Black holes beyond GR

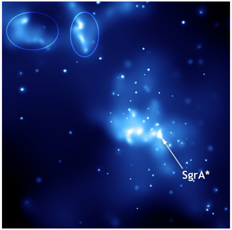
Christos Charmousis

IJCLab-CNRS

Corfu 2021 Summer Institute

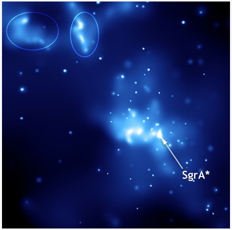
Collaborators : T. Anson, O. Baake, E. Babichev, M. Crisostomi, R. Gregory, M. Hassaine, M. San Juan, A Lehébel, N. Stergioulas

Black holes and neutron stars, breakthrough in observational data



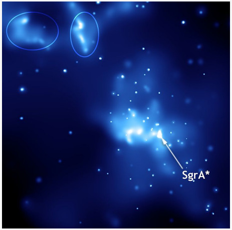
- GW binaries and their ringdown phase : GW170817 neutron star merger, GW190814 and the large mass secondary at $2.59^{+0.08}_{-0.09}\ M_{\odot}$
- Array of radio telescopes, EHT : image of M87 black hole with its light ring, Gravity : observation of star trajectories orbiting SgrA central black hole : orbit characteristics give us tests of strong gravity which get better as precision increases.
- X-ray telescopes and timing observations of pulsars, (eg NICER aiming to measure EoS for neutron stars).
- What is the maximal mass of neutron stars? What is their equation of state? How rapid is their rotation?
- Is the compact secondary the heaviest neutron star or the lightest astrophysical black hole?
- Can we find pulsars in the vicinity of SgrA?
- Can we find alternatives to GR black holes as precise rulers of departure from GR?

Black holes and neutron stars, breakthrough in observational data



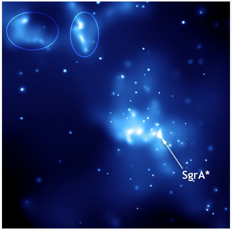
- GW binaries and their ringdown phase : GW170817 neutron star merger, GW190814 and the large mass secondary at $2.59^{+0.08}_{-0.09}\ M_\odot$
- Array of radio telescopes, EHT : image of M87 black hole with its light ring, Gravity : observation of star trajectories orbiting SgrA central black hole : orbit characteristics give us tests of strong gravity which get better as precision increases.
- X-ray telescopes and timing observations of pulsars, (eg NICER aiming to measure EoS for neutron stars).
- What is the maximal mass of neutron stars? What is their equation of state? How rapid is their rotation?
- Is the compact secondary the heaviest neutron star or the lightest astrophysical black hole?
- Can we find pulsars in the vicinity of SgrA?
- Can we find alternatives to GR black holes as precise rulers of departure from GR?

Black holes and neutron stars, breakthrough in observational data



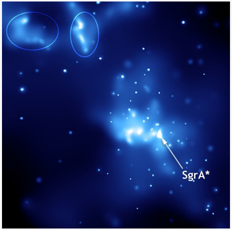
- GW binaries and their ringdown phase : GW170817 neutron star merger, GW190814 and the large mass secondary at $2.59^{+0.08}_{-0.09}\ M_{\odot}$
- Array of radio telescopes, EHT : image of M87 black hole with its light ring, Gravity : observation of star trajectories orbiting SgrA central black hole : orbit characteristics give us tests of strong gravity which get better as precision increases.
- X-ray telescopes and timing observations of pulsars, (eg NICER aiming to measure EoS for neutron stars).
- What is the maximal mass of neutron stars? What is their equation of state? How rapid is their rotation?
- Is the compact secondary the heaviest neutron star or the lightest astrophysical black hole?
- Can we find pulsars in the vicinity of SgrA?
- Can we find alternatives to GR black holes as precise rulers of departure from GR?

