Intermittency analysis in heavy ion collisions: a review of the current status and challenges.

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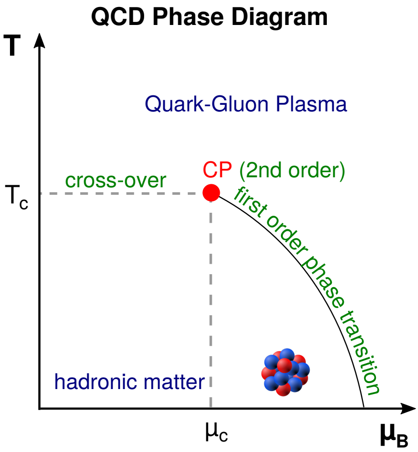
Outline

The phase diagram of QCD

- Phase diagram of strongly interacting matter in T and μ_B $\Rightarrow$
- Phase transitions from hadronic matter to quark-gluon plasma:
  - Low μ_B & high T $\rightarrow$ cross-over (lattice QCD)
  - High μ_B & low T $\rightarrow$ 1st order (effective models)

  $\Rightarrow$ 1st order transition line ends at Critical Point (CP) $\rightarrow$ 2nd order transition
- At the CP: scale-invariance, universality, collective modes $\Rightarrow$ good physics signatures

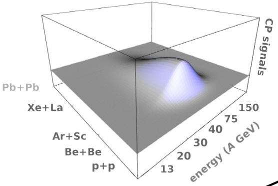


- Detection of the QCD Critical Point (CP): Main goal of many heavy-ion collision experiments (in particular the SPS NA61/SHINE experiment)
- Look for observables tailored for the CP; Scan phase diagram by varying energy and size of collision system.

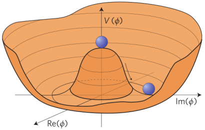
Critical Observables & the Order Parameter (OP)

CP observables

Event-by-event (global) fluctuations:
Variance, skewness, kurtosis –
sensitive to experimental acceptance



Local:
density fluctuations of OP
in transverse space
(stochastic fractal)



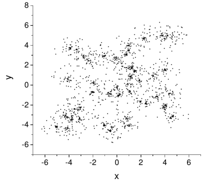
Order parameter

A quantity that:
- is $= 0$ in disordered phase (QGP)
- is $\neq 0$ in ordered phase (hadrons)

Chiral condensate
$\sigma(\boldsymbol{x}) = \langle \bar{q}(\boldsymbol{x}) q(\boldsymbol{x}) \rangle$

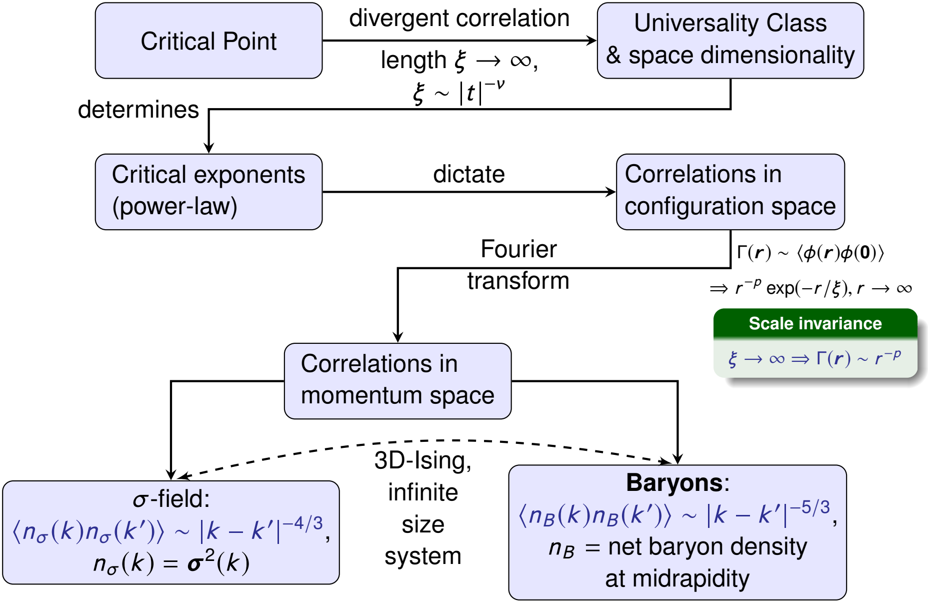
coupling ⇕ induced critical fluctuations*

Net baryon density
$n_B(\boldsymbol{x})$



*[Y. Hatta and M. A. Stephanov, PRL**91**, 102003 (2003)]

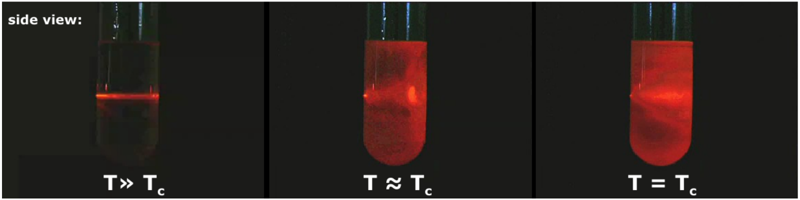
Self-similar density fluctuations near the CP



[Antoniou *et al*, Nucl. Phys. A **693** 799–824 (2001)]

[Antoniou *et al*, PRL **97**, 032002 (2006)]

Observing power-law fluctuations through intermittency



[Csorgo, Tamas, PoS CPOD2009 (2009) 035]

Experimental observation of local, power-law distributed fluctuations of net baryon density

⇓

Intermittency in transverse momentum space at mid-rapidity (Critical opalescence in ion collisions)

[F.K. Diakonos, N.G. Antoniou and G. Mavromanolakis, PoS (CPOD2006) 010, Florence]

- Net proton density carries the same critical fluctuations as the net baryon density, and can be substituted for it.

