Emergent Geometries from the BMN Matrix Model

Yuhma Asano (KEK)
21 Sep 2019 @Corfu

Based on:
Y.A., V. Filev, S. Kovacik, D. O'Connor, JHEP 1807, 152 (2018).

and based on collaborations with
G. Ishiki, T. Okada, S. Shimasaki, and S. Terashima.

1. Introduction

BMN Matrix Model (aka Plane Wave Matrix Model)

[Berenstein-Maldacena-Nastase '02]

1D super quantum mechanics ($SU(N)$ gauge)

- 16 supersymmetries (32 if you include kinematical susy)
- mass parameter (many discrete vacua)
- dimensional reduction of $\mathcal{N}$=4 Super Yang–Mills on $R\times S^3$ to 1D

(should reproduce $\mathcal{N}$=4 SYM around a special vacuum)

[Ishiki-Shimasaki-Takayama-Tsuchiya '06, …]

Action:

$$S = N\int dt\,\mathrm{Tr}\Big[\frac{1}{2}(D_t X^a)^2+\frac{1}{2}(D_t X^m)^2-\frac{1}{4}\left(\frac{\mu}{3}\epsilon_{abc}X^c - i[X^a,X^b]\right)^2+\frac{1}{2}[X^a,X^n]^2 + \frac{1}{4}[X^m,X^n]^2 - \frac{\mu^2}{72}X^m X^m + \text{fermions}\Big]$$

$a,b=1,2,3,\quad m,n=4,\dots,9$

Symmetry: $R\times SO(3)\times SO(6)$

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- Matrix-regularised membrane theory on the 11D plane-wave geometry [deWit-Hoppe-Nicolai '88]

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- Gauge/Gravity dual to 11D/IIA bubbling geometry

BMN matrix model ⟷ 11D/IIA SUGRA

[Lin-Lunin-Maldacena '04, Lin-Maldacena '05]

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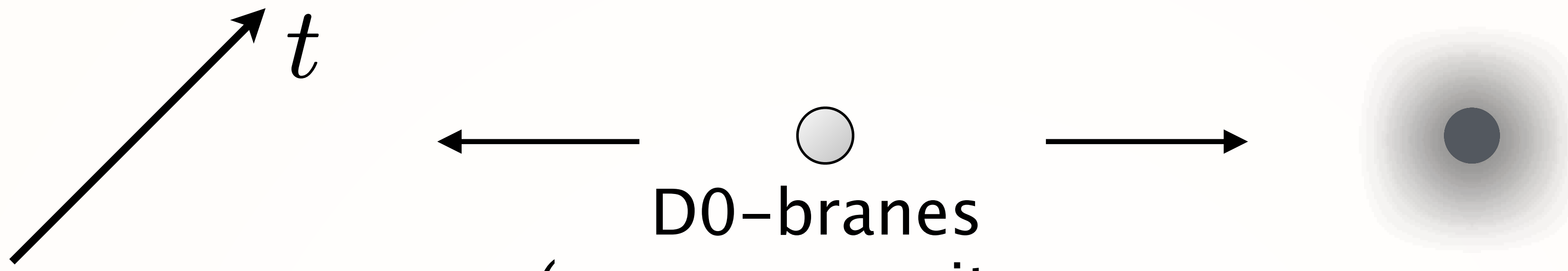


D0−branes
(or supergravitons
on plane−wave B.G.)

BMN matrix model ⟷ **11D/IIA SUGRA**

symmetry: $R\times SO(3)\times SO(6)$ $\xleftrightarrow{SU(2|4)}$ isometry: $R\times SO(3)\times SO(6)$

vacua ($SU(2)$ rep.) ⟷ bubbling geometries

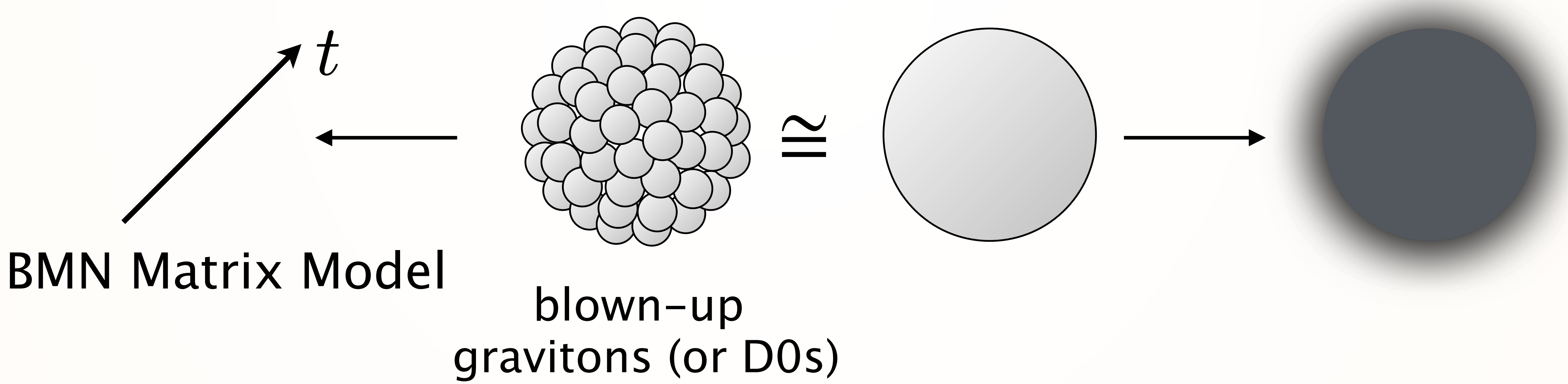
- dim. of irreducible rep. — M5/NS5 charge N_5
- multiplicity of irred. rep. — M2/D2 charge N_2

[Lin-Lunin-Maldacena '04, Lin-Maldacena '05]

2. Emergent Geometries

Fuzzy sphere vacuum
BMN matrix model

M-brane solution
11D SUGRA



2. Emergent Geometries

Realisation of geometries was found by susy localisation calculation of the BPS sector in which ϕ is invariant:

$$\phi(t) = X^3(t) + i(\sin(t)X^8(t) + \cos(t)X^9(t))$$

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$\Longleftrightarrow$

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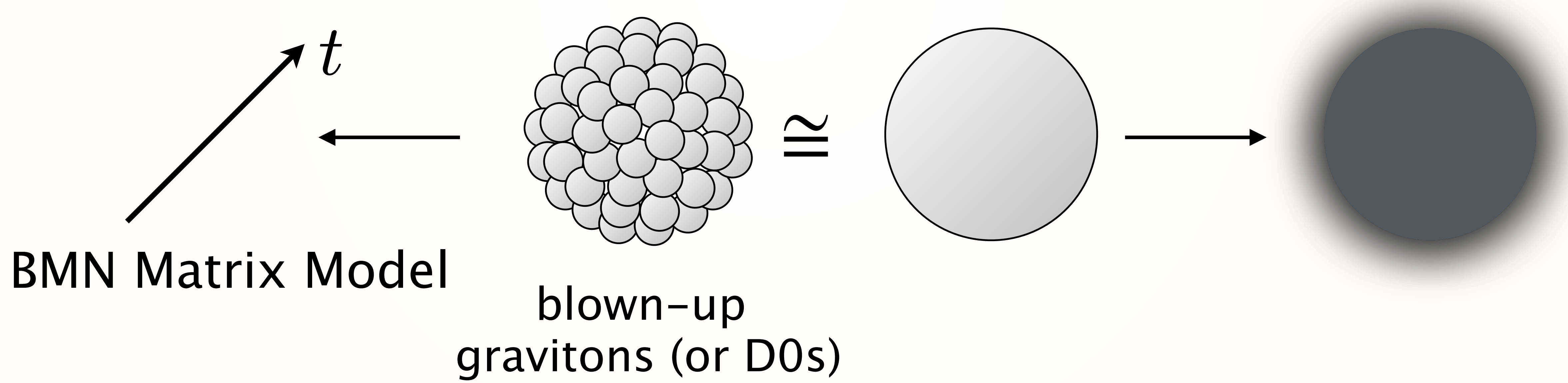

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A BPS operator can be calculated by a simpler matrix integral:

$$\left\langle \prod_I \mathrm{Tr}\, f_I(\phi(t_I)) \right\rangle = \left\langle \prod_I \mathrm{Tr}\, f_I(\tfrac{\mu}{6}(-2L_3 + iM)) \right\rangle_{MM}$$

where $< >_{MM}$ is a vev computed by an integral of matrix M, L_a is proportional to the $SO(3)$ matrices in the vacuum, and L_a and M satisfy $\quad [L_a, L_b] = i\epsilon_{abc}L_c\,, \qquad [L_a, M] = 0$

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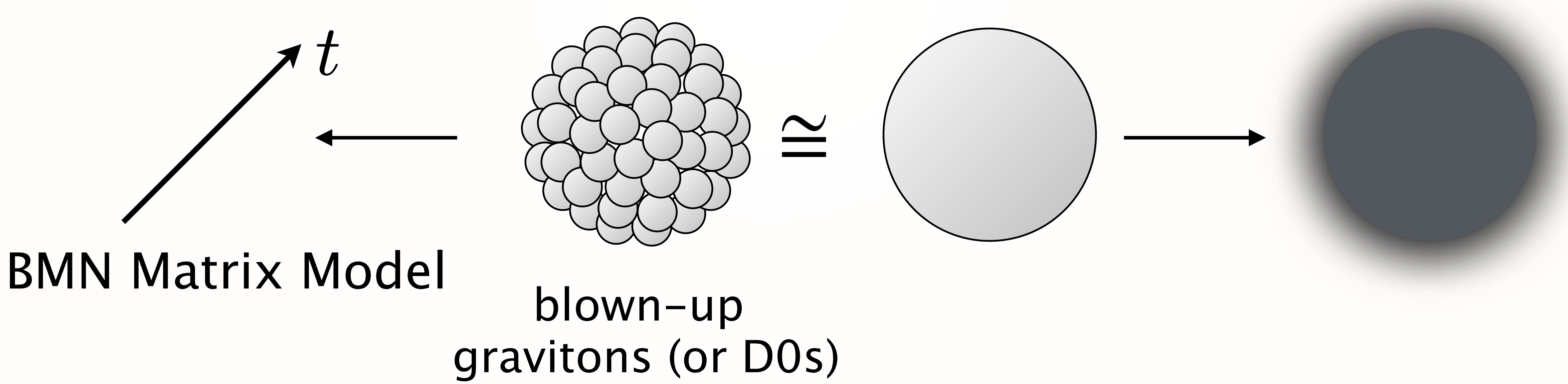