Plan and keywords




- GR black holes
- Scalar tensor : stealth solutions and Carter's work on HJ and Kerr geodesics
- Kerr Schild method and Disformal transformations will permit
- Regular black holes and stationary black holes beyond GR

Plan and keywords




- GR black holes
- Scalar tensor : stealth solutions and Carter's work on HJ and Kerr geodesics
- Kerr Schild method and Disformal transformations will permit
- Regular black holes and stationary black holes beyond GR

GR black holes

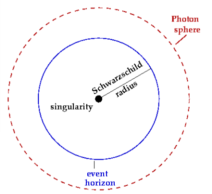
In GR black holes are characterised by a finite number of charges

They are relatively simple solutions-they have no hair, $Q^2 = -J^2/M$

- During collapse, black holes lose their hair and relax to some stationary state of large symmetry. They are (mostly) vacuum solutions of Einstein's eqs, $G_{\mu\nu} = 0$
- Static and spherically symmetric Schwarzschild solution :

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

  with $f(r) = 1 - \frac{2M}{r}$
- Far away spacetime is asymptotically flat $f(r) \underset{r\to\infty}{\longrightarrow} 1$
- The zero(s) of $f(r)$ are coordinate and not curvature singularities, they are the horizon(s) of the black hole ($r_h = 2M$).
- An event horizon determines an absolute surface of no return. It defines the trapped region of the black hole. It hides the central curvature singularity at $r = 0$



GR black holes

In GR black holes are characterised by a finite number of charges

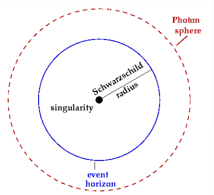
They are relatively simple solutions-they have no hair, $Q^2 = -J^2/M$

- During collapse, black holes lose their hair and relax to some stationary state of large symmetry. They are (mostly) vacuum solutions of Einstein's eqs, $G_{\mu\nu} = 0$
- Static and spherically symmetric Schwarzschild solution :

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

with $f(r) = 1 - \frac{2M}{r}$

- Far away spacetime is asymptotically flat $f(r) \underset{r\to\infty}{\longrightarrow} 1$
- The zero(s) of $f(r)$ are coordinate and not curvature singularities, they are the horizon(s) of the black hole ($r_h = 2M$).
- An event horizon determines an absolute surface of no return. It defines the trapped region of the black hole. It hides the central curvature singularity at $r = 0$



In GR black holes are characterised by a finite number of charges

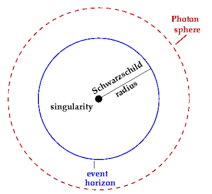
They are relatively simple solutions-they have no hair, $Q^2 = -J^2/M$

- During collapse, black holes lose their hair and relax to some stationary state of large symmetry. They are (mostly) vacuum solutions of Einstein's eqs, $G_{\mu\nu} = 0$
- Static and spherically symmetric Schwarzschild solution :

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

with $f(r) = 1 - \frac{2M}{r}$

- Far away spacetime is asymptotically flat $f(r) \underset{r\to\infty}{\longrightarrow} 1$
- The zero(s) of $f(r)$ are coordinate and not curvature singularities, they are the horizon(s) of the black hole ($r_h = 2M$).
- An event horizon determines an absolute surface of no return. It defines the trapped region of the black hole. It hides the central curvature singularity at $r = 0$



In GR black holes are characterised by a finite number of charges

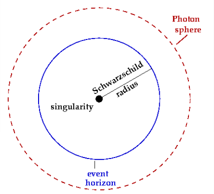
They are relatively simple solutions-they have no hair, $Q^2 = -J^2/M$

- During collapse, black holes lose their hair and relax to some stationary state of large symmetry. They are (mostly) vacuum solutions of Einstein's eqs, $G_{\mu\nu} = 0$
- Static and spherically symmetric Schwarzschild solution :

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

with $f(r) = 1 - \frac{2M}{r}$

- Far away spacetime is asymptotically flat $f(r) \underset{r\to\infty}{\longrightarrow} 1$
- The zero(s) of $f(r)$ are coordinate and not curvature singularities, they are the horizon(s) of the black hole ($r_h = 2M$).
- An event horizon determines an absolute surface of no return. It defines the trapped region of the black hole. It hides the central curvature singularity at $r = 0$



GR black holes

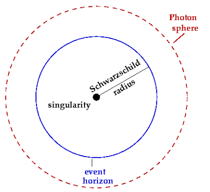
In GR black holes are characterised by a finite number of charges

They are relatively simple solutions-they have no hair, $Q^2 = -J^2/M$

- During collapse, black holes lose their hair and relax to some stationary state of large symmetry. They are (mostly) vacuum solutions of Einstein's eqs, $G_{\mu\nu} = 0$
- Static and spherically symmetric Schwarzschild solution :

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

  with $f(r) = 1 - \frac{2M}{r}$
- Far away spacetime is asymptotically flat $f(r) \underset{r\to\infty}{\longrightarrow} 1$
- The zero(s) of $f(r)$ are coordinate and not curvature singularities, they are the horizon(s) of the black hole ($r_h = 2M$).
- An event horizon determines an absolute surface of no return. It defines the trapped region of the black hole. It hides the central curvature singularity at $r = 0$