  [Y. Hatta and M. A. Stephanov, PRL**91**, 102003 (2003)]

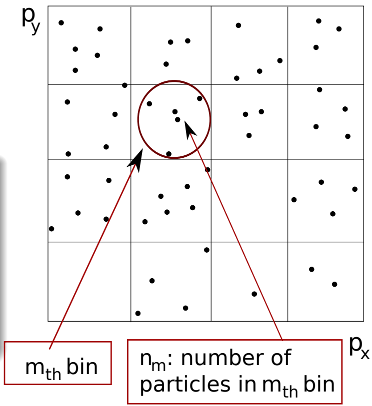
- Furthermore, antiprotons can be ignored (their multiplicity is negligible compared to protons), and we can analyze just the proton density.

Observing power-law fluctuations: Factorial moments

- Pioneered by Białas and others, as a method to detect non-trivial dynamical fluctuations in high energy nuclear collisions
- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size $\Leftrightarrow$ number of cells M:

$$F_2(M) \equiv \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i-1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2},$$

where $\langle \ldots \rangle$ denotes averaging over events.



[A. Bialas and R. Peschanski, *Nucl. Phys. B* **273** (1986) 703-718]
[A. Bialas and R. Peschanski, *Nucl. Phys. B* **308** (1988) 857-867]
[J. Wosiek, *Acta Phys. Polon.* **B 19** (1988) 863-869]
[A. Bialas and R. Hwa, *Phys. Lett.* **B 253** (1991) 436-438]
[Z. Burda, K. Zalewski, R. Peschanski, J. Wosiek, *Phys. Lett. B* **314** (1993) 74-78]

$p_{x,y}$ range in present analysis:
$-1.5 \le p_{x,y} \le 1.5$ GeV/c
$M^2 \sim 10\,000$

Background subtraction – the correlator $\Delta F_2(M)$

- Background of non-critical pairs must be subtracted from experimental data;

Partitioning of pairs into critical/background

$$\langle n(n-1)\rangle = \underbrace{\langle n_c(n_c-1)\rangle}_{\text{critical}} + \underbrace{\langle n_b(n_b-1)\rangle}_{\text{background}} + \underbrace{2\langle n_b n_c\rangle}_{\text{cross term}}$$

$$\underbrace{\Delta F_2(M)}_{\text{correlator}} = \underbrace{F_2^{(d)}(M)}_{\text{data}} - \lambda(M)^2 \cdot \underbrace{F_2^{(b)}(M)}_{\text{background}} - 2 \cdot \underbrace{\lambda(M)}_{\text{ratio } \frac{<n>_b}{<n>_d}} \cdot (1-\lambda(M))\, f_{bc}$$

- If $\lambda(M) \lesssim 1$ (dominant background) $\Rightarrow$
  cross term negligible & $F_2^{(b)}(M) \sim F_2^{\text{mix}}(M)$ (Critical Monte Carlo* simulations)
  then:

$$\Delta F_2(M) \simeq F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

Intermittency restored in $\Delta F_2(M)$:

$$\Delta F_2(M) \sim \left(M^2\right)^{\varphi_2}, \; M \gg 1$$

$\Longrightarrow$

φ_2: intermittency index

Theoretical prediction* for φ_2

$$\varphi_{2,cr}^{(p)} = \frac{5}{6} \; (0.833\ldots)$$

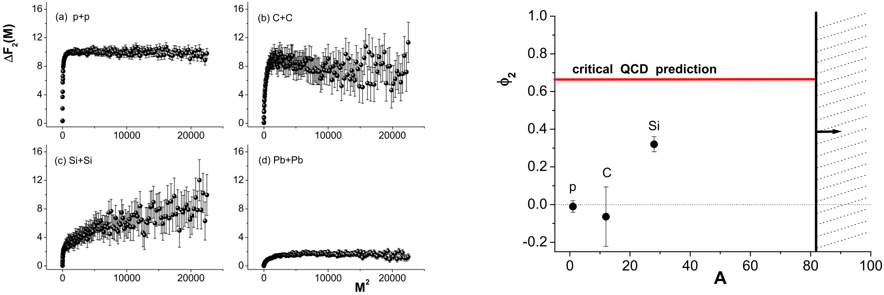
*[Antoniou *et al*, PRL **97**, 032002 (2006)]

NA49 C+C, Si+Si, Pb+Pb @ $\sqrt{s_{NN}} \simeq 17$ GeV – dipions

- **3 sets of NA49 collision systems** at **158A GeV/c** ($\sqrt{s_{NN}} \simeq 17$ **GeV**)

  [T. Anticic *et al*, Phys. Rev. C **81**, 064907 (2010); T. Anticic *et al*., Eur. Phys. J. C 75:587 (2015)]

- **Intermittent behaviour** ($\phi_2^{(\sigma)} \simeq 0.35$) of **dipion pairs** ($\pi^+, \pi^-$) in **transverse momentum space** observed in **central Si+Si collisions** at **158A GeV**.

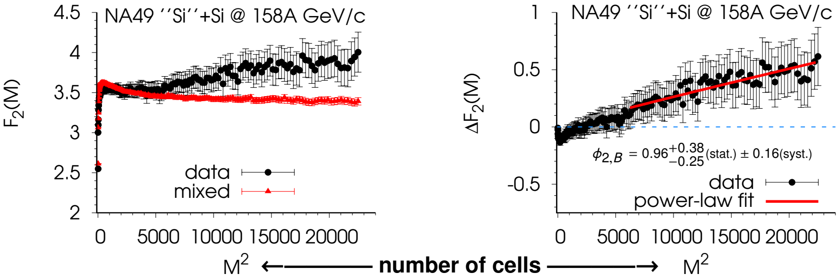


[T. Anticic *et al*, Phys. Rev. C **81**, 064907 (2010)]

- **No such** power-law behaviour observed in **central C+C and Pb+Pb** collisions at **the same energy**.

