# Celestial Amplitudes from UV to IR

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Based on hep-th/2012.04208 with N. Arkani-Hamed, A. Raclariu and A. Strominger

## A Brief Digression ...

# "Celestial Operator Product Expansions and $\mathrm{w}_{1+\infty}$ Symmetry for All Spins"

**2108.07763 with Elizabeth (Mina) Himwich and Kyle Singh**

1. Use **Poincaré symmetry** to fix OPE coefficients between massless Mellin primaries of **any spin**:

$$\mathcal{O}_{h_1,\bar{h}_1}(z,\bar{z})\mathcal{O}_{h_2,\bar{h}_2}(0,0) \sim \frac{1}{z}\sum_{p}\sum_{m=0}^{\infty}\frac{\gamma_p^{s_1,s_2}}{m!}B(2\bar{h}_1+p+m,2\bar{h}_2+p)\bar{z}^{p+m}\bar{\partial}^m\mathcal{O}_{h_1+h_2-1,\bar{h}_1+\bar{h}_2+p}(0,0),$$

$$p = d_V - 4 = s_1 + s_2 - s_3 - 1.$$

2. Construct **celestial currents** from light-transforms of conformally soft gravitons that **generate the action of $\mathrm{w}_{1+\infty}$** on massless particles:

$$\left[\widehat{\mathrm{w}}\,^q_n, \mathcal{O}_{h,\bar{h}}(z,\bar{z})\right] = \frac{1}{2}\sum_{\ell=0}^{2q-3}\binom{q+n-1}{\ell}\frac{(2q-2-\ell)\Gamma(2\bar{h}+1)}{\Gamma(2\bar{h}+1-\ell)}\bar{z}^{q+n-1-\ell}\partial_{\bar{z}}^{2q-3-\ell}\mathcal{O}_{h+2-q,\bar{h}+2-q}(z,\bar{z}).$$

$$q = 1,\frac{3}{2},2,\frac{5}{2},\cdots.$$

3. Verify OPE coefficients also respect $\mathrm{w}_{1+\infty}$.

[see also Hongliang Jiang's talk and paper 2108.08799]

# Motivation

- **Overarching Goal:** add to our understanding of quantum gravity
- **Prevailing Idea:** holography
  - Quantum gravity can be described by known frameworks (such as QFT) provided we find an appropriate recasting in terms of a holographically dual theory
- **Challenge:** find (and justify) the dual theory

# Scattering in Asymptotically Flat Spacetimes

- The scattering problem is a natural question in quantum gravitational theories in asymptotically flat spacetimes and readily admits a holographic interpretation.
- First, the data characterizing the scattering problem resides in a slightly different space than the gravitational theory.
    - Namely, scattering data is specified at the past and future boundaries of spacetime where gravitational effects are weak and perturbative.
- Second, scattering data in 4D spacetime is organized by symmetries which include the global conformal symmetry of theories in two dimensions.

➡ A 2D theory with conformal symmetry is a natural candidate for a holographic dual of quantum gravity in 4D asymptotically flat spacetimes.

# Celestial Amplitudes

- To investigate the merits of the proposal, it is helpful to work in a basis in which the 2D conformal symmetry is manifest.
- Lorentz symmetry $SO(3,1) \cong SL(2,\mathbb{C})$ is the 4D interpretation of the 2D global conformal symmetry.
- States which diagonalize a maximal number of the Lorentz generators transform most simply under 2D global conformal symmetry.
- Can simultaneously diagonalize 1 boost & 1 rotation

  $\Rightarrow$ Boost & helicity eigenstates

# Celestial Amplitudes

- Boost + helicity eigenstates are related to momentum eigenstates by a change of basis.
- For example, for massless particles $p^\mu = \omega\left(1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z}\right)$ and

$$|\,\Delta, s, z, \bar{z}\rangle = \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta\,|\,\omega, s, z, \bar{z}\rangle.$$

