Holographic Chiral Algebra: Supersymmetry, Infinite Ward Identities, and EFTs

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Aug. 31, 2021

Celestial Amplitudes and Flat Space Holography

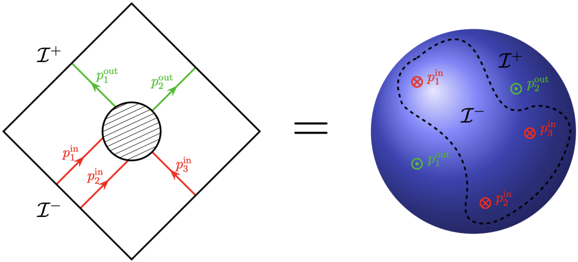
Based on 2108.08799, 2105.10269

This work is largely motivated by

[Guevara, Himwich, Pate, Strominger '21], [Strominger '21]

The goal is to understand symmetries and their consequences in celestial holography.

Motivation: why celestial holography ?



I will focus on the 4D massless case.

[figure adapted from Strominger '17]

Celestial holography [Cheung,Fuente,Sundrum '16, Pasterski,Shao,Strominger '16]

4D S-matrix = 2D CFT correlator

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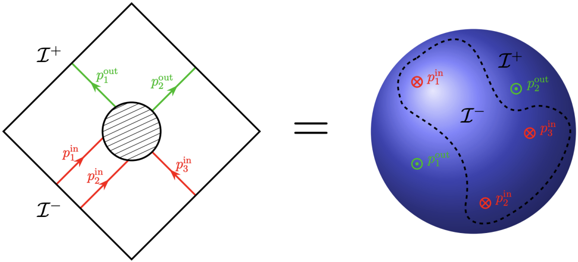

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4D S-matrix = 2D CFT correlator

- A new paradigm for flat holography beyond AdS/CFT
- A new program of reformulating scattering amplitude
- New insights into classical GR and quantum gravity

Motivation: why symmetries in celestial holography?

Symmetry is one of the most important guiding principles in physics!

The whole story of celestial holography is about symmetry.

It was motivated from and began with symmetries ....

soft theorems $\leftrightarrow$ BMS asymptotic symmetries

momentum space amplitude $\leftrightarrow$ manifest translational symmetry

celestial amplitude $\leftrightarrow$ manifest Lorentz symmetry

...but symmetries are still not fully understood!

Outline

1. Introduction & Toolkit
2. Holographic chiral algebra: soft-soft OPE
3. Ward identity: soft-hard OPE
4. EFT correction
   ★General formulae for OPE and Ward identitiy
5. Outlook

Celestial Amplitude

Celestial sphere variables:

$$p^\mu = \varepsilon\omega q^\mu, \qquad q^\mu = \Big(1 + z\bar{z}, \quad z + \bar{z}, \quad -i(z - \bar{z}), \quad 1 - z\bar{z}\Big)\ .$$

- Mellin transformation gives celestial amplitude

[Pasterski, Shao, Strominger '17]

$$\mathcal{A}_n(J_i, p_i^\mu) = A_n(J_i, p_i^\mu)\delta(\sum_i p_i^\mu)$$

$$\Big\downarrow \prod_{j=1}^n \int_0^\infty d\omega_j\ \omega_j^{\Delta_j - 1}$$

$$\mathcal{M}_n(\Delta_i, J_i, z_i, \bar{z}_i) = \langle \mathcal{O}^{\varepsilon_1}_{\Delta_1, J_1}(z_1, \bar{z}_1) \cdots \mathcal{O}^{\varepsilon_n}_{\Delta_n, J_n}(z_n, \bar{z}_n)\rangle_{\text{CFT}}$$

$\Delta_i = h_i + \bar{h}_i$
$J_i = h_i - \bar{h}_i =$ 2D spin $=$ 4D helicity

only focus on out-going $\varepsilon = +1$...

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only focus on out-going $\varepsilon = +1$...

