

Beyond the Standard Model- The still elusive neutrinos

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Partial list of neutrino topics

- History of neutrino
- General Properties of Neutrinos.
- The neutrino as the finger print of the weak interaction \leftrightarrow violation of many sacred conservation laws.
- Neutrino mass and neutrino mixing.
- Neutrino Oscillations.
- The elusive neutrino mass scale and experiments searching for it (astrophysics, decay experiments double beta decay)
- Neutrinos as a means for exploring the sky.

Ia: Some important stages in the life of neutrino

- It is elusive, but it does not really escape :

Indirect observation in neutron decay: $n \rightarrow p + e^- + \nu_e$ (Pauli 1930, 1932, Fermi 1934).

- Bethe (1939) finds for it a role in our world. Without it our sun and the “stars cannot shine”. :

Proton fusion (Symbolically) : $4p \rightarrow \text{He} + 2\nu_e + 2e^+ + \text{light}$

- Its direct discovery (Reines & Cowan (1953)

- $\text{anti-}\nu_e + p \rightarrow n + e^+$

The neutrino is different from its antineutrino (Davis, 1955):

$\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$ (yes!),

- $\text{anti-}\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$ (No!)

- Neutrino violates sacred laws: The fall of parity (Lee & Yang, Wu et al 1956)

- Neutrino is prejudiced (always left handed!) (Goldhaber et al 1958)

- A new neutrino is born in 1947 (ν_μ since it always accompanies μ^-)

- This new neutrino is different from the old: $\nu_\mu \neq \nu_e$ (Brookhaven experiments 1962)

- One more neutrino, ν_τ , is born. it appears always together with the lepton τ^- (SPEAR, SLAC, 1976). It was shown that $\nu_\tau \neq \nu_e$, $\nu_\tau \neq \nu_\mu$ (Fermilab 1997)

- These discoveries played a role in the formulation of the standard model (GSW, 1967-1971)

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- Measurement of θ_{13} :T2K 2011 (upper and lower limits); definite result, Daya Bay, march 2012
- Nobel prize (2015) for establishing neutrino oscillation, **McDonald and Kajita**.

Detection (2013) of very energetic neutrinos 10^{15} eV= 10^4 GeV by IceCube neutrino observatory originating from AGN galaxies, i.e. containing AGN (active galactic nuclei). Signal of new physics?

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$$\nu_\alpha(0) = \cos\theta\nu_1 - \sin\theta\nu_2, \quad \nu_\beta(t) = \sin\theta\nu_1 e^{-iE_1 t} + \cos\theta\nu_2 e^{-iE_2 t}$$

The conversion probability is

$$P(\nu_\alpha(0) \rightarrow \nu_\beta(t)) = |\langle \nu_\alpha(0) | \nu_\beta(t) \rangle|^2 = \sin^2 \theta \cos^2 \theta |e^{-iE_1 t} - e^{-iE_2 t}|^2 = \sin^2 2\theta \sin^2 \Delta_{12} t$$

$$\Delta_{12} = \frac{1}{2}(E_1 - E_2) \approx \frac{m_1^2 - m_2^2}{4E_\nu} \Rightarrow$$

$$P(\nu_\alpha(0) \rightarrow \nu_\beta(L)) = \sin^2 2\theta \sin^2 \pi \frac{L}{\ell_{12}}, \quad \ell_{12} = \frac{4\pi E_\nu}{\Delta m_{12}^2} = \text{oscillation length}$$

$$\Delta m_{12}^2 = m_1^2 - m_2^2, \quad L = ct = \text{source detector distance}$$

The survival probability of species ν_α is

$$P(\nu_\alpha(0) \rightarrow \nu_\alpha(L)) = 1 - \sin^2 2\theta \sin^2 \pi \frac{L}{\ell_{12}}$$

Neutrino Oscillations-

Appearance (solid curves)-disappearance (dot, dash)

