\frac{1}{8}g^{ac}g^{bd}\mathrm{Tr}\left\{(f_{ab}^C\phi_C - ig[\phi_a,\phi_b])(f_{cd}^D\phi_D - ig[\phi_c,\phi_d])\right\}$$

$V(\phi)$ still only formal since ϕ_a must satisfy one more constraint.

- If $G \supset S \Rightarrow H$ breaks to $K = C_G(S)$:

$$\begin{array}{cl} G \supset S \times K \leftarrow & \text{gauge group after SSB} \\ \quad\;\; \cup \quad \cap & \\ G \supset R \times H \leftarrow & \text{gauge group in 4 dims} \end{array}$$

Harnad, Shnider, Tafel (1980)

Reduction of 10D, $N = 1$ E_8 over $S/R = SU(3)/U(1) \times U(1)$

Manousselis, Zoupanos (2001-2004)

The non-symmetric (nearly-Kähler) coset space $SU(3)/U(1) \times U(1)$:

- admits torsion and may have different radii
- naturally produces soft supersymmetry breaking terms
- preserves the supersymmetric multiplets

We use the decomposition

$$E_8 \supset E_6 \times SU(3) \supset E_6 \times U(1)_A \times U(1)_B$$

and choose $R = U(1)_A \times U(1)_B$

$$\rightarrow H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B$$

Since $S \subset G$, H breaks to

$$K = C_G(S) = E_6 \times [U(1) \times U(1)]_{\text{global}}$$

- $N = 1,\ E_6 \times U(1)_A \times U(1)_B$ gauge group
- Three chiral supermultiplets $A^i : 27_{(3,1/2)}\ ,\ B^i : 27_{(-3,1/2)}\ ,\ C^i : 27_{(0,-1)}$
- Three chiral supermultiplets $A : 1_{(3,1/2)}\ ,\ B : 1_{(-3,1/2)}\ ,\ C : 1_{(0,-1)}$
- Gaugino mass $M = (1 + 3\tau)\frac{R_1^2+R_2^2+R_3^2}{8\sqrt{R_1^2 R_2^2 R_3^2}}$

$$V = \frac{g^2}{5}\left(\frac{1}{R_1^4} + \frac{1}{R_2^4} + \frac{1}{R_3^4}\right) + V_F + V_D + V_{soft}$$

$$\frac{2}{g^2}V_D = \frac{1}{2}D^\alpha D^\alpha + \frac{1}{2}D_1 D_1 + \frac{1}{2}D_2 D_2 \qquad \frac{2}{g^2}V_F = \sum_s |F_s|^2\,, \quad F_s = \frac{\partial \mathcal{W}}{\partial s}$$

$$\frac{2}{g^2}V_{soft} = \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2}\right)\left(\alpha^i\alpha_i + \bar{\alpha}\alpha\right) + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2}\right)\left(\beta^i\beta_i + \bar{\beta}\beta\right) + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_2^3}\right)\left(\gamma^i\gamma_i + \bar{\gamma}\gamma\right)$$
$$+ \sqrt{280}\left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_2R_1}\right)\left(d_{ijk}\alpha^i\beta^j\gamma^k + \alpha\beta\gamma + h.c.\right)$$

where

$$D^\alpha = \frac{1}{\sqrt{3}}\left(\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j\right)$$

$$D_1 = \frac{\sqrt{10}}{3}\left(\alpha^i(3\delta_i^j)\alpha_j + \bar{\alpha}(3)\alpha + \beta^i(-3\delta_i^j)\beta_j + \bar{\beta}(-3)\beta\right)$$

$$D_2 = \frac{\sqrt{40}}{3}\left(\alpha^i(\frac{1}{2}\delta_i^j)\alpha_j + \bar{\alpha}(\frac{1}{2})\alpha + \beta^i(\frac{1}{2}\delta_i^j)\beta_j + \bar{\beta}(\frac{1}{2})\beta + \gamma^i(-1\delta_i^j)\gamma_j + \bar{\gamma}(-1)\gamma\right)$$

$$\mathcal{W} = \sqrt{40}d_{ijk}A^iB^jC^k + \sqrt{40}ABC$$

Singlets α and β are chosen to acquire vevs $\rightarrow E_6 \times U(1)_A \times U(1)_B \rightarrow E_6$

$\rightarrow U(1)_A \times U(1)_B$ remain as global symmetries

The Wilson Flux Breaking

Hosotani (1983); Witten (1985); Zoupanos (1988);
Kozimirov, Kuzmin, Tkachev (1989); Kapetanakis, Zoupanos (1989)

$$M^4 \times B_0 \rightarrow M^4 \times B, \qquad B = B_0 / F^{S/R}$$

– $F^{S/R}$ is a freely acting discrete symmetry of B_0.