- Soft theorem $\to$ Ward identity
- Collinear factorization $\to$ OPE
- One can also introduce fermionic coordinates $\rightsquigarrow$ celestial superamplitudes

[Jiang '21; Brandhuber, Brown, Gowdy, Spence, Travaglini '21]

Celestial operators: soft v.s. hard

$$\mathcal{O}_{\Delta,J}$$

Here $\Delta \in 1 + i\mathbb{R}$ to form a basis, but can be analytically continue

- Soft operator: $\mathcal{O}_{k,J}$ with $k = |J|, |J| - 1, \cdots$ $\rightsquigarrow$ symmetry
  - ★ Soft current:

$$R^{k,J}(z,\bar{z}) = \lim_{\Delta \to k} (\Delta - k)\mathcal{O}_{\Delta,J}(z,\bar{z})$$

- Hard operator: the rest

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  ★ Soft current:

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- Hard operator: the rest

- **Caveat**: $\mathcal{O}_{2,\pm 2}$ and $\mathcal{O}_{\frac{3}{2},\pm\frac{3}{2}}$ are actually hard, but useful to discuss together with soft operators

- Ambiguities arise when scattering **soft** particles of mixed helicities
  $\rightsquigarrow$ only consider **positive** helicity soft particles with $J > 0$

Mode decomposition of soft current

- The soft currents admit decomposition into chiral currents

[Banerjee, Ghosh, Paul 20; Guevara, Himwich, Pate, Strominger '21]

$$
\begin{aligned}
R^{k,J}(z,\bar{z}) &= \sum_{n=\frac{k-J}{2}}^{\frac{J-k}{2}} \frac{R_n^{k,J}(z)}{\bar{z}^{n+\frac{k-J}{2}}} \\
&= \bar{z}^{J-k} R^{k,J}_{\frac{k-J}{2}}(z) + \bar{z}^{J-k-1} R^{k,J}_{\frac{k-J+2}{2}}(z) + \cdots + R^{k,J}_{\frac{J-k}{2}}(z)
\end{aligned}
$$

Chiral currents $R_n^{k,J}(z)$ transform in the $(J+1-k)$-dim representation of $\overline{SL(2,\mathbb{R})}$.

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$$\begin{aligned} R^{k,J}(z,\bar{z}) &= \sum_{n=\frac{k-J}{2}}^{\frac{J-k}{2}} \frac{R_n^{k,J}(z)}{\bar{z}^{n+\frac{k-J}{2}}} \\ &= \bar{z}^{J-k} R^{k,J}_{\frac{k-J}{2}}(z) + \bar{z}^{J-k-1} R^{k,J}_{\frac{k-J+2}{2}}(z) + \cdots + R^{k,J}_{\frac{J-k}{2}}(z) \end{aligned}$$

Chiral currents $R_n^{k,J}(z)$ transform in the $(J+1-k)$-dim representation of $\overline{SL(2,\mathbb{R})}$.

- It is convenient to rescale the chiral currents: [Strominger '21]

$$\mathcal{R}_n^{i,J} = (i-1-n)!(i-1+n)!R_n^{J+2-2i,J}$$

$$n = 1-i, 2-i, \cdots i-1, \quad i = \frac{J-k}{2} + 1 = 1, \frac{3}{2}, 2, \cdots \ .$$

Light transformation of soft operator

$$\mathcal{R}_n^{i,J} = (i-1-n)!(i-1+n)!R_n^{J+2-2i,J}$$

Physically, this can be understood as light transformation: [Strominger '21]

$$\bar{\mathbf{L}}[\mathcal{O}](z,\bar{w}) = \int d\bar{z}\,(\bar{w}-\bar{z})^{2\bar{h}-2}\mathcal{O}_{(h,\bar{h})}(z,\bar{z})$$

[Kravchuk, Simmons-Duffin '18]

Plugging mode expansion gives

$$\epsilon\bar{\mathbf{L}}[\mathcal{O}_{k+\epsilon,J}](z,\bar{w}) = \bar{\mathbf{L}}[R^{k,J}](z,\bar{w})$$

$$= \sum_{n=\frac{k-J}{2}}^{\frac{J-k}{2}} R_n^{k,J}(z)\int d\bar{z}\,(\bar{w}-\bar{z})^{k+\epsilon-J-2}\bar{z}^{-n+\frac{J-k-\epsilon}{2}}$$

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$$= \sum_{n=\frac{k-J}{2}}^{\frac{J-k}{2}} R_n^{k,J}(z) \int d\bar{z}\, (\bar{w}-\bar{z})^{k+\epsilon-J-2} \bar{z}^{-n+\frac{J-k-\epsilon}{2}}$$

As a result,

$$\boxed{\bar{\mathbf{L}}[\mathcal{O}_{J+2-2i,J}](z,\bar{z}) = \pi \mathrm{i} \frac{(-)^{2i}}{\Gamma(2i)} \sum_{n=1-i}^{i-1} \frac{\mathcal{R}_n^{i,J}}{(-\bar{z})^{i+n}}}$$