- **Celestial amplitudes** are constructed by Mellin-transforming each external massless particle state:

$$\mathscr{A}(\Delta_i, s_i, z_i, \bar{z}_i) = \left(\prod_{j=1}^{n}\int_0^\infty \frac{d\omega_j}{\omega_j}\omega_j^{\Delta_j}\right)\mathbf{A}(\omega_i, s_i, z_i, \bar{z}_i).$$

[Kapec, Mitra, Raclariu, & Strominger, hep-th/1609.00282; Cheung, de la Fuente & Sundrum, hep-th/1609.00732]

# Celestial Amplitudes

- Celestial amplitudes transform under Lorentz like correlation functions of primary operators:

$$z \to z' = \frac{az+b}{cz+d}, \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{C}),$$

$$\mathscr{A}(\Delta_i, s_i, z_i, \bar{z}_i) \to \mathscr{A}(\Delta_i, s_i, z_i', \bar{z}_i') = \left( \prod_{j=1}^{n} (cz_j + d)^{2h_j} (c\bar{z}_j + d)^{2\bar{h}_j} \right) \mathscr{A}(\Delta_i, s_i, z_i, \bar{z}_i).$$

- This supports the hope that quantum gravity in asymptotically flat spacetimes might be amenable to standard field theory techniques, applied to this auxiliary space (a.k.a. the "celestial sphere").

[Kapec, Mitra, Raclariu, & Strominger, hep-th/1609.00282; Cheung, de la Fuente & Sundrum, hep-th/1609.00732]

# Celestial Conformal Field Theory

- Ideally would like to know if CCFT is a field theory.
- More modest question:

What are the implications of known field theoretic (or non-field theoretic) behavior of 4D gravitational scattering amplitudes for CCFT?

- Comments (on why field theory behavior is important):
  - Can provide good perturbative framework.
  - Physical phenomena are known to be encoded in certain analytic behavior.
  - Field theory formalizes decoupling of physics at long & short distances.

# UV and IR Aspects of Celestial Amplitudes

- **Focus of the talk:** implications of known or assumed UV and IR behavior of 4D gravitational scattering amplitudes for celestial amplitudes

1. EFT expansion
2. Soft factorization

[See also Kevin's and Prahar's talks and references therein for more recent developments]

In momentum space, these formalize the insensitivity of low energy physics to the details of UV completion.

These properties are manifest in a basis in which we have diagonalized the maximal number of Poincaré generators.

How do they manifest in the celestial amplitudes?

# Outline

1. Review of massless scalar 4-point celestial amplitude
2. EFT expansion from celestial amplitude
3. Soft factorization in celestial amplitudes
4. Open questions

# Massless Scalar 4-point Celestial Amplitude

- First, must identify variables that parametrize celestial amplitudes.
- Consider Poincaré constrained 4-point massless scalar celestial amplitude.

1. Lorentz = Global conformal symmetry

$\Rightarrow$ Only non-trivial function of conformal cross ratios $z$ and $\bar{z}$, where $z = \dfrac{z_{13}z_{24}}{z_{12}z_{34}}$.

2. 4D Translations

$\Rightarrow$ Only non-trivial function of real part of $z$ and $\beta = \sum_i (\Delta_i - 1)$.

[Zlotnikov & Law, hep-th/1910.04356; Arkani-Hamed, MP, Raclariu, & Strominger, hep-th/2012.04208]

# Massless Scalar 4-point Amplitude

- Derive Poincaré-constrained celestial amplitude by directly transforming momentum space amplitude.
- In momentum space, $2 \to 2$ massless scalar scattering is constrained by