NA49 C+C, Si+Si, Pb+Pb @ $\sqrt{s_{NN}} \simeq 17$ GeV – protons

- **Factorial moments of proton transverse momenta** analyzed at **mid-rapidity**



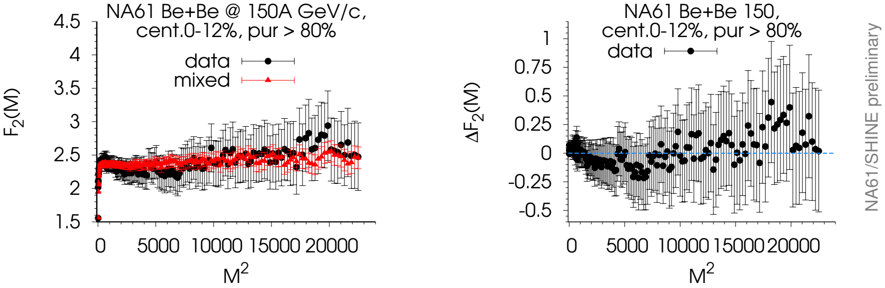
- $F_2(M)$, $\Delta F_2(M)$ errors estimated by the **bootstrap method**

  [W.J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).]

- Fit with $\Delta F_2^{(e)}(M\,;\,C,\phi_2) = 10^C \cdot \left(\frac{M^2}{M_0^2}\right)^{\phi_2}$, for $M^2 \geq 6000$ ($M_0^2 \equiv 10^4$)
- **Evidence** for intermittency in "Si"+Si – but **large statistical errors**.

NA61/SHINE intermittency: Be+Be @ $\sqrt{s_{NN}} \simeq 17$ GeV

- Intermittency analysis is pursued within the framework of the **NA61/SHINE experiment**, inspired by the **positive**, if ambiguous, **NA49 Si+Si** result.
  [T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]
- **Two NA61/SHINE systems** were initially examined:
  ^{7}Be + ^{9}Be and **^{40}Ar + ^{45}Sc @ 150A GeV/c ($\sqrt{s_{NN}} \simeq 17$ GeV)**

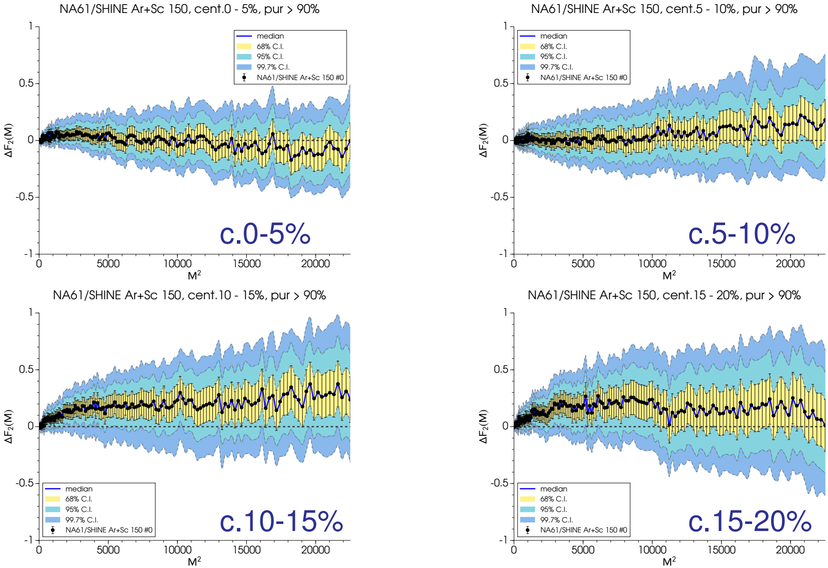


- $F_2(M)$ of data and mixed events **overlap** ⇒
- Subtracted moments $\Delta F_2(M)$ **fluctuate around zero** ⇒
  No intermittency effect is observed in **Be+Be**.

NA61/SHINE $^{40}Ar + ^{45}Sc$ @ $\sqrt{s_{NN}} \simeq 17$ GeV

- **First indication** of **intermittency** in **mid-central Ar+Sc 150A GeV/c** collisions presented at **CPOD2018**; In **2019**, an **extended event statistics set** was analysed;
- A **scan in centrality** was performed, in the **0-20% range**, in **5% and 10% intervals**, as centrality may influence the system's **freeze-out temperature**;
- **Event statistics** were of the order of $\sim$ **400K events** per **10% centrality interval**;
- **Bootstrap confidence intervals** are calculated for $\Delta F_2(M)$ values;
- Due to **M-bin correlations**, determining **confidence intervals** for ϕ_2 is **challenging**; **Various approaches** to the problem are being investigated, such as **model-weighting**;
- **Ar+Sc** system is still **inconclusive**.

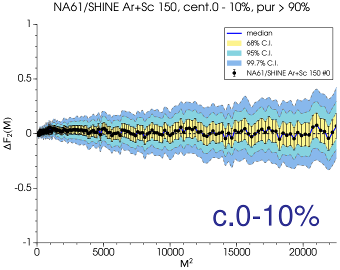
NA61/SHINE Ar+Sc @150A GeV/c: 5% cent. intervals

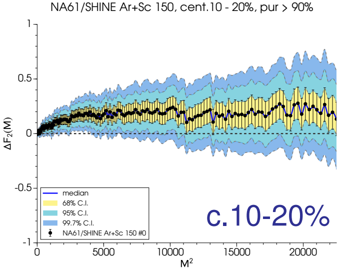


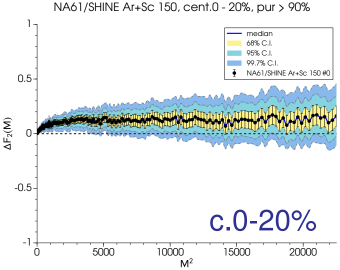
NA61/SHINE preliminary

- **No signal** in **c.0-5%, 5-10%**; **weak signal** in **c.10-15%, 15-20%**.

