Celestial IR divergences and the effective action of supertranslation modes

based on arXiv:2105.10526 in collaboration with Jakob Salzer

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Motivation

Soft factorization of scattering amplitudes:

$$\mathcal{A} = \mathcal{A}_{\mathsf{soft}}\, \mathcal{A}_{\mathsf{hard}}\,.$$

For n hard external particles and arbitrary number of soft virtual gravitons [Weinberg '65]:

$$\mathcal{A}_{\mathsf{soft}} = \exp\left[-\frac{2G\log\Lambda_0}{\pi}\sum_{i,j=1}^{n}\eta_i\eta_j\omega_i\omega_j\,|x_i-x_j|^2\ln|x_i-x_j|^2\right]\,,$$

where Λ_0 is an IR regulator and $x_i \in \mathcal{CS}$.

Motivation

Formulation in terms of the supertranslation field $C(x)$
[Himwich et al '20, Arkani-Hamed et al '20]

$$\mathcal{A}_{\text{soft}}(p_1, .., p_n) = \langle \mathcal{W}_i \ldots \mathcal{W}_n \rangle \,, \qquad \mathcal{W}_i = e^{i\eta_i \omega_i C(x_i)} \,,$$

such that

$$\mathcal{A}_{\text{soft}} = \exp\left[-\frac{1}{2}\sum_{i\neq j}^{n} \eta_i \eta_j \omega_i \omega_j \langle C(x_i) C(x_j)\rangle\right] \,.$$

By comparison with Weinberg's result, one can infer [Himwich et al '20]

$$\langle C(x) C(y)\rangle = \frac{4G\log\Lambda_0}{\pi} \, |x-y|^2 \log|x-y|^2 \,.$$

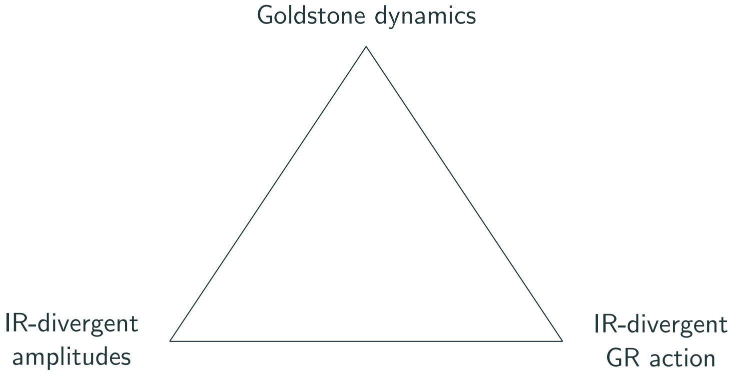
Objectives

Basic question

Is there a more direct way to derive the expression of $\langle C(x)C(y)\rangle$?

- Dynamics of the supertranslation Goldstone mode $C(x)$?
- Effective boundary action? (for superrotations see [KN-Salzer '20, KN '21])
- Relation to IR divergences?
- Intrinsic formulation of the celestial CFT?
- Formulation of celestial holography for nonlinear gravity, *à la* AdS/CFT?

The IR-divergent triangle



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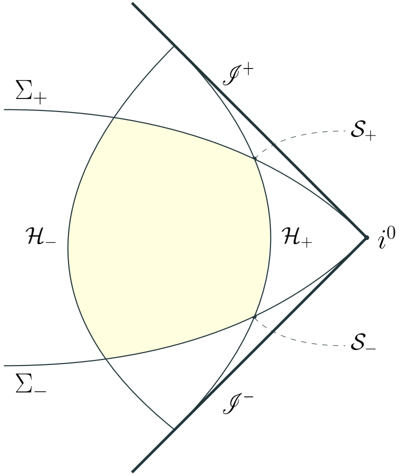

Figure 1: Spacetime near spatial infinity i^0. The de Sitter hyperboloids $\mathcal{H}_\pm$ are surfaces of constant $\rho = \Lambda_\pm$.

Asymptotic configuration space

Beig–Schmidt gauge [Beig-Schmidt '82]

$$ds^2 = N^2\, d\rho^2 + H_{ab}\,(N^a\, d\rho + dx^a)\,(N^b\, d\rho + dx^b)\,,$$

with

$$\begin{aligned} N &= 1 + \frac{\sigma}{\rho}\,, \\ H_{ab}N^b &= o(\rho^{-1})\,, \\ H_{ab} &= \rho^2 \left(h_{ab} + \rho^{-1} h^{(1)}_{ab} + \frac{\log\rho}{\rho^2}\, i_{ab} + \rho^{-2} h^{(2)}_{ab} + o(\rho^{-2}) \right)\,. \end{aligned}$$