1. Lorentz symmetry

$$\mathbf{A} = \mathbf{A}(p_i \cdot p_j).$$

2. Translations ⇒ *only two* $p_i \cdot p_j$ are independent

$$\mathbf{A} = \mathbf{M}(s, t)\ \delta^{(4)}\Big(\sum_{i=1}^{4} p_i\Big).$$

Center of mass energy: $s = -(p_1 + p_2)^2 = -2p_1 \cdot p_2,$

Momentum transfer: $t = -(p_1 + p_3)^2 = -2p_1 \cdot p_3.$

# Massless Scalar 4-point Celestial Amplitude

$$s = \omega^2, \quad t = -z\omega^2$$

Parametrize Mandelstam invariants by center of mass energy $\omega$ and conformal cross ratio $z$

$$\mathcal{A}(\Delta_i, z_i, \bar{z}_i) = \left(\prod_{j=1}^{4} \int_0^\infty \frac{d\omega_j}{\omega_j} \omega_j^{\Delta_j}\right) \mathbf{M}(s,t)\ \delta^{(4)}\left(\sum_{k=1}^{4} p_k\right)$$

$$\sim \int_0^\infty \frac{d\omega}{\omega} \omega^\beta \prod_{j=2}^{4} \int_0^\infty \frac{d\sigma_j}{\sigma_j} \sigma_j^{\Delta_j}$$

$$\beta = \sum_{i=1}^{4} \left(\Delta_i - 1\right)$$

$$\sim \delta(z - \bar{z}) \prod_{i=2}^{4} \delta\big(\sigma_i - f_i(z_j, \bar{z}_j)\big), \quad \sigma_i = \frac{\omega_i}{\omega}$$

$\Rightarrow$ Momentum-conserving delta function localizes three of four integrals

# Massless Scalar 4-point Celestial Amplitude

- Arrive at the following decomposition:

$$\mathscr{A}(\Delta_i, z_i, \bar{z}_i) \sim \left(\prod_{i<j} z_{ij}^{\frac{1}{3}h-h_i-h_j}\bar{z}_{ij}^{\frac{1}{3}\bar{h}-\bar{h}_i-\bar{h}_j}\right)\delta(z-\bar{z})\int_0^\infty \frac{d\omega}{\omega}\omega^\beta \mathbf{M}(\omega^2, -z\omega^2)$$

Fixed by kinematics

$\equiv \mathscr{M}(\beta, z)$

Captures dynamics

- Conformal cross ratio replaces ratio of Mandelstam invariants.
- Center of mass energy $\omega$ is traded for sum of conformal dimensions $\beta + 4$
- Dynamical content is contained in $\mathscr{M}$ and related to momentum space matrix element (at fixed angle) by a *single* Mellin transform.

# Effective Field Theory Expansion

**Momentum space scattering amplitudes admit an effective field theory expansion.**

- Consider scattering of massless scalars mediated by a massive exchange:

$$\mathbf{M}(s) \sim \lambda \frac{M^2}{s - M^2}.$$

- In momentum space, amplitude admits low-energy (EFT) expansion:

$$\mathbf{M}(\omega) \sim -\lambda \left( 1 + \frac{\omega^2}{M^2} + \frac{\omega^4}{M^4} + \cdots \right), \qquad s = \omega^2.$$

- Consider celestial amplitude for leading term:

$$\mathscr{M}(\beta) \sim \int_0^\infty \frac{d\omega}{\omega} \omega^\beta (-\lambda) \overset{\beta = ib}{=} -2\pi\lambda\delta(b).$$

- Subleading corrections ruin marginal convergence (diverges at upper limit).

⇒ Celestial amplitudes *don't exist* for truncated EFT's!

# Beyond the Wilsonian Paradigm

**Wilsonian paradigm: low-energy physics is insensitive to the details of the UV.**

- Simple result for full (not truncated) amplitude:

$$\mathbf{M}(s) = \lambda \frac{M^2}{s - M^2} \qquad \Rightarrow \qquad \mathcal{M}(\beta) = \lambda M^\beta \frac{i\pi e^{i\pi\beta}}{1 - e^{i\pi\beta}}.$$

- Celestial amplitudes are sensitive to UV physics
  - Drastically different result if truncate EFT expansion
  - Really only exist for consistent UV complete theories
  - UV sensitivity is a consequence of scattering boost eigenstates, which contain contributions from arbitrarily high-energy modes.