The rotating Kerr black hole

- Kerr black hole

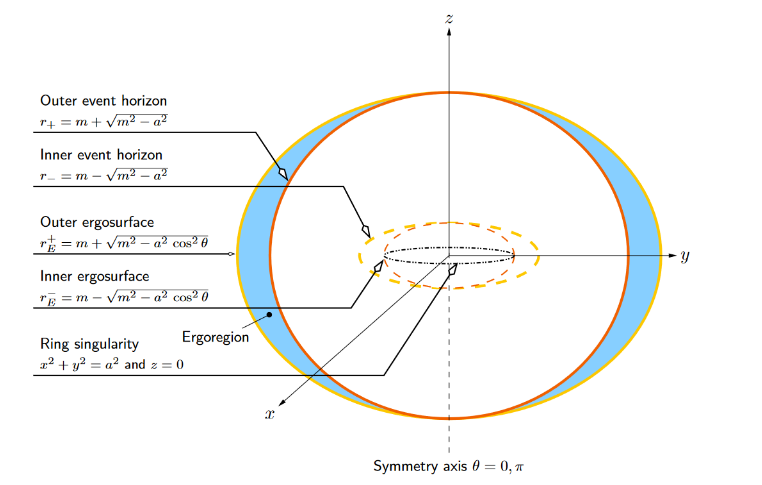
$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4aMr\sin^2\theta}{\rho^2} dt d\varphi + \frac{\sin^2\theta}{\rho^2} \left[(r^2 + a^2)^2 - a^2\Delta\sin^2\theta\right] d\varphi^2$$
$$+ \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2$$

where M is the mass, a is the angular momentum per unit mass, and

$$\rho^2 = r^2 + a^2\cos^2\theta, \ \Delta = r^2 + a^2 - 2Mr \ .$$

- Stationary and axisymmetric spacetime : two Killing vectors $\partial_t, \partial_\varphi$
- Spacetime is circular : $(-t, -\varphi) \leftrightarrow (t, \varphi)$
- Geodesics are integrable : In 4 dimensions we need 4 constants of motion to describe test particles : L_z, E, m, Q.
  Geodesic equation is given as a first order diff eq using HJ functional S,

$$\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2$$



[Visser, 2007]

- ∂_t, $\partial_t + \omega\partial_\varphi$ define static and stationary observers.
- Kerr is a causal spacetime as long as it is a black hole!
- timelike and null geodesics dictate trajectories of test particles or light in the vicinity of the black hole : light ring, black hole shadow etc.

Scalar tensor theories : a robust measurable departure from GR

Simplest modified gravity theory with a single scalar degree of freedom

limit of most modified gravity theories
Examples : BD theory,..., Horndeski,..., beyond Horndeski,..., DHOST theories
([Noui, Langlois, Crisostomi, Koyama et al])

- simplest ST have only GR black hole solutions (no hair theorems)
- For hairy black holes we need to have higher derivative theories... Horndeski, Beyond and DHOST (most general well defined theory with 3 degrees of freedom)
- Nothing fundamental about ST theories, they are just sane and measurable departures from GR.
- They are limits of more complex fundamental theories
- They are parametrized by 6 functions of scalar and its kinetic energy, $f, K, G_3, A_{3,4,5} = A_{3,4,5}(\phi, X)$.

Scalar tensor theories : a robust measurable departure from GR

Simplest modified gravity theory with a single scalar degree of freedom

limit of most modified gravity theories
Examples : BD theory,..., Horndeski,..., beyond Horndeski,..., DHOST theories
([Noui, Langlois, Crisostomi, Koyama et al])

- simplest ST have only GR black hole solutions (no hair theorems)
- For hairy black holes we need to have higher derivative theories... Horndeski, Beyond and DHOST (most general well defined theory with 3 degrees of freedom)
- Nothing fundamental about ST theories, they are just sane and measurable departures from GR.
- They are limits of more complex fundamental theories
- They are parametrized by 6 functions of scalar and its kinetic energy, $f, K, G_3, A_{3,4,5} = A_{3,4,5}(\phi, X)$.