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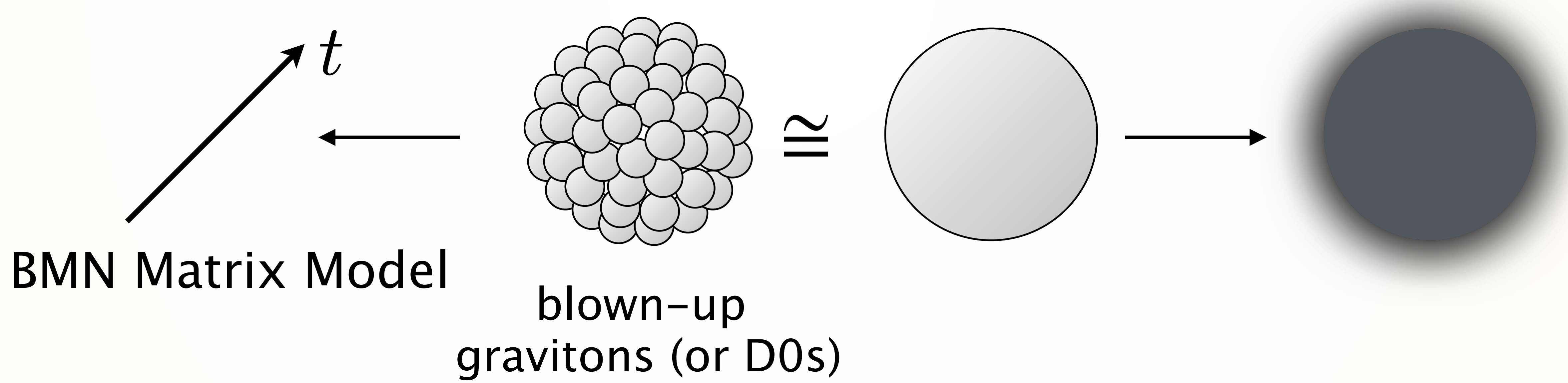

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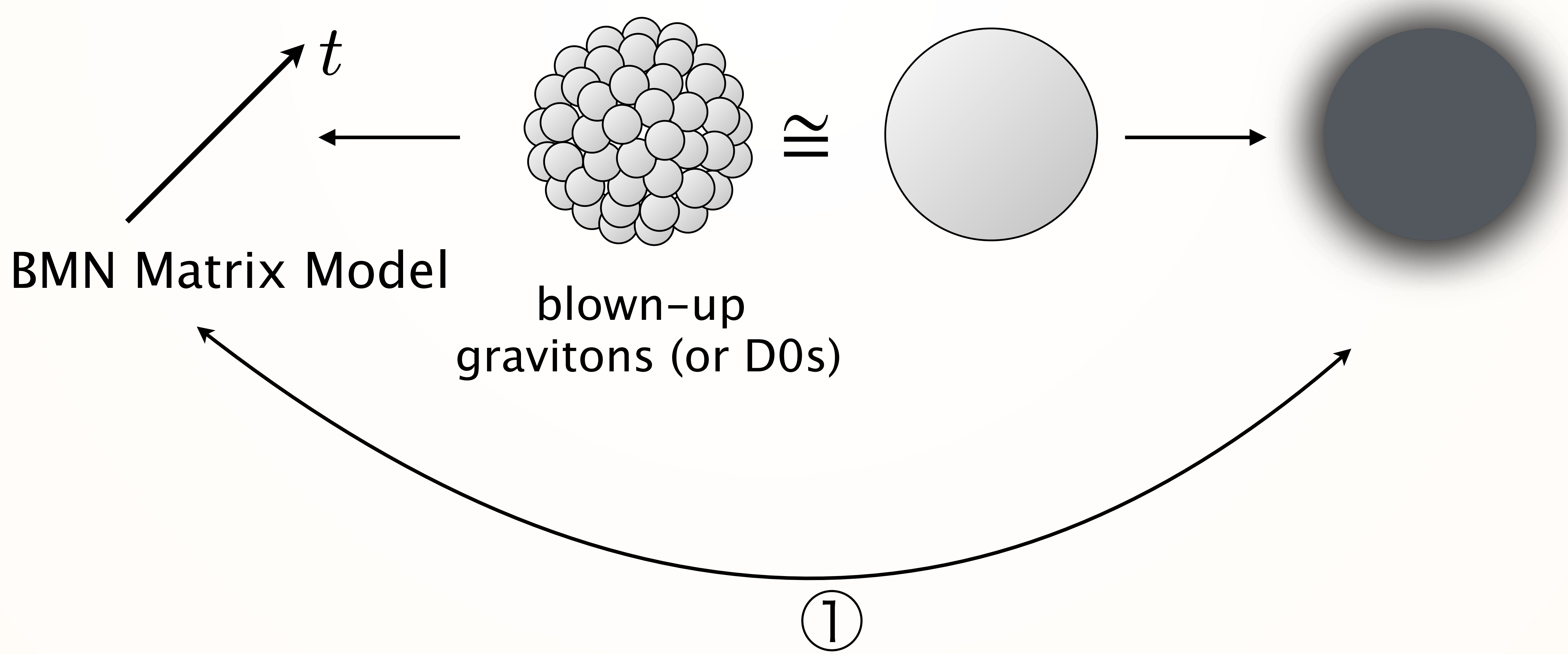
① Part of Einstein's eq. was obtained by the eigenvalue density of ϕ

nontrivial part in terms of the symmetry [Y.A.-Okada-Ishiki-Shimasaki '14]



2. Gauge/Gravity Duality & Emergent Geometries

E.g. vacuum corresponding to a stack of M5s

① A saddle point eq. for the BPS operator is obtained.

$\rho(q)$: eigenvalue dist. of M , where $\phi \sim (\mu/6)(-2L_3 + iM)$

$$\boxed{\rho(q) - \frac{1}{\pi}\int_{-q_m}^{q_m} dq' \frac{2N_5}{(2N_5)^2 + (q-q')^2}\rho(q') = -\frac{2\mu^3 N_5^2}{\pi} q^2 + \mathrm{const.}}$$

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On the gravity side,
the metric is written only by a single function $f(x)$.

The Killing spinor eq. for the geometry reduces to

$$\boxed{f(x) - \frac{1}{\pi}\int_{-R}^{R} du \frac{\pi N_5}{(\pi N_5)^2 + (x-u)^2} f(u) = -V_0 N_5 x^2 + \text{const.}}$$

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Completely the same equations with identification

$$f(x) = \frac{\pi}{4}\rho\left(\frac{2}{\pi}x\right) \qquad R_{S^5}^2 = 4R = 2\pi q_m \qquad V_0 = \frac{2}{\pi^2 g^2}$$

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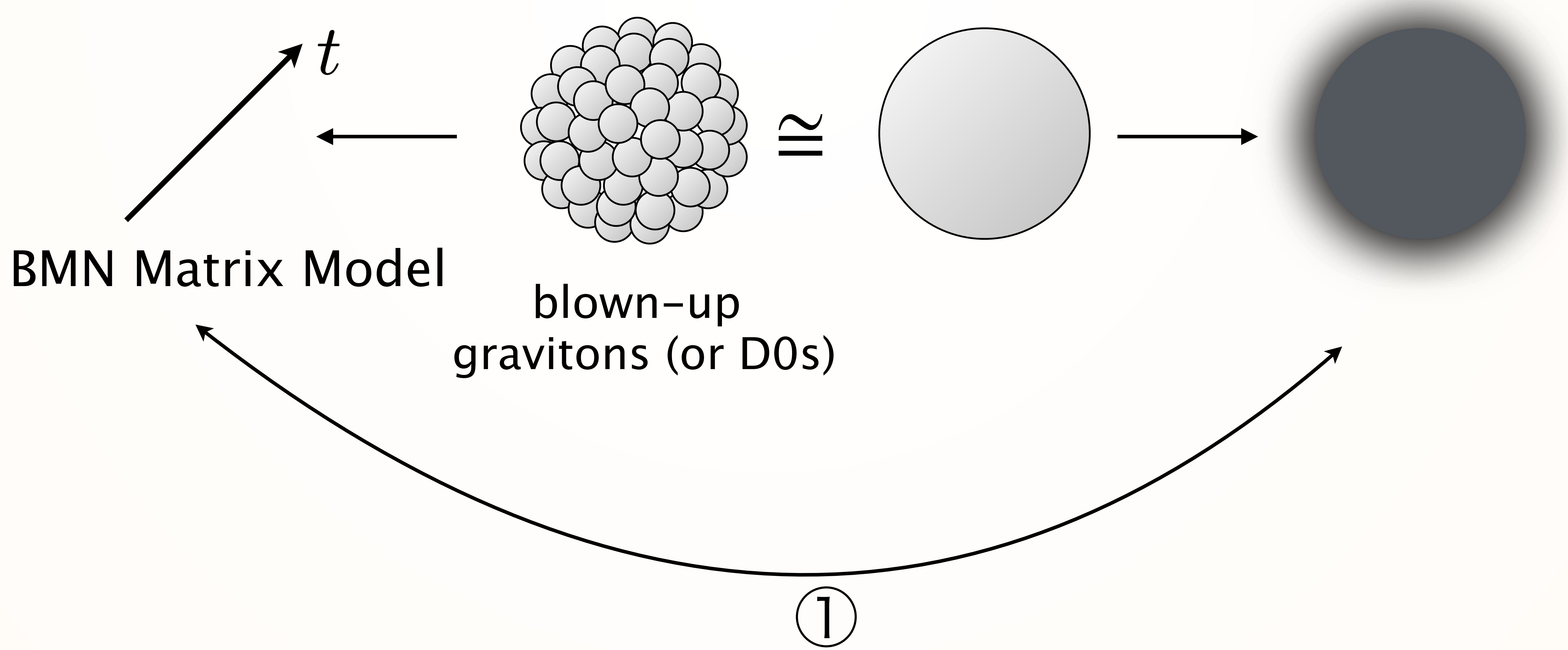