Summing over $\overline{SL(2,\mathbb{R})}$ descendants

- General OPE takes the form:

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim C_{\mathcal{O}_1\mathcal{O}_2}^{\mathcal{O}_P} \frac{\mathcal{O}_{\Delta_P,J_P}(z_2,\bar{z}_2)}{(z_{12}\bar{z}_{12})^{\frac{\Delta_1+\Delta_2-\Delta_P}{2}}(z_{12}/\bar{z}_{12})^{\frac{J_1+J_2-J_P}{2}}} + \cdots$$

where $\mathcal{O}_{\Delta_P,J_P}$ is primary, dots are descendants.

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- Want to sum over *all* $\overline{SL(2,\mathbb{R})}$ descendants $\rightsquigarrow$ $\overline{SL(2,\mathbb{R})}$ OPE block

[Czech, Lamprou, McCandlish, Mosk, Sully '16]

$$\int_{\bar{z}_2}^{\bar{z}_1} d\bar{z}_3\ \mathcal{O}_{\bar{h}_P}(\bar{z}_3)\ \langle \mathcal{O}_{\bar{h}_1}(\bar{z}_1)\mathcal{O}_{\bar{h}_2}(\bar{z}_2)\tilde{\mathcal{O}}_{1-\bar{h}_P}(\bar{z}_3)\rangle$$

$$= \int_{\bar{z}_2}^{\bar{z}_1} \frac{d\bar{z}_3\ \mathcal{O}_{\bar{h}_P}(\bar{z}_3)}{\bar{z}_{12}^{\bar{h}_1+\bar{h}_2+\bar{h}_P-1}\,\bar{z}_{32}^{\bar{h}_2-\bar{h}_1-\bar{h}_P+1}\,\bar{z}_{13}^{\bar{h}_1-\bar{h}_2-\bar{h}_P+1}}$$

Summing over all $\overline{SL(2,\mathbb{R})}$ descendants

[Guevara, Himwich, Pate, Strominger '21]

OPE with all $\overline{SL(2,\mathbb{R})}$ descendants

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim \mathcal{N}^{\mathcal{O}_P}_{\mathcal{O}_1\mathcal{O}_2}\frac{\bar{z}_{12}^{N-M}}{z_{12}^{N+M}} \times$$

$$\int_0^1 dt\, \mathcal{O}_{\Delta_P,J_P}(z_2,\bar{z}_2+t\bar{z}_{12})\, t^{\Delta_1-J_1-M+N-1}(1-t)^{\Delta_2-J_2-M+N-1}$$

where

$$M = \frac{\Delta_1+\Delta_2-\Delta_P}{2}\,, \qquad N = \frac{J_1+J_2-J_P}{2}\,,$$

$$\mathcal{N}^{\mathcal{O}_P}_{\mathcal{O}_1\mathcal{O}_2} = \frac{C^{\mathcal{O}_P}_{\mathcal{O}_1\mathcal{O}_2}}{B(\Delta_1-J_1-M+N,\ \Delta_2-J_2-M+N)}\,.$$

Celestial OPE

- In GR, the OPE between two graviton operators:

[Fan, Fotopoulos, Taylor 19; Pate, Raclariu, Strominger, Yuan '19]

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1-1,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)$$

- More generally, the minimal universal gravitational coupling gives

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1-J_1+1,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,J_1}(z_2,\bar{z}_2)$$

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- Supergravity $\rightarrow$ super-OPE [Jiang '21]

$$W_{\Delta_1}(z_1,\bar{z}_1,\eta_1)W_{\Delta_2}(z_2,\bar{z}_2,\eta_2) \sim -\frac{\bar{z}_{12}}{z_{12}} W_{\Delta_1+\Delta_2}(z_2,\bar{z}_2,\eta_1 e^{\frac{1}{2}\partial_{\Delta_1}}+\eta_2 e^{\frac{1}{2}\partial_{\Delta_2}})B(\Delta_1-1,\Delta_2-1)$$

where the celestial on-shell sueprfield/super-operator is

$$W_\Delta(z,\bar{z},\eta) = \mathcal{O}_{\Delta,+2}(z,\bar{z}) + \eta\mathcal{O}_{\Delta,+\frac{3}{2}}(z,\bar{z})$$

