Field Theory on Random Geometry

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CNRS UMR8627 and Université Paris-Sud, Orsay
Quantum Geometry, Field Theory and Gravity
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Introduction

This talk is based on arXiv1905.12783, joint work with [Nicolas Delporte].
Some weird speculations however, are (especially if wrong) entirely mine.

At the Planck scale we expect space and time to change drastically as gravity becomes quantized.

Why random trees ?

- Quantum gravity: space-time as random geometry
- Random trees: simplest example of non-trivial random geometry, with effective dimension $d_s = 4/3$
- Random trees naturally label the melonic approximation (MA) characteristic of tensor models (TM) and of the Sachdev-Ye-Kitaev model (SYK). This quantum model saturates the [Maldacena-Shenker-Stanford] (MSS) bound, hence is maximally-chaotic and a serious toy model for quantum black-holes.

$$S_{SYK} = \int dt \left(\frac{1}{2}\psi_i \partial_t \psi_i + \sum_{1 \le i_1 < \cdots < i_q \le N} J_{i_1,\cdots,i_q} \psi_{i_1} \cdots \psi_{i_q} \right)$$

with $J_{i_1,\cdots,i_q}$ a quenched iid random tensor and ψ_i a real Fermionic vector.

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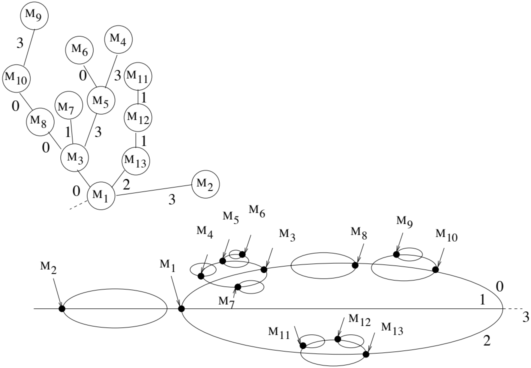
The Tree structure of Melonic Graphs

Melonic graphs are obtained by finitely many recursive edge-insertions of "elementary melons" within themselves.

=>



General remarks on Quantum Gravity approaches

Particle physics centered approaches: superstring theory...

General-relativity centered approaches: background independence, space emergence... Is time also emergent? Is causality fundamental?

In the first case one may want to reduce the dimension, eg from 10 to 4, using eg compactification.

In the second case one may want to increase the dimension, eg from 0 to 4. For this one might use holography, which means boundary => bulk.

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Quantum Gravity and Discrete Random Geometry

Typical holography is AdS/CFT. But there are no CFT in $d = 0, 1$, so how to start?

Tensor models are 0-dimensional. The SYK model is a 1-dimensional model, not truly quantum, in which time is given at the start.

Witten proposed to improve SYK into a true quantum tensor model (Gurau-Witten). A typical uncolored [Bonzom, Gurau, R.] action is the following one, found by [Klebanov, Tarnopolsky] using earlier work by [Carrozza,Tanasa]

$$\mathbf{S}_{\mathrm{CTKT}}[\psi] = \int \mathrm{d}t \left(\frac{1}{2} \psi_{abc} \partial_t \psi_{abc} + \frac{\lambda}{N^{3/2}} \psi_{a_1 a_2 a_3} \psi_{a_1 b_2 b_3} \psi_{b_1 a_2 b_3} \psi_{b_1 b_2 a_3} \right).$$

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How to organize sub-dominant terms (beyond melons)?

The standard interpretation of the melonic approximation (MA) is branched polymers (random trees). How to go beyond this random geometry?

Several avenues:

- multiple scalings [Bonzom, Gurau, Dartois, Lionni, R. Schaeffer...]
- tensor field theories and their renormalization group trajectories [Benedetti, Ben Geloun, Carrozza, Eichhorn, Koslowski, Lumma, Martini, Oriti, Pereira, R., Toriumi...] This is the tensor generalization of the Kontsevich, Grosse and Wulkenhaar matrix models
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Random Spaces

Probabllity measure on the space of all spaces (Gromov-Hausdorff space).