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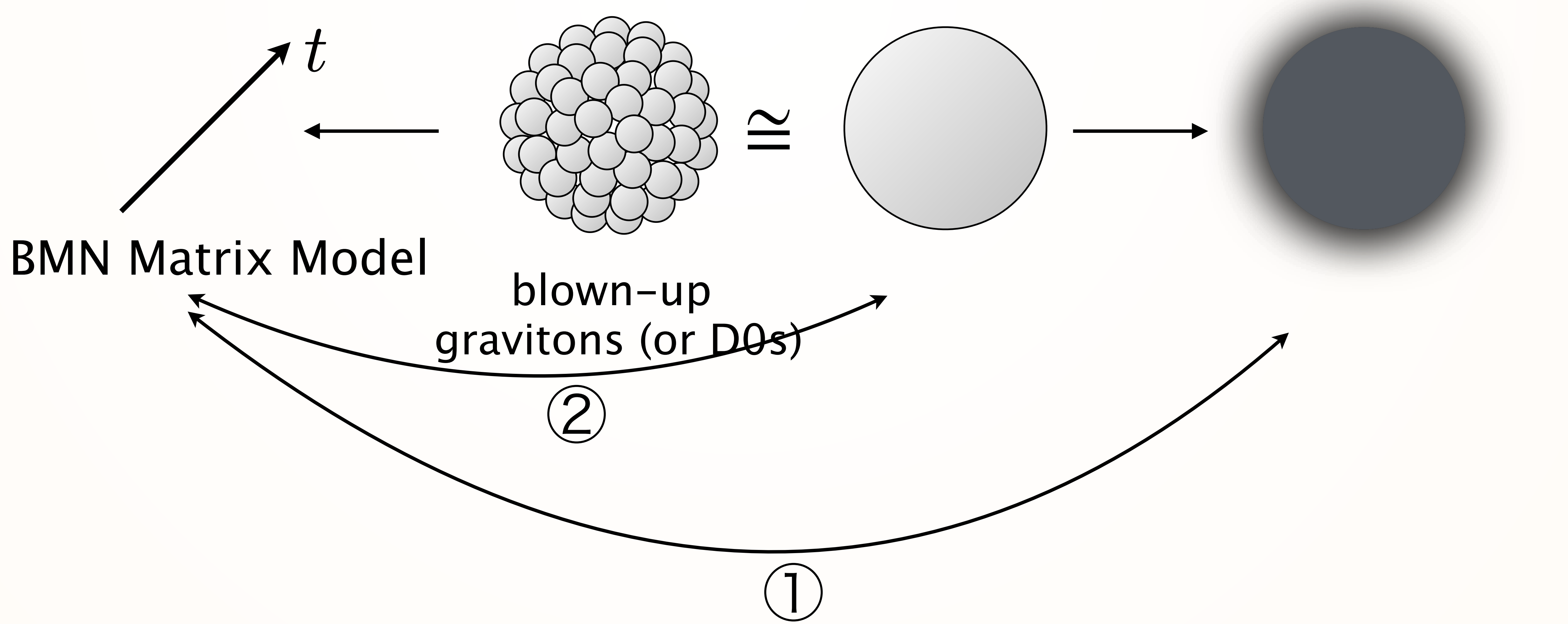
[Y.A.-Okada-Ishiki-Shimasaki '14]

② It reproduces M2 & M5 geometries with the correct radii.

[Y.A.-Ishiki-Shimasaki-Terashima '17]



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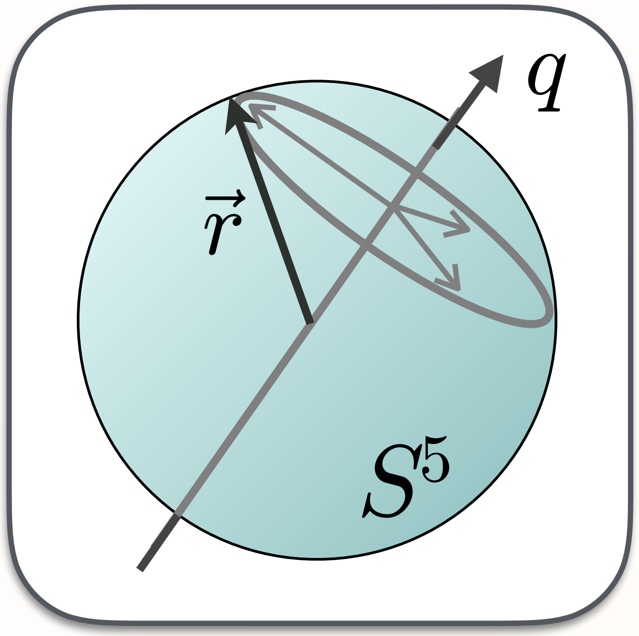
S^5 radius

$SO(6)$ symmetric uplift to 6 dim.

with $\int d^6\vec{r}\,\hat{\rho}(\vec{r})q^n = \int_{-q_m}^{q_m} dq\,\rho(q)q^n$



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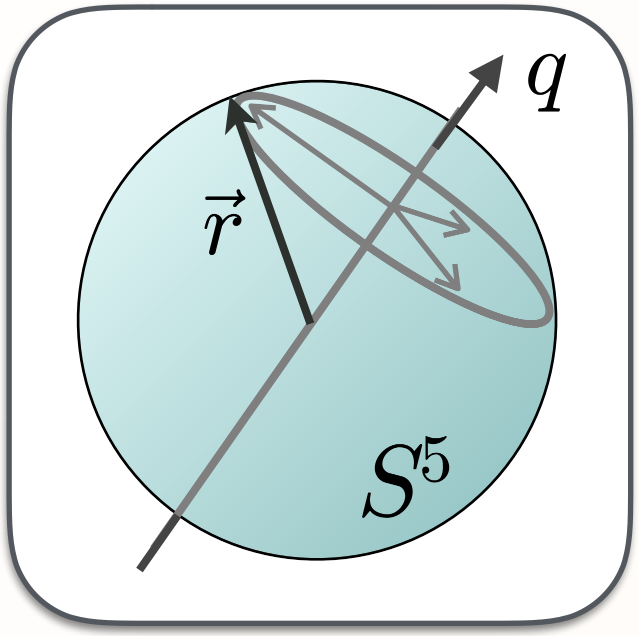
Spherical shell distribution
with the expected radius



2. Emergent Geometries

We now have a fairly good picture of emergent geometries.

① The infinitely many discrete vacua in the large-N BMN model correspond to gravity solutions characterised by droplets of smeared M2 and M5 charges.
The gravity solution, Lin-Maldacena geometry, is described by a single function. This function is equivalent to the eigenvalue density of the BPS operator ϕ on the gauge-theory side.

[Y.A.-Okada-Ishiki-Shimasaki '14]

② The eigenvalue density of this BPS operator also reproduces the spheres wrapped by M2 & M5 in the brane picture.

[Y.A.-Ishiki-Shimasaki-Terashima '17]

※ A set of matrices that gives M5's S^5 as a non-commutative sphere has not been obtained yet. But the BPS operator ϕ describes the S^5 and M2's S^2 by its eigenvalues in a consistent way.

2. Emergent Geometries

How does the geometry emerge when we scale the Temperature?

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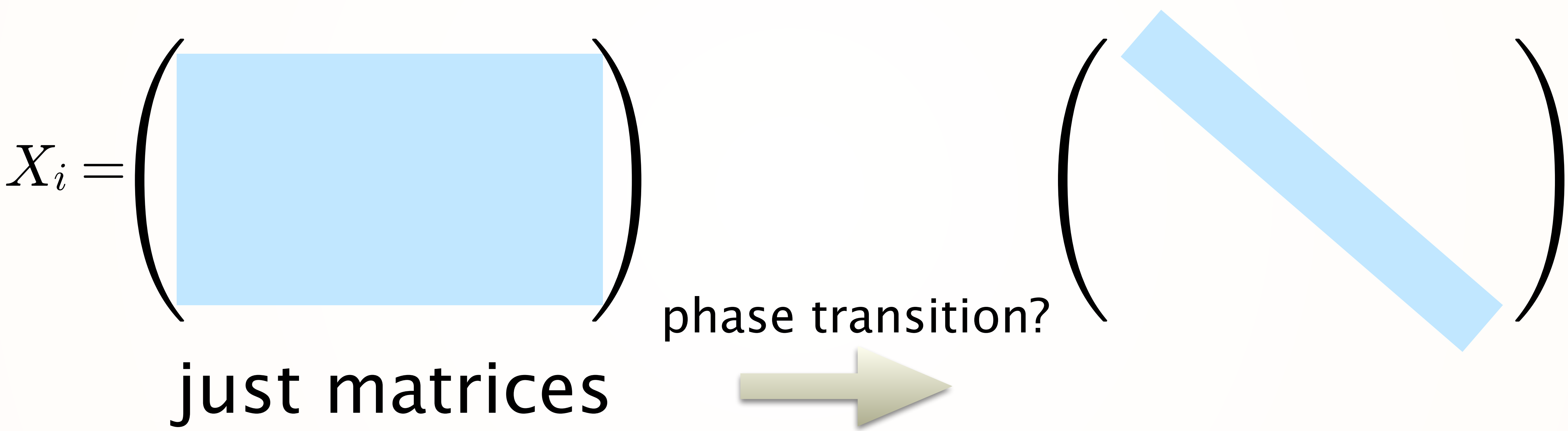

high energy scale

low energy scale

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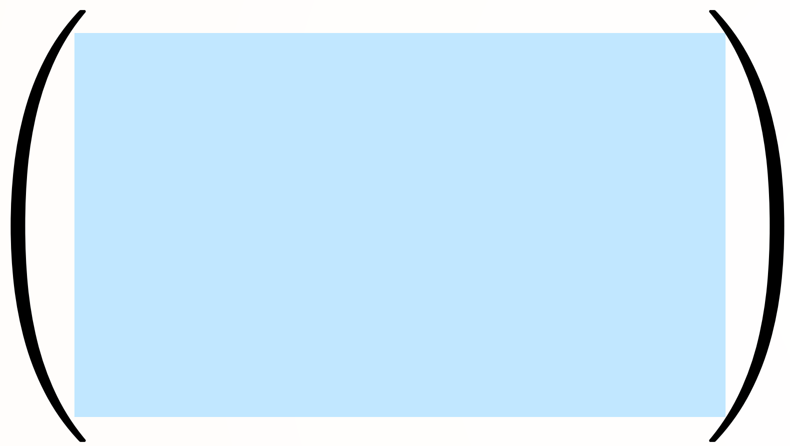
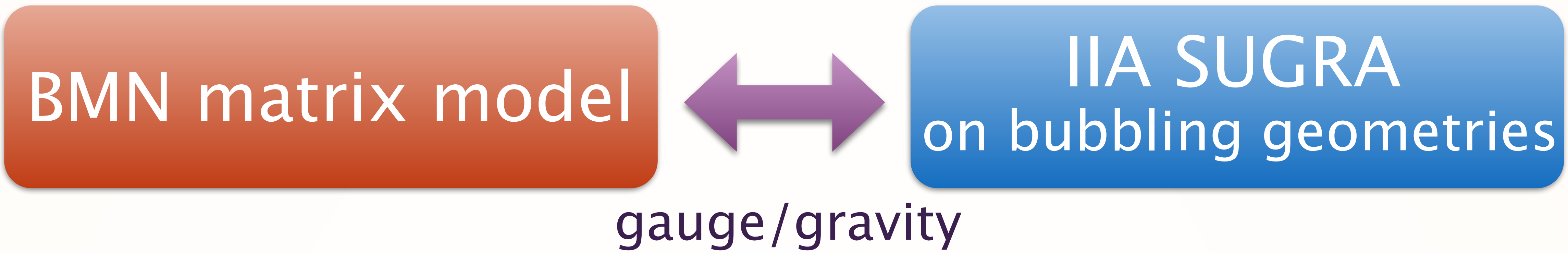
$X_i =$ 

phase transition?