$$S = M_P^2 \int \mathrm{d}^4x \sqrt{-g} \left(f(\phi, X) R + K(\phi, X) - G_3(\phi, X) \Box\phi + \sum_{i=1}^{5} A_i(\phi, X) \mathcal{L}_i \right)$$

$$+ S_\mathrm{m} \left[g_{\mu\nu}, \psi_\mathrm{m} \right]$$

$$\mathcal{L}_1 = \partial_{\mu\nu}\partial^{\mu\nu}, \quad \mathcal{L}_2 = (\Box\phi)^2, \quad \mathcal{L}_3 = \phi_{\mu\nu}\partial^\mu\partial^\nu\Box\phi,$$

$$\mathcal{L}_4 = \phi_\mu\phi^\nu\phi^{\mu\alpha}\phi_{\nu\alpha}, \quad \mathcal{L}_5 = \left(\phi_{\mu\nu}\partial^\mu\partial^\nu\right)^2$$

$$X = \phi^\mu\phi_\mu$$

Scalar tensor theories

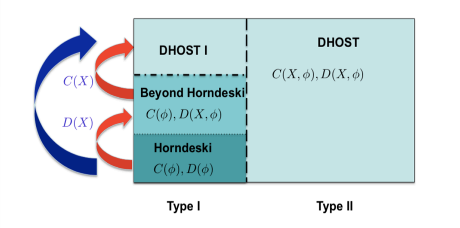
Limits of numerous modified gravity theories
Example :

$$S = \int d^4x \sqrt{-g} \left[R - 2\Lambda_b - X + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] ,$$

- Kinetic term is $X = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ and theories are shift symmetric.
- Conformal and Disformal transformations are internal maps of DHOST theories. They permit us to relate the different versions of ST theories.
- Conformal and disformal map :

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X) \nabla_\mu \phi \nabla_\nu \phi$$

  for given (regular) functions C and D.
- Aim : Construct black hole solutions



[Langlois, 2018]

Scalar tensor theories

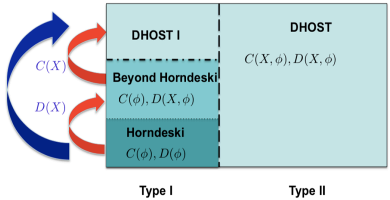
Limits of numerous modified gravity theories
Example :

$$S = \int d^4x \sqrt{-g} \left[R - 2\Lambda_b - X + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right],$$

- Kinetic term is $X = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ and theories are shift symmetric.
- Conformal and Disformal transformations are internal maps of DHOST theories. They permit us to relate the different versions of ST theories.
- Conformal and disformal map :

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X) \nabla_\mu \phi \nabla_\nu \phi$$

  for given (regular) functions C and D.
- Aim : Construct black hole solutions



[Langlois, 2018]

Solution of spherical symmetry

- Example Horndeski theory [Babichev, CC]

$$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda_b - \eta X + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right],$$

- We find the general spherically symmetric solutions, $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, $\phi = \phi(t, r)$,
- simple (stealth) solution reads

$$f = h = 1 - \frac{2\mu}{r} + \frac{\eta}{3\beta} r^2$$

,

$$\phi = qt \pm \int dr \, \frac{q}{h} \sqrt{1 - h}$$

with $q^2 = \frac{\zeta\eta + \Lambda_b \beta}{\beta\eta}$.

- A disformal transformation will take us to a new solution for a different theory,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2} X} \phi_\mu \phi_\nu .$$

- The disformed metric is still a stealth black hole

Solution of spherical symmetry

- Example Horndeski theory [Babichev, CC]

$$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda_b - \eta X + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right],$$

- We find the general spherically symmetric solutions, $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, $\phi = \phi(t, r)$,
- simple (stealth) solution reads

$$f = h = 1 - \frac{2\mu}{r} + \frac{\eta}{3\beta} r^2$$

,

$$\phi = qt \pm \int dr \, \frac{q}{h} \sqrt{1 - h}$$

with $q^2 = \frac{\zeta\eta + \Lambda_b \beta}{\beta\eta}$.

- A disformal transformation will take us to a new solution for a different theory,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2} X} \phi_\mu \phi_\nu .$$

- The disformed metric is still a stealth black hole

Solution of spherical symmetry

- Example Horndeski theory [Babichev, CC]

$$S = \int d^4x \sqrt{-g}\left[\zeta R - 2\Lambda_b - \eta X + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

- We find the general spherically symmetric solutions, $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, $\phi = \phi(t, r)$,
- simple (stealth) solution reads

$$f = h = 1 - \frac{2\mu}{r} + \frac{\eta}{3\beta}r^2$$

,

$$\phi = qt \pm \int dr\ \frac{q}{h}\sqrt{1-h}$$

with $q^2 = \frac{\zeta\eta + \Lambda_b\beta}{\beta\eta}$.