Right now (2019) probabilists understand analytically essentially two universal non-trivial "continuous" random spaces:

- The Continuous Random Tree [Aldous, $\simeq$ 1990]
- The Brownian Sphere [Le Gall, Miermont $\simeq$ 2011]

The second space can be thought of as a set of random labels living on the first.

In practice: these spaces can be discretized as random graphs.

- Infinite trees with single spine
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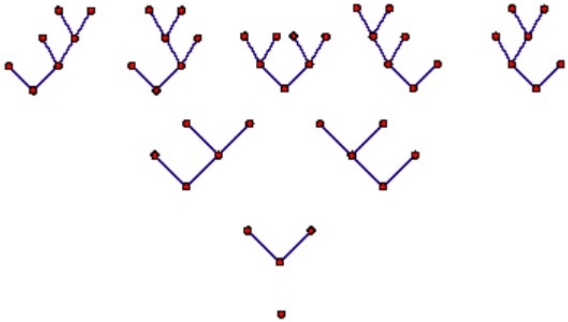
Galton-Watson trees

Galton-Watson processes are another way to understand random trees.



GW trees have independent probabilities p_k to have k offsprings at each vertex.

Galton-Watson trees

In the simplest case (binary trees) the critical Galton-Watson process corresponds to offspring probabilities $p_0 = p_2 = \frac{1}{2}$, $p_k = 0$ for $k \neq 0, 2$. .

The generating function for such trees obeys the simple Catalan equation $Z(\zeta) = \zeta(1 + Z^2(\zeta))$, which solves to $Z = \frac{1-\sqrt{1-4\zeta^2}}{2\zeta}$.

Infinite random trees: critical Galton-Watson trees with fixed branching rate conditioned on non-extinction.

In physics, such random trees are often called branched polymers.

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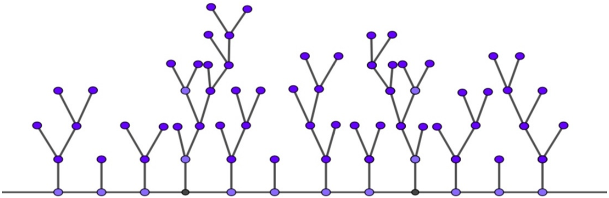

The condition of non-extinction generically leads to infinite random trees characterized by *a single infinite spine* $\mathcal{S} = \mathbb{N}$ or $\mathbb{Z}$ decorated at each node v by an independent finite Galton-Watson branch T_v. The corresponding measure is

$$d\nu(\Gamma) = \prod_{v \in \mathcal{S}} d\nu_{GW}(T_v)$$

The Hausdorff dimension d_H is defined by the average volume/diameter relation: $V \simeq cD^{d_H}$. For random trees we have $d_H(Random\ Tree) = 2$.

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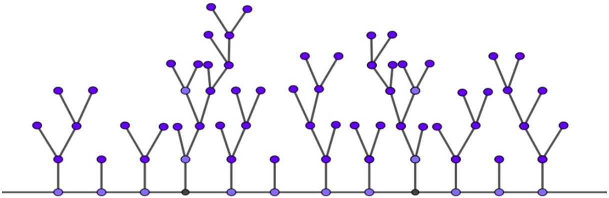

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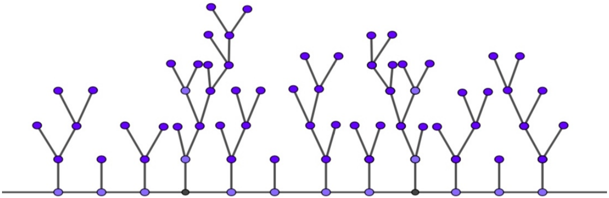

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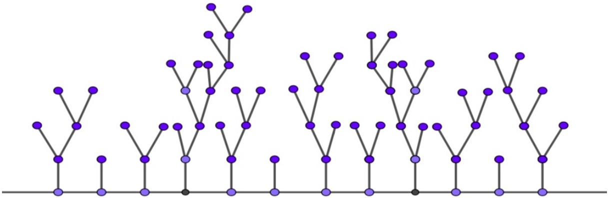

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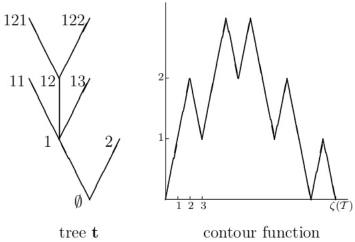
Why $d_H(\textit{Random Tree}) = 2$?