just matrices → geometry

high energy scale → low energy scale

3. Thermal BMN Matrix Model



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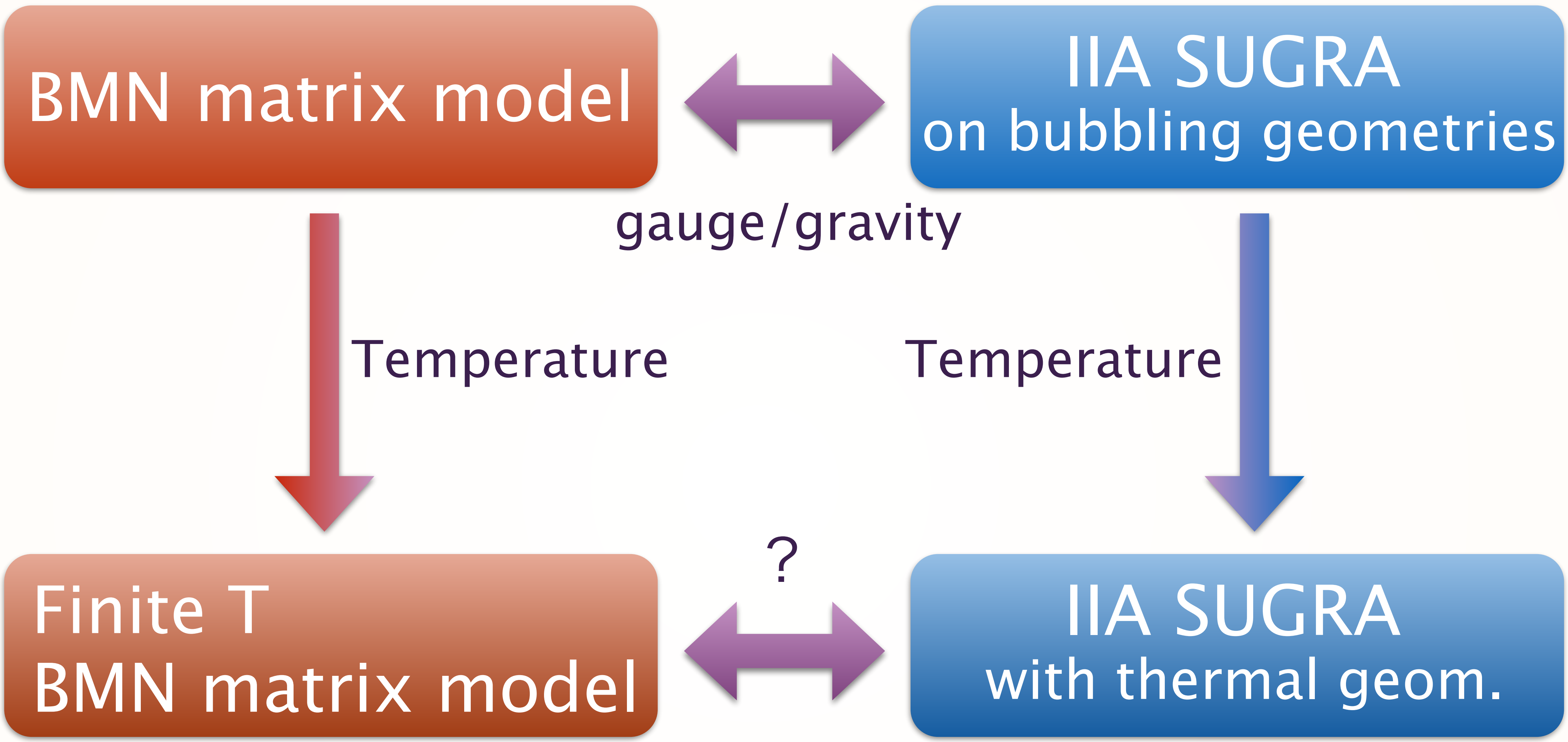


perturbatively computed in the case where the black hole horizon has the simplest topology

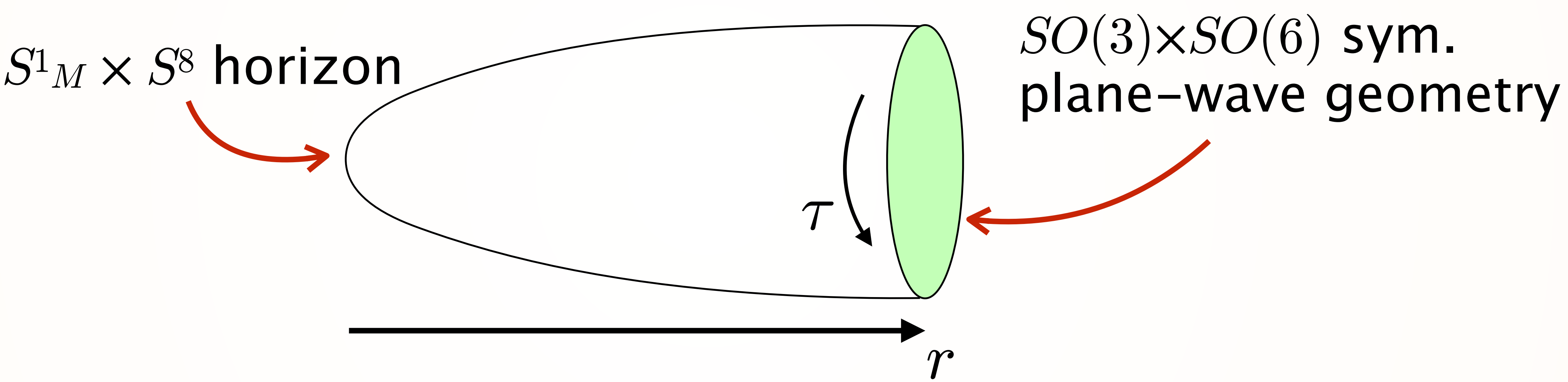
[Costa-Greenspan-Penedones-Santos '14]

3. Thermal BMN Matrix Model

[Costa-Greenspan-Penedones-Santos '14]

At small μ & high T $\approx$ non-extremal black 0-brane

$\sim$ $R\times SO(3)\times SO(6)$ plane-wave geom. at infinity



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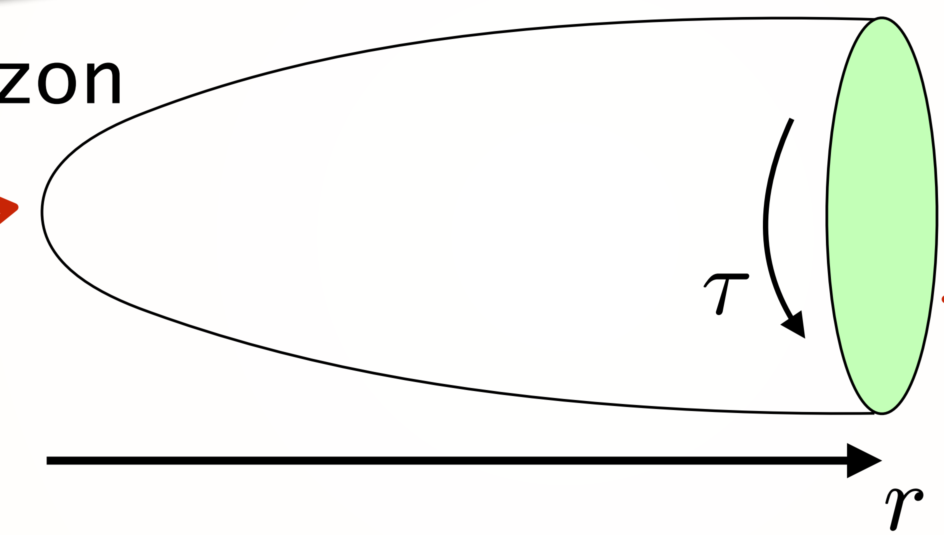

$SO(3){\times}SO(6)$ sym. plane-wave geometry

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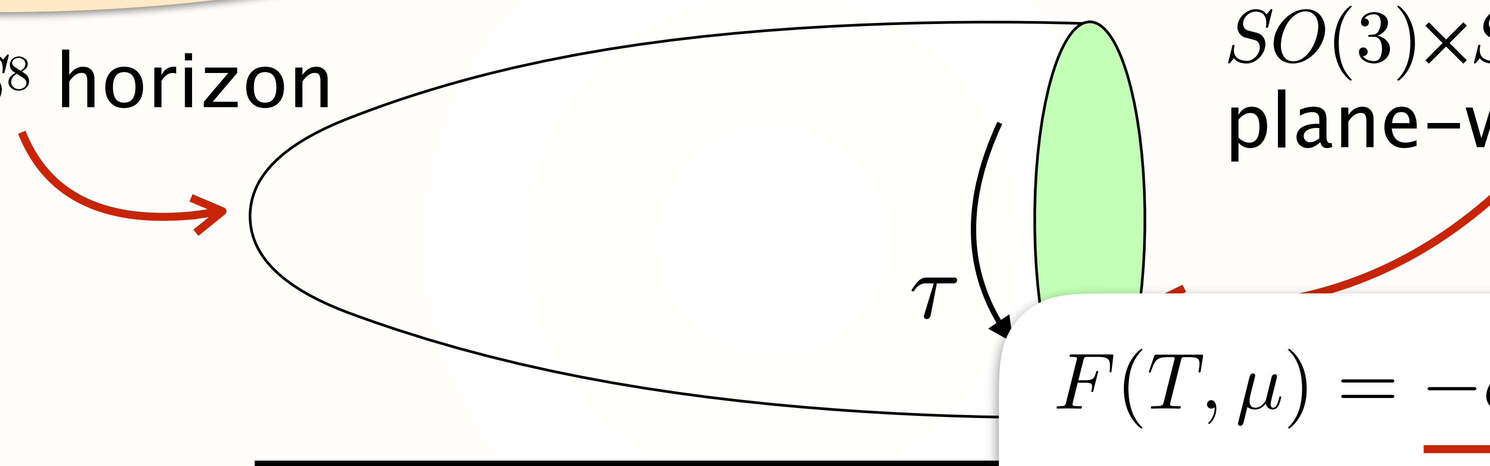
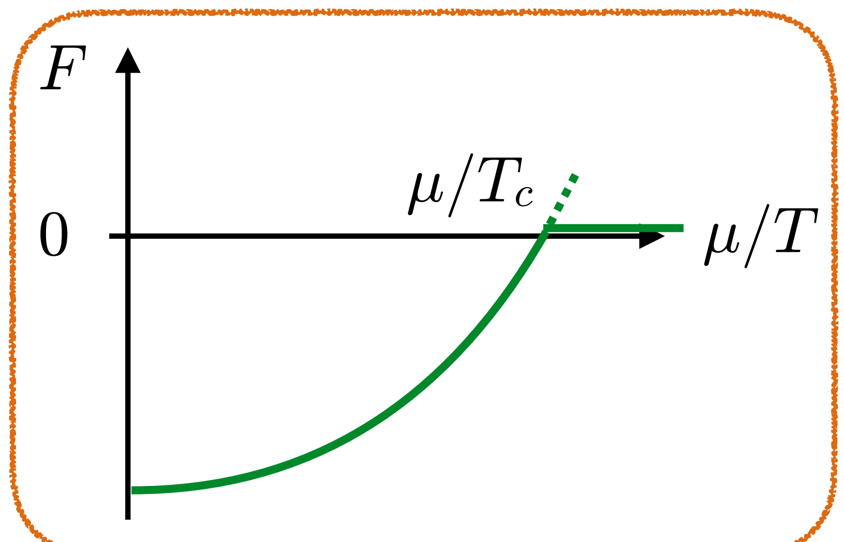