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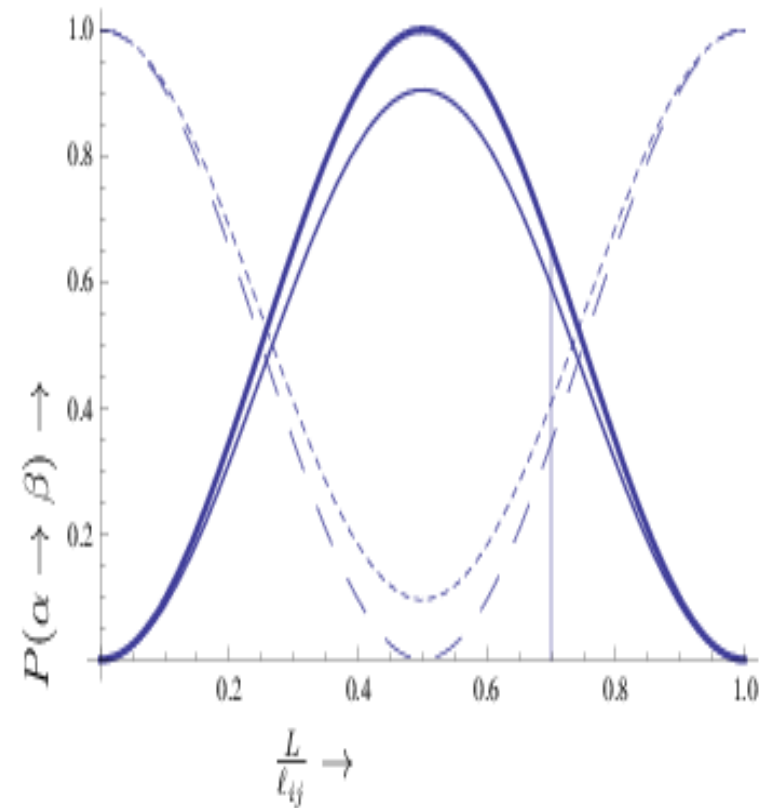
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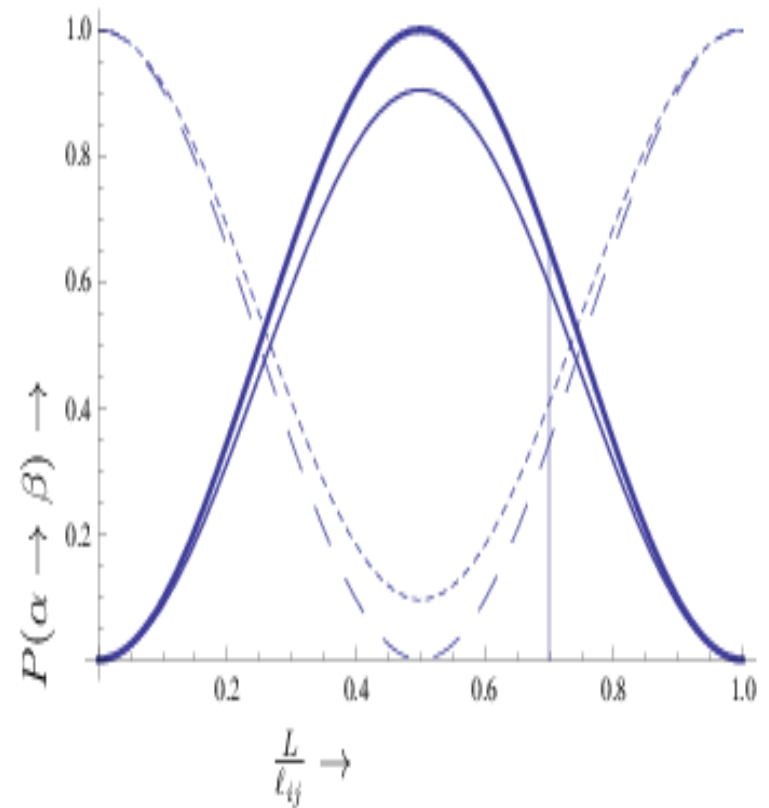
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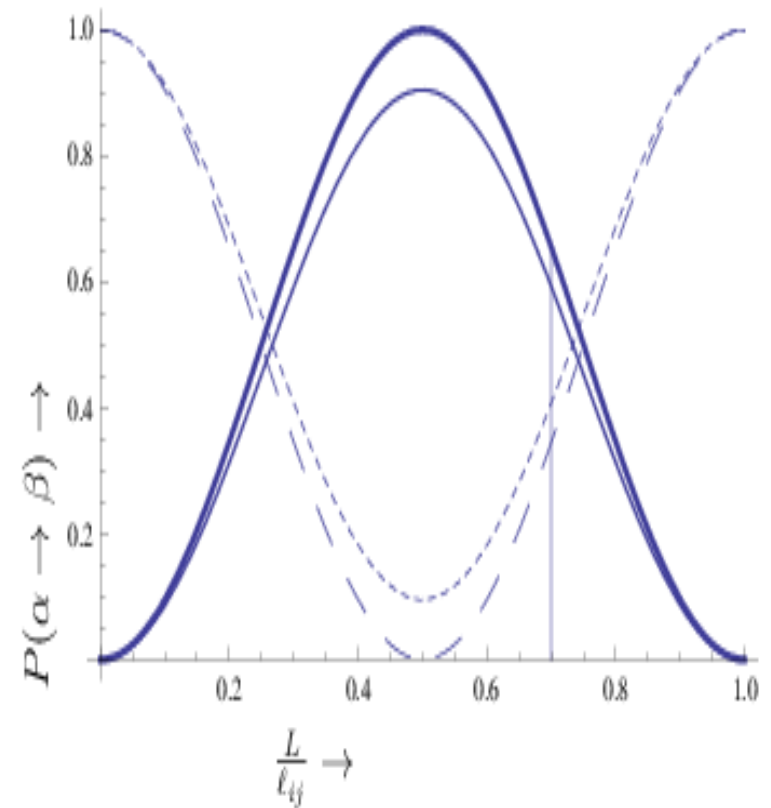
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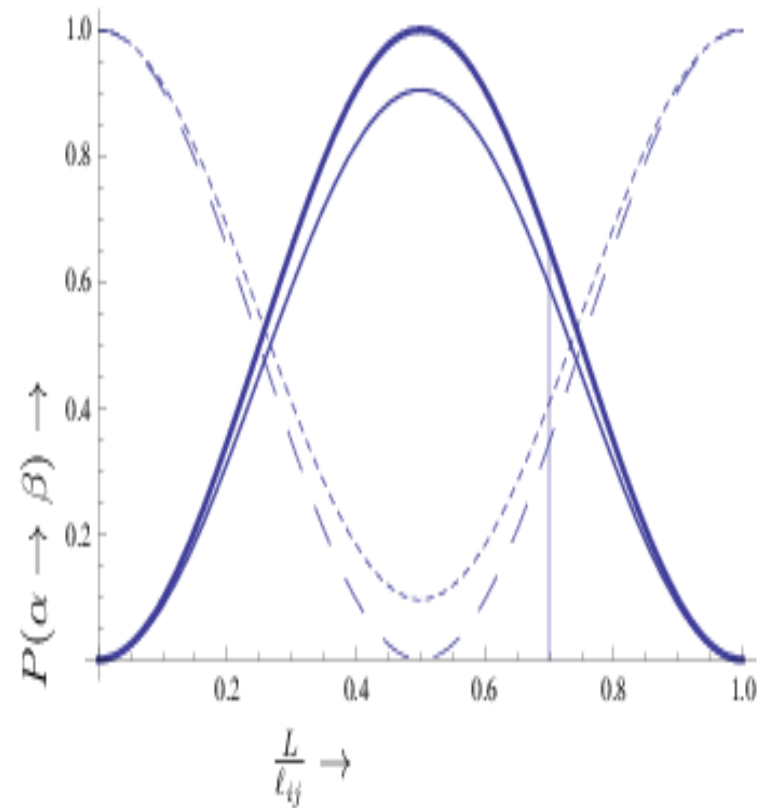
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the value at $L=0$ (no oscillation)
- Can we see the full wave? Not so easy! It will not fit the detector!
- For $\Delta m^2=10^{-3} \text{ eV}^2$, $E_\nu=1 \text{ MeV}$ \rightarrow
- The detector size ought to be $d=2.5 \text{ km}$!



Aspects of neutrino mass: I Dirac mass

In the discussion of the SM we have seen that the neutrinos cannot acquire a mass like all the other fermions because the right handed neutrino does not exist. If the right handed neutrino exists then the neutrino can acquire a mass through the Yukawa coupling, similarly to the up quarks:

$$\mathcal{L}_D = y_D (\bar{\nu}_L, \bar{e}_L) \begin{pmatrix} \phi^{*0} \\ -\phi^- \end{pmatrix} \nu_R + HC = y_D \bar{\nu}_L \nu_R \phi^{*0} - \bar{e} \nu_R \phi^- + HC \quad (12.3)$$

In the unitary gauge we get

$$\mathcal{L}_D = y_D \bar{\nu}_L \nu_R \frac{\eta + v}{\sqrt{2}} + HC = \bar{\nu}_L \mathcal{M}_D \nu_R + \frac{\eta}{v} \bar{\nu}_L \mathcal{M}_D \nu_R \quad (12.4)$$

where

$$\mathcal{M}_D = \frac{v}{\sqrt{2}} y_D \quad (12.5)$$

Aspects of neutrino mass II: Majorana mass

- connecting a left handed neutrino with a right handed antineutrino in the presence of lepton number violating interactions:

$$\mathcal{M}_\nu = m_{\alpha,\beta} \bar{\nu}_L^0 \nu_R^{0c}, \quad m_{\alpha,\beta} \text{ symmetric not necessarily real}$$