Holographic chiral algebra: soft-soft OPE

- Let's start with OPE between graviton and matter

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - J_1 + 1, \Delta_2 - 1)\mathcal{O}_{\Delta_1+\Delta_2,J_1}(z_2,\bar{z}_2)$$

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- Applying the $\overline{SL(2,\mathbb{R})}$ descendant summation formula leads to

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- Taking both operators soft $\rightsquigarrow$ OPE between two soft currents

$$\Delta_1 \to k = J_1, J_1 - 1, \cdots, \qquad \Delta_2 \to l = 2,\ 1, \cdots$$

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- Taking both operators soft $\rightsquigarrow$ OPE between two soft currents

$$\Delta_1 \to k = J_1, J_1 - 1, \cdots, \qquad \Delta_2 \to l = 2,\ 1, \cdots$$

- Picking up specific chiral component in each soft current
  $\rightsquigarrow$ OPE between two chiral currents

$$\boxed{\mathcal{R}^{i,J_1}_{n}(z_1)\mathcal{R}^{j,+2}_{m}(z_2) \sim -\frac{2}{z_{12}}\Big(m(i-1) - n(j-1)\Big)\mathcal{R}^{i+j-2,J_1}_{n+m}(z_2)}$$

Holographic chiral algebra in supersymmetric EYM theory

The same procedure gives all the chiral OPEs in super-EYM theory:

$$
\begin{aligned}
\text{graviton} \;\rightarrow\quad \mathcal{H}^i_n(z)\mathcal{H}^j_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{H}^{i+j-2}_{n+m}(0)\ ,\\
\text{gravitino} \;\rightarrow\quad \mathcal{I}^i_n(z)\mathcal{H}^j_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{I}^{i+j-2}_{n+m}(0)\ ,\\
\text{gluon} \;\rightarrow\quad \mathcal{K}^{i,a}_n(z)\mathcal{H}^j_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{K}^{i+j-2,a}_{n+m}(0)\ ,\\
\text{gluino} \;\rightarrow\quad \mathcal{L}^{i,a}_n(z)\mathcal{H}^j_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{L}^{i+j-2,a}_{n+m}(0)\ ,\\
\mathcal{K}^{i,a}_n(z)\mathcal{K}^{j,b}_m(0) &\sim \frac{f^{abc}}{z}\mathcal{K}^{i+j-1,c}_{n+m}(0)\ ,\\
\mathcal{K}^{i,a}_n(z)\mathcal{L}^{j,b}_m(0) &\sim \frac{f^{abc}}{z}\mathcal{L}^{i+j-1,c}_{n+m}(0)\ ,\\
\mathcal{I}^i_n(z)\mathcal{K}^{j,a}_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{L}^{i+j-2,a}_{n+m}(0)\ ,\\
\mathcal{I}\mathcal{I},\mathcal{L}\mathcal{I},\mathcal{L}\mathcal{L} &\sim 0\ .
\end{aligned}
$$

[Guevara, Himwich, Pate, Strominger '21][Jiang '21]

Since this algebra is generated by infinite chiral currents, I will call it **holographic chiral algebra (HCA)**.

Holographic chiral algebra

$$\mathcal{H}^i_n(z)\mathcal{H}^j_m(0) \sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{H}^{i+j-2}_{n+m}(0)$$

Using the formula

$$[A,B](z) = \oint_z \frac{dw}{2\pi i} A(w)B(z)$$

one gets the commutator

$$[\mathcal{H}^i_n\,,\ \mathcal{H}^j_m] = -2\Big(m(i-1)-n(j-1)\Big)\mathcal{H}^{i+j-2}_{n+m}$$

$w_{1+\infty}$ algebra as observed by [Strominger '21]

- $\mathcal{H}^1_0$ is a central term: $[\mathcal{H}^1_0, \mathcal{H}^i_n] = 0$.
- The soft gravitons up to sub-sub-leading order, $\mathcal{H}^{\frac{3}{2}}_n$, $\mathcal{H}^2_n$, $\mathcal{H}^{\frac{5}{2}}_n$, generate the whole tower of chiral currents $\mathcal{H}^i_n$ and thus also the HCA.

Questions?

What is the implication of the ∞-dim symmetries?

Especially, what are the corresponding Ward identities?