$$F(T,\mu) = \underline{-c_1 T^{\frac{14}{5}}}\, f(\mu/T)$$

BFSS free energy



Hawking-Page-like

Critical temperature (gravity side)

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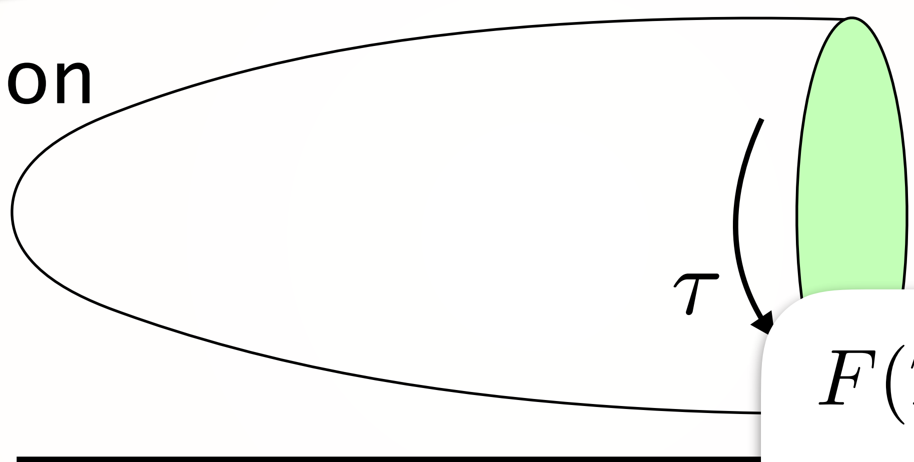

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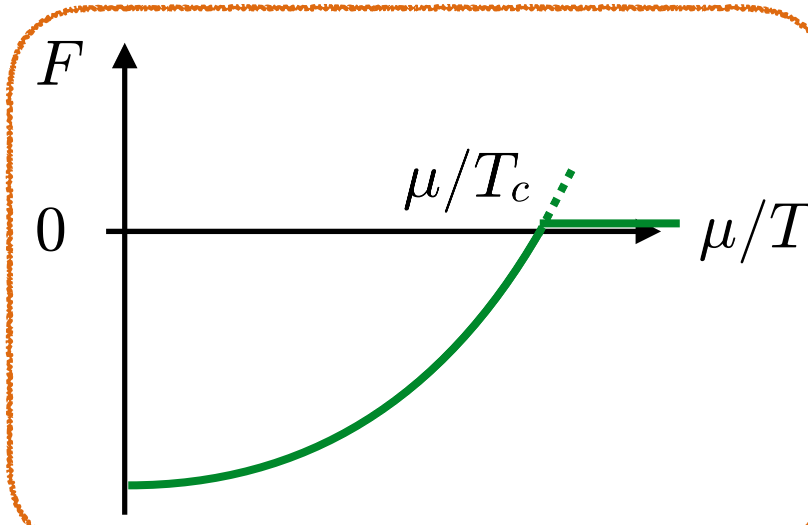

Hawking-Page-like

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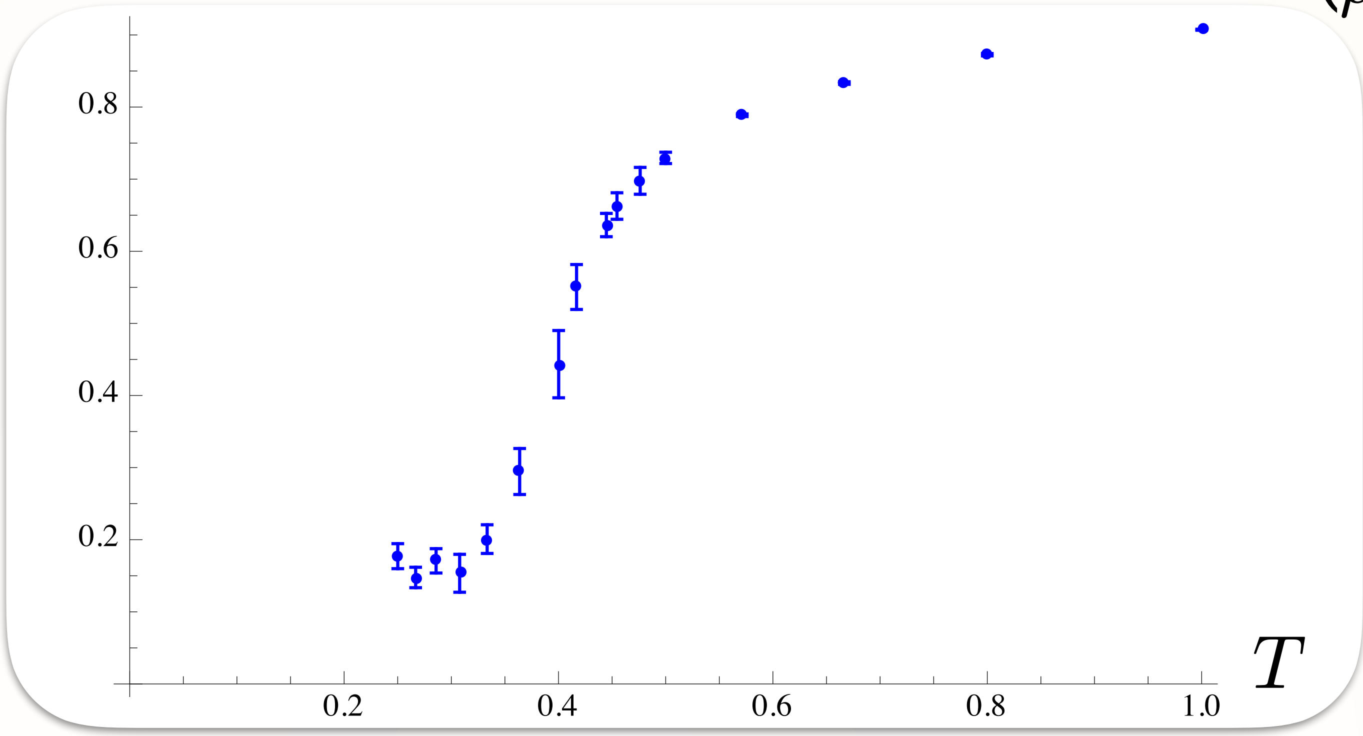
$$\frac{T_c}{\mu} = 0.105905(57)$$

4. Lattice Simulation

$(\mu=5,\ \Lambda=24,\ N=11)$

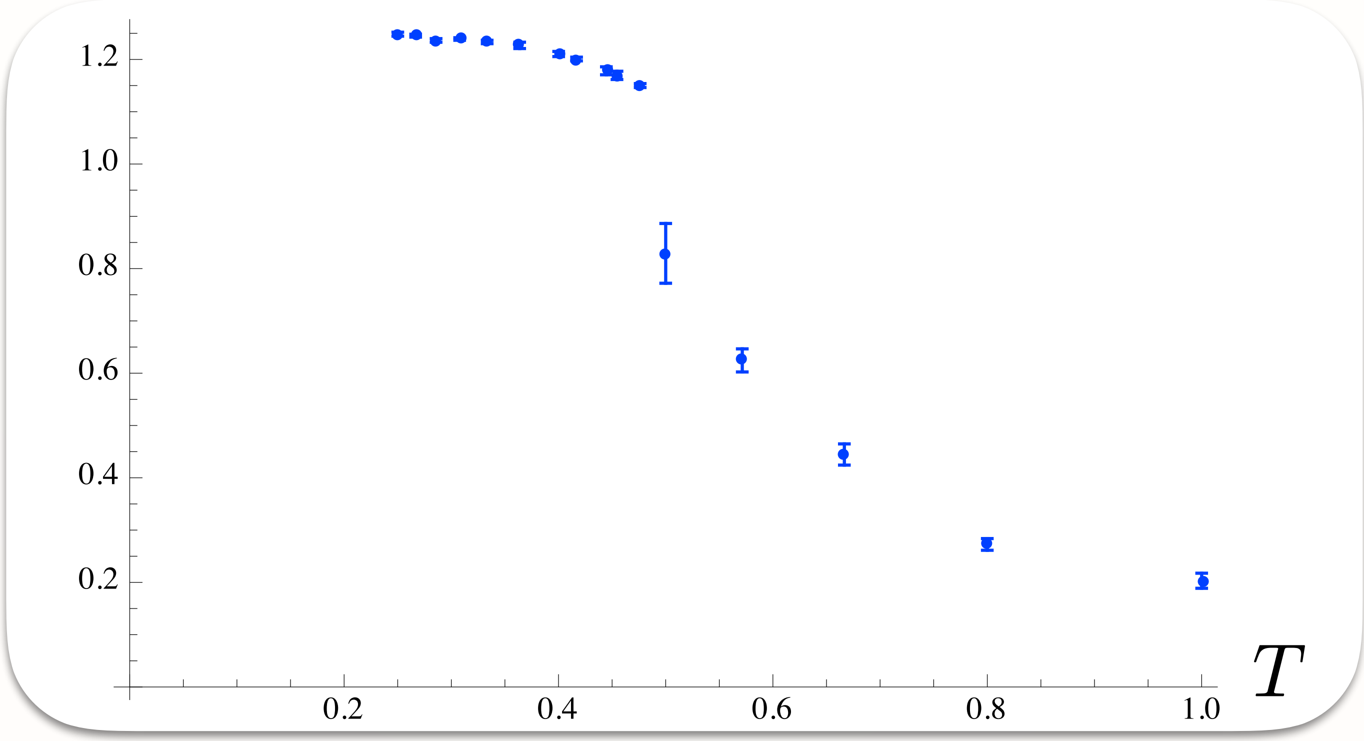
Polyakov loop:

$\langle |P| \rangle$



Myers term:

$\sim \langle \mathrm{Tr}\,(iX_1[X_2, X_3]) \rangle$



[Y.A.-Filev-Kovacik-O'Connor '18]

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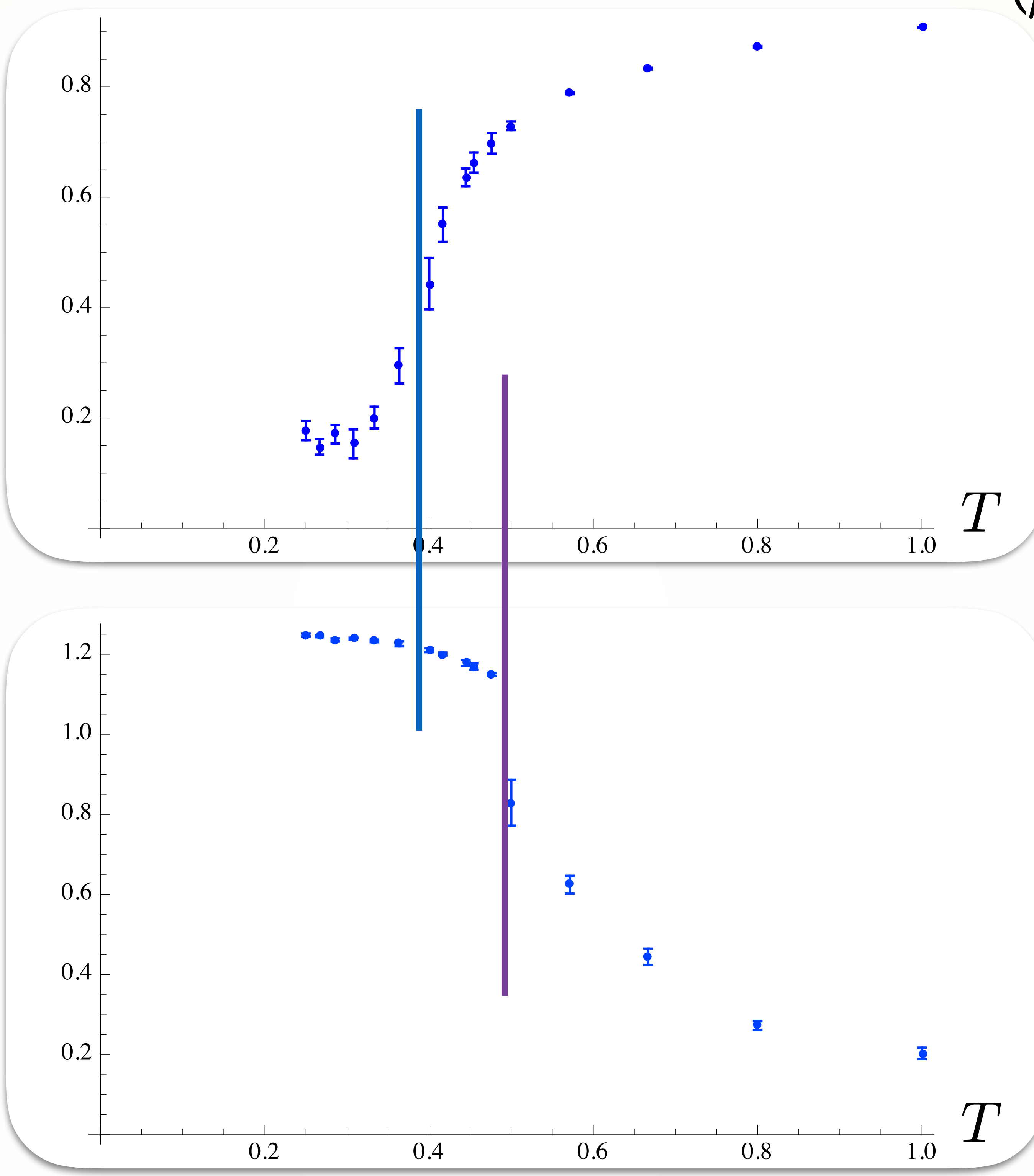
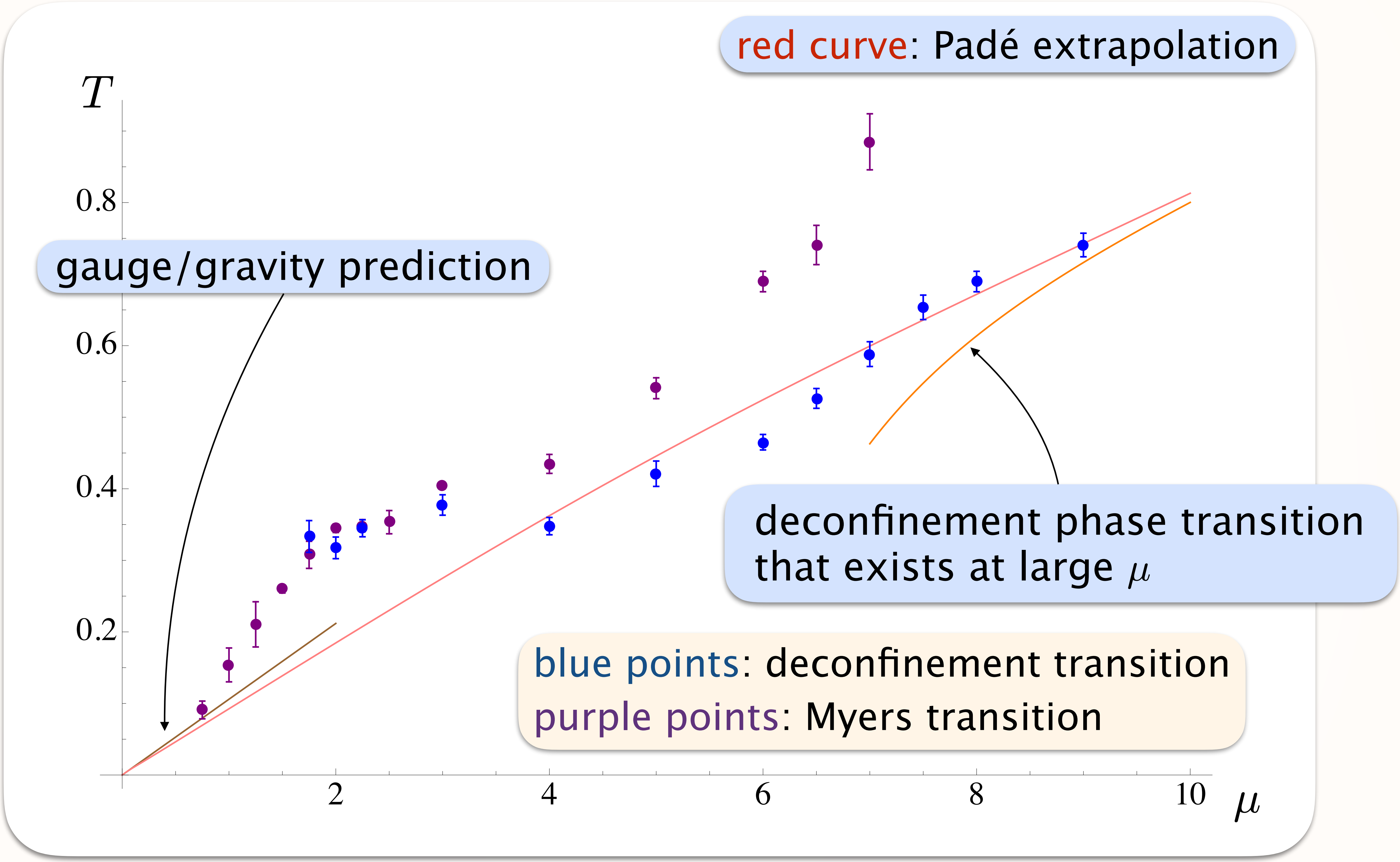


[Y.A.–Filev–Kovacik–O’Connor ’18]

4. Lattice Simulation

(Λ=24, N=8)

