Spacetime structure near generic horizons and soft hair

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Der Wissenschaftsfonds.

Introduction - Boundary conditions (BC)

General relativity: invariant under diffeomorphisms

Spacetime has boundary and the fall-offs cannot be anything
E.g.: Fall-offs of the field crucially enter in the computations of charges

Boundary conditions: field fall-offs such that the charges are finite

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Phase space = Action + BC

Examples

Ex 1: Electromagnetism

a) Fall off of the potential in $1/r$

b) Electric conductor has Dirichlet boundary conditions

Ex 2:

Symmetry algebra of asympt. flat 4D spacetimes at null infinity

$\mathbf{BMS}_4 =$ semi direct sum of superrotations with supertranslations

Supertranslations= angle dependent translations
Superrotations= conformal Killing vectors of the 2 sphere

Introduction - Black Holes (BH)

BH have **horizons**, separating two causally disconnected regions

Boundary conditions at the horizon

BH thermodynamics: $S = \frac{A}{4}$ - What are the microstates?

Physical processes from the perspective of the horizon

Goal

$\boxed{\text{Explore boundary conditions on non extremal BH horizons in } D \geq 3}$

Outline

1. Near horizon expansion
2. Boundary conditions
3. BMS-like symmetries
4. Heisenberg-like symmetries

Near horizon expansion

Starting point: Horizon ($\rho = 0$) line element in a Rindler form

$$\mathrm{d}s^2 = -\kappa^2 \rho^2 \,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \Omega_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b + \dots$$

where κ is the surface gravity, $a = 1, ..., D-2$.

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where κ is the surface gravity, $a = 1, ..., D-2$.

Near horizon expansion (in a co-rotating frame):

$$\begin{aligned} g_{tt} &= -\kappa^2 \rho^2 + \mathcal{O}(\rho^3) & g_{t\rho} &= \mathcal{O}(\rho^2) & g_{ta} &= f_{ta} \, \rho^2 + \mathcal{O}(\rho^3) \\ g_{\rho\rho} &= 1 + \mathcal{O}(\rho) & g_{\rho a} &= f_{\rho a} \, \rho + \mathcal{O}(\rho^2) & g_{ab} &= \Omega_{ab} + \mathcal{O}(\rho^2) \end{aligned}$$

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Near horizon symmetries: $\mathcal{L}_\xi g = \mathcal{O}(g)$

$$\xi^t = \frac{\eta}{\kappa} + \mathcal{O}(\rho), \qquad \xi^\rho = \mathcal{O}(\rho^2), \qquad \xi^a = \eta^a + \mathcal{O}(\rho^2)$$

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Charges

$$\delta Q[\eta,\, \eta^a] = \int \mathrm{d}^{D-2}x \left[\eta\, \delta\mathcal{P} + \eta^a\, \delta\mathcal{J}_a\right]$$

$$\mathcal{P} := \frac{\sqrt{\Omega}}{8\pi G} \qquad \mathcal{J}_a := \frac{\sqrt{\Omega}}{16\pi G\kappa}\left(\partial_t f_{\rho a} - 2 f_{ta}\right)$$

Integrability in Hamiltonian formalism

Restrict to integrable charges i.e. no fluxes through the horizon and a well defined variational principle

ADM decomposition:

$$N = \mathcal{N}\rho + \mathcal{O}(\rho^2), \qquad N^a = \mathcal{N}^a + \mathcal{O}(\rho^2), \qquad N^\rho = \mathcal{O}(\rho^2).$$

Co-rotating frame: $\mathcal{N}^a = 0$, $\mathcal{N} = \kappa$.

$$\delta I_B = -\int \mathrm{d}t\, \mathrm{d}^{D-2}x \left(\mathcal{N}\,\delta\mathcal{P} + \mathcal{N}^a\,\delta\mathcal{J}_a\right)$$

Integrability

$$\mathcal{N} = \frac{\delta\mathcal{F}}{\delta\mathcal{P}}, \quad \mathcal{N}^a = \frac{\delta\mathcal{F}}{\delta\mathcal{J}_a} \qquad \text{for some } \mathcal{F}(\mathcal{P}, \mathcal{J}_a)$$

BC are given by $\mathcal{F}(\mathcal{P}, \mathcal{J}_a)$

Compatibility of time-evolution with near horizon symmetries
$\Rightarrow \mathcal{N} \to \eta$ and $\mathcal{N}^a \to \eta^a$ have the same field dependence.