B becomes multiply connected $\rightarrow$ breaking of H to $K' = C_H(T^H)$

T^H is the image of the homomorphism of $F^{S/R}$ into H

In our case

- $F^{S/R} = \mathbb{Z}_3 \rightarrow B = SU(3)/U(1) \times U(1) \times \mathbb{Z}_3$
- $H = E_6 \times U(1)_A \times U(1)_B$
- $K' = SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B$, still $N = 1$

Matter fields invariant under $F^{S/R} \oplus T^H$ survive

$\rightarrow \gamma_3 = \text{diag}(\mathbf{1}, \omega \mathbf{1}, \omega^2 \mathbf{1}), \quad \omega = e^{2i\pi/3} \in \mathbb{Z}_3$

Irges, Zoupanos (2011)

The surviving gauge fields satisfy the condition:

- $[A_M, \gamma_3] = 0 \quad \Rightarrow A_M = \gamma_3 A_M \gamma_3^{-1}$

The matter counterpart of the above equation is:

- $A^i = \omega\gamma_3 A^i, \quad B^i = \omega^2\gamma_3 B^i, \quad C^i = \omega^3\gamma_3 C^i, \quad A = \omega A, \quad B = \omega^2 B, \quad C = \omega^3 C$

$$E_6 \supset SU(3)_c \times SU(3)_L \times SU(3)_R \qquad 27 = (1,3,\bar{3}) \oplus (3,\bar{3},1) \oplus (\bar{3},1,3)$$

Surviving matter content of the projected theory:

- $A_3 \equiv \Psi_1 \equiv q^c \sim (\bar{3},1,3)_{(3,\frac{1}{2})}, \quad B_2 \equiv \Psi_2 \equiv Q \sim (3,\bar{3},1)_{(-3,\frac{1}{2})},$
  $C_1 \equiv \Psi_3 \equiv L \sim (1,3,\bar{3})_{(0,-1)}, \quad C \equiv \theta \sim (1,1,1)_{(0,-1)}$

Non-trivial monopole charges in $R \rightarrow$ three generations: $\Psi_1^{(l)},\ \Psi_2^{(l)},\ \Psi_3^{(l)},\ C^{(l)}$

Dolan (2003)

$$q^c = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, \quad Q = \begin{pmatrix} -d_L^1 & -d_L^2 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}, \quad L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}$$

For one generation:

$$D^A = \frac{1}{\sqrt{3}}\langle\Psi_i|G^A|\Psi_i\rangle$$

$$D_1 = 3\sqrt{\frac{10}{3}}(\langle\Psi_1|\Psi_1\rangle - \langle\Psi_2|\Psi_2\rangle)$$

$$D_2 = \sqrt{\frac{10}{3}}(\langle\Psi_1|\Psi_1\rangle + \langle\Psi_2|\Psi_2\rangle - 2\langle\Psi_3|\Psi_3\rangle - 2|\theta|^2)$$

$$\frac{2}{g^2}V_F = 360\mathrm{tr}(\hat{q}^{c^\dagger}\hat{q}^c + \hat{Q}^\dagger\hat{Q} + \hat{L}^\dagger\hat{L})$$

$$\begin{aligned}\frac{2}{g^2}V_{soft} &= \left(\frac{4R_1^2}{R_2^2R_3^2} - \frac{8}{R_1^2}\right)\langle\Psi_1|\Psi_1\rangle + \left(\frac{4R_2^2}{R_1^2R_3^2} - \frac{8}{R_2^2}\right)\langle\Psi_2|\Psi_2\rangle \\ &+ \left(\frac{4R_3^2}{R_1^2R_2^2} - \frac{8}{R_3^2}\right)(\langle\Psi_3|\Psi_3\rangle + |\theta|^2) \\ &+ 80\sqrt{2}\left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_1R_2}\right)(d_{abc}\Psi_1^a\Psi_2^b\Psi_3^c + h.c) \\ &= m_1^2\langle\Psi_1|\Psi_1\rangle + m_2^2\langle\Psi_2|\Psi_2\rangle + m_3^2\left(\langle\Psi_3|\Psi_3\rangle + |\theta|^2\right) + (\alpha_{abc}\Psi_1^a\Psi_2^b\Psi_3^c + h.c)\end{aligned}$$

Note: Supersymmetry is broken by D- and F-terms, in addition to the soft breaking.