# EFT Expansion in Celestial Amplitudes

- Can we recover the EFT expansion directly from celestial amplitudes?
  - *What is the signature of the EFT expansion in celestial amplitudes?*
- **Strategy:** Use EFT expansion to approximate the amplitude in the lower range of integration

$$\mathbf{M}(s,t) \sim \sum_{p,q} a_{p,q} s^p t^q \sim \sum_n a_n(z)\omega^{2n} \qquad \Rightarrow \qquad \mathscr{M}(\beta) \supset \int_0 \frac{d\omega}{\omega}\omega^\beta \sum_n a_n(z)\omega^{2n} \sim \sum_n \frac{a_n(z)}{\beta + 2n}$$

$\Rightarrow$ Residues of poles at negative even integer $\beta$ give coefficients in EFT expansion.

# EFT Expansion from Celestial Amplitudes

**Residues of poles at negative even integer $\beta$ give coefficients in EFT expansion.**

- Recall example:

$$\mathbf{M}(\omega) = \lambda \frac{M^2}{\omega^2 - M^2} \quad \longleftrightarrow \quad \mathscr{M}(\beta) = \lambda M^\beta \frac{i\pi e^{i\pi\beta}}{1 - e^{i\pi\beta}}$$

EFT expansion ↓      ↓ Residue at negative even integer

$$\mathbf{M}(\omega) = -\lambda \sum_n \frac{\omega^{2n}}{M^{2n}} \quad \longleftrightarrow \quad \text{Res}[\mathscr{M}(\beta)]_{\beta=-2n} = -\frac{\lambda}{M^{2n}}$$

# Factorization of Infrared Divergences

## Soft factorization of momentum space amplitudes in QED with massless charges

$$\mathbf{A}(p_i) = e^B \mathbf{A}_0(p_i)$$

Infrared divergent

Infrared finite

Universal soft factor containing all infrared divergences:

$$B = -\frac{e^2}{4\pi^2} \ln \Lambda_{IR} \sum_{i<j} Q_i Q_j \ln \left( \omega_i \omega_j z_{ij} \bar{z}_{ij} \right)$$

$$\sim p_i \cdot p_j$$

[Weinberg 1965]

# Soft Factor as 2D Correlator

- $(z_i, \bar{z}_i)$ dependence is entirely in terms of pairwise distances on the 2D plane

  $\Rightarrow$ Express $(z_i, \bar{z}_i)$ contribution as 2-point correlation function on the 2D plane

$$B = -\frac{e^2}{4\pi^2} \ln \Lambda_{IR} \sum_{i<j} Q_i Q_j \ln(\omega_i \omega_j) - \sum_{i<j} Q_i Q_j \langle \Phi(z_i, \bar{z}_i) \Phi(z_j, \bar{z}_j) \rangle,$$

$$\langle \Phi(z_i, \bar{z}_i) \Phi(z_j, \bar{z}_j) \rangle = \frac{e^2}{4\pi^2} \ln \Lambda_{IR} \ln |z_{ij}|^2 .$$

- Exploit pairwise structure to express entirely as correlation of free field:

$$e^B = \left( \prod_i \omega_i^{\frac{e^2}{4\pi^2} Q_i^2 \ln \Lambda_{IR}} \right) \langle e^{iQ_1 \Phi(z_1, \bar{z}_1)} \ldots e^{iQ_n \Phi(z_n, \bar{z}_n)} \rangle.$$

[Nande, MP & Strominger, hep-th/1705.00608]