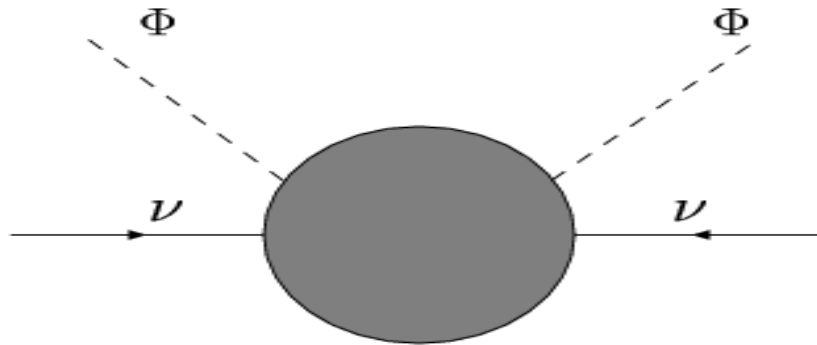
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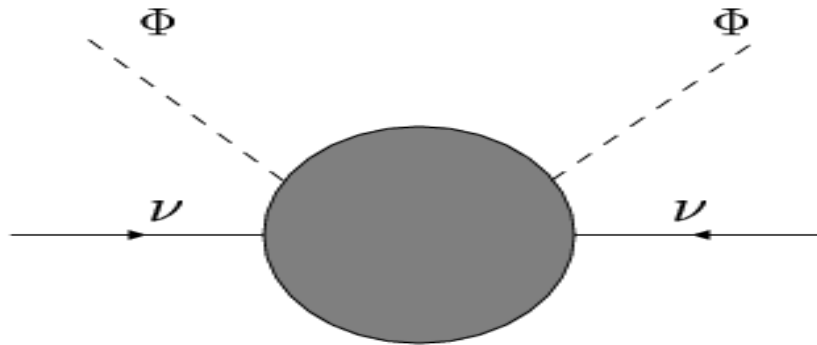
basic dim-5 operator 



unknown scale and flavour structure

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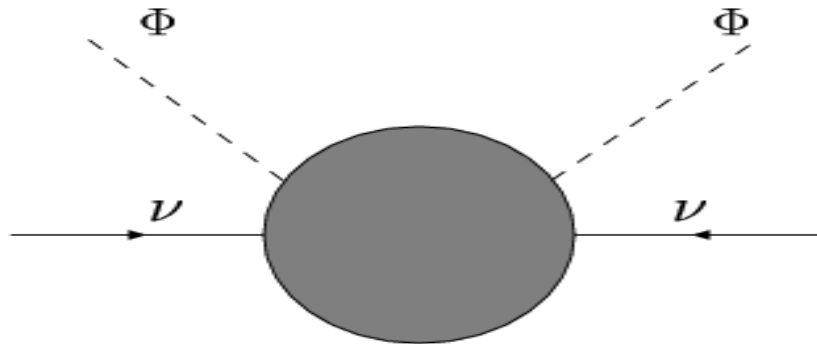


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Realization:

Weinberg's extension of the SM: two $I=1/2$ Higgs \rightarrow
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basic dim-5 operator ●



unknown scale and flavour structure

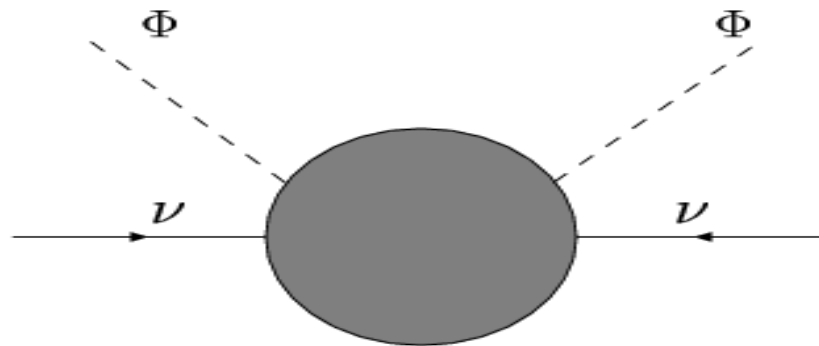
Realization:

$$\mathcal{L}_W = -\frac{\lambda_{\alpha\beta}}{\Lambda} \ell_{\alpha L}^i \varepsilon^{ij} H^j C \ell_{\beta L}^k \varepsilon^{kl} H^l + \text{h.c.}, \quad (1)$$

where $\ell_L = (\nu_L, l_L)^T$ in the $SU(2)_L$ gauge space, $\lambda_{\alpha\beta} = \lambda_{\beta\alpha}$ are effective Yukawa couplings with flavour indices $\alpha, \beta = e, \mu, \tau$ and C is the charge conjugation matrix. \forall

Weinberg's extension of the SM: two $I=1/2$ Higgs \rightarrow Violates lepton number by two units \rightarrow Majorana Mass

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Realization:

$$(m_\nu)_{\alpha\beta} = (\langle H_0 \rangle^2 / 2\Lambda) \lambda_{\alpha\beta}$$

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Majorana mass: I=1 Higgs triplet->

Majorana mass (violates lepton number)

The Lagrangian takes the form (each entry is 3x3 matrix):

$$\mathcal{L}_{m_\nu} = f_{m_\nu} (\bar{\nu}_L, \bar{e}_L) \begin{pmatrix} T^0 & \frac{1}{\sqrt{2}}T^- \\ \frac{1}{\sqrt{2}}T^- & T^{--} \end{pmatrix} \begin{pmatrix} \nu_R^c \\ -e_R^c \end{pmatrix} + HC \quad (12.6)$$

Now if the isotriplet acquires a vacuum expectation value v_T we get the majorana mass matrix:

$$\mathcal{L}_{m_\nu} = \bar{\nu}_L \mathcal{M}_{m_\nu} \nu_R^c + HC, \quad \mathcal{M}_{m_\nu} = f_{m_\nu} v_T \quad (12.7)$$

The above expression is the celebrated light neutrino Majorana mass matrix. Again as in the case of the other fermion masses, the models cannot predict the numerical values of the entries of this matrix. One can arrange them to fit the data, if the vacuum expectation value is sufficiently small. This is all fine, except that neither the isotriplet has been found nor other effects attributed to it have yet been observed.

The standard see-saw mechanism

Isosinglet Majorana mass (with right handed ν_R)

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Once the right handed neutrinos have been introduced, one can easily construct a Majorana mass term for them since they do not carry any SM quantum numbers (they are isosinglets). This will be of the form:

$$\mathcal{L}_{m_N} = \bar{\nu}_L^c \mathcal{M}_{m_N} \nu_R + HC, \quad \mathcal{M}_{m_N} = \int m_N \psi_S$$

Combining this with the Dirac Mass we obtain the 6×6 matrix:

$$\mathcal{L}_m = (\bar{\nu}_L \bar{\nu}_L^c) \begin{pmatrix} 0 & \mathcal{M}_D \\ \mathcal{M}_D^T & \mathcal{M}_{m_N} \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} \quad (12.8)$$

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Assuming now that the Majorana mass has eigenvalues which are all much larger than the entries of the Dirac mass we obtain an effective light Majorana mass of the form

$$(\mathcal{M}_{m_\nu})_{eff} = -\mathcal{M}_D^T \mathcal{M}_N^{-1} \mathcal{M}_D \quad (12.9)$$

similar in form with that obtained in the case of the isotriplet. In fact one can see that even if the Dirac mass entries are of the order of 100 MeV, the resulting neutrinos can be quite light, if the isosinglet neutrinos are extremely heavy. E.g.:

$$\frac{m_D^2}{M_N} = \frac{(100\text{MeV})^2}{10^{10}\text{MeV}} = 10^{-6}\text{MeV} = 1\text{eV},$$

The leptonic mixing matrix: U^{PNMS}

(Pontecorvo-Maki-Nakagawa-Sakata)