Einstein's equation imply that h_{ab} is 3d de Sitter metric,

$$R\,[h]_{ab} = 2h_{ab}\,.$$

We define the magnetic potential

$$k_{ab} \equiv h^{(1)}_{ab} + 2\sigma h_{ab} \,,$$

such that the electric and magnetic Weyl tensors are

$$E^{(1)}_{ab} = -\left(D_a D_b + h_{ab}\right)\sigma \,, \qquad B^{(1)}_{ab} = \frac{1}{2}\epsilon_a{}^{cd} D_c k_{db} \,.$$

Boundary conditions for a well-defined action principle
[Compère-Dehouck '11, Virmani '11]:

$$k \equiv h^{ab} k_{ab} = 0, \qquad D^a k_{ab} = 0 \,.$$

Supertranslations

ASG contains spi-supertranslations:

$$\rho \mapsto \rho + \omega(x^a) + o(\rho^0)\,, \qquad \left(D^2+3\right)\omega = 0\,,$$
$$x^a \mapsto x^a + \rho^{-1} D^a \omega + o(\rho^{-1})\,.$$

that act as

$$\delta_\omega \sigma = \delta_\omega h_{ab} = 0, \qquad \delta_\omega k_{ab} = 2(D_a D_b + h_{ab})\omega\,.$$

They have been mapped to supertranslations at $\mathscr{I}$ [Troessart '17, Prabhu '19].
Non-radiative spacetime:

$$k_{ab} = 2\left(D_a D_b + h_{ab}\right)\Phi\,, \qquad \left(D^2+3\right)\Phi = 0\,,$$

where Φ is the spi-supertranslation Goldstone mode,

$$\delta_\omega \Phi = \omega\,.$$

Matching to null infinity

Bondi gauge near $\mathscr{I}^+$:

$$ds^2 = -\,du^2 - 2\,du\,dr + r^2\gamma_{AB}\,dx^A\,dx^B \\ + \frac{2m}{r}\,du^2 + r\,C_{AB}\,dx^A\,dx^B + D^A C_{AB}\,du\,dx^B + \ldots\,,$$

Non-radiative spacetimes:

$$C_{AB} = -2D_A D_B C + \gamma_{AB} D^2 C\,,$$

where C is the supertranslation Goldstone mode,

$$\delta_T C = T\,.$$

Building on [Troessart '17], we can show

$$C \sim \lim_{\mathscr{I}^+} \Phi\,, \qquad T \sim \lim_{\mathscr{I}^+} \omega\,.$$

and derive the antipodal matching condition [Strominger '13].

Infrared divergences

Basic action [Mann-Marolf '05]

$$S = S_{\mathsf{EH}} + S_{\mathcal{H}_+} \,,$$

with

$$S_{\mathcal{H}_+} = \frac{1}{8\pi G} \int_{\mathcal{H}_+} d^3x \, \sqrt{-H} \, (K - \hat{K}) \,.$$

Its onshell variation localizes at the corners $\mathcal{S}_\pm$,

$$\delta S = \pm \frac{\log \Lambda_+}{16\pi G} \int_{\mathcal{S}_\pm} dS_a \left(4\delta\sigma D^a \sigma + \frac{1}{2} \delta k_{bc} \, D^a k^{bc} - \delta k_{bc} \, D^c k^{ab} \right) \\ + \delta \left(\Lambda_+ \, \mathcal{R}_{\mathcal{S}_\pm} + \log \Lambda_+ \, \mathcal{R}^{(log)}_{\mathcal{S}_\pm} + \log^2 \Lambda_+ \, \mathcal{R}^{(log\, 2)}_{\mathcal{S}_\pm} \right) + O(\Lambda_+^0) \,.$$

Infrared divergences

The first line is responsible for a logarithmically divergent symplectic structure! [Compère-Dehouck '11]

Renormalized action

Total renormalized action:

$$S_{\text{total}} = S - \Lambda_+ \, \Delta\mathcal{R}_{\mathcal{S}} - \log\Lambda_+ \, \Delta\mathcal{R}_{\mathcal{S}}^{(log)} - \log^2\Lambda_+ \, \Delta\mathcal{R}_{\mathcal{S}}^{(log\,2)} + S_{\text{CD}} \,,$$

with [Compère-Dehouck '11]

$$S_{\text{CD}} = \frac{\log\Lambda_+}{4\pi G}\left(S^{(\sigma)} + S^{(k)}\right) \,,$$

$$S^{(\sigma)} = -\frac{1}{2}\int_{\mathcal{H}} d^3x\,\sqrt{h}\,\left(D^a\sigma D_a\sigma - 3\sigma^2\right) \,,$$

$$S^{(k)} = -\frac{1}{8}\int_{\mathcal{H}} d^3x\,\sqrt{h}\,\left(\frac{1}{2}D_a k_{bc}\,D^a k^{bc} - D_a k_{bc} D^b k^{ac} - \frac{3}{2}k^{ab}k_{ab}\right) \,.$$

The renormalized onshell action is infrared finite!