- $X = g^{\mu\nu}\phi_\mu\phi_\nu = -\frac{q^2}{h} + q^2\frac{f(1-h)}{h^2} = -q^2$ is constant.
- A disformal transformation will take us to a new solution for a different theory,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2}X}\phi_\mu\phi_\nu .$$

- The disformed metric is still a stealth black hole

- Example Horndeski theory [Babichev, CC]

$$S = \int d^4x\sqrt{-g}\left[\zeta R - 2\Lambda_b - \eta X + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

- We find the general spherically symmetric solutions, $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, $\phi = \phi(t,r)$,
- simple (stealth) solution reads

$$f = h = 1 - \frac{2\mu}{r} + \frac{\eta}{3\beta}r^2$$

,

$$\phi = qt \pm \int dr\ \frac{q}{h}\sqrt{1-h}$$

with $q^2 = \frac{\zeta\eta+\Lambda_b\beta}{\beta\eta}$.

- A disformal transformation will take us to a new solution for a different theory,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2}X}\,\phi_\mu\phi_\nu.$$

- The disformed metric is still a stealth black hole

For spherical symmetry we can find numerous solutions

Stealth solutions with X constant are generic in DHOST theories

- The real difficulty is how to implement rotation.
- Can we construct stealth rotating solutions?
- For spherical symmetry we have a GR metric and $X = -q^2$.
  Can we obtain the same for a Kerr metric?
- Questions : What is then the scalar field? What is the theory permitting such a solution?
- The key is understanding what $X = -q^2$ signifies geometrically.
  Kerr : Geodesic equation is given as a first order diff eq using HJ functional S,

$$\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2$$

  The *HJ* potential is the scalar field!
- Result : for a certain class of DHOST theories, Kerr with $X = -q^2$ is solution.
- We then find a congruence of geodesics such that the HJ potential is regular everywhere
- Stealth Kerr black hole in DHOST theory [Crisostomi, CC, Gregory, Stergioulas]

Going beyond spherical symmetry

For spherical symmetry we can find numerous solutions

Stealth solutions with X constant are generic in DHOST theories

- The real difficulty is how to implement rotation.
- Can we construct stealth rotating solutions?
- For spherical symmetry we have a GR metric and $X = -q^2$.
  Can we obtain the same for a Kerr metric?
- Questions : What is then the scalar field? What is the theory permitting such a solution?
- The key is understanding what $X = -q^2$ signifies geometrically.
  Kerr : Geodesic equation is given as a first order diff eq using HJ functional S,

$$\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2$$

  The *HJ* potential is the scalar field!
- Result : for a certain class of DHOST theories, Kerr with $X = -q^2$ is solution.
- We then find a congruence of geodesics such that the HJ potential is regular everywhere
- Stealth Kerr black hole in DHOST theory [Crisostomi, CC, Gregory, Stergioulas]

Going beyond spherical symmetry

For spherical symmetry we can find numerous solutions

Stealth solutions with X constant are generic in DHOST theories

- The real difficulty is how to implement rotation.
- Can we construct stealth rotating solutions?
- For spherical symmetry we have a GR metric and $X = -q^2$.
  Can we obtain the same for a Kerr metric?
- Questions : What is then the scalar field? What is the theory permitting such a solution?
- The key is understanding what $X = -q^2$ signifies geometrically.
  Kerr : Geodesic equation is given as a first order diff eq using HJ functional S,

$$\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2$$

  The HJ potential is the scalar field!
- Result : for a certain class of DHOST theories, Kerr with $X = -q^2$ is solution.
- We then find a congruence of geodesics such that the HJ potential is regular everywhere
- Stealth Kerr black hole in DHOST theory [Crisostomi, CC, Gregory, Stergioulas]

Going beyond spherical symmetry

For spherical symmetry we can find numerous solutions

Stealth solutions with X constant are generic in DHOST theories

- The real difficulty is how to implement rotation.
- Can we construct stealth rotating solutions?
- For spherical symmetry we have a GR metric and $X = -q^2$.
  Can we obtain the same for a Kerr metric?
- Questions : What is then the scalar field? What is the theory permitting such a solution?
- The key is understanding what $X = -q^2$ signifies geometrically.
  Kerr : Geodesic equation is given as a first order diff eq using HJ functional S,

$$\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2$$

  The *HJ* potential is the scalar field!
- Result : for a certain class of DHOST theories, Kerr with $X = -q^2$ is solution.
- We then find a congruence of geodesics such that the HJ potential is regular everywhere
- Stealth Kerr black hole in DHOST theory [Crisostomi, CC, Gregory, Stergioulas]