NA61/SHINE Ar+Sc @150A GeV/c: 10/20% cent. intervals





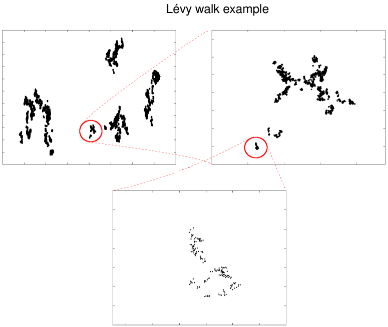


NA61/SHINE preliminary

- **Centrality dependence** is evident; **Some signal indication** in **c.10-20%**.

Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC* code:
  - Only protons produced
  - One cluster per event, produced by random Lévy walk:
    $\tilde{d}_F^{(B,2)} = 1/3 \Rightarrow \phi_2 = 5/6$
  - Lower / upper bounds of Lévy walks $p_{min,max}$ plugged in.
  - Cluster center exponential in p_T, slope adjusted by T_c parameter.
  - Poissonian proton multiplicity distribution.

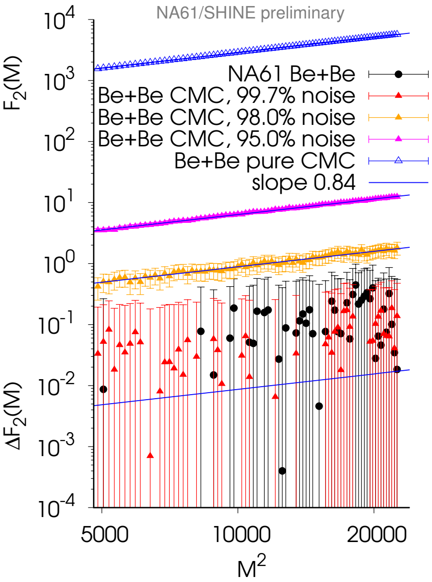


Input parameters

Parameter	p_{min} (MeV)	p_{max} (MeV)	λ_{Poisson}	T_c (MeV)
Value	$0.1 \to 1$	$800 \to 1200$	$\langle p \rangle_{\text{non-empty}}$	163

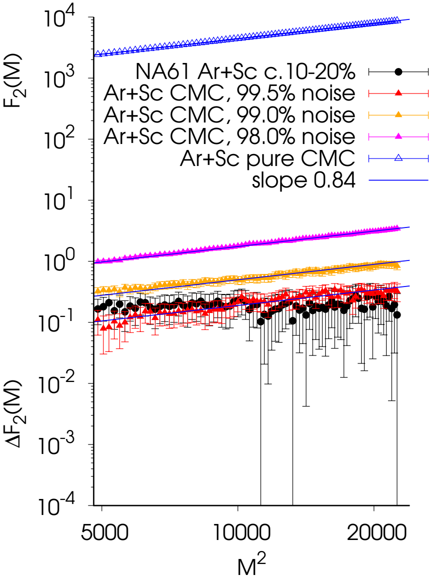
*[Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]

Be+Be – Critical Monte Carlo toy model



- **Critical Monte Carlo (CMC)** + **random background** in transverse momentum space;
- $\Delta F_2(M)$ **retains critical behaviour** of **pure CMC ($\lambda = 0$)**, even when their moments differ by orders of magnitude!
- Preliminary analysis with CMC simulation indicates an upper limit of **$\sim$ 0.3% critical protons ($\lambda \approx 0.997$)** [PoS(CPOD2017) 054]
- CMC results show our approximation (dominant background) is **reasonable**.

Ar+Sc – Critical Monte Carlo



- Preliminary analysis with CMC simulation indicates an upper limit of **$\sim$ 0.5% critical protons**

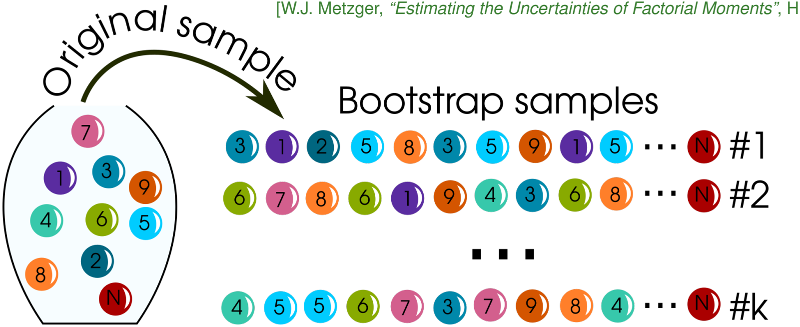
Challenges in proton intermittency analysis

1. **Particle species**, especially **protons**, **cannot** be **perfectly identified** experimentally; candidates will always contain a **small percentage of impurities**;
2. **Experimental momentum resolution** sets a **limit** to how **small a bin size (large M)** we can probe;
3. A **finite (small) number** of **usable events** is available for analysis; the **"infinite statistics" behaviour** of $\Delta F_2(M)$ must be **extracted** from these;
4. **Proton multiplicity** for medium-size systems is **low** (typically $\sim$ **2 – 3 protons** per event, in the window of analysis) – and the demand for **high proton purity lowers** it still more;
5. M-bins are **correlated** – the **same events** are used to calculate **all** $F_2(M)$! This **biases fits** for the **intermittency index** ϕ_2, and makes **confidence interval estimation hard**.