One can understand the metric properties of a large random tree via a nice one-to-one map.

The Dyck walk turns around the tree to identify the tree to its contour function quotiented by an equivalence relation. The contour function is exactly a Brownian excursion.



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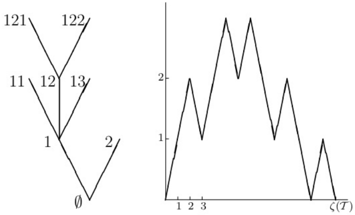

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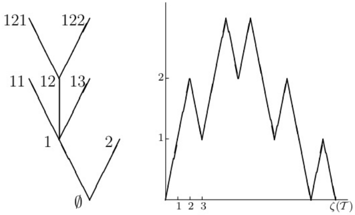

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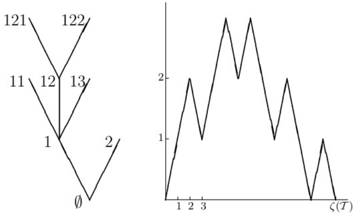

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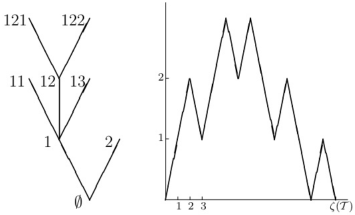

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Random Walks on a Graph

On a graph Γ

- we have no longer translation and rotation invariance, Fourier analysis and the notion of momenta are lost...
- what remains: the Laplace operator. $\mathcal{L}_\Gamma = D_\Gamma - A_\Gamma$ (D_Γ: degree matrix ; A_Γ: incidence matrix). It inverse has the random path expansion:

$$C_\Gamma^m(x,y) = \sum_{\omega: x \to y} \prod_{v \in \Gamma} \left[\frac{1}{d_v + m^2}\right]^{n_v(\omega)} \sim \int_0^\infty \mathrm{d}t\; e^{-m^2 t} p_t(x,y),$$

Spectral dimension d_s: if $p_t(x,y)$ is the probability for a random walk starting at x to be at y after t steps, then d_s is defined through

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It is the infrared scaling of what physicists call the tadpole.
For random trees $d_s(\textit{Random Tree}) = 4/3$ [Durhuus, Johnson, Wheather]

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Diffusion time

How to intuitively understand that $d_s(\textit{Random Tree}) = 4/3$? It is because in time t a random path typically explores up to distance $t^{1/3}$.

Consider a random walk starting at the tree root killed when it first reaches height L at time T.

The corresponding conditioned heat kernel $p^L(r,x)$ is harmonic (except at the root) hence constant on the branches and linear on the walk from x to r

So $p^L(r,r) \simeq cL$, and in fact $p^L(r,x) \leq c(L - d(x,r))$

Then

$$< T >= \sum_x p^L(r,x) \simeq L \times L^2 \simeq L^3.$$

Since the path explores in time t distances up to $t^{1/3}$, hence volumes $t^{2/3}$ the normalization should be in $t^{-2/3}$ hence $d_s = 4/3$, to fit with the usual (d integer) formula $p_t(x,y) = t^{-d/2} e^{-\frac{(x-y)^2}{4t}}$.