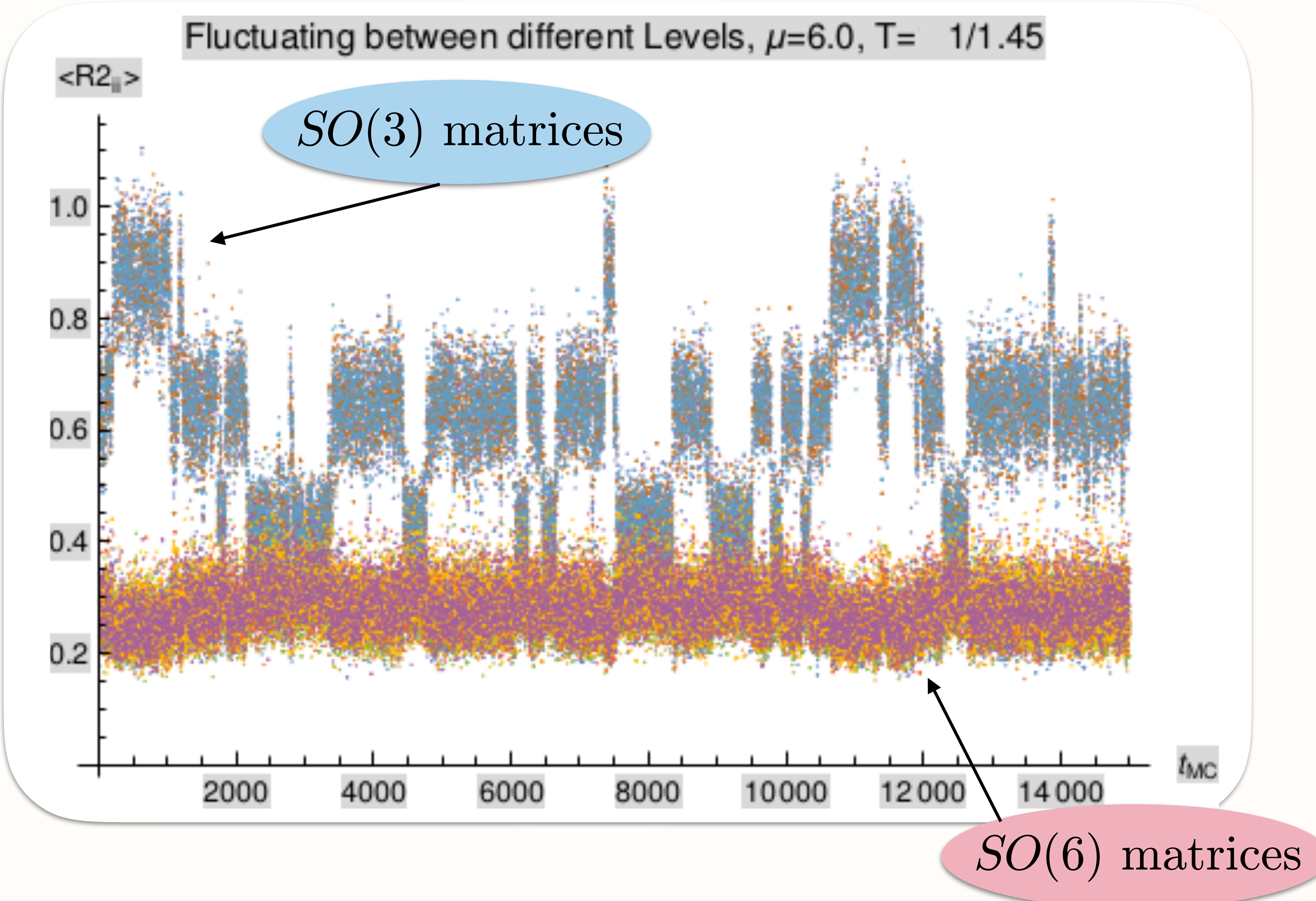


The simulation results AGREE with theoretical predictions.

[Y.A.–Filev–Kovacik–O'Connor '18]

4. Lattice Simulation

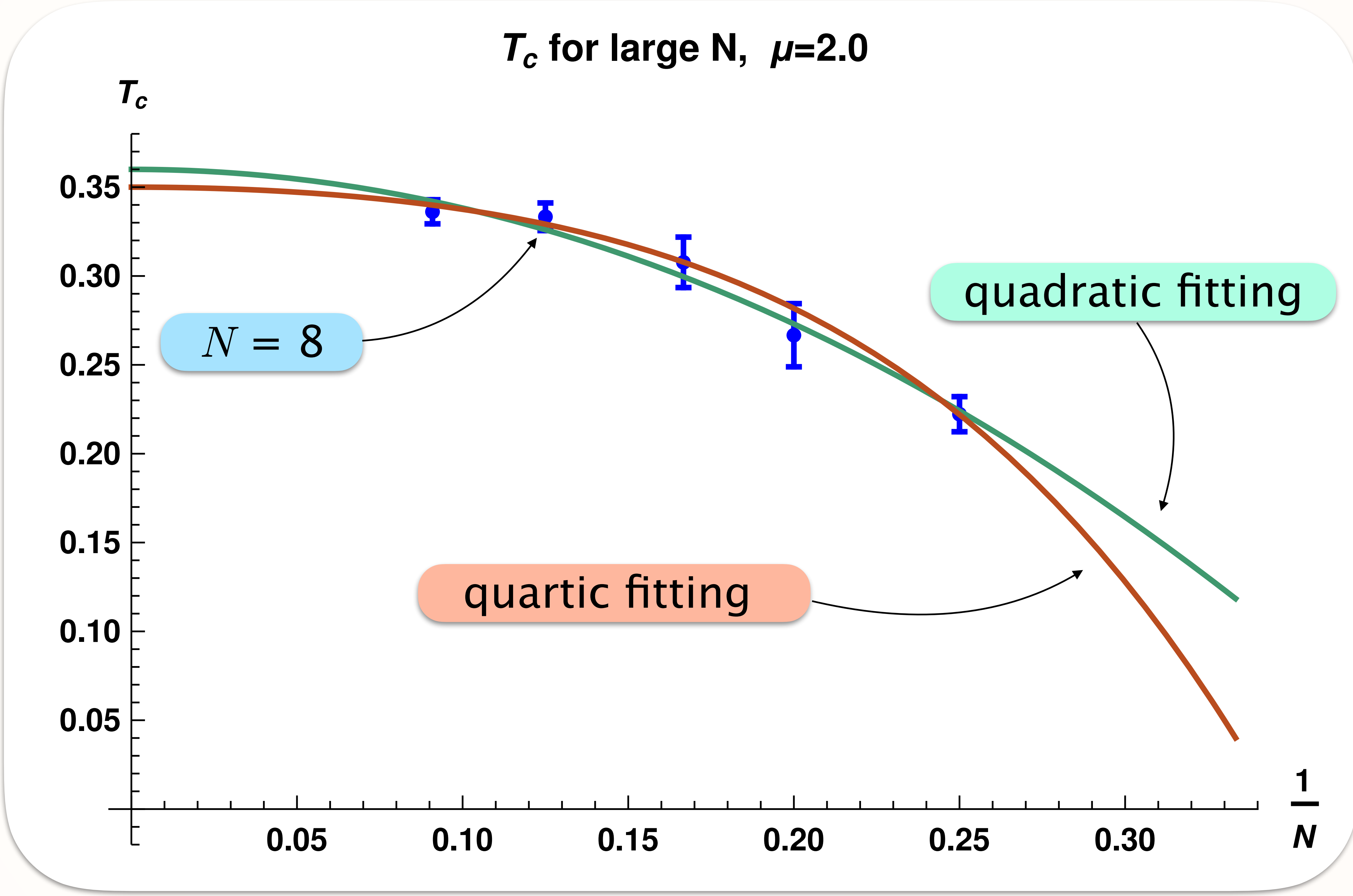
~ $\mathrm{Tr}[X_i X_i]/N$ (μ=6, β=1.45, Λ=24, N=8)



[Y.A.–Filev–Kovacik–O'Connor '18]

4. Lattice Simulation

Large-N extrapolation of the critical T (μ=2, Λ=24)



[Y.A.–Filev–Kovacik–O'Connor '18]

5. Summary

- Observed two phase transitions:

  deconfinement transition and Myers transition

  They are consistent with the gravity prediction so far, and don't merge at least on the lattice with finite N at $3 \lesssim \mu \lesssim 6$.

Geometrical interpretation

- Myers transition looks like "no geometry → geometry".

Gauge/Gravity duality

- We found the critical temperature of the deconfinement transition is dependent on $SU(2)$ representations.

  Keep the state at the trivial vac. under the transition

  $\longrightarrow$ Much closer to the gravity prediction

- Since the gravity dual at zero temperature is the droplet solutions, we expect a richer structure at lower temperatures, which should reflect geometrical information.

5. Summary

Bosonic BMN model

- There are some interesting things found about the transitions in the bosonic version.

Samuel's talk

Longitudinal M5-branes

- M5-branes appearing in the BMN model are transverse to the M-theory direction. To study longitudinal M5-branes properly, we can tackle the Berkooz-Douglas model, which is the BFSS model (D0s) probing D4-branes. [Berkooz-Douglas '96]

- There have been numerical results of the BD model. The simulations showed the agreement in the gauge/gravity duality. [Filev-O'Connor '15, Y.A.-Filev-Kovacik-O'Connor '16]

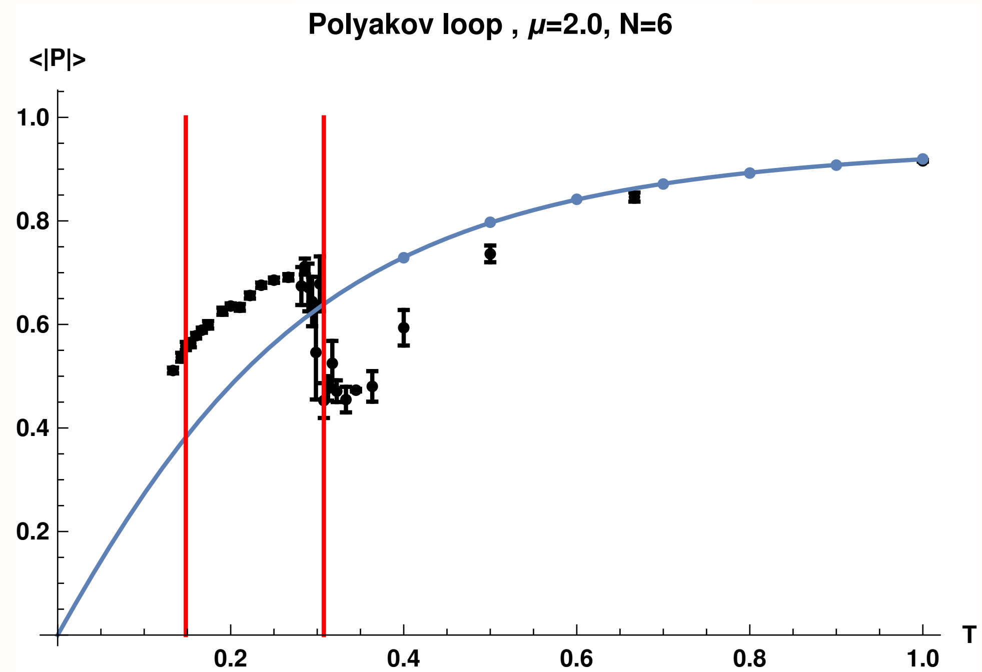
- And furthermore, a BMN-like mass-deformed version of the BD model exists, namely membranes on the plane-wave background. [Kim-Lee-Yi '02]