Intermittency analysis tools: the bootstrap

- Random **sampling** of events, **with replacement**, from the original set of events;
- k bootstrap samples ($k \sim 1000$) of the **same number of events** as the original sample;
- Each **statistic** ($\Delta F_2(M)$, ϕ_2) **calculated for bootstrap** samples as for the **original**; [B. Efron, *The Annals of Statistics* **7**,1 (1979)]
- **Variance of bootstrap values** estimates **standard error** of **statistic**.

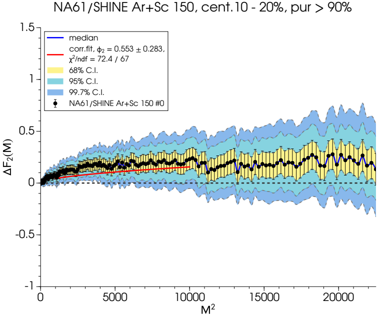
[W.J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).]



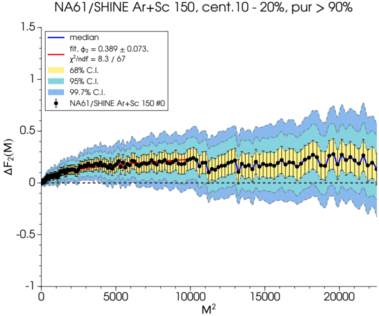
Intermittency analysis tools: correlated fit

- Possible to perform **correlated fits for ϕ_2**, with M-correlation matrix **estimated via bootstrap**;

Correlated fit



Uncorrelated fit



- **Replication of events** means bootstrap sets are **not independent** of the original: **magnitude of variance and covariance** estimates **can be trusted**, but central values will be **biased** to the original sample;
- **Correlated fits for ϕ_2** are known to be **unstable**;

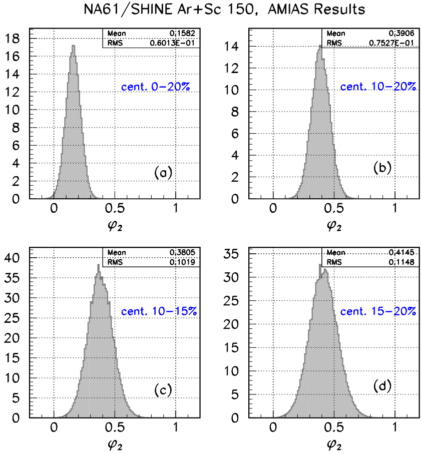
  [B. Wosiek, APP B21, 1021 (1990); C. Michael, PRD49, 2616 (1994)]

Intermittency analysis tools: the AMIAS scheme

Avoid fitting,
use model weighting!

- **AMIAS algorithm** (A Model-Independent Analysis Scheme) can be used to extract **model-parameter distributions** such as ϕ_2 from sets of data;
  [C. N. Papanicolas, E. Stiliaris, arXiv:1205.6505 (2012); AIP Conf. Proc. **904**, 257 (2007)]
- It works by sampling parameter space at random, then weighting selected models by goodness-of-fit function ($e^{-\chi^2/2}$) to the dataset.

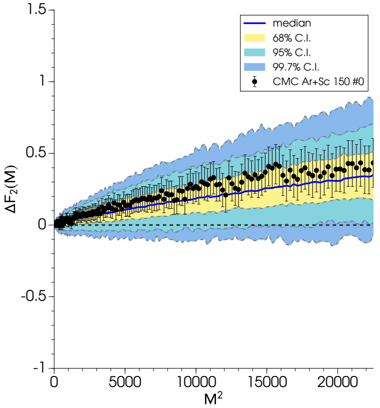
Model: $\Delta F_2(M; a_0, \phi_2) = 10^{a_0} \left(\frac{M^2}{10^4}\right)^{\phi_2}$



[N. G. Antoniou *et al*, Nucl. Phys. A **1003** 122018 (2020).]

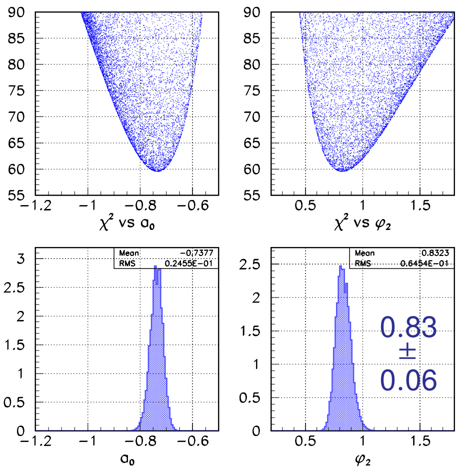
AMIAS scheme example: Critical Monte Carlo

CMC Ar+Sc 150, cent.10 - 20%, bkg. = 99.3%



- Testing **millions** of **possible solutions** on 400 independent $F_2(M)$ samples!
- Also works on **bootstrap** samples (although **not independent**).

CMC Ar+Sc 150, AMIAS Results



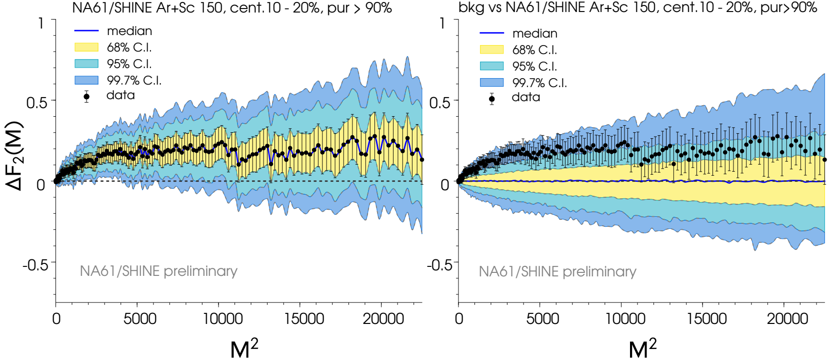
[N. G. Antoniou *et al*, Nucl. Phys. A **1003** 122018 (2020).]