# Factorization Revisited

- Can now express factorization as a statement pertaining to asymptotic states:

$$\mathbf{A}(p_i) = e^B \mathbf{A}_0(p_i),$$

$$e^B = \Big(\prod_i \omega_i^{\frac{e^2}{4\pi^2} Q_i^2 \ln \Lambda_{IR}}\Big) \langle e^{iQ_1 \Phi(z_1,\bar{z}_1)} \ldots e^{iQ_n \Phi(z_n,\bar{z}_n)} \rangle.$$

$$\Rightarrow \quad |p_k\rangle = \omega_k^{\frac{e^2}{4\pi^2} Q_k^2 \ln \Lambda_{IR}} e^{iQ_k \Phi(z_k,\bar{z}_k)} \widehat{|p_k\rangle}$$

[Nande, MP & Strominger, hep-th/1705.00608]

# Interpretation of Factorization

$$|p_k\rangle = \omega_k^{\frac{e^2}{4\pi^2}Q_k^2 \ln \Lambda_{IR}} e^{iQ_k\Phi(z_k,\bar{z}_k)} \widehat{|p_k\rangle}$$

$$\Phi(z,\bar{z}) \to \Phi(z,\bar{z}) + \varepsilon(z,\bar{z})$$

Transformation of Goldstone boson

$$e^{iQ_k\Phi(z_k,\bar{z}_k)} \to e^{iQ_k\varepsilon(z_k,\bar{z}_k)} e^{iQ_k\Phi(z_k,\bar{z}_k)}$$

Transformation of charge $Q_k$ state under large gauge symmetry

⇒ Under (Goldstone) shift transformation of $\Phi$, IR divergent factor fully captures non-trivial transformation of asymptotic particles under large gauge symmetry.

[Nande, MP & Strominger, hep-th/1705.00608]

# Factorization of Celestial Amplitudes

$$\mathscr{A}(\Delta_i, z_i, \bar{z}_i) = \left(\prod_{k=1}^{n} \int_0^\infty \frac{d\omega_k}{\omega_k} \omega_k^{\Delta_k}\right) e^B \mathbf{A}_0(p_i)$$

$$= \underbrace{\langle e^{iQ_1\Phi(z_1,\bar{z}_1)} \cdots e^{iQ_n\Phi(z_n,\bar{z}_n)}\rangle}_{\mathscr{A}_{\text{soft}}} \underbrace{\left(\prod_{k=1}^{n} \int_0^\infty \frac{d\omega_k}{\omega_k} \omega_k^{\Delta_k + \frac{e^2}{4\pi^2} Q_k^2 \ln \Lambda_{IR}}\right) \mathbf{A}_0(p_i)}_{\mathscr{A}_{\text{hard}}}$$

$$\langle \Phi(z_i, \bar{z}_i) \Phi(z_j, \bar{z}_j) \rangle = \frac{e^2}{4\pi^2} \ln \Lambda_{IR} \ln |z_{ij}|^2$$

$\Rightarrow$ $e^{iQ_k\Phi(z_k,\bar{z}_k)}$ has conformal weight $\Delta = -\frac{e^2}{4\pi^2} Q_k^2 \ln \Lambda_{IR}$

} Renormalization of dimensions in $\mathscr{A}_{\text{hard}}$ is needed to account for these non-trivial weights.

# Infrared Safe Scattering Amplitudes

- To obtain IR safe amplitudes in momentum space, dress charged particles

$$W_k[f]\,|\,p_k, Q_k\rangle,$$

$$W_k[f] = \exp\left[-eQ_k \int \frac{d^3\vec{q}}{(2\pi)^3}\frac{f(\vec{q})}{2q^0}\left(\frac{p_k\cdot\varepsilon^{*\alpha}}{p_k\cdot q}a_\alpha(\vec{q}) - \frac{p_k\cdot\varepsilon^{\alpha}}{p_k\cdot q}a_\alpha^\dagger(\vec{q})\right)\right],$$

Photons

$$f(0) = 1.$$

[Faddeev & Kulish 1970;
Kapec, Perry, Raclariu & Strominger, hep-th/1703.05448]