Soft-hard OPE

- As before, we start with OPE between graviton and matter

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1-1,\Delta_2-J_2+1)\mathcal{O}_{\Delta_1+\Delta_2,J_2}(z_2,\bar{z}_2) \ ,$$

- then sum over all $\overline{SL(2,\mathbb{R})}$ descendants

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} \int_0^1 dt \ \mathcal{O}_{\Delta_1+\Delta_2,J_2}(z_2,\bar{z}_2+t\bar{z}_{12}) \ t^{\Delta_1-2}(1-t)^{\Delta_2-J_2} \ .$$

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- Taking $\mathcal{O}_{\Delta_1,+2}$ soft, while keeping $\mathcal{O}_{\Delta_2,J_2}$ hard

$$\Delta_1 \to l = 2, 1 \cdots$$

$\rightsquigarrow$ OPE between soft current and hard operator

$$\begin{aligned} &H^l(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \\ \sim \quad & -\frac{\bar{z}_{12}}{z_{12}} \sum_{s=0}^{1-l} \frac{(\bar{z}_{12})^s}{s!} \bar{\partial}^s \mathcal{O}_{\Delta_2+l,+J_2}(z_2,\bar{z}_2)\ \frac{(-1)^{-s+1-l}}{(-s+1-l)!} \frac{\Gamma(\Delta_2-J_2+1)}{\Gamma(\Delta_2-J_2+l+s)} \ , \end{aligned}$$

Ward identities from soft-hard OPE

$$
\begin{aligned}
& H^l(z_1,\bar z_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar z_2) \\
\sim \quad & -\frac{\bar z_{12}}{z_{12}}\sum_{s=0}^{1-l}\frac{(\bar z_{12})^s}{s!}\partial^s\mathcal{O}_{\Delta_2+l,+J_2}(z_2,\bar z_2)\,\frac{(-1)^{-s+1-l}}{(-s+1-l)!}\,\frac{\Gamma(\Delta_2-J_2+1)}{\Gamma(\Delta_2-J_2+l+s)}
\end{aligned}
$$

The main claim is:

∞ Graviton Ward identities [Jiang '21]

$$
\begin{aligned}
& \langle H^l(z,\bar z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle \\
= \quad & \sum_{k=1}^{m}\sum_{s=0}^{1-l}\frac{(\bar z-\bar z_k)^{s+1}}{z-z_k}\,\frac{(-1)^{-s-l}}{s!(-s+1-l)!}\,\frac{\Gamma(2\bar h_k+1)}{\Gamma(2\bar h_k+l+s)} \\
& \times\bar\partial_k^s\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_k+l,J_k}(z_2,\bar z_2)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle
\end{aligned}
$$

Ward identities up to sub-sub-leading order

[Adamo, Mason, Sharma '19; Puhm '19; Guevara '19...]

- leading order $l = 1$:

$$
\begin{aligned}
&\langle H^1(z,\bar z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle \\
= &-\sum_{k=1}^{m}\frac{\bar z-\bar z_k}{z-z_k}\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_k+1,J_k}(z_k,\bar z_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle
\end{aligned}
$$

- sub-leading order $l = 0$:

$$
\begin{aligned}
&\langle H^0(z,\bar z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle \\
= &\sum_{k=1}^{m}\frac{(\bar z-\bar z_k)^2}{z-z_k}\left[\frac{2\bar h_k}{\bar z-\bar z_k}-\bar\partial_k\right]\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle
\end{aligned}
$$

- sub-sub-leading order $l = -1$:

$$
\begin{aligned}
&\langle H^{-1}(z,\bar z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle \\
= &-\frac{1}{2}\sum_{k=1}^{m}\frac{(\bar z-\bar z_k)^3}{z-z_k}\left[\frac{2\bar h_k(2\bar h_k-1)}{(\bar z-\bar z_k)^2}-\frac{4\bar h_k\bar\partial_k}{\bar z-\bar z_k}+\bar\partial_k^2\right] \\
&\times\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_k-1,J_k}(z_k,\bar z_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle
\end{aligned}
$$

Graviton Ward identities

Graviton Ward identities [Jiang '21]

$$\langle H^l(z,\bar z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle$$

$$= \sum_{k=1}^{m}\sum_{s=0}^{1-l}\frac{(\bar z-\bar z_k)^{s+1}}{z-z_k}\frac{(-1)^{-s-l}}{s!(-s+1-l)!}\frac{\Gamma(2\bar h_k+1)}{\Gamma(2\bar h_k+l+s)}$$

$$\times\bar\partial_k^s\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_k+l,J_k}(z_k,\bar z_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle$$

- Since we include all $\overline{SL(2,\mathbb{R})}$ descendants, the $\bar z$ dependence is supposed to be exact.
- The z dependence is a set of simple poles in the OPE limit $z\to z_i$ $\rightsquigarrow$ meromorphic fcn with only simple poles is determined by the poles and residues.

