S-matrix bootstrap and (asymptotic) symmetries

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Introduction

- In four dimensional flat space time the asymptotic symmetries are infinite dimensional. (Bondi-van der Burg-Metzner, Sachs, Barnich-Troessaert,..)

- They act on the S-matrix and the Ward identities for these symmetries are the soft theorems.
(Strominger, He-Lysov-Mitra-Strominger, Strominger-Ziboedov, Campiglia-Laddha, Kapec-Lysov-Pasterski-Strominger, Kapec-Mitra-Raclariu-Strominger,...)

- Can one bootstrap S-matrix using the infinite symmetries ?

- In other words, to what extent can one solve for the S-matrix using general properties of the holographic dual theory ?

An Example

- I will give an example.
- Consider tree level MHV graviton scattering amplitudes in GR.
- Using the infinite dimensional asymptotic symmetries (one copy of Vir , $\overline{SL(2,\mathbb{C})}$ current algebra, supertranslations and an infinite number of other soft symmetries) one can write down differential equations for MHV amplitudes,

- $$\left(\mathcal{L}_{-1}\mathcal{P}_{-1,-1} + 2\mathcal{J}^0_{-1}\mathcal{P}_{-1,-1} - (\Delta+1)\mathcal{P}_{-2,-1} - \bar{\mathcal{L}}_{-1}\mathcal{P}_{-2,0}\right) \times \left\langle G(\epsilon\omega, z, \bar{z}, \sigma=+2) \prod_i G(\epsilon_i\omega_i, z_i, \bar{z}_i, \sigma_i) \right\rangle_{MHV} = 0 \tag{1}$$

  where

  $$\begin{aligned} \mathcal{L}_{-1}\mathcal{P}_{-1,-1} &= \epsilon\omega \frac{\partial}{\partial z} \\ \mathcal{J}^0_{-1}\mathcal{P}_{-1,-1} &= -\left(\sum_i \frac{\frac{1}{2}\left(-\omega_i \frac{\partial}{\partial\omega_i} - \sigma_i\right) + (\bar{z}_i - \bar{z})\bar{\partial}_i}{z_i - z}\right)\omega \\ (\Delta+1)\mathcal{P}_{-2,-1} &= -\left(-\omega\frac{\partial}{\partial\omega} + 1\right)\left(\sum_i \frac{\epsilon_i\omega_i}{z_i - z}\right) \\ \bar{\mathcal{L}}_{-1}\mathcal{P}_{-2,0} &= -\frac{\partial}{\partial\bar{z}}\left(\sum_i \frac{\bar{z}_i - \bar{z}}{z_i - z}\epsilon_i\omega_i\right) \end{aligned} \tag{2}$$

- There are $(n-2)$ such equations, corresponding to $(n-2)$ positive helicity gravitons, for an n-point MHV amplitude.
  (Banerjee - Ghosh - paul)

- These equations arise due to the decoupling of primary descendants of the infinite dimensional symmetry algebra.
- The holographic dual theory which computes the MHV amplitudes is a Chiral CFT_2 with infinite dimensional global symmetries (w_∞ symmetries......).
- This CFT_2 can be successfully bootstrapped like Minimal models.
- Note that this is bootstrapping tree level massless scattering amplitudes.
- The same thing can be done for MHV gluon scattering amplitudes. (Banerjee - Ghosh ; Hu - Ren - Srikant - Volovich)

Difference with standard CFT bootstrap

- In celestial CFT the scaling dimensions of primary operators do not need to be bootstrapped because they are arbitrary (complex) numbers.
- This is a major difference which, hopefully, makes the job of bootstrapping celestial CFTs simpler.
- It will be very useful if one can develop techniques of incorporating the asymptotic symmetries in the standard framework of S-matrix bootstrap.