Including the Majorana phases, which do not contribute to ν -oscillations

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$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}, \quad (12.10)$$

with

$$s_{ij} = \sin \theta_{ij}, \quad c_{ij} = \cos \theta_{ij}, \quad \alpha_{21} = \alpha_2 - \alpha_1, \quad \alpha_{31} = \alpha_3 - \alpha_1 \quad (12.11)$$

or

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Three generation ν -oscillations

$$P(\nu_\alpha, \nu_\beta) = |\langle \nu_\beta(0) | \nu_\alpha(t) \rangle|^2, \quad \langle \nu_\beta(0) | \nu_\alpha(t) \rangle = \sum_j U_{\beta_j}^* U_{\alpha_j} e^{-iE_j t}, \quad (12.20)$$

$$P(\nu_\alpha, \nu_\beta) = |\langle \nu_\beta(0) | \nu_\alpha(t) \rangle|^2 = \sum_{j,k} U_{\beta_j}^* U_{\alpha_j} U_{\beta_k} U_{\alpha_k}^* e^{2i\Delta_{jk}L}, \quad \Delta_{jk} = \frac{E_j - E_k}{2}, \quad L = ct.$$

or

$$P(\nu_\alpha, \nu_\beta) = \delta_{\alpha\beta} - 4\text{Re} \left\{ \sum_{j < k} U_{\beta_j}^* U_{\alpha_j} U_{\beta_k} U_{\alpha_k}^* \sin^2 \pi \frac{L}{L_{jk}} \right\} - 2 \text{Im} \left\{ \sum_{j < k} U_{\beta_j}^* U_{\alpha_j} U_{\beta_k} U_{\alpha_k}^* \sin \left(2\pi \frac{L}{L_{jk}} \right) \right\}.$$

with ℓ_{jk} conveniently written as:

$$\ell_{jk} = 2.476 \text{km} \frac{E_\nu / 1 \text{GeV}}{|\Delta m_{jk}^2| / 1 \text{eV}^2} = 2.476 \text{m} \frac{E_\nu / 1 \text{MeV}}{|\Delta m_{jk}^2| / 1 \text{eV}^2} = 2.476 \text{m} \frac{E_\nu / 1 \text{keV}}{|\Delta m_{jk}^2| / 10^{-3} \text{eV}^2}. \quad (1)$$

Neutrino Oscillation Experiments

- Atmospheric Neutrinos, originating from cosmic rays (of moderately high energies).
- Reactor neutrinos, following nuclear decay. Low energy of the order of a few MeV* (only disappearance of electron antineutrinos. The energy is below threshold for muon production)
- Accelerator neutrinos. Produced in accelerators. In the multi GeV range (both appearance and disappearance).
- *These experiments confirmed the solar neutrino deficit. Solar neutrino deficit originated the idea of oscillations. Such oscillations, however, could not be seen explicitly as such.

The Daya Bay reactor experiment

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- Recall:
- $|\nu_e(0)\rangle = \cos\theta_{13}(\cos\theta_{12}|\nu_1\rangle + \sin\theta_{12}|\nu_2\rangle) + \sin\theta_{13}|\nu_3\rangle$

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In the special case of $\ell_{31} \approx \ell_{32}$ and $\theta_{13} \ll 1$, we find

$$P(\nu_e \rightarrow \nu_e) = 1 - \left[\sin^2 2\theta_{12} \sin^2 \left(\pi \frac{L}{\ell_{21}} \right) + \sin^2 2\theta_{13} \sin^2 \left(\pi \frac{L}{\ell_{32}} \right) \right],$$

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$$\sin^2 2\theta_{13} = 0.092 \pm 0.016 \text{ (stat)} \pm 0.005 \text{ (syst)}$$

Information extracted from the standard neutrino oscillation experiments

- The three mixing **angles**: θ_{12} , θ_{13} , θ_{23}
- **The absolute value** of the mass squared differences, e.g.

$$\Delta m_{21}^2 = m_2^2 - m_1^2, \quad \Delta m_{32}^2 = m_3^2 - m_2^2$$

- One sign, e.g. the first, can arbitrarily be chosen positive.
- The other* can only be determined from the difference between neutrinos and antineutrinos. Not yet achieved
- So two scenarios emerged: Normal Hierarchy (NH) and Inverted Hierarchy (IH)

* The third difference is not independent: $\Delta_{13} = \Delta_{12} + \Delta_{23}$

Lepton mixing matrix is established.

It is accurately known. Approximately:

Quark mixing matrix

The CKM matrix

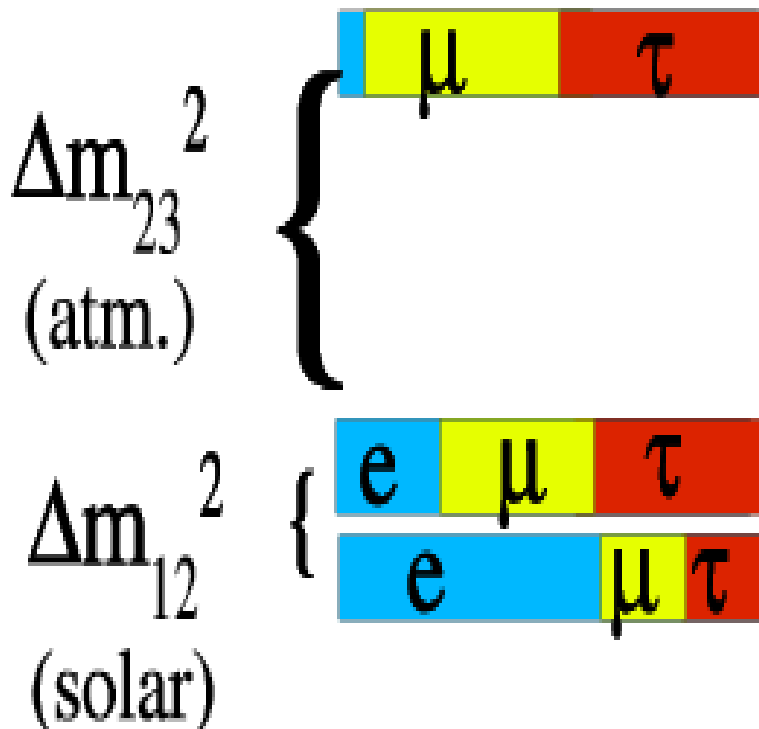
$$\begin{pmatrix} 0.975 & 0.221 & 0.003 \\ 0.221 & 0.975 & 0.040 \\ 0.009 & 0.039 & 0.999 \end{pmatrix}$$

Lepton mixing matrix PNMS

$$\begin{pmatrix} 0.82 & 0.55 & 0.15 \\ -0.49 & 0.52 & 0.70 \\ 0.30 & -0.65 & 0.70 \end{pmatrix}$$

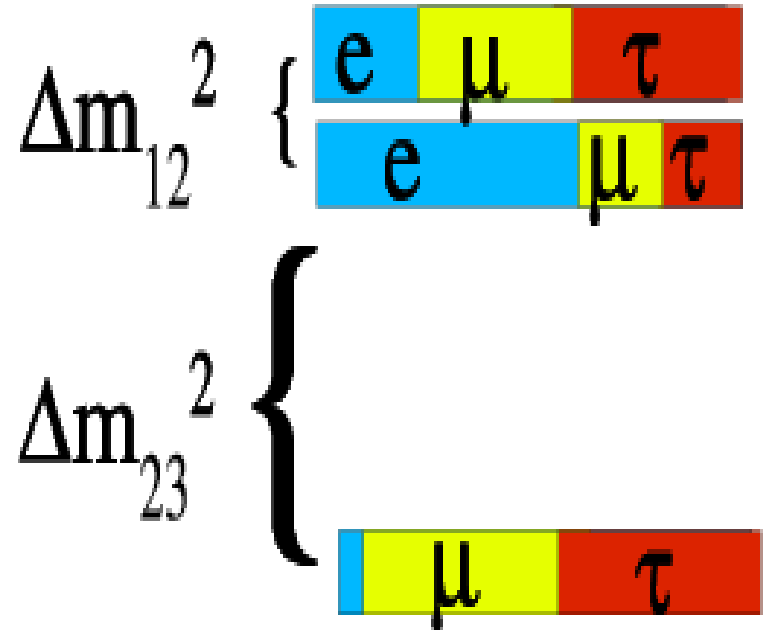
One of the needed signs of Δm^2 is not known
 -> Two scenarios: Normal Hierarchy (NH)
 on the left and Inverted (IH) on the right.