Graviton Ward identities

Graviton Ward identities [Jiang '21]

$$\begin{aligned}
&\langle H^l(z,\bar z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle \\
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&\times\bar\partial_k^s\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_k+l,J_k}(z_k,\bar z_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle
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- Only the first few Ward identities up to sub-sub-leading orders are independent, as H^1, H^0, H^{-1} generate the rest of tower of soft currents.

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\end{aligned}
$$

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- Only the first few Ward identities up to sub-sub-leading orders are independent, as H^1, H^0, H^{-1} generate the rest of tower of soft currents.
- Equivalent to the infinite (conformally) soft theorems studied before?

[Hamada, Shiu '18; Li, Lin, Zhang '18; Guevara '19]

Graviton chiral Ward identities

The mode expansion of soft current is compatible with Ward identity: comparing coefficient of $\bar{z}^n$ $\rightsquigarrow$ chiral Ward identities

Graviton chiral Ward identities [Jiang '21]

$$
\begin{aligned}
&\langle \mathcal{H}^i_n(z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle \\
=\;& \sum_{k=1}^{m}\frac{(-1)^{n+i}}{z-z_k}\sum_{r=\max(0,n-i+2)}^{n+i-1}\binom{i+n-1}{r}\frac{(r-n+i-1)\Gamma(2\bar{h}_k+1)}{\Gamma(2\bar{h}_k+r-n-i+2)} \\
&\times \bar{z}_k^r\,\bar{\partial}_k^{r-n+i-2}\langle \mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k-2i+4,J_k}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle
\end{aligned}
$$

Shadow Ward identities

Shadow transformation in 2D CFT: [Osborn '12]

$$\widetilde{O}_{(1-h,1-\bar{h})}(w,\bar{w}) = \int d^2z\,(z-w)^{2h-2}(\bar{z}-\bar{w})^{2\bar{h}-2}O_{(h,\bar{h})}(z,\bar{z})$$

Shadow graviton Ward identities [Jiang '21]

$$\begin{aligned}&\langle \widetilde{H^l}(w,\bar{w})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle\\ =\;& \frac{(-1)^l\pi}{(3-l)!}\sum_{k=1}^{m}\sum_{s=0}^{1-l}\frac{(s+1)\Gamma(2\bar{h}_k+1)}{\Gamma(2\bar{h}_k+l+s)}\times(w-z_k)^l(\bar{w}-\bar{z}_k)^{l+s-2}\\ &\times\bar{\partial}_k^s\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k+l,J_k}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle\end{aligned}$$

Shadow graviton Ward identities

- leading order $l = 1$:

$$\begin{aligned}&\langle \widetilde{H^1}(w,\bar{w})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle\\ = \quad & -\frac{\pi}{2}\sum_{k=1}^{m}\frac{w-z_k}{\bar{w}-\bar{z}_k}\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k+1,+J_k}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle\end{aligned}$$

- sub-leading order $l = 0$:

$$\begin{aligned}&\langle\frac{3}{\pi}\widetilde{H^0}(w,\bar{w})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle\\ = \quad & \sum_{k=1}^{m}\left[\frac{\bar{h}_k}{(\bar{w}-\bar{z}_k)^2}+\frac{\bar{\partial}_k}{\bar{w}-\bar{z}_k}\right]\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle\end{aligned}$$

$\rightsquigarrow$ stress tensor Ward identity [Kapec, Mitra, Raclariu, Strominger '16]

- sub-sub-leading order $l = -1$:

$$\begin{aligned}&\langle\widetilde{H^{-1}}(w,\bar{w})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle\\ = \quad & \sum_{k=1}^{m}\frac{1}{w-z_k}\left[\frac{2\bar{h}_k(2\bar{h}_k-1)}{(\bar{w}-\bar{z}_k)^3}+\frac{4\bar{h}_k\bar{\partial}_k}{(\bar{w}-\bar{z}_k)^2}+\frac{3\bar{\partial}_k^2}{\bar{w}-\bar{z}_k}\right]\\ & \times\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k-1,J_k}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle\end{aligned}$$

Question?

How robust are the Ward identities?

Let us first derive some general results!