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Field theory and Observables

Partition function on a graph Γ:

$$Z(\Gamma;\lambda) = \int e^{-\lambda \sum_{x\in V_\Gamma} \phi^{q}(x)} d\mu_{C_\Gamma}(\phi) = \int d\nu_\Gamma(\phi).$$

Correlation functions:

$$S_N(\Gamma; z_1,\ldots,z_N) = \int \phi(z_1)\ldots\phi(z_N)\ d\nu_\Gamma(\phi) = \sum_{V=0}^{\infty} \frac{(-\lambda)^V}{V!} \sum_G A_G(\Gamma; z_1,\ldots,z_N).$$

The spine is common to all $\Gamma \in \mathcal{T}$. Hence we can define the observables as averaged Schwinger functions with arguments $\{z_1,\ldots,z_N\} \in \mathcal{S}$ on this spine

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- We identify the fractional power of the Laplacian which makes the theory just-renormalizable
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- We combine this analysis with precise heat kernel estimates of [Barlow, Kumagai]
- We obtain uniform bounds on convergent graphs and localization estimates for the divergent subgraphs which require renormalization
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Some weird conjectures

- Critical point for melons of 0-d TM can be interpreted as emergence of a $d = 4/3$ dendritic gravi-time
- The spine of this gravi-time can be interpreted as an ordinary $d = 1$ classical time.
- Gravithermal quantum-mechanics is the Euclidean $U(1)$ compact version of the gravi-time. It is generated by critical unicycles. Fermions and Bosons can be defined as usually by periodic/antiperiodic bc along the single cycle.
- Conjecture: SYK, Gurau-Witten, CTKT models on this gravitime still saturate the MSS bound
- If true, it should lead to some new $BNCFT_{4/3}/BAdS_2$ random holographic correspondence to de defined. (B meaning "brownian").

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Conclusion

Is time a random tree?

L'arbre Eternité vit, sans faîte et sans racines — Victor Hugo

Propagating the matter field

The propagator is the inverse of the Lapacian

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with an IR regulator m.

We then use the Euler β-function identity:

$$\mathcal{L}^{-\alpha} = \frac{\sin \pi\alpha}{\pi} \int_0^\infty \mathrm{d}m \frac{2m^{1-2\alpha}}{\mathcal{L} + m^2},$$

$(0 < \alpha \leq 1)$ to define the rescaled propagator as

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Divergence degree

The standard degree of divergence for a ϕ^q Feynman graph G in dimension d (V vertices, L internal edges and E external legs, $qV = 2L + E$) is:

$$\omega(G) = (d - 2\alpha)L - d(V - 1) = (d - 2\alpha)(qV - E)/2 - d(V - 1),$$

The just-renormalizable case occurs for

$$\alpha = \frac{d}{2} - \frac{d}{q}, \quad \alpha = \frac{1}{3} \text{ for } d = \frac{4}{3}, q = 4.$$

Then the divergence degree depends only on N:

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$$\omega(G) = (d - 2\alpha)L - d(V - 1) = (d - 2\alpha)(qV - E)/2 - d(V - 1),$$

The just-renormalizable case occurs for

$$\alpha = \frac{d}{2} - \frac{d}{q}, \quad \alpha = \frac{1}{3} \text{ for } d = \frac{4}{3}, q = 4.$$

Then the divergence degree depends only on N:

$$\omega(G) = d\left(1 - \frac{E}{q}\right).$$

For $q = 4$, $E = 2$ and $E = 4$, i.e 2- and 4-point functions need renormalization.

RG: multiscale analysis (towards the IR)

(1) Decompose the propagators into "proper time" scales $I_j = [M^{2(j-1)}, M^{2j}]$: $C = \sum_{j=0}^{\rho} C^j$ (note that $j = 0$ is the UV scale in our setting; for simplicity, external propagators are taken at IR cutoff scale ρ).
Each amplitude becomes a sum over all scale assignments μ.

(2) Identify superficial degree of divergence ω and divergent graphs.
Given μ, high subgraphs (quasi-local) control the divergences:

$$HS: \quad \text{(scales of internal legs)} < \text{(scales of external legs)}$$

$$|A_{G,\mu}| \leq \prod_{G_i \in HS} M^{\omega(G_i)}.$$

(3) Expand the divergent subgraphs around some reference point (localization of external propagators). Kill the first diverging terms by (local) counterterms.