Model: $\Delta F_2(M; a_0, \phi_2) = 10^{a_0}\left(\frac{M^2}{10^4}\right)^{\phi_2}$

Ar+Sc 150 $\Delta F_2(M)$ – statistical significance of signal

- **Plan:** compare $\Delta F_2(M)$ bootstrap distribution of **Ar+Sc data** to **uncorrelated proton moments** of the same event statistics.

1. **$\Delta F_2(M)$, NA61/SHINE Ar+Sc @ 150 GeV/c bootstrap distributions**
2. **$\Delta F_2(M)$, random background sub-sample distributions**



1. ~ **85 – 95%** of $\Delta F_2(M)$ values **above zero** in Ar+Sc 150
2. ~ **85 – 95%** of **random background $\Delta F_2(M)$** values **below** Ar+Sc 150 average

- Roughly **5 – 15% chance** of random background producing a spurious "effect"

Conclusions & Outlook

- **Intermittency analysis** of proton density is a **promising strategy** for detecting the **Critical Point**;
- However, this analysis is **challenging** in the context of an **actual heavy-ion collision experiment**, always **constrained** in terms of **available statistics, particle multiplicity**, and **proton identification**;
- **New techniques** were developed to **better determine $\Delta F_2(M)$ and ϕ_2 uncertainties** (**bootstrap errors**, **AMIAS weighting**);
- **Detailed exploration** of **refined models** with **critical & non-critical** components is certainly needed, in order to **assess experimental data**;
- **Analysis** of **different systems** and **collision energies** is ongoing.

Thank You!

Acknowledgements

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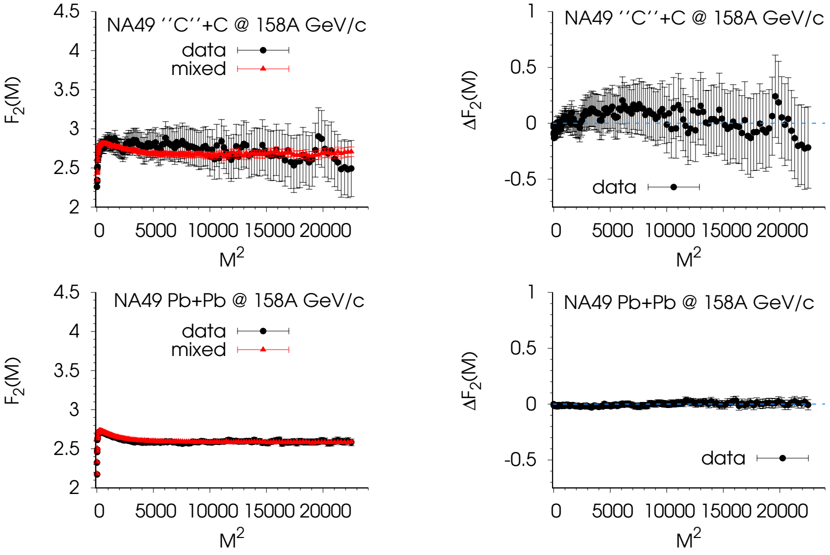
Backup Slides

Backup Slides Outline

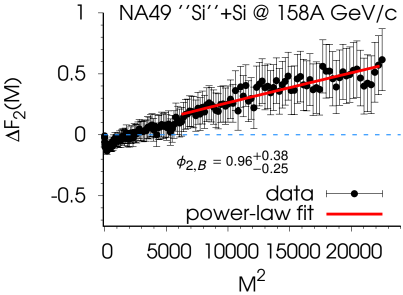
NA49 C+C, Si+Si, Pb+Pb @ $\sqrt{s_{NN}} \simeq 17$ GeV

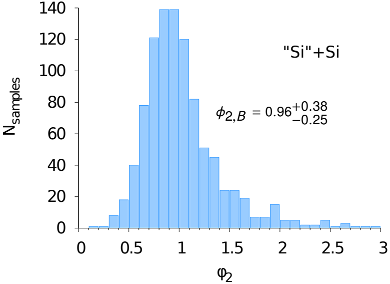


- No intermittency detected in the “C”+C, Pb+Pb datasets.

NA49 C+C, Si+Si, Pb+Pb @ $\sqrt{s_{NN}} \simeq 17$ GeV

- Evidence for intermittency in “Si”+Si – but large statistical errors.
- Distribution of φ_2 values, $P(\varphi_2)$, and confidence intervals for φ_2 obtained by fitting individual bootstrap samples [B. Efron, *The Annals of Statistics* **7**,1 (1979)]

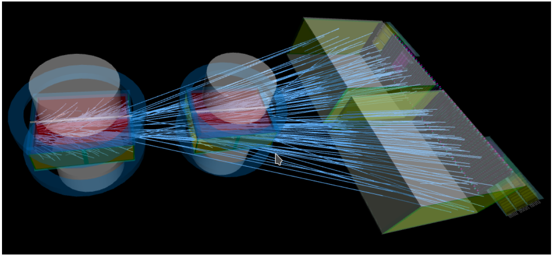




- Bootstrap distribution of ϕ_2 values is highly asymmetric (due to closeness of $F_2^{(d)}(M)$ to $F_2^{(m)}(M)$).
- Uncorrelated fits used, but errors between M are correlated!
- Estimated intermittency index: $\phi_{2,B} = 0.96^{+0.38}_{-0.25}$(stat.) $\pm$ 0.16(syst.)