# Dressing with Boost Eigenstate Photons

- Choose conformally invariant dressing $f(\vec{q}) = 1$.

$$W_k[f=1] = \exp\left[-\frac{eQ_k}{\sqrt{2}(2\pi)^3}\int d^2z\left(\frac{1}{\bar{z}-\bar{z}_k}\int_0^\infty d\omega\left(a_+(\omega,z,\bar{z}) - a_-^\dagger(\omega,z,\bar{z})\right) + h.c.\right)\right]$$

$$\int_0^\infty \frac{d\omega}{\omega}\omega^\Delta a_+(\omega,z,\bar{z})\bigg|_{\Delta=1}$$

$\Rightarrow$ Dressing involves photon in boost eigenstate with boost weight $\Delta = 1$!

# Boost Weight $\Delta = 1$ Photons

- Two photon modes with boost weight $\Delta = 1$:
  1. Generator of large gauge symmetry
  2. Its symplectic partner

[Donnay, Puhm & Strominger, hep-th/1810.05219]

Soft photon theorem = Ward identity for large gauge symmetry

[He, Mitra, Porfyriadis & Strominger, hep-th/1407.3789;
Kapec, MP & Strominger, hep-th/1506.02906]

Soft photons generate large gauge symmetry

$$\lim_{\omega \to 0} \omega a_+(\omega, z, \bar{z}) \sim \int_0^\infty d\omega \ \delta(\omega)\omega \ a_+(\omega, z, \bar{z})$$

$$\sim \lim_{\Delta \to 1} (\Delta - 1) \int_0^\infty \frac{d\omega}{\omega} \omega^\Delta a_+(\omega, z, \bar{z})$$

Conformally soft currents involve residues in conformal dimension, similar to EFT expansion

[Cheung, de la Fuente,& Sundrum, hep-th/1609.00732;
Fan, Fotopoulos & Taylor, hep-th/1903.01676;
MP, Raclariu, & Strominger, hep-th/1904.10831 ]

# Boost Weight $\Delta = 1$ Photons

Generator of large gauge transformations:

$$J_z \sim \lim_{\omega \to 0} \omega \left[a_+(\omega, z, \bar{z}) + a_-^\dagger(\omega, z, \bar{z})\right]$$

$$\sim \lim_{\Delta \to 1} (\Delta - 1) \int_0^\infty \frac{d\omega}{\omega} \omega^\Delta \left[a_+(\omega, z, \bar{z}) + a_-^\dagger(\omega, z, \bar{z})\right]$$

$\Delta = 1$ mode in dressing:

$$S_z \sim \int_0^\infty \frac{d\omega}{\omega} \omega^\Delta \left[a_+(\omega, z, \bar{z}) - a_-^\dagger(\omega, z, \bar{z})\right]\Bigg|_{\Delta=1}$$

$$\left[a(\omega, z, \bar{z}), a^\dagger(\omega', z', \bar{z}')\right] \sim \frac{\delta(\omega - \omega')}{\omega} \delta^{(2)}(z - z')$$

$\Rightarrow$

$$\left[J_z, S_{\bar{w}}\right] \sim \delta^{(2)}(z - w)$$

$\Rightarrow$ $J$ and $S$ are symplectic partners

$$S_z = i\partial_z \Phi$$

$\Rightarrow$ Identify $S$ with Goldstone boson!

[Donnay, Puhm & Strominger, hep-th/1810.05219]

# Dressing with Boost Eigenstate Photons

- Return to dressing with boost eigenstate photons:

$$W_k[f=1] \sim \exp\left[-Q_k \int \frac{d^2z}{2\pi}\left(\frac{1}{\bar{z}-\bar{z}_k}\int_0^\infty d\omega\left(a_+(\omega,z,\bar{z}) - a_-^\dagger(\omega,z,\bar{z})\right) + h.c.\right)\right]$$

$$\int_0^\infty \frac{d\omega}{\omega}\omega^\Delta\left[a_+(\omega,z,\bar{z}) - a_-^\dagger(\omega,z,\bar{z})\right]\Big|_{\Delta=1} \sim -\frac{1}{2}i\partial_z\Phi$$

$$\Rightarrow \quad \boxed{W_k[f=1] = e^{-iQ_k\Phi(z_k,\bar{z}_k)}}$$

*Precisely* cancels IR divergent factor previously found in factorization!