General spinning OPE

- In the collinear limit, $p_1 // p_2$, $P = p_1 + p_2$, amplitudes factorize

$$A_n(1^{s_1}, 2^{s_2}, \cdots) \xrightarrow{p_1//p_2} \sum_{s_3} \underbrace{A_3(1^{s_1} + 2^{s_2}, -P^{s_3}) \frac{1}{P^2}}_{\text{Split}(1^{s_1}+2^{s_2}\to P^{-s_3})} A_{n-1}(P^{-s_3}, \cdots)$$

General spinning OPE

- In the collinear limit, $p_1 // p_2$, $P = p_1 + p_2$, amplitudes factorize

$$A_n(1^{s_1}, 2^{s_2}, \cdots) \xrightarrow{p_1//p_2} \sum_{s_3} \underbrace{A_3(1^{s_1} + 2^{s_2}, -P^{s_3}) \frac{1}{P^2}}_{\text{Split}(1^{s_1}+2^{s_2}\to P^{-s_3})} A_{n-1}(P^{-s_3}, \cdots)$$

- Three-point on-shell amplitude is fixed by locality and symmetry!

$$\begin{aligned}
\text{Split}(1^{s_1} + 2^{s_2} \to P^{-s_3}) &= \frac{1}{P^2} A_3(1^{s_1}, 2^{s_2}, -P^{s_3}) \\
&\propto \frac{1}{\langle 12\rangle[12]} [12]^{s_1+s_2-s_3} [1P]^{s_1+s_3-s_2} [2P]^{s_2+s_3-s_1} \\
&\propto \frac{[12]^{s_1+s_2+s_3-1}}{\langle 12\rangle} (\sqrt{x})^{s_1+s_3-s_2} (\sqrt{1-x})^{s_2+s_3-s_1}
\end{aligned}$$

where $s_1 + s_2 + s_3 \geq 0$ and $p_1 = xP, p_2 = (1-x)P$.

General spinning OPE

- In terms of celestial variables,

$$\text{Split}(1^{s_1} + 2^{s_2} \to P^{-s_3}) \propto \frac{\bar{z}_{12}^{s_1+s_2+s_3-1}}{z_{12}} \omega_1^{s_2+s_3-1} \omega_2^{s_1+s_3-1} \omega_P^{-s_3}$$

- Mellin transformation gives the general OPE:

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2)$$
$$\sim c_{J_1J_2J_3} \frac{\bar{z}_{12}^{J_1+J_2+J_3-1}}{z_{12}} B\Big(\Delta_1 + J_2 + J_3 - 1, \Delta_2 + J_1 + J_3 - 1\Big)\mathcal{O}_{\Delta_3,-J_3}(z_2,\bar{z}_2)$$

where $\Delta_3 = \Delta_1 + \Delta_2 + J_1 + J_2 + J_3 - 2$.

- Including all the $\overline{SL(2,\mathbb{R})}$ descendants: [Jiang '21] [Himwich, Pate, Singh '21]

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim \frac{c_{J_1J_2J_3}}{z_{12}} \sum_{s=0}^{\infty} \frac{(\bar{z}_{12})^{J_1+J_2+J_3+s-1}}{s!}$$
$$\times \bar{\partial}^s \mathcal{O}_{\Delta_3,-J_3}(z_2,\bar{z}_2)\, B(\Delta_1 + s + J_2 + J_3 - 1, \Delta_2 + J_1 + J_3 - 1)$$

General Ward identities in CCFT

General formula for Ward identities [Jiang '21]

$$
\begin{aligned}
&\langle R^{l,J}(z,\bar z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle \\
=\;& \sum_{k=1}^{m}(-1)^{\nu(\nu_1+\cdots+\nu_{k-1})}c_{JJ_kJ'_k}\sum_{s=0}^{1-l-J_k-J'_k}\frac{(\bar z-\bar z_k)^{J+J_k+J'_k+s-1}}{z-z_k} \\
&\times\frac{(-1)^{(1-l-s-J_k-J'_k)}}{s!(1-l-s-J_k-J'_k)!}\frac{\Gamma(\Delta_k+J+J'_k-1)}{\Gamma(\Delta_k+J+J_k+2J'_k+l+s-2)} \\
&\times\bar\partial_k^s\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_k+l+J+J_k+J'_k-2,-J'_k}(z_k,\bar z_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle
\end{aligned}
$$

where $\nu_i=0,1$ for bosonic and fermionc operators, respectively.