(4) A renormalizable theory is defined at scale i by a finite number of parameters, with all parameters associated to lower scales $j < i$ having been integrated out. ($\rightarrow$ RG flow)

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Probabilistic estimates

For a parameter $\lambda \geq 1$, the ball $B(x, r)$ is said λ–good if (essentially):

$$r^2 \lambda^{-2} \leq |B(x, r)| \leq r^2 \lambda.$$

Crucially, [Barlow, Kumagai] showed that λ–good balls occur more and more likely for larger and larger λ:

$$\mathbb{P}[B(x, r) \text{ is not } \lambda\text{–good}] \leq c_1 e^{-c_2 \lambda}.$$

Then, they obtained the (quenched) bounds:
Given $r > 0$ and that $B(x, r)$ is λ–good, if $t \in [r^3 \lambda^{-6}, r^3 \lambda^{-5}]$, then

- for any $K \geq 0$ and any $y \in T$ with $d(x, y) \leq K t^{1/3}$

$$p_t(x, y) \leq c \left(1 + \sqrt{K}\right) t^{-2/3} \lambda^3 ,$$

- for any $y \in T$ with $d(x, y) \leq c_1 r \lambda^{-19}$

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Propagators

Slicing the propagator into proper time slices $I_j = [M^{2(j-1)}, M^{2j}]$

$$C_T^j(x,y) \underset{u=m^2}{=} \int_0^\infty \mathrm{d}u \; u^{-\alpha} \int_{I_j} \mathrm{d}t \; p_t(x,y) e^{-ut} = \Gamma(1-\alpha) \int_{I_j} \mathrm{d}t \; p_t(x,y) t^{\alpha-1}$$

Lemma (Single Line)

- $cM^{-2j/3} \le \mathbb{E}\left[C_T^j(x,x)\right] \le c' M^{-2j/3}$,
- $cM^{2j/3} \le \mathbb{E}\left[\sum_y C_T^j(x,y)\right] \le c' M^{2j/3}$.

Interpretation: The propagator scales as

$$\int_{I_j} t^{-2/3} t^{-2/3} dt \simeq cM^{-2j/3}.$$

The integration volume $\sum_y$ costs $c' M^{4j/3}$ as in a naive dimension $d = 4/3$. c, c' are dummy names for inessential constants.

Idea of proof: slice the y-sum into rings and sum over all possible volumes inside with associated λ–not-goodness probability factors.

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Convergent graphs

Theorem

For a completely convergent graph (no 2- or 4- point subgraphs) G of order $V(G) = n$, the limit as $\lim_{\rho\to\infty} \mathbb{E}(A_G)$ of the averaged amplitude exists and obeys the uniform bound

$$\mathbb{E}(A_G) \leq c^n (n!)^\beta$$

where $\beta = \frac{52}{3}$.

Comment: essentially uses the bounds above, Cauchy-Schwarz (for loops) and again slicing the space into rings that are asked to be λ-good.

However intersecting rings don't have independent probabilities. This is what leads to the (non-optimal) factorial growth.

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Divergent graphs I

We want to know how an amplitude changes when moving an external leg from one point z to a close point y:

Lemma

Defining $\Delta^j_T(x;y,z) := \left| C^j_T(x,y) - C^j_T(x,z) \right|$, *we obtain*

$$\mathbb{E}[\Delta^j_T(x;y,z)] \leq cM^{-2j/3} M^{-j/3} \sqrt{d(y,z)}.$$

Comment: uniform in x and the factor $M^{-j/3}\sqrt{d(y,z)}$ is the gain, provided $d(y,z) \ll r_j = M^{2j/3}$. The precise inequality for $y, z \in T$ is

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For $j_m \ll j_M$, we want to compare the "bare" amplitude

$$A_T^{bare}(x,z) := \sum_{y \in T} C_T^{j_M}(x,y) C_T^{j_m}(y,z)$$

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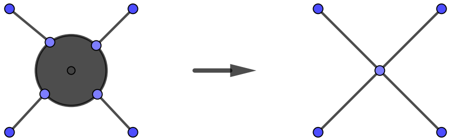
The previous lemma allows to write 4-point subgraphs as a local 4-vertex, plus corrections unseen by the external scale, defining hence an effective amplitude A^{eff}:



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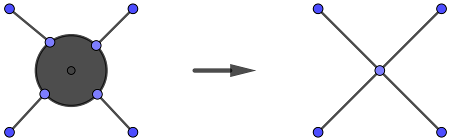

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