Donnay Giribet Gonzalez Pino (DGGP) BC

$$\mathcal{F}(\mathcal{P},\, \mathcal{J}_a) = \left[\mathcal{N}\,\mathcal{P} + \mathcal{N}^a\,\mathcal{J}_a\right] \text{ where } \delta\mathcal{N} = 0\,, \delta\mathcal{N}^a = 0$$

$\delta\mathcal{N} = 0 \Rightarrow \delta\kappa = 0$ is kept fixed on the phase space

$$\begin{aligned} \frac{\delta\mathcal{F}}{\delta\mathcal{J}_a} =& \mathcal{N}^a & \Rightarrow& \delta\eta^a = 0 \\ \frac{\delta\mathcal{F}}{\delta\mathcal{P}} =& \mathcal{N} & \Rightarrow& \delta\eta = 0\,. \end{aligned}$$

$$\delta Q[\eta,\, \eta^a] = \int \mathrm{d}^{D-2}x\,\left[\eta\,\delta\mathcal{P} + \eta^a\,\delta\mathcal{J}_a\right] \Rightarrow Q[\eta,\, \eta^a] = \int \mathrm{d}^{D-2}x\,\left[\eta\,\mathcal{P} + \eta^a\,\mathcal{J}_a\right]$$

Family of boundary conditions labelled by $s = r(D-2)$,

$$\mathcal{F}(\mathcal{P},\, \mathcal{J}_a) = \mathcal{N}_{(s)}\, \frac{\mathcal{P}^{r+1}}{r+1} + \mathcal{N}^a \mathcal{J}_a \quad \text{with } \delta\mathcal{N}_{(s)} = 0\,, \delta\mathcal{N}^a = 0$$

$$\begin{aligned} \frac{\delta \mathcal{F}}{\delta \mathcal{J}_a} &= \mathcal{N}^a & \Rightarrow \eta^a &= \eta^a \\ \frac{\delta \mathcal{F}}{\delta \mathcal{P}} &= \mathcal{N} = \mathcal{N}_{(s)} \mathcal{P}^r & \Rightarrow \eta &= \eta_{(s)} \mathcal{P}^r\,. \end{aligned}$$

$\delta Q[\eta,\, \eta^a] = \int \mathrm{d}^{D-2}x \left[\eta\, \delta\mathcal{P} + \eta^a\, \delta\mathcal{J}_a\right]$

$$Q[\eta_{(s)},\, \eta^a] = \int \mathrm{d}^{D-2}x \left[\eta_{(s)} \mathcal{P}_{(s)} + \eta^a \mathcal{J}_a\right] \quad \text{with } \mathcal{P}_{(s)} = \mathcal{P}^{r+1}/(r+1)$$

Poisson bracket algebra is

$$\{\mathcal{J}_a(x), \mathcal{P}_{(s)}(y)\} = \left(r\mathcal{P}_{(s)}(y) \frac{\partial}{\partial x^a} - \mathcal{P}_{(s)}(x) \frac{\partial}{\partial y^a}\right) \delta(x-y)$$

$$\{\mathcal{P}_{(s)}(x), \mathcal{P}_{(s)}(y)\} = 0$$

$$\{\mathcal{J}_a(x), \mathcal{J}_b(y)\} = \left(\mathcal{J}_a(y) \frac{\partial}{\partial x^b} - \mathcal{J}_b(x) \frac{\partial}{\partial y^a}\right) \delta(x-y)$$

4D and conformal coordinates for Ω_{ab}

Conformal coordinates for $\Omega_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b = \Omega^2(|\zeta|)\,\mathrm{d}\zeta\,\mathrm{d}\bar{\zeta}$

$$\mathcal{P}^{(s)}_{(m,n)} = \int d\zeta d\bar{\zeta}\,\zeta^m\bar{\zeta}^n\mathcal{P}^{(s)}\,, \quad \mathcal{J}_k = -\int d\zeta d\bar{\zeta}\,\zeta^{k+1}\mathcal{J}\,, \quad \bar{\mathcal{J}}_k = -\int d\zeta d\bar{\zeta}\,\bar{\zeta}^{k+1}\bar{\mathcal{J}}$$

Algebra: Semi direct sum of superrotations with spin-s supertranslations

$$\begin{aligned}
\{\mathcal{J}_k,\,\mathcal{P}^{(s)}_{(m,n)}\} &= \left(\frac{s}{2}(k+1)-m\right)\,\mathcal{P}^{(s)}_{(k+m,n)}\\
\{\bar{\mathcal{J}}_k,\,\mathcal{P}^{(s)}_{(m,n)}\} &= \left(\frac{s}{2}(k+1)-n\right)\,\mathcal{P}^{(s)}_{(m,k+n)}\\
\{\mathcal{J}_n,\,\mathcal{J}_m\} &= (n-m)\,\mathcal{J}_{n+m}\\
\{\bar{\mathcal{J}}_n,\,\bar{\mathcal{J}}_m\} &= (n-m)\,\bar{\mathcal{J}}_{n+m}\\
\{\mathcal{J}_n,\,\bar{\mathcal{J}}_m\} &= \{\mathcal{P}^{(s)}_{(m,n)},\,\mathcal{P}^{(s)}_{(m',n')}\} = 0\,.
\end{aligned}$$