- Applicable to massless theories controlled by cubic interactions, including fermionic fields.
- Shadow Ward identities for $\widetilde{R^{l,J}}$ and Chiral Ward identities for $\mathcal{R}^i_n$ can be similarly derived in a straightforward way.

Now we can apply the general results to discuss the EFT corrections to graviton Ward identities...

EFT correction to soft graviton theorem

$$\mathcal{A}_{n+1} \xlongequal{p_s \to 0} \left(S^{(0)} + S^{(1)} + S^{(2)} \right) \mathcal{A}_n + \mathscr{O}(p_s^2)$$

- $S^{(0)}$ leading soft factor: exact
- $S^{(1)}$ sub-leading soft factor: no EFT correction, but has 1-loop quantum correction
- $S^{(2)}$ sub-sub-leading soft factor: EFT correction

EFT correction to soft graviton theorem

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- $S^{(0)}$ leading soft factor: exact
- $S^{(1)}$ sub-leading soft factor: no EFT correction, but has 1-loop quantum correction
- $S^{(2)}$ sub-sub-leading soft factor: EFT correction

[Elvang, Jones, Naculich '16]

$$\mathcal{A}_{n+1} \xlongequal{p_s \to 0} \left(S^{(0)} + S^{(1)} + S^{(2)} \right) \mathcal{A}_n + \tilde{S}^{(2)} \mathcal{A}_n + \mathscr{O}(p_s^2)$$

$$\tilde{S}^{(2)} \mathcal{A}_n = \sum_k g_k \frac{[sk]^3}{\langle sk \rangle} \tilde{\mathcal{A}}_n^{(k)}$$

EFT correction to soft graviton theorem

Consider cubic vertex of three particles with helicities s_1, s_2, s_3.

[Elvang, Jones, Naculich '16]

$$\beta = s_1 - s_2 - s_3 + 1$$

For soft graviton $s_1 = +2$: local unitary EFT $\rightsquigarrow$ $\beta < 4$

$$\beta = 3, \qquad \cancel{2}, \qquad 1, \qquad \cdots$$

$\beta = 3$: minimal gravitational coupling

$\beta = 1$: correction to soft factor

For soft photon $\beta \leq 2$. As a result, the total helicity $s_1 + s_2 + s_3$ can not vanish in theories involving graviton, gravitino, or photon

$$\rightsquigarrow \frac{1}{z_{12}\bar{z}_{12}} \text{ is forbidden in OPE}$$

EFT correction to graviton Ward identities

$$\beta = 1 : \qquad \phi R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \ , \quad R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \ , \quad R^{\mu\nu\rho\sigma} \bar{\psi}_\rho \gamma_{\mu\nu} \partial_\sigma \chi$$

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}^3}{z_{12}} B(\Delta_1+1, \Delta_2 - J_2 + 3)\mathcal{O}_{\Delta_1+\Delta_2+2,J_2-2}(z_2,\bar{z}_2)$$

The resulting Ward identity is

$$\begin{aligned}
&\langle H^l(z,\bar{z})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle \\
&= \sum_{k=1}^{m}\sum_{s=0}^{-l-1} \frac{(\bar{z}-\bar{z}_k)^{s+3}}{z-z_k} \frac{(-1)^{-l-s-1}}{s!(-l-s-1)!} \frac{\Gamma(\Delta_k - J_k + 3)}{\Gamma(\Delta_k - J_k + l + s + 4)} \\
&\qquad \times \bar{\partial}_k^s \langle \mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k+l+2,J_k-2}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle
\end{aligned}$$

Non-trivial correction starts at sub-sub-leading order $l = -1$:

$$\begin{aligned}
&\langle H^{-1}(z,\bar{z})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle \\
&= \sum_{k=1}^{m} \frac{(\bar{z}-\bar{z}_k)^3}{z-z_k} \langle \mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k+1,J_k-2}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle
\end{aligned}$$

Outlook

- Based on [Guevara, Himwich, Pate, Strominger '21], we propose a procedure to study Ward identities associated with holographic chiral symmetries in celestial holography:

$$\text{celestial OPE} \ \rightarrow \ \text{procedure} \rightarrow \text{HCA} + \text{Ward identities}$$

- We also derive a general formula for OPE arising from cubic interaction of three spinning massless particles:

$$\text{triple } (J_1, J_2, J_3) \rightarrow \ \text{celestial OPE}$$

- Negative helicity soft particles?
- Quantum corrections?
- Higher dims?
- Multiple soft theorems?
- ...

Thank you!