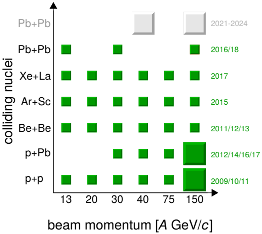
[T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]

The NA61/SHINE experiment



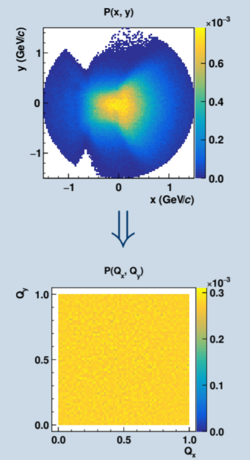
- Fixed-target, high-energy collision experiment at CERN SPS;
- Reconstruction & identification of emitted protons in an extended regime of rapidity, with precise evaluation of their momentum vector;
- Centrality of the collision measured by a forward Projectile Spectator Detector (PSD);

- Direct continuation of NA49
- Search for Critical Point signatures



Independent bin analysis with cumulative variables

- **M-bin correlations** complicate uncertainties estimations for $\Delta F_2(M)$ & ϕ_2; one way around this problem is to use **independent bins** – a **different subset** of events is used to calculate $F_2(M)$ for **each** M;
- **Advantage:** correlations are no longer a problem; **Disadvantage:** we **break up statistics**, and can only **calculate** $F_2(M)$ for **a handful of bins**.
- Furthermore, instead of p_x and p_y, one can use **cumulative quantities:** [Bialas, Gazdzicki, PLB 252 (1990) 483]



(example for 0-5% Ar+Sc at 150A GeV/c)

$$Q_x(x) = \int_{min}^{x} P(x)dx \Big/ \int_{min}^{max} P(x)dx;$$

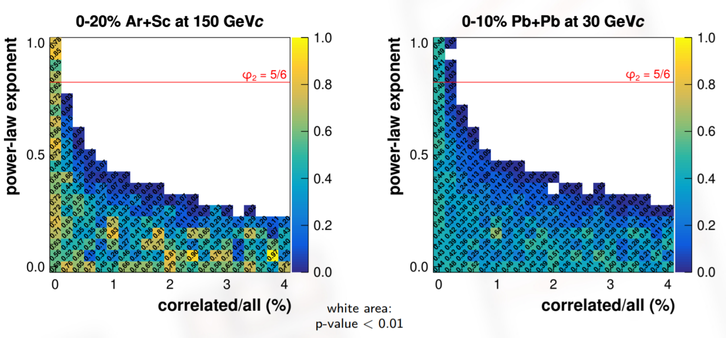
$$Q_y(x,y) = \int_{ymin}^{y} P(x,y)dy \Big/ P(x)$$

- transform any distribution into **uniform** one $(0,1)$;
- **remove the dependence** of F_2 on the shape of the **single-particle distribution**;
- approximately **preserves ideal power-law** correlation function. [Antoniou, Diakonos, https://indico.cern.ch/event/818624/]

Pb+Pb @ 30 GeV/c analysis ($\sqrt{s_{NN}} \simeq 7.6$ GeV)

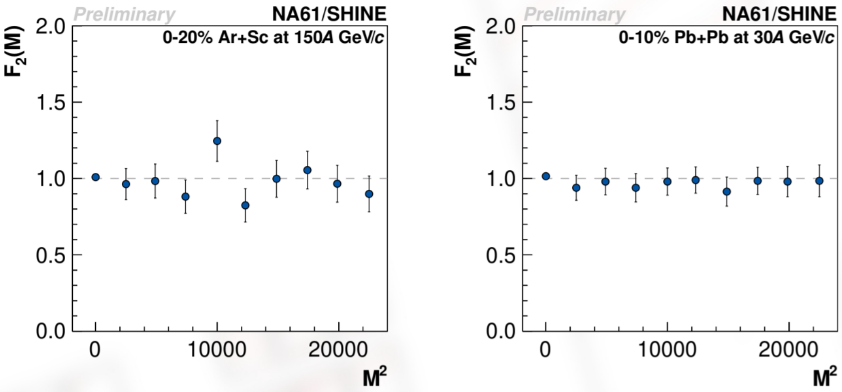
- **NA61/SHINE** presently **undergoes the effort** of a **concentrated analysis** of **new Pb + Pb data** at the energies of $\sqrt{s_{NN}}$ **= 5.1 – 7.6 GeV**, to verify the **latest STAR results** (arXiv: 2001.02852) claimed as a **possible signal** of the **Critical Point**;
- So far, preliminary analysis shows **no indication of intermittency** in Pb+Pb.

Exclusion plots: Ar+Sc, Pb+Pb



[T. Czopowicz, *Search for critical point via intermittency analysis in NA61/SHINE*, C.P.O.D. 2021 Online]

Independent bin analysis – Ar+Sc & Pb+Pb results



statistical uncertainties only

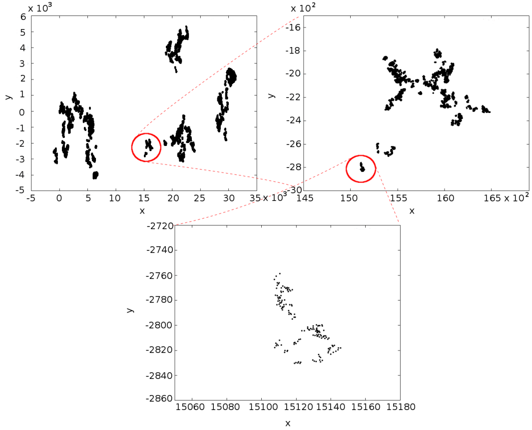
No indication for power-law increase with bin size