Going beyond spherical symmetry

For spherical symmetry we can find numerous solutions

Stealth solutions with X constant are generic in DHOST theories

- The real difficulty is how to implement rotation.
- Can we construct stealth rotating solutions?
- For spherical symmetry we have a GR metric and $X = -q^2$.
  Can we obtain the same for a Kerr metric?
- Questions : What is then the scalar field? What is the theory permitting such a solution?
- The key is understanding what $X = -q^2$ signifies geometrically.
  Kerr : Geodesic equation is given as a first order diff eq using HJ functional S,

$$\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2$$

  The *HJ* potential is the scalar field!
- Result : for a certain class of DHOST theories, Kerr with $X = -q^2$ is solution.
- We then find a congruence of geodesics such that the HJ potential is regular everywhere
- Stealth Kerr black hole in DHOST theory [Crisostomi, CC, Gregory, Stergioulas]

Going beyond spherical symmetry

For spherical symmetry we can find numerous solutions

Stealth solutions with X constant are generic in DHOST theories

- The real difficulty is how to implement rotation.
- Can we construct stealth rotating solutions?
- For spherical symmetry we have a GR metric and $X = -q^2$.
  Can we obtain the same for a Kerr metric?
- Questions : What is then the scalar field? What is the theory permitting such a solution?
- The key is understanding what $X = -q^2$ signifies geometrically.
  Kerr : Geodesic equation is given as a first order diff eq using HJ functional S,
  $$\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -m^2$$
  The *HJ* potential is the scalar field!
- Result : for a certain class of DHOST theories, Kerr with $X = -q^2$ is solution.
- We then find a congruence of geodesics such that the HJ potential is regular everywhere
- Stealth Kerr black hole in DHOST theory [Crisostomi, CC, Gregory, Stergioulas]

Stealth Kerr solution

- Metric is Kerr

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left[dt - a\sin^2\theta d\varphi\right]^2 + \rho^2\left(\frac{dr^2}{\Delta_r} + d\theta^2\right) + \frac{\sin^2\theta}{\rho^2}\left[a\,dt - \left(r^2 + a^2\right)d\varphi\right]^2 ,$$

$$\Delta_r = \left(r^2 + a^2\right) - 2\mu r\,, \quad \rho^2 = r^2 + a^2\cos^2\theta\,,$$

- Black hole parameters a, μ. What is the scalar field painting this spacetime?
- Carter found separable HJ potential $S = -Et + L_z\varphi + S_r(r) + S_\theta(\theta)$ for which

$$\partial_\mu S\,\partial_\nu S\,g^{\mu\nu}_{Kerr} = -m^2$$

S depends on E, L_z, m, Q, the trajectory parameters of an arbitrary timelike test particle.

- Scalar is given by $\phi = S$. But now ϕ needs to be defined everywhere in spacetime (Geodesics do not cover all of spacetime necessarily!)

$$\phi(t,r) = -q\,t \pm \int \frac{\sqrt{q^2(r^2+a^2)2Mr}}{\Delta_r}dr\,,$$

for $E = m = q$, $L_z = 0$, $Q = ..$

Stealth Kerr solution

- Metric is Kerr

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left[dt - a\sin^2\theta d\varphi\right]^2 + \rho^2\left(\frac{dr^2}{\Delta_r} + d\theta^2\right) + \frac{\sin^2\theta}{\rho^2}\left[a\,dt - \left(r^2 + a^2\right)d\varphi\right]^2 ,$$

$$\Delta_r = \left(r^2 + a^2\right) - 2\mu r\,,\quad \rho^2 = r^2 + a^2\cos^2\theta\,,$$

- Black hole parameters a, μ. What is the scalar field painting this spacetime?
- Carter found separable HJ potential $S = -Et + L_z\varphi + S_r(r) + S_\theta(\theta)$ for which

$$\partial_\mu S\,\partial_\nu S\,g^{\mu\nu}_{Kerr} = -m^2$$

  S depends on E, L_z, m, Q, the trajectory parameters of an arbitrary timelike test particle.
- Scalar is given by $\phi = S$. But now ϕ needs to be defined everywhere in spacetime (Geodesics do not cover all of spacetime necessarily!)

$$\phi(t,r) = -q\,t \pm \int \frac{\sqrt{q^2(r^2+a^2)2Mr}}{\Delta_r}dr\,,$$

  for $E = m = q, L_z = 0, Q = ..$

- By considering an arbitrary disformal transformation we can construct stationary metrics which are not Kerr metrics.
- In fact, the disformed Kerr metrics even with X constant are not trivial at all!