Further Gauge Breaking of $SU(3)^3$

Babu, He, Pakvasa (1986); Ma, Mondragon, Zoupanos (2004); Leontaris, Rizos (2006); Sayre, Wiesenfeldt, Willenbrock (2006)

Two generations of L acquire vevs that break the GUT:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V \end{pmatrix}, \quad \langle L_s^{(2)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V & 0 & 0 \end{pmatrix}$$

each one alone is not enough to produce the (MS)SM gauge group:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)$$
$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)'_R \times U(1)'$$

Their combination gives the desired breaking:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

electroweak breaking then proceeds by:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} \upsilon_d & 0 & 0 \\ 0 & \upsilon_u & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Choice of Radii

Manolakos, Patellis, Zoupanos (2020)

— Soft trilinear terms $\sim \frac{1}{R_i}$
— Soft scalar masses $\sim \frac{1}{R_i^2}$

Two main possible directions:

- Large $R_i \rightarrow$ calculation of the Kaluza-Klein contributions of the 4D theory
  $\times$ Eigenvalues of the Dirac and Laplace operators unknown.

- Small $R_i \rightarrow$ high scale SUSY breaking
- Small $R_i \sim \frac{1}{M_{GUT}}$ with R_3 slightly different in a specific configuration

$$\rightarrow \quad m_3^2 \sim -\mathcal{O}(TeV^2), \quad m_{1,2}^2 \sim -\mathcal{O}(M_{GUT}^2), \quad a_{abc} \gtrsim M_{GUT}$$

  — supermassive squarks
  — TeV-scale sleptons
  — TeV-scale soft Higgs squared masses

Reminder: in this scenario $M_C = M_{GUT}$

Lepton Yukawas and μ terms

After the GUT breaking, the potential at the minimum becomes

$$\frac{2}{g^2} V_{scalar}^{GUT} = \frac{6}{5}\left(\frac{1}{R_1^4}+\frac{1}{R_2^4}+\frac{1}{R_3^4}\right)+\frac{V^4}{9}$$
$$+\frac{20}{3}\left[(V^2+(\theta_0^{(3)})^2)^2+(V^2+(\theta_0^{(2)})^2)^2+(\theta_0^{(1)})^4\right]$$
$$+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\left(2V^2+|\theta_0^{(3)}|^2+|\theta_0^{(2)}|^2+|\theta_0^{(1)}|^2\right)$$

$$\rightarrow \quad \langle\theta^{(3)}\rangle \sim \mathcal{O}(TeV)\,, \quad \langle\theta^{(1,2)}\rangle \sim \mathcal{O}(M_{GUT})$$

- The two global $U(1)$s forbid Yukawa terms for leptons
  $\rightarrow$ introduce higher-dimensional operators: $L\overline{e}H_d\left(\frac{\overline{K}}{M}\right)^3$
- μ terms for each generation of Higgs doublets are absent
  $\rightarrow$ solution through higher-dimensional operators: $H_u^{(i)}H_d^{(i)}\overline{\theta}^{(i)}\frac{\overline{K}}{M}$

— $\overline{K}$ is the vev of the conjugate scalar component of either $S^{(i)}$, $\nu_R^{(i)}$ or $\theta^{(i)}$, or any combination of them

Approximate Scale of Parameters

Parameter	Scale
soft trilinear couplings	$\mathcal{O}(GUT)$
squark masses	$\mathcal{O}(GUT)$
slepton masses	$\mathcal{O}(TeV)$
soft Higgs masses	$\mathcal{O}(TeV)$
$\mu^{(3)}$	$\mathcal{O}(TeV)$
$\mu^{(1,2)}$	$\mathcal{O}(GUT)$
unified gaugino mass M_U	$\mathcal{O}(TeV)$

Gauge Unification

Since many SUSY parameters are comparable to M_{GUT}, we consider them to decouple at an intermediate scale M_{int}.