# Infrared Safe Celestial Amplitudes

- Obtain natural construction of IR safe celestial amplitudes
- Dressing with boost weight $\Delta = 1$ photons precisely cancels IR divergent correlation of Goldstone bosons.

$$\mathscr{A}_{\text{dressed}} = \mathscr{A}_{\text{hard}},$$

$$\mathscr{A}_{\text{hard}} = \left( \prod_{k=1}^{n} \int_0^\infty \frac{d\omega_k}{\omega_k} \omega_k^{\Delta'_k} \right) \mathbf{A}_0(p_i).$$

# Factorization in Gravity

- Soft factorization of massless particle momentum-space amplitudes in **gravity**:

$$\mathbf{A}(p_i) = e^B \mathbf{A}_0(p_i),$$

where the universal soft factor containing all IR divergences is given by

$$e^B = \exp\left[ -\frac{1}{\epsilon} \frac{G}{2\pi} \sum_{i,j=1}^{n} p_i \cdot p_j \ln\left( \frac{2 p_i \cdot p_j}{\mu^2} \right) \right].$$

$d = 4 - 2\epsilon$,
$\epsilon$ regulates the IR divergences

[Weinberg 1965; Naculich & Schnitzer hep-th/1101.1524]

# Soft Factor as a 2D Correlator

- Parametrize by energies and points on 2D plane:

$$e^B = \exp\left[\frac{1}{\epsilon}\frac{G}{\pi}\sum_{i\neq j}^{n}\eta_i\eta_j\omega_i\omega_j\mathcal{G}(z_i,\bar{z}_i;z_j,\bar{z}_j)\right],$$

$$p_i^\mu = \eta_i\omega_i(1+z_i\bar{z}_i,\cdots),$$

$$\text{out/in}: \eta_i = \pm 1$$

$$\mathcal{G}(z_i,\bar{z}_i;z_j,\bar{z}_j) = z_{ij}\bar{z}_{ij}\log(z_{ij}\bar{z}_{ij})\,.$$

- Expressed as a correlation of free fields:

$$e^B = \left\langle e^{i\eta_1\omega_1 C(z_1,\bar{z}_1)}\cdots e^{i\eta_n\omega_n C(z_n,\bar{z}_n)}\right\rangle,$$

$$\langle C(z_i,\bar{z}_i)C(z_j,\bar{z}_j)\rangle = -\frac{1}{\epsilon}\frac{2G}{\pi}\mathcal{G}(z_i,\bar{z}_i;z_j,\bar{z}_j).$$

[Himwich, Narayanan, MP, Paul & Strominger, hep-th/2005.13433]

# Interpretation of Factorization

- Factorization as a statement pertaining to asymptotic states:

$$|p_k\rangle = e^{i\omega_k C(z_k,\bar{z}_k)}|\widehat{p_k}\rangle.$$

- Identify factorization as decomposition according to supertranslation symmetry:

$$\delta_f C(z,\bar{z}) = f(z,\bar{z}) \qquad \Rightarrow \qquad -i\delta_f e^{i\omega_k C(z_k,\bar{z}_k)} = \omega_k f(z_k,\bar{z}_k) e^{i\omega_k C(z_k,\bar{z}_k)}.$$

$C$ transforms like a Goldstone boson associated supertranslation symmetry

Reproduces net infinitesimal transformation of single particle states under supertranslations:

$$-i\delta_f|p_k\rangle = \omega_k f(z_k,\bar{z}_k)|p_k\rangle.$$

[Himwich, Narayanan, MP, Paul & Strominger, hep-th/2005.13433]