$$g_{\mu\nu}^{Kerr} \longrightarrow \tilde{g}_{\mu\nu} = g_{\mu\nu}^{Kerr} + D(X)\nabla_\mu\phi\nabla_\nu\phi$$

for given D. Rotation creates a solution which has similar characteristics but is completely distinct from the Kerr solution.

$$ds^2 = -\left(1 - \frac{2\tilde{M}r}{\rho^2}\right)dt^2 - \frac{4\sqrt{1+D}\tilde{M}ar\sin^2\theta}{\rho^2}dtd\varphi + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta\right]d\varphi^2$$
$$+ \frac{\rho^2\Delta - 2\tilde{M}(1+D)rD(a^2+r^2)}{\Delta^2}dr^2 - 2D\frac{\sqrt{2\tilde{M}r(a^2+r^2)}}{\Delta}dtdr + \rho^2d\theta^2 .$$

For $D \neq 0$ not an Einstein metric!

Properties of disformed Kerr

$\tilde{g}_{\mu\nu} = g^{Kerr}_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$

For each D we have a new stationary solution. D measures the departure from Kerr

Properties :

- In the absence of rotation the disformal map is a coordinate transformation.
- When the metric is rotating the metric is not an Einstein metric
- Metric has a ring singularity, and an ergoregion. It is a causal spacetime with an event horizon !
- However stationary observers cease to exist before hitting the event horizon (it is not a Killing horizon)!
- Spacetime is not circular!
- Geodesics are not integrable
- Asymptotically we have,

$$d\tilde{s}^2 = ds^2_{\mathrm{Kerr}} + \frac{D}{1+D}\left[\mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}}{r^3}\right)dT^2 + \mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}^{3/2}}{r^{7/2}}\right)\alpha_i dT dx^i + \mathcal{O}\left(\frac{\tilde{a}^2}{r^2}\right)\beta_{ij}dx^i dx^j\right] .$$

Properties of disformed Kerr

$\tilde{g}_{\mu\nu} = g_{\mu\nu}^{Kerr} + D(X)\nabla_\mu\phi\nabla_\nu\phi$

For each D we have a new stationary solution. D measures the departure from Kerr

Properties :

- In the absence of rotation the disformal map is a coordinate transformation.
- When the metric is rotating the metric is not an Einstein metric
- Metric has a ring singularity, and an ergoregion. It is a causal spacetime with an event horizon !
- However stationary observers cease to exist before hitting the event horizon (it is not a Killing horizon)!
- Spacetime is not circular!
- Geodesics are not integrable
- Asymptotically we have,

$$d\tilde{s}^2 = ds^2_{\text{Kerr}} + \frac{D}{1+D}\left[\mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}}{r^3}\right)dT^2 + \mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}^{3/2}}{r^{7/2}}\right)\alpha_i dT dx^i + \mathcal{O}\left(\frac{\tilde{a}^2}{r^2}\right)\beta_{ij}dx^i dx^j\right] .$$

Properties of disformed Kerr

$\tilde{g}_{\mu\nu} = g^{Kerr}_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$

For each D we have a new stationary solution. D measures the departure from Kerr

Properties :

- In the absence of rotation the disformal map is a coordinate transformation.
- When the metric is rotating the metric is not an Einstein metric
- Metric has a ring singularity, and an ergoregion. It is a causal spacetime with an event horizon !
- However stationary observers cease to exist before hitting the event horizon (it is not a Killing horizon)!
- Spacetime is not circular!
- Geodesics are not integrable
- Asymptotically we have,

$$d\tilde{s}^2 = ds^2_{\rm Kerr} + \frac{D}{1+D}\left[\mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}}{r^3}\right)dT^2 + \mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}^{3/2}}{r^{7/2}}\right)\alpha_i dT dx^i + \mathcal{O}\left(\frac{\tilde{a}^2}{r^2}\right)\beta_{ij}dx^i dx^j\right].$$

Properties of disformed Kerr

$\tilde{g}_{\mu\nu} = g^{Kerr}_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$

For each D we have a new stationary solution. D measures the departure from Kerr

Properties :