The 1-loop gauge β-functions are:

$$2\pi\beta_i = b_i\alpha_i^2$$

— b_i depends on the particle content

Scale	b_1	b_2	b_3
M_{EW}-M_{TeV}	$\frac{21}{5}$	-3	-7
M_{TeV}-M_{int}	$\frac{11}{2}$	$-\frac{1}{2}$	-5
M_{int}-M_{GUT}	$\frac{39}{5}$	3	-3

- $\alpha_{1,2}$ are used as input to determine M_{GUT}
- 0.3% uncertainty at the unification boundary

$\rightarrow$ α_3 is predicted within 2σ of the experimental value

$$a_s(M_Z) = 0.1218$$

$$a_s^{EXP}(M_Z) = 0.1187 \pm 0.0016$$

Scale	GeV
M_{GUT}	$\sim 1.7 \times 10^{15}$
M_{int}	$\sim 9 \times 10^{13}$
M_{TeV}	~ 1500

✓ No proton decay problem: $U(1)_A = -\frac{1}{9}B$

Higgs Potential

The analysis is restricted to the third generation

$$\begin{aligned} V_{Higgs} = & \left(3|\mu^{(3)}|^2 + m_3^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) + \left(3|\mu^{(3)}|^2 + m_3^2\right)\left(|H_u^0|^2 + |H_u^+|^2\right) \\ & + b^{(3)}\left[(H_u^+ H_D^- - H_u^0 H_D^0) + c.c.\right] \\ & + \frac{10}{3} g^2 \Big[|H_d^0|^4 + |H_d^-|^4 + |H_u^0|^4 + |H_u^+|^4 + \\ & \qquad 2|H_d^0|^2|H_d^-|^2 + 2|H_d^-|^2|H_u^0|^2 + 2|H_d^0|^2|H_u^+|^2 + 2|H_u^0|^2|H_u^+|^2\Big] \\ & + \frac{20}{3} g^2 \left[|H_d^0|^2|H_u^0|^2 + |H_d^-|^2|H_u^+|^2\right] - 20g^2\left[\overline{H_d^0} H_d^- \overline{H_u^0} H_u^+ + c.c.\right] \end{aligned}$$

$\rightarrow$ Comparison with standard 2 Higgs doublet potential gives:

- $\lambda_1 = \lambda_2 = \lambda_3 = \frac{20}{3} g^2$
- $\lambda_4 = 20g^2$
- $\lambda_5 = \lambda_6 = \lambda_7 = 0$

– $\lambda_{5,6,7} = 0$ in a (even broken) SUSY theory
– These relations are boundary conditions at M_{GUT}

Boundary Conditions and Uncertainties

At the unification scale we have the following boundary conditions and their respective uncertainties due to threshold corrections (such uncertainties also appear at the TeV boundary): Kubo, Mondragon, Olechowski, Zoupanos (1996)

GUT BC	GUT Unc.	TeV Unc.
$g_3 = g$	0.3%	
$Y_{t,b} = g$	6%	2%
$\lambda_{1,2} = \frac{20}{3} g^2$	8%	8%
$\lambda_3 = \frac{20}{3} g^2$	7%	5%
$\lambda_4 = 20 g^2$	7%	5%

The τ lepton Yukawa emerges from a higher-dimensional operator and has significantly wider freedom. The standard τ lepton mass is used as input.

1-loop Results

1-loop β-functions used throughout the analysis that change between the three landmark scales M_{GUT}, M_{int} and M_{TeV}.

$\rightarrow$ $m_b(M_Z)$ and $\hat{m}_t$ are predicted within 2σ of the experimental values

- $m_b(M_Z) = 3.00 \ \text{GeV}$ $\quad$ $m_b^{EXP}(M_Z) = 2.83 \pm 0.10 \ \text{GeV}$
- $\hat{m}_t = 171.6 \ \text{GeV}$ $\quad$ $\hat{m}_t^{EXP} = 172.4 \pm 0.7 \ \text{GeV}$