# Factorization of Celestial Amplitudes

$$\mathscr{A}(\Delta_i, z_i, \bar{z}_i) = \left(\prod_{k=1}^{n} \int_0^\infty \frac{d\omega_k}{\omega_k}\omega_k^{\Delta_k}\right) \left\langle e^{i\eta_1\omega_1 C(z_1,\bar{z}_1)} \cdots e^{i\eta_n\omega_n C(z_n,\bar{z}_n)} \right\rangle \mathbf{A}_0(p_i)$$

$$= \underbrace{\left\langle e^{iP_1 C(z_1,\bar{z}_1)} \cdots e^{iP_n C(z_n,\bar{z}_n)} \right\rangle}_{\mathscr{A}_{\text{soft}}} \underbrace{\left(\prod_{k=1}^{n} \int_0^\infty \frac{d\omega_k}{\omega_k}\omega_k^{\Delta_k}\right) \mathbf{A}_0(p_i)}_{\mathscr{A}_{\text{hard}}}$$

$$P_k | p_k \rangle = \eta_k \omega_k | p_k \rangle$$

$$P_k | \Delta_k, z_k, \bar{z}_k \rangle = \int_0^\infty \frac{d\omega_k}{\omega_k}\omega_k^{\Delta_k} P_k | \omega_k, z_k, \bar{z}_k \rangle = \eta_k | \Delta_k + 1, z_k, \bar{z}_k \rangle$$

- Soft component is an *operator* for celestial amplitudes!

# Infrared Safe Scattering Amplitudes

- Obtain infrared-safe amplitudes in *momentum space* by dressing particles with coherent clouds of gravitons

$$W_k[f]\,|\,p_k\rangle, \qquad W_k[f] = \exp\left[-\frac{\kappa}{2}\int \frac{d\vec{q}}{(2\pi)^3}\frac{f(\vec{q})}{2q^0}\frac{p_k^\mu p_k^\nu}{p_k\cdot q}\left(\varepsilon^{*\alpha}_{\mu\nu}a_\alpha(\vec{q}) - \varepsilon^{\alpha}_{\mu\nu}a^\dagger_\alpha(\vec{q})\right)\right].$$

[Choi, Kol & Akhoury, hep-th/1708.05717; Choi & Akhoury, hep-th/1712.04551]

- Conformally invariant choice $f(\vec{q}) = 1$ can be identified with exponentiated Goldstone boson:

$$W_k[f=1] = e^{-i\eta_k\omega_k C(z_k,\bar{z}_k)}.$$

# Infrared Safe Celestial Amplitudes

- Conformally invariant graviton dressing *precisely cancels* IR divergent correlator of Goldstone bosons $C$:

$$\boxed{\mathscr{A}_{\text{dressed}} = \mathscr{A}_{\text{hard}},}$$

$$\mathscr{A}_{\text{hard}} = \left(\prod_{k=1}^{n} \int_0^{\infty} \frac{d\omega_k}{\omega_k} \omega_k^{\Delta_k}\right) \mathbf{A}_0(p_i).$$

# Summary

- Identified Poincaré-constrained 4-point celestial amplitude (for Mellin primaries)
  - Non-trivial dependence on real part of conformal cross ratio $z$ and sum of conformal dimensions $\beta + 4$
- Identified EFT expansion implies poles at negative even integer $\beta$
- Soft factorization in momentum space is current algebra factorization in celestial amplitudes
- IR safe celestial amplitudes obtained by dressing with (conformal primary) Goldstone bosons.

# Open Questions

1. Minimal number of variables (analogues of $\beta$ and $z$) for higher-point Poincaré-constrained celestial amplitudes.
2. Precise relation between causality in 4D and supertranslation symmetry of scattering amplitudes.

# Thank You!