Infrared effective action

Infrared sector

The Compère–Dehouck boundary action S_{CD} controls the IR divergent sector of General Relativity.

- Can it teach us something about the supertranslation Goldstone mode?

Strategy

In the spirit of AdS/CFT correspondence, let us evaluate S_{CD} onshell, in the absence of radiation for simplicity.

- S_{CD} is defined on a 3d de Sitter hyperboloid $\mathcal{H}$
- Onshell, it should reduce to a 2d theory on the celestial sphere $\mathcal{CS}$

Onshell reduction

We have to solve

$$(D^2+3)\Phi = (D^2+3)\sigma = 0\,.$$

Let's use planar coordinates on $\mathcal{H}$,

$$ds^2_{\mathcal{H}} = \eta^{-2}\left(-\,d\eta^2 + \delta_{ij}\,dx^i\,dx^j\right)\,, \qquad i,j=1,2\,.$$

Familiar from the (A)dS/CFT correspondence, we can expand close to $\mathcal{CS}$,

$$\Phi(\eta,x) = \eta^{-1}\left(\Phi_{(0)} + \eta^2\,\Phi_{(2)} + \eta^4\ln\eta\,\tilde{\Phi} + \eta^4\,\Phi_{(4)} + O(\eta^6)\right)\,,$$

and solve in terms of $\Phi_{(0)}$ and $\Phi_{(4)}$. Importantly, we can show

$$C_{\mathsf{plane}}(x^i) = \Phi_{(0)}(x^i)\,.$$

The infrared effective action

Result

$$S_{\mathsf{IR}} \equiv S_{\mathsf{CD}}\big|_{\mathsf{onshell}} = -\frac{\log\Lambda}{32\pi G}\int_{\mathcal{CS}} d^2x\,(\Box C_{\mathsf{plane}})^2\;, \qquad \Lambda \equiv \Lambda_+/\Lambda_-\,.$$

- Invariant under global SL(2,$\mathbb{C}$), not under Virasoro
- Extremized by "translations" modes $1, z, \bar{z}, z\bar{z}$, i.e.,

$$\Box^2 C_{\mathsf{plane}} = 0\,.$$

- Two-point function

$$\langle C_{\mathsf{plane}}(x) C_{\mathsf{plane}}(0)\rangle \sim G\,|x|^2 \log|x|^2\,.$$

Path integral formulation of celestial CFT

Celestial derivation of the soft factor

Recall [Himwich et al '20, Arkani-Hamed et al '20]

$$\mathcal{A}_{\text{soft}}(p_1,..,p_n) = \langle \mathcal{W}_i \ldots \mathcal{W}_n \rangle \,, \qquad \mathcal{W}_i = e^{i\eta_i \omega_i C(x_i)} \,.$$

Path integral representation:

$$\langle \mathcal{W}_i \ldots \mathcal{W}_n \rangle \equiv \int \mathcal{D}C_{\text{plane}} \, \mathcal{W}_i \ldots \mathcal{W}_n \, e^{-S_{\text{IR}}[C_{\text{plane}}]/\log^2 \Lambda} \,.$$

Explicit evaluation yields Weinberg's result [Weinberg '65]

$$\mathcal{A}_{\text{soft}}(p_1,..,p_n) \sim \exp\left[-\frac{2G\log\Lambda}{\pi} \sum_{i,j=1}^{n} \eta_i \eta_j \omega_i \omega_j \, |x_i - x_j|^2 \ln |x_i - x_j|^2 \right] \,.$$

Summary and open questions

Summary and open questions

Summary:

- Setup appropriate to describe nonlinear gravity
- S_{CD} controls and regulates the IR divergences
- S_{IR} describes the IR-divergent sector of scattering amplitudes/CCFT
- Intrinsic path integral formulation of CCFT

Open questions:

- Full action is IR-finite $\overset{?}{\leftrightarrow}$ dressed IR-safe amplitudes [Faddeev-Kulish '70]
- Natural use of dS_3/CFT_2 technology (prediction: $\Delta = 3$ operator $\Phi_{(4)}$ $\overset{?}{\leftrightarrow}$ Goldstone diamond [Pasterski-Puhm-Trevisani '21])
- Inclusion of radiation $\rightarrow$ Goldstone-radiation coupling
- ...

The IR-divergent triangle