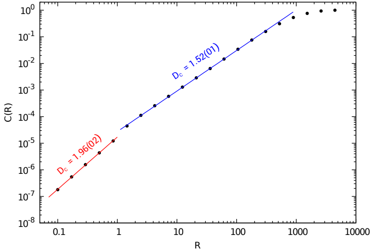
[T. Czopowicz, *Search for critical point via intermittency analysis in NA61/SHINE*, C.P.O.D. 2021 Online]

Simulating fractal sets through random Lévy walks

- In D-dimensional space, we can simulate a fractal set of dimension d_F, $D-1 < d_F < D$, through a random walk with step size Δr distribution:

$$Pr(\Delta r > \Delta r_0) = \begin{cases} 1, & \text{for } \Delta r_0 < \Delta r_d \\ C\,\Delta {r_0}^{-d_F}, & \text{for } \Delta r_d \leq \Delta r_0 \leq \Delta r_u \\ 0, & \text{for } \Delta r_0 > \Delta r_u \end{cases}$$

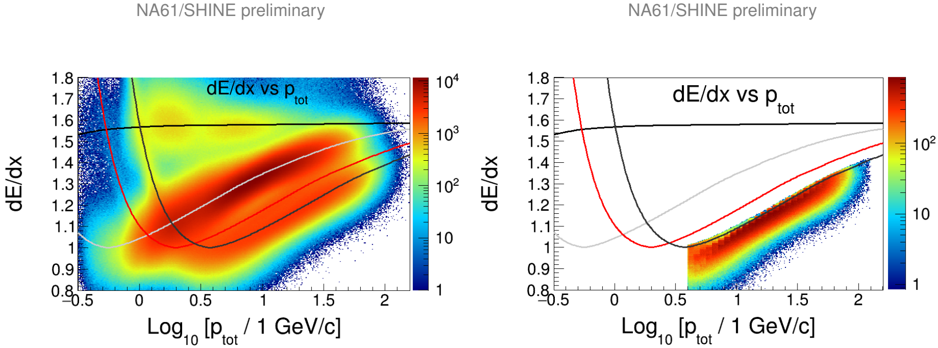




- The result is a set of fractal correlation dimension,

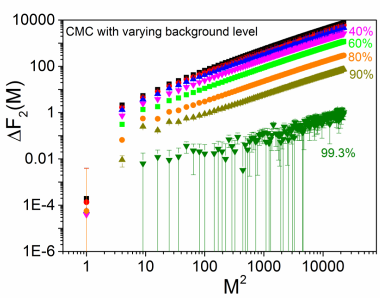
$$C(R) = \frac{2}{N(N-1)} \sum_{\substack{i,j \\ i<j}} \Theta(R - |\boldsymbol{x}_i - \boldsymbol{x}_j|)$$

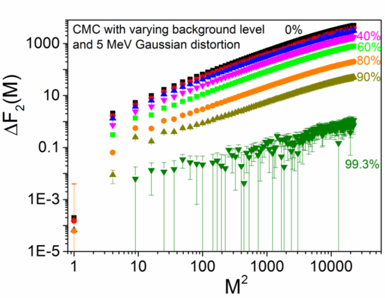
Proton selection

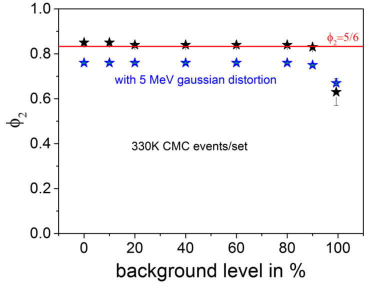


- Particle ID through energy loss dE/dx in the Time Projection Chambers (TPCs);
- Employ p_{tot} region where Bethe-Bloch bands do not overlap (3.98 GeV/c $\leq p_{tot} \leq$ 126 GeV/c);
- Mid-rapidity region ($|y_{CM}| < 0.75$) selected for present analysis.

Momentum resolution: effect on intermittency



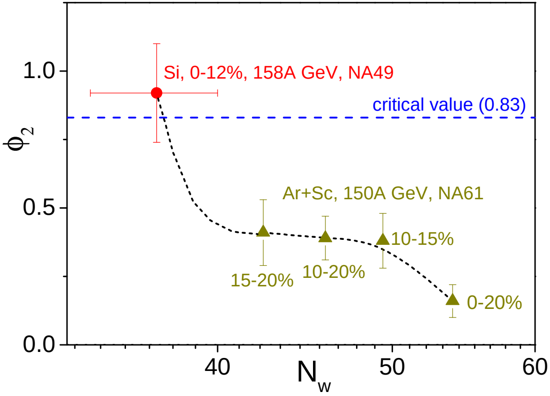




- **CMC + background + Gaussian noise** (**5 MeV radius**);
- A **5 MeV Gaussian error** in $\boldsymbol{p_x, p_y}$ leads to $\sim$ **10% discrepancy** in the **value of $\boldsymbol{\phi_2}$**.
- For **very large backround** values **(> 99%)**, momentum resolution **matters little** to the **overall distortion**.

AMIAS on NA49 & NA61/SHINE data – ϕ_2 vs $N_{wounded}$

- ϕ_2 AMIAS confidence intervals calculated for NA49 & NA61/SHINE systems with indications of intermittency
- Corresponding mean number of participating ("wounded") nucleons N_w estimated via geometrical Glauber model simulation



[N. G. Antoniou *et al*, Nucl. Phys. A **1003** 122018 (2020)]

- Peripheral Ar+Sc collisions approach Si + Si criticality $\Rightarrow$ insight of how the critical region looks as a function of baryon density μ_B.
- Check theoretical predictions* for **narrow critical scaling region in T & μ_B**

*[F. Becattini *et. al.*, arXiv:1405.0710v3 [nucl-th] (2014); N. G. Antoniou, F. K. Diakonos, arXiv:1802.05857v1 [hep-ph] (2018)]