"Normal" hierarchy



OR

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- To settle the controversy of Dirac vs Majorana :
Neutrinoless double beta decay

- PART IV
- THE ELUSIVE SCALE OF NEUTRINO MASS

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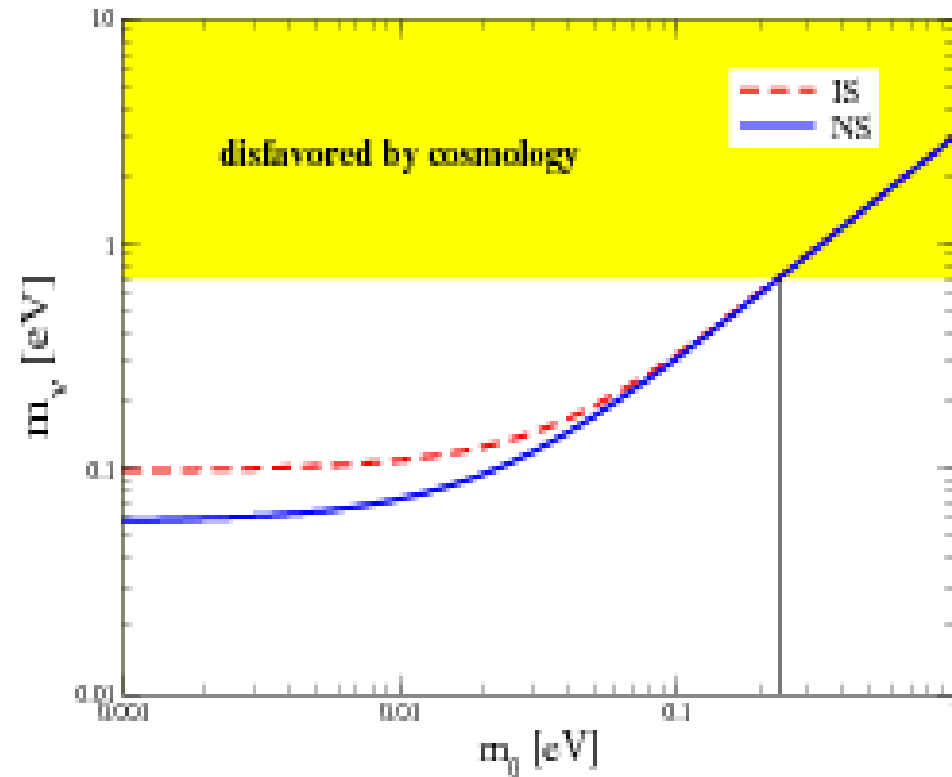
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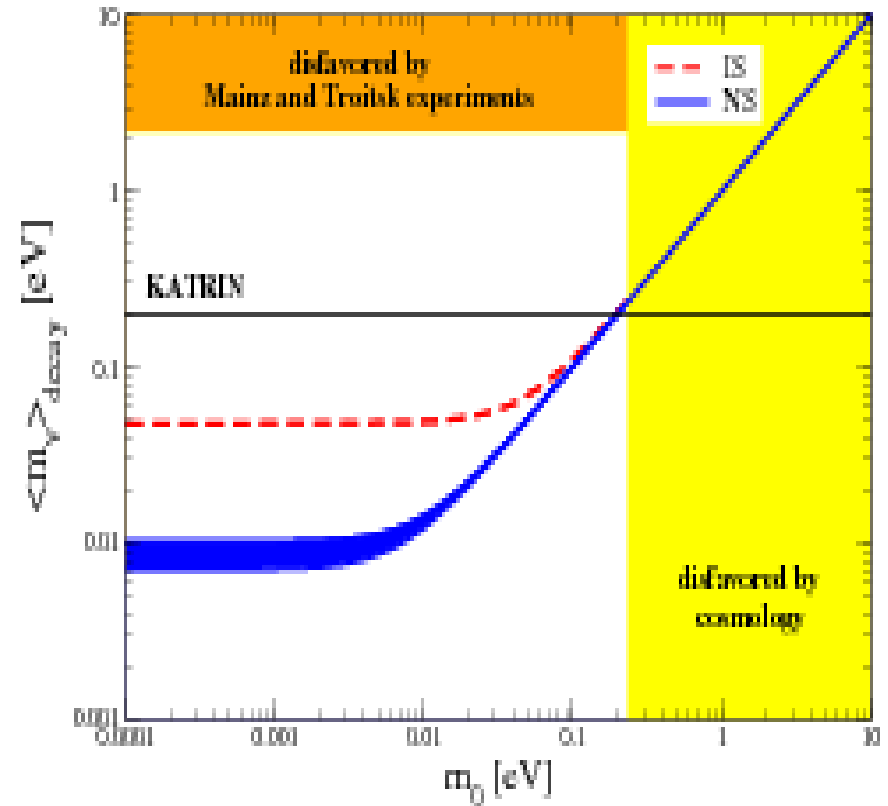
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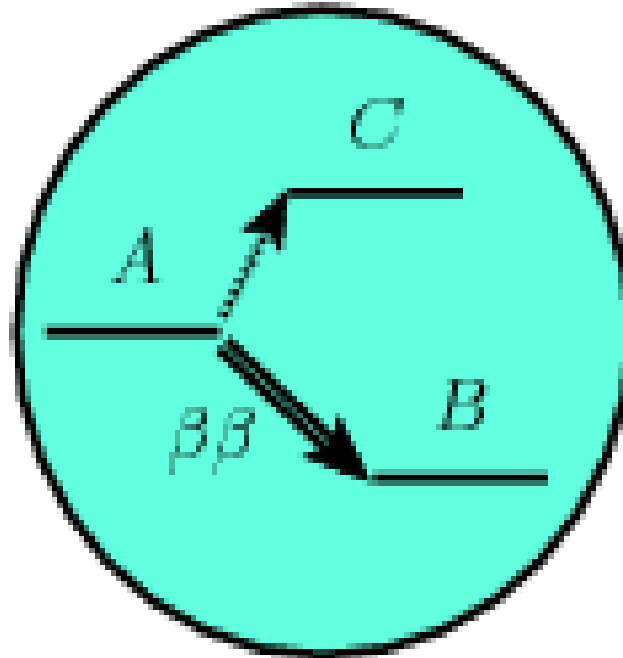
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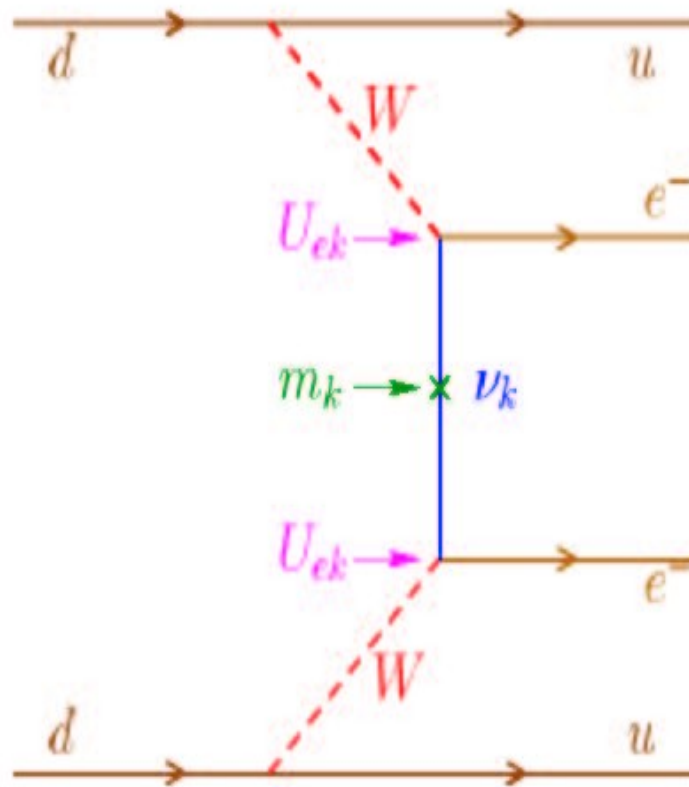
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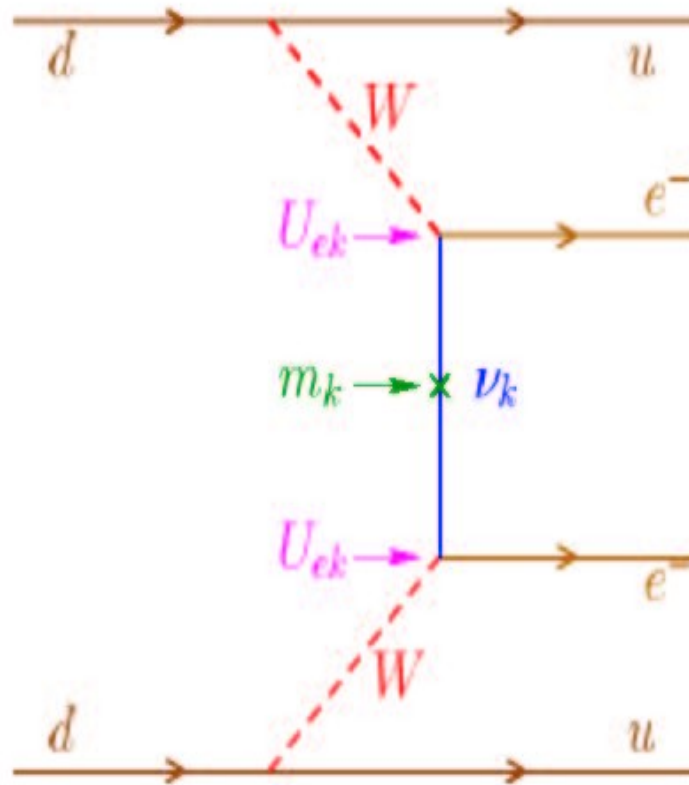
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It proceeds only if the neutrino is a Majorana fermion:

For light neutrino the amplitude is proportional to: $\sum_k (U^{ek})^2 m_k$



lower neutrino mass bound from $0\nu\beta\beta$ -decay: $|\langle m_\nu \rangle|$

(JDV, Ejiri, Simkovic, IJMPE; [arXiv:1205.0649](https://arxiv.org/abs/1205.0649))

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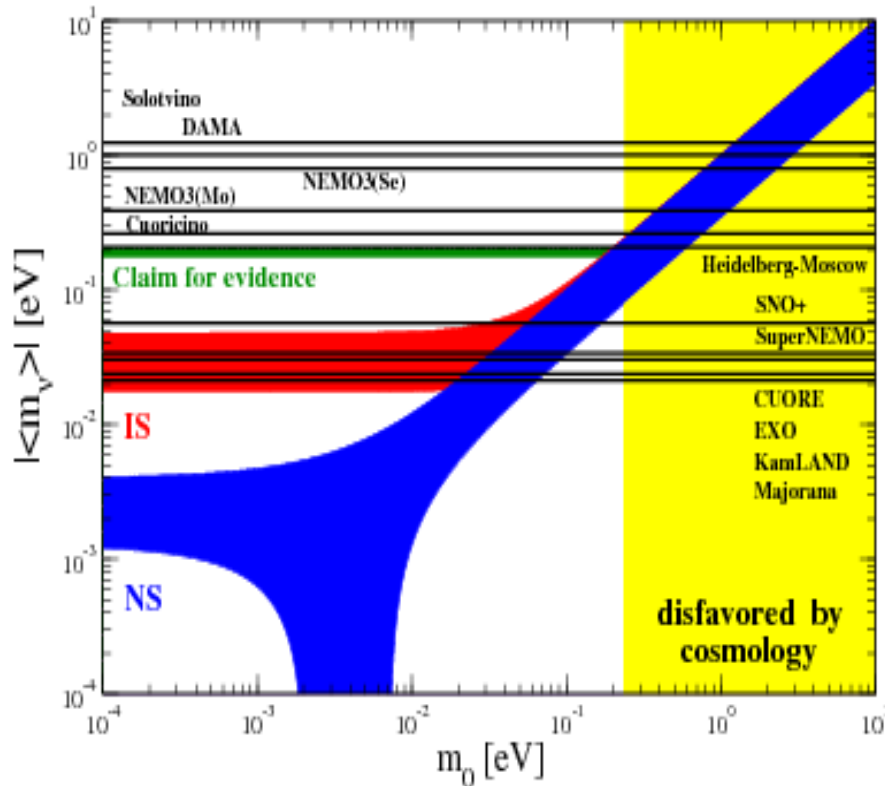
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- Whether sterile neutrinos can be the source of dark matter