$\rightarrow$ m_h is predicted within 1σ of the experimental value

- $m_h = 125.18 \ \text{GeV}$ $\quad$ $m_h^{EXP} = 125.10 \pm 0.14 \ \text{GeV}$

– Large $\tan\beta \sim 48$

– $M_A \sim 2000 - 3000 \ \text{GeV}$ ✓

– LSP $\sim 1500 \ \text{GeV}$

Conclusions

- Special choice of coset radii for split-like SUSY 2HDM
- $M_{GUT} \sim 10^{15}$ GeV — no proton decay
- 1-loop predictions agree with LHC measurements
- LSP $\gtrsim$ 1500 GeV
- $M_A \sim 2000 - 3000$ GeV

Work in preparation/planned

- Full (light) SUSY spectrum
- 2-loop analysis
- Application of B-physics constraints

o Calculation of CDM relic density

o Investigation of discovery potential at existing and future colliders

— Examination of high energy potential $\rightarrow$ test agreement with observed value of cosmological constant

Thank you for your attention!

...a few more things

Chiral Fermions

The $(SU(2) \times SU(2)) \times SO(d)$ branching rule is:

$$\sigma_D = (2,1;\sigma_d) + (1,2;\sigma'_d) \qquad \sigma'_D = (2,1;\sigma'_d) + (1,2;\sigma_d)$$

where σ_D, σ'_D are non-self conjugate spinors of $SO(1, D-1)$.

Starting with Dirac fermions $\rightarrow$ equal number of LH and RH reps of H.

- Odd D : Weyl condition cannot be applied $\rightarrow$ equal number of LH and RH reps of H.
- Even D : Weyl condition selects either σ_D or σ'_D .

$$\Gamma^{D+1}\Psi_{\pm} = \pm\Psi_{\pm} \qquad \text{Weyl condition}$$

$$\Psi = \Psi_{+} \oplus \Psi_{-} = \sigma_D + \sigma'_D \,,$$

- $D = 4n+2$: We start with a vector-like rep F $\rightarrow$ σ_d is non-self-conjugate and $\sigma'_d = \bar{\sigma}_d$.

$$SO(d) \supset R: \qquad \sigma_d = \sum \sigma_k \,, \quad \bar{\sigma}_d = \sum \bar{\sigma}_k$$

$$G \supset R_G \times H: \qquad F = \sum_i (r_i, h_i)$$

(r_i, h_i) either self-conjugate or have a partner $(\bar{r}_i, \bar{h}_i)$

Chiral Fermions

– Rule from $\sigma_d \rightarrow$ 4D LH fermions $f_L = \sum h_k^L$

σ_d is non-self-conjugate $\rightarrow f_L$ is non-self-conjugate.

– Similarly from $\bar{\sigma}_d \rightarrow$ 4D RH rep $\sum \bar{h}_k^R = \sum h_k^L$

F vector-like $\rightarrow \bar{h}_k^R \sim h_k^L \rightarrow H$ is chiral theory with double spectrum.

- Majorana condition $\rightarrow$ eliminate the doubling of the fermion spectrum.

Majorana condition forces f_R to be the charge conjugate of f_L.

All together:

- $D = 4n + 2$
- Fermions in vector-like rep F of the 10D G gauge group
- Weyl + Majorana conditions imposed

$\rightarrow$ chiral 4D theory!

2HDM General Potential

$$
\begin{aligned}
V =&(|\mu|^2 + m_{H_1}^2)(H_1^\dagger H_1) + (|\mu|^2 + m_{H_2}^2)(H_2^\dagger H_2) + b(H_1^\dagger H_2 + \text{hc}) \\
&+ \frac{1}{2}\lambda_1 (H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2 (H_2^\dagger H_2)^2 \\
&+ \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\
&+ \{\frac{1}{2}\lambda_5 (H_1^\dagger H_2)^2 + [\lambda_6 (H_1^\dagger H_1) + \lambda_7 (H_2^\dagger H_2)](H_1^\dagger H_2) + \text{hc}\}
\end{aligned}
$$

Supersymmetry imposes tree level relations among couplings

$$
\begin{aligned}
\lambda_1 = \lambda_2 &= \frac{1}{4}(g^2 + g'^2) \\
\lambda_3 &= \frac{1}{4}(g^2 - g'^2)\ , \ \lambda_4 = -\frac{1}{4}g^2 \\
\lambda_5 = \lambda_6 &= \lambda_7 = 0
\end{aligned}
$$

Gunion, Haber (1986); Quiros (1997);
Branco, Ferreira, Lavoura, Rebelo, Sher, Silva - Review (2011)