- In the absence of rotation the disformal map is a coordinate transformation.
- When the metric is rotating the metric is not an Einstein metric
- Metric has a ring singularity, and an ergoregion. It is a causal spacetime with an event horizon !
- However stationary observers cease to exist before hitting the event horizon (it is not a Killing horizon)!
- Spacetime is not circular!
- Geodesics are not integrable
- Asymptotically we have,

$$d\tilde{s}^2 = ds^2_{\text{Kerr}} + \frac{D}{1+D}\left[\mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}}{r^3}\right)dT^2 + \mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}^{3/2}}{r^{7/2}}\right)\alpha_i dT dx^i + \mathcal{O}\left(\frac{\tilde{a}^2}{r^2}\right)\beta_{ij}dx^i dx^j\right] .$$

Properties of disformed Kerr

$\tilde{g}_{\mu\nu} = g^{Kerr}_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$

For each D we have a new stationary solution. D measures the departure from Kerr

Properties :

- In the absence of rotation the disformal map is a coordinate transformation.
- When the metric is rotating the metric is not an Einstein metric
- Metric has a ring singularity, and an ergoregion. It is a causal spacetime with an event horizon !
- However stationary observers cease to exist before hitting the event horizon (it is not a Killing horizon)!
- Spacetime is not circular!
- Geodesics are not integrable
- Asymptotically we have,

$$d\tilde{s}^2 = ds^2_{\text{Kerr}} + \frac{D}{1+D}\left[\mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}}{r^3}\right)dT^2 + \mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}^{3/2}}{r^{7/2}}\right)\alpha_i dT dx^i + \mathcal{O}\left(\frac{\tilde{a}^2}{r^2}\right)\beta_{ij}dx^i dx^j\right].$$

Properties of disformed Kerr

$\tilde{g}_{\mu\nu} = g^{Kerr}_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$

For each D we have a new stationary solution. D measures the departure from Kerr

Properties :

- In the absence of rotation the disformal map is a coordinate transformation.
- When the metric is rotating the metric is not an Einstein metric
- Metric has a ring singularity, and an ergoregion. It is a causal spacetime with an event horizon !
- However stationary observers cease to exist before hitting the event horizon (it is not a Killing horizon)!
- Spacetime is not circular!
- Geodesics are not integrable
- Asymptotically we have,

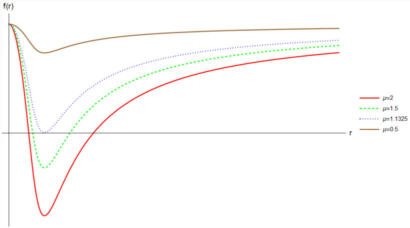
$$d\tilde{s}^2 = ds^2_{\text{Kerr}} + \frac{D}{1+D}\left[\mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}}{r^3}\right)dT^2 + \mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}^{3/2}}{r^{7/2}}\right)\alpha_i dT dx^i + \mathcal{O}\left(\frac{\tilde{a}^2}{r^2}\right)\beta_{ij}dx^i dx^j\right].$$

Construct exotic objects (non existant or problematic in GR)?

Regular black holes, wormholes...

- Solution generating methods in GR : Kerr Schild method
- Extending the Kerr-Schild construction. Metric is everywhere regular-genuine particle like solution
- Inner and outer event horizon, No horizon for small enough mass. Black hole→ soliton

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \text{ with } f(r) = 1 - \frac{4\mu \arctan(\frac{\pi r^3}{2\sigma^2})}{r\pi} \text{ and } X(r) = \frac{2}{\pi}\arctan(\frac{\pi r^3}{2\sigma^2})$$

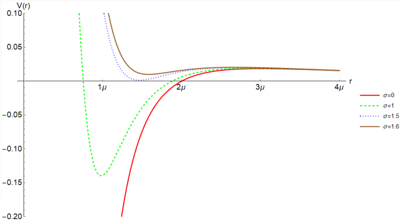


Construct exotic objects (non existant or problematic in GR)?

Regular black holes, wormholes...

- Solution generating methods in GR : Kerr Schild method
- Extending the Kerr-Schild construction. Metric is everywhere regular-genuine particle like solution
- Effective potential for light geodesics grazing the black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \text{ with } f(r) = 1 - \frac{4\mu \arctan(\frac{\pi r^3}{2\sigma^2})}{r\pi} \text{ and } X(r) = \frac{2}{\pi}\arctan(\frac{\pi r^3}{2\sigma^2})$$



Conclusions

- We have seen how to construct non trivial ST black holes which are well behaved
- Using classical results from GR and mathematical symmetries we can construct an armada of phenomenologically interesting solutions
- We can construct exotic solutions like regular black holes, wormholes
- GW, EHT give certain constraints on coupling constant parameters but a lot more to come in the future with key differences from GR