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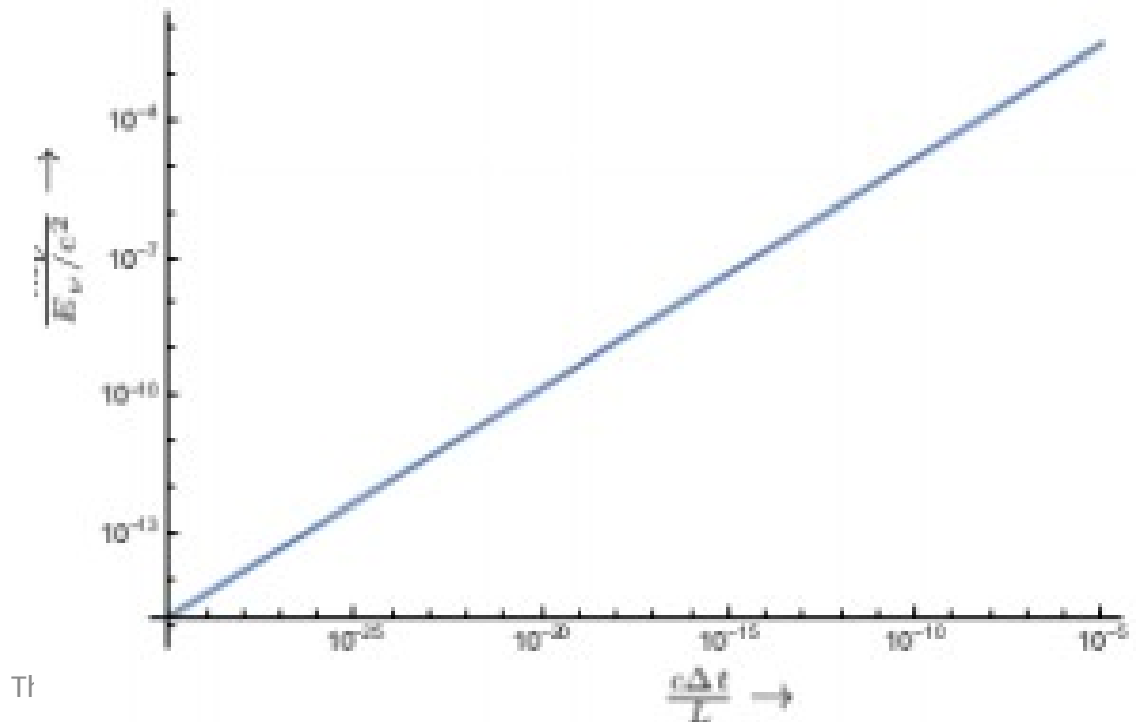
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● THE END

In all these cases we have neutrino mixing

Then we can distinguish between two types of neutrino states

- The weak eigenstates ν_L^0 produced in weak interactions
- And the eigenstates ν_L of the Hamiltonian, which are stationary states.

They are connected by a unitary transformation

$$\nu_L^0 = U \nu_L$$

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G_{01} kinematical function fairly well known.

$M^{0\nu\beta\beta}$ nuclear matrix element. Not very well determined since:

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G_{01} kinematical function fairly well known.

$M^{0\nu\beta\beta}$ nuclear matrix element. Not very well determined since:

- 1) The nuclei involved have complicated structure
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We are now going to discuss bounds on $\langle m_\nu \rangle$

The best fit values of all neutrino oscillation data, solar, atmospheric and reactor (KamLAND , CHOOZE, K2K, Daya Bay); From Forero, Tortola & Valle (2014).

parameter	best fit $\pm 1\sigma$	2σ range	3σ range
$\Delta m_{21}^2 [10^{-5} \text{eV}^2]$	$7.60^{+0.19}_{-0.18}$	7.26–7.99	7.11–8.18
$ \Delta m_{31}^2 [10^{-3} \text{eV}^2]$ (NH)	$2.48^{+0.05}_{-0.07}$	2.35–2.59	2.30–2.65
$ \Delta m_{31}^2 [10^{-3} \text{eV}^2]$ (IH)	$2.38^{+0.05}_{-0.06}$	2.26–2.48	2.20–2.54
$\sin^2 \theta_{12}/10^{-1}$	3.23 ± 0.16	2.92–3.57	2.78–3.75
$\theta_{12}/^\circ$	34.6 ± 1.0	32.7–36.7	31.8–37.8
$\sin^2 \theta_{23}/10^{-1}$ (NH)	$5.67^{+0.32}_{-1.24}$ ^a	4.14–6.23	3.93–6.43
$\theta_{23}/^\circ$	$48.9^{+1.8}_{-7.2}$	40.0–52.1	38.8–53.3
$\sin^2 \theta_{23}/10^{-1}$ (IH)	$5.73^{+0.25}_{-0.39}$	4.35–6.21	4.03–6.40
$\theta_{23}/^\circ$	$49.2^{+1.5}_{-2.3}$	41.3–52.0	39.4–53.1
$\sin^2 \theta_{13}/10^{-2}$ (NH)	2.26 ± 0.12	2.02–2.50	1.90–2.62
$\theta_{13}/^\circ$	$8.6^{+0.3}_{-0.2}$	8.2–9.1	7.9–9.3
$\sin^2 \theta_{13}/10^{-2}$ (IH)	2.29 ± 0.12	2.05–2.52	1.93–2.65
$\theta_{13}/^\circ$	8.7 ± 0.2	8.2–9.1	8.0–9.4
δ/π (NH)	$1.41^{+0.55}_{-0.40}$	0.0–2.0	0.0–2.0
$\delta/^\circ$	254^{+99}_{-72}	0–360	0–360
δ/π (IH)	1.48 ± 0.31	0.00–0.09 & 0.86–2.0	0.0–2.0
$\delta/^\circ$	266 ± 56	0–16 & 155–360	0–360

^aThere is a local minimum in the first octant, at $\sin^2 \theta_{23} = 0.473$ with $\Delta\chi^2 = 0.36$ with respect to the global minimum

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.7$)	
		bf $\pm 1\sigma$	3σ range	bf $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2 \theta_{12}$	$0.310^{+0.013}_{-0.012}$	0.275 \rightarrow 0.350	$0.310^{+0.013}_{-0.012}$	0.275 \rightarrow 0.350
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	31.61 \rightarrow 36.27	$33.82^{+0.78}_{-0.76}$	31.61 \rightarrow 36.27
	$\sin^2 \theta_{23}$	$0.580^{+0.017}_{-0.021}$	0.418 \rightarrow 0.627	$0.584^{+0.016}_{-0.020}$	0.423 \rightarrow 0.629
	$\theta_{23}/^\circ$	$49.6^{+1.0}_{-1.2}$	40.3 \rightarrow 52.4	$49.8^{+1.0}_{-1.1}$	40.6 \rightarrow 52.5
	$\sin^2 \theta_{13}$	$0.02241^{+0.00065}_{-0.00065}$	0.02045 \rightarrow 0.02439	$0.02264^{+0.00066}_{-0.00066}$	0.02068 \rightarrow 0.02463
	$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	8.22 \rightarrow 8.99	$8.65^{+0.13}_{-0.13}$	8.27 \rightarrow 9.03
	$\delta_{CP}/^\circ$	215^{+40}_{-29}	125 \rightarrow 392	284^{+27}_{-29}	196 \rightarrow 360
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.39^{+0.21}_{-0.20}$	6.79 \rightarrow 8.01	$7.39^{+0.21}_{-0.20}$	6.79 \rightarrow 8.01
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.525^{+0.033}_{-0.032}$	$+2.427 \rightarrow +2.625$	$-2.512^{+0.034}_{-0.032}$	$-2.611 \rightarrow -2.412$	
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.3$)	
		bf $\pm 1\sigma$	3σ range	bf $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2 \theta_{12}$	$0.310^{+0.013}_{-0.012}$	0.275 \rightarrow 0.350	$0.310^{+0.013}_{-0.012}$	0.275 \rightarrow 0.350
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	31.61 \rightarrow 36.27	$33.82^{+0.78}_{-0.75}$	31.62 \rightarrow 36.27
	$\sin^2 \theta_{23}$	$0.582^{+0.015}_{-0.019}$	0.428 \rightarrow 0.624	$0.582^{+0.015}_{-0.018}$	0.433 \rightarrow 0.623
	$\theta_{23}/^\circ$	$49.7^{+0.9}_{-1.1}$	40.9 \rightarrow 52.2	$49.7^{+0.9}_{-1.0}$	41.2 \rightarrow 52.1
	$\sin^2 \theta_{13}$	$0.02240^{+0.00065}_{-0.00066}$	0.02044 \rightarrow 0.02437	$0.02263^{+0.00065}_{-0.00066}$	0.02067 \rightarrow 0.02461
	$\theta_{13}/^\circ$	$8.61^{+0.12}_{-0.13}$	8.22 \rightarrow 8.98	$8.65^{+0.12}_{-0.13}$	8.27 \rightarrow 9.03
	$\delta_{CP}/^\circ$	217^{+40}_{-28}	135 \rightarrow 366	280^{+25}_{-28}	196 \rightarrow 351
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.39^{+0.21}_{-0.20}$	6.79 \rightarrow 8.01	$7.39^{+0.21}_{-0.20}$	6.79 \rightarrow 8.01
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.525^{+0.033}_{-0.031}$	$+2.431 \rightarrow +2.622$	$-2.512^{+0.034}_{-0.031}$	$-2.606 \rightarrow -2.413$	