Backups

5. Summary

Other remarkable points

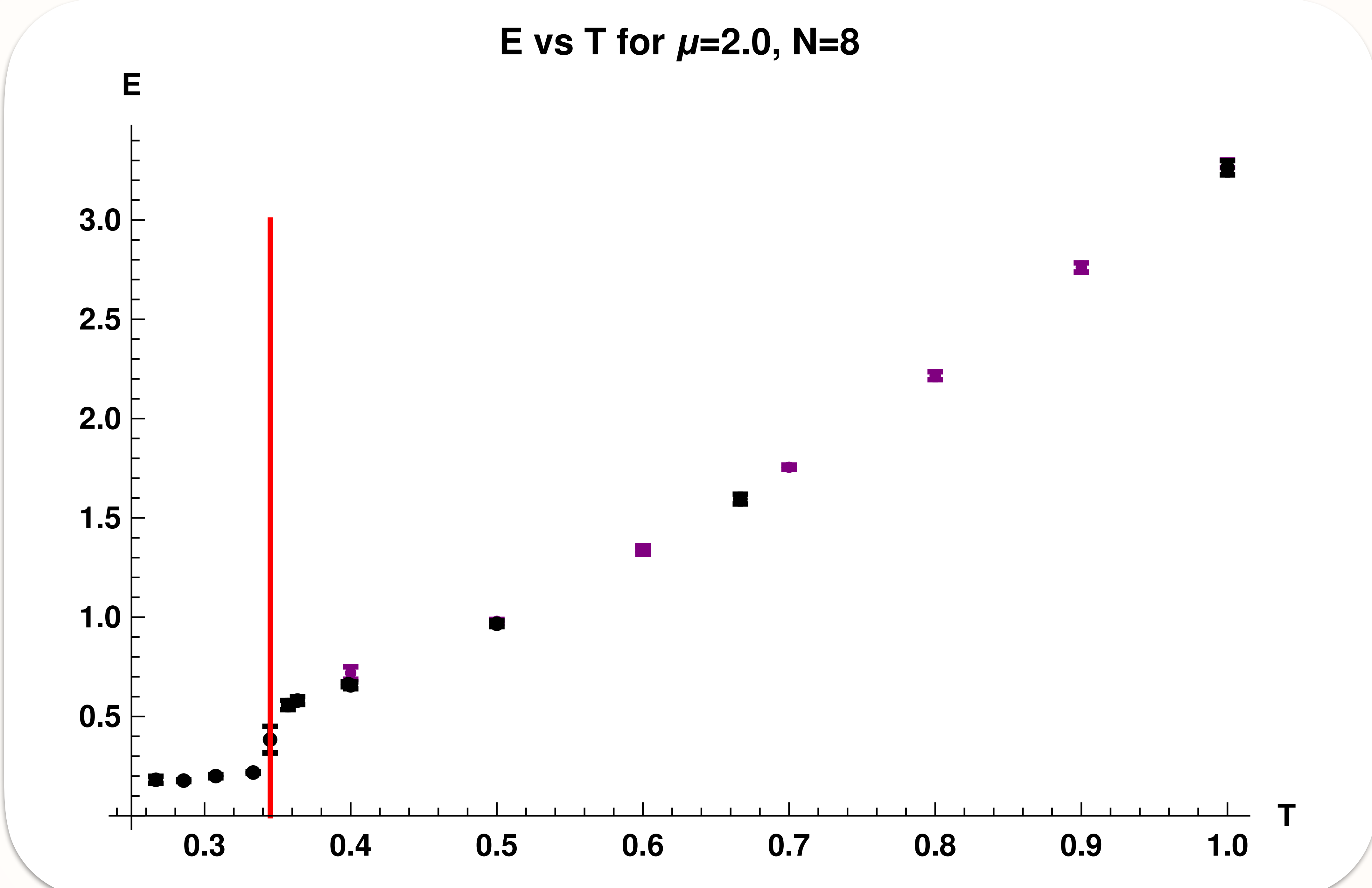
- The Polyakov loop shows a sharp, narrow transition at $\mu \gtrsim 6.0$, while it gets wider and non-monotonic of T, due to the Myers transition; different fuzzy spheres give different critical T of the deconfinement transition.



- The Myers term has a finite value at $\mu = 2.0$ in the large-N extrapolation. This is realised e.g. when the dimension of a typical fuzzy sphere representation is finite and fixed: a state of 5-branes.

4. Lattice Simulation

The transitions are of the first order. $(\mu=2,\ \Lambda=24)$



[Y.A.-Filev-Kovacik-O'Connor '18]

Localisation

1) Identify the appropriate BPS sector.

Emergence of S^2 and S^5 is rather trivial.

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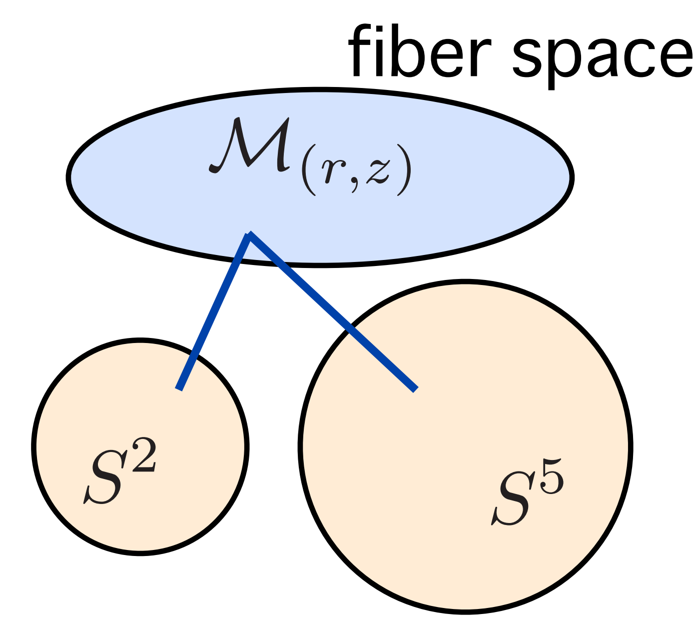
$$\langle\phi(\tau)\rangle \Rightarrow R\times SO(3)\times SO(6) \rightarrow SO(2)\times SO(5) \Rightarrow$$



Perform the localisation method.

Applying the localisation method to the BMN model

[Y.A.-Ishiki-Okada-Shimasaki '12]

- SUSY: quarter BPS sector such that ϕ is invariant.
- B.C.: all fields approach to the same vacuum configurations at $\tau\to\pm\infty$.
- Deformation $\delta_s\mathscr{V}$: SUSY-invariant and positive-definite.

$$Z(t) := \int \mathcal{D}X\, e^{-S[X]-t\delta_s\mathcal{V}}$$

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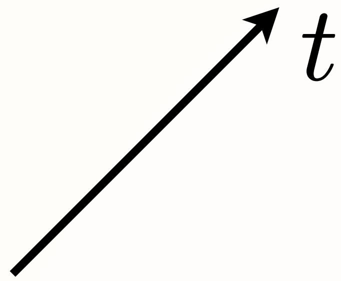
$$\Longrightarrow \int \mathcal{D}X \, e^{-S[X]} = Z(0) = Z(\infty)$$

Localised around $\delta_s \mathscr{V} = 0$!



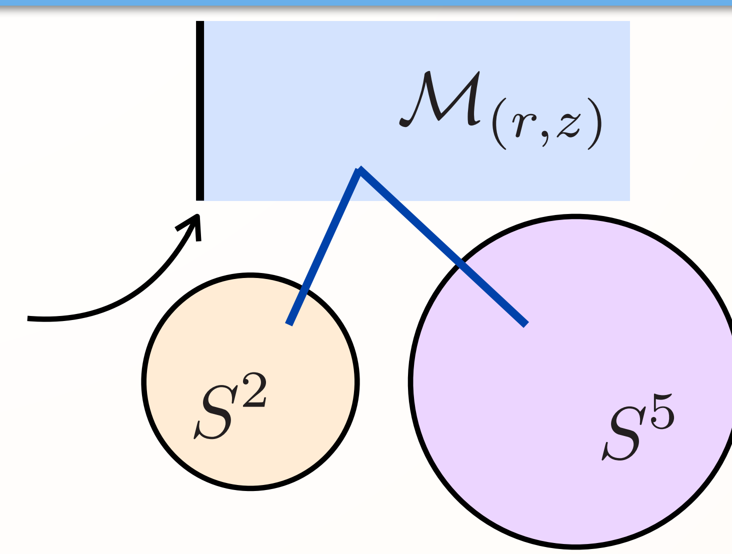
supergravitons
on plane-wave B.G.
or D0-branes

[Lin-Lunin-Maldacena '04, Lin-Maldacena '05]

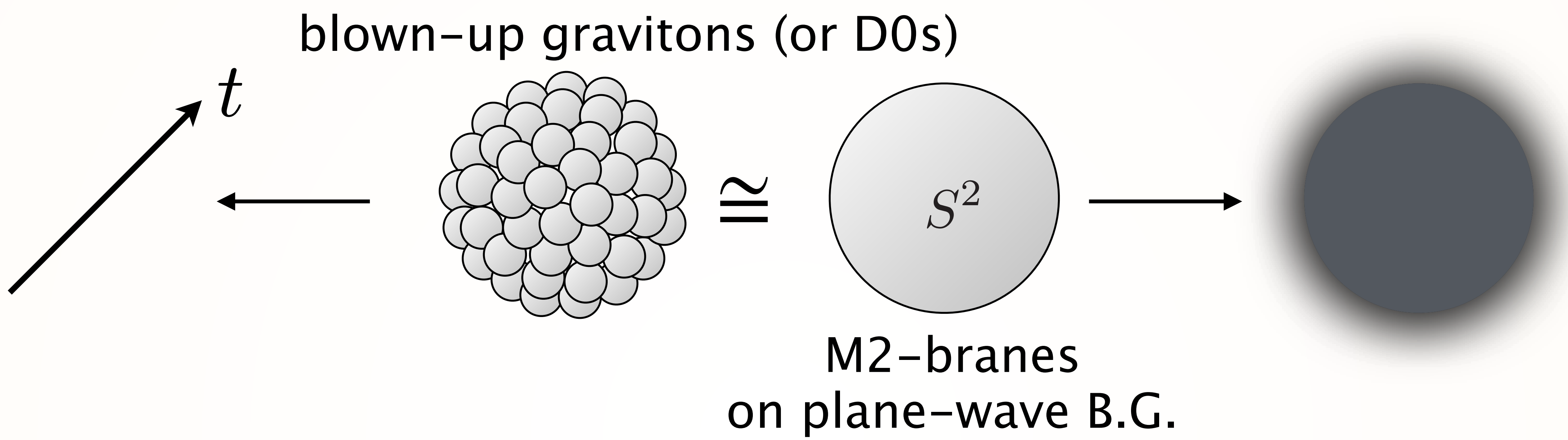
BMN matrix model ⟷ **11D/IIA SUGRA**

BMN matrix model		11D/IIA SUGRA
symmetry: $R\times SO(3)\times SO(6)$	$\xleftrightarrow{SU(2\|4)}$	isometry: $R\times SO(3)\times SO(6)$
vacua ($SU(2)$ rep.) – dim. of irreducible rep. – multiplicity of irred. rep.	⟷	bubbling geometries – M5/NS5 charge N_5 – M2/D2 charge N_2

2- & 5-brane charges are located on
the edge



Duality of M2 in Matrix Model



Large fuzzy S^2 vacuum
BMN matrix model

$\Longleftrightarrow$

Smeared M2 solution
11D SUGRA

NB: The limit to obtain M2-brane theory

Decoupling limit: Large N_5

Strong coupling of D2 theory: $1/(\mu^3 N_5) \gg N_5$