- $s=0$ - DGGP algebra - $\mathcal{P}^{(0)} = \mathcal{P}$
- $s=1$ - BMS_4 algebra
- $\cdots$

$$\delta Q[\eta,\, \eta^a] = \int \mathrm{d}^{D-2}x \left[\eta\, \delta\mathcal{P} + \eta^a\, \delta\mathcal{J}_a\right], \quad \mathcal{P} := \frac{\sqrt{\Omega}}{8\pi G} \quad \mathcal{J}_a := \frac{\sqrt{\Omega}}{16\pi G\kappa}\left(\partial_t f_{\rho a} - 2f_{ta}\right)$$

Swap the density/vector role of η^a, $\mathcal{J}_a$.

$$\mathcal{F}(\mathcal{P},\, \mathcal{J}_a) = \mathcal{N}_{\mathrm{H}}\mathcal{P} + \mathcal{N}^a_{\mathrm{H}}\, \mathcal{J}_a\, \mathcal{P}^{-1} \text{ with } \delta\mathcal{N}_{\mathrm{H}} = 0,\ \delta\mathcal{N}^a_{\mathrm{H}} = 0$$

$$\frac{\delta\mathcal{F}}{\delta\mathcal{J}_a} = \mathcal{N}^a = \mathcal{N}^a_{\mathrm{H}}\mathcal{P}^{-1} \qquad \Rightarrow \eta^a = \eta^a_{\mathrm{H}}\mathcal{P}^{-1}$$

$$\frac{\delta\mathcal{F}}{\delta\mathcal{P}} = \mathcal{N} = \mathcal{N}_{\mathrm{H}} - \mathcal{N}^a_{\mathrm{H}}\, \mathcal{J}_a\, \mathcal{P}^{-2} \qquad \Rightarrow \eta = \eta_{\mathrm{H}} - \eta^a_{\mathrm{H}}\, \mathcal{J}_a\, \mathcal{P}^{-2}\,.$$

$$Q_{\mathrm{H}}[\eta_{\mathrm{H}},\, \eta^a_{\mathrm{H}}] = \int \mathrm{d}^{D-2}x \left[\eta_{\mathrm{H}}\, \mathcal{P} + \eta^a_{\mathrm{H}}\, \mathcal{J}^{\mathrm{H}}_a\right] \qquad \text{with } \mathcal{J}^{\mathrm{H}}_a := \mathcal{J}_a\mathcal{P}^{-1}$$

Heisenberg-like symmetries

$$\{\mathcal{J}_a^{\mathrm{H}}(x)\,,\,\mathcal{P}(y)\} = \frac{\partial}{\partial x^a}\delta(x-y)$$

$$\{\mathcal{P}(x)\,,\,\mathcal{P}(y)\} = 0$$

$$\{\mathcal{J}_a^{\mathrm{H}}(x)\,,\,\mathcal{J}_b^{\mathrm{H}}(y)\} = \mathcal{P}^{-1}(x)F_{ba}(x)\,\delta(x-y) \qquad \text{with } F := \mathrm{d}\mathcal{J}^{\mathrm{H}}$$

•When $F = 0$, $\mathcal{J}_a^{\mathrm{H}} =: 8\pi G\partial_a\mathcal{Q}$. Heisenberg algebra

$$\{\mathcal{Q}(x),\,\mathcal{P}(y)\} = \frac{1}{8\pi G}\,\delta(x-y)$$

For ex.: 3D, Schwarzschild black holes, ...
• The Hamiltonian Q_{∂_t} commutes with everything
→ All the excitations are soft (do not change the energy)
• Entropy

$$S = 2\pi\mathcal{P}_0 \qquad \text{with } \mathcal{P}_0 = \int \mathrm{d}^{D-2}x\,\mathcal{P}$$

Soft hair as microstates? - fluff proposal in 3D

Conclusions

Recap' Exploration of horizon BCs with integrable charges

- Each BC corresponds to fix different chemical potentials
- First realization of BMS algebra in the near horizon perspective
- Heisenberg-like algebra and soft hair
- Can be applied to cosmological horizons

Further directions:

- Near horizon soft hair as microstates?
- Relation between asymp. and near horizon symm.?
- BC with non integrable charges and fluxes through the horizon

$\rightarrow$ work in progress with H. Adami, D. Grumiller, S. Sadeghian M.M. Sheikh-Jabbari, CZ

- Discussion of the information loss paradox


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