Table 1. Three-flavour oscillation parameters from our fit to global data. The numbers in the 1st (2nd) column are obtained assuming NO (IO), i.e., relative to the respective local minimum. Note that $\Delta m_{3\ell}^2 \equiv \Delta m_{31}^2 > 0$ for NO and $\Delta m_{3\ell}^2 \equiv \Delta m_{32}^2 < 0$ for IO. The results shown in the upper (lower) table are without (with) adding the tabulated SK-atm $\Delta\chi^2$.

The leptonic mixing matrix: U^{PNMS} (Pontecorvo-Maki-Nakagawa-Sakata)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}, \quad (12.10)$$

with

$$s_{ij} = \sin \theta_{ij}, \quad c_{ij} = \cos \theta_{ij}, \quad \alpha_{21} = \alpha_2 - \alpha_1, \quad \alpha_{31} = \alpha_3 - \alpha_1 \quad (12.11)$$

or

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{i\delta}s_{13} \\ -c_{23}s_{12} - e^{-i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{-i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{-i\delta}c_{12}c_{23}s_{13} & -e^{-i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}. \quad (12.12)$$

The Majorana phases do not appear in neutrino oscillations or in other lepton flavor changing processes. They appear only in lepton number violating processes, neutrinoless double beta decay etc.

The best fit to the mixing matrix, 3σ . The best fit for δ_{CP} is 215°

$$\rightarrow \sin(\delta_{CP}) = -0.574$$

$$U_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.797 \rightarrow 0.842 & 0.518 \rightarrow 0.585 & 0.143 \rightarrow 0.156 \\ 0.233 \rightarrow 0.495 & 0.448 \rightarrow 0.679 & 0.639 \rightarrow 0.783 \\ 0.287 \rightarrow 0.532 & 0.486 \rightarrow 0.706 & 0.604 \rightarrow 0.754 \end{pmatrix}$$

$$U_{3\sigma}^{\text{with SK-atm}} = \begin{pmatrix} 0.797 \rightarrow 0.842 & 0.518 \rightarrow 0.585 & 0.143 \rightarrow 0.156 \\ 0.235 \rightarrow 0.484 & 0.458 \rightarrow 0.671 & 0.647 \rightarrow 0.781 \\ 0.304 \rightarrow 0.531 & 0.497 \rightarrow 0.699 & 0.607 \rightarrow 0.747 \end{pmatrix}$$

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-- We have learned from neutrino oscillations that:

- the neutrinos are massive. We know mass squared differences, except for a sign.
- The neutrinos are admixed. We know all three mixing angles.

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$$\begin{aligned}
 &P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) = \alpha \\
 &-2 \sin(\delta) \sin(\theta_{12}) \sin(\theta_{13}) \sin(\theta_{23}) (2 \cos(\theta_{23}) \cos^2(\theta_{12}) \sin(\Delta(1, 3))) \\
 &+ 2 \cos(\theta_{23}) \cos(\theta_{12}) \sin(\Delta(1, 2)) + \cos(2\theta_{12}) \cos(2\theta_{23}) \sin(\Delta(2, 3)) + \sin(\Delta(2, 3))) \\
 &P(\nu_e \rightarrow \nu_\tau) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) = \alpha \\
 &-2 \sin(\delta) \sin(\theta_{12}) \sin(\theta_{13}) \sin(\theta_{23}) \\
 &(\cos(\theta_{23}) \cos^2(\theta_{12}) \sin(\Delta(1, 2)) + \cos^2(\theta_{23}) \cos(\theta_{12}) \sin(\Delta(2, 3)) + \cos(\theta_{23}) \cos(\theta_{12}) \sin(\Delta(1, 3))) \\
 &P(\nu_\mu \rightarrow \nu_\tau) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau) = \alpha \\
 &-4 \sin(\delta) \sin(\theta_{12}) \sin(\theta_{13}) \sin(\theta_{23}) \\
 &\cos(\theta_{12}) \cos(\theta_{23}) (\cos(\theta_{12}) \cos(\theta_{23}) \sin(\Delta(1, 2)) + \cos(\theta_{23}) \sin(\Delta(1, 3)) + \sin(\Delta(2, 3)))
 \end{aligned}$$

II: After many attempts (V-A theory etc.) there came

The Standard Model: Symmetry and particle content

3.6 The particle content of the SM

According to the symmetry of the SM:

$$SU_I(2) \otimes U_Y(1) \otimes SU_c(3)$$

we need the following set of gauge fields:

- Three fields A_μ^i , $i = 1, 2, 3$ transforming as an $I = 1$ under $SU_I(2)$ (isotriplet).
- one field B_μ associated with $U_Y(1)$, which is scalar under $SU_I(2)$ and $SU_c(3)$.
- 8 gauge fields G_μ^a transforming as the regular representation of $SU_c(3)$, which are scalars under $SU_I(2)$.

The rest of the fields are not dictated by the symmetry and were put in by hand. The fermion fields were put in chiral multiplets:

$$f_L = \frac{1}{2}(1 - \gamma_5)f, \quad f_R = \frac{1}{2}(1 + \gamma_5)f \text{ such that } \gamma_5 f_L = -f_L, \quad \gamma_5 f_R = f_R$$

The left handed are put in isodoublets ($I = 1/2$) and the right handed in isosinglets ($I = 0$).

The fermions of the Standard model

1. leptons.

$$\ell_L = \left(\begin{array}{c} \nu_e \\ e^- \end{array} \right)_L, \left(\begin{array}{c} \nu_\mu \\ \mu^- \end{array} \right)_L, \left(\begin{array}{c} \nu_\tau \\ \tau^- \end{array} \right)_L \text{ with } I = \frac{1}{2}, Y = -1,$$

$$\ell_R = (e_R^-, \mu_R^-, \tau_R^-) \text{ with } I = 0, Y = -2.$$

The absence of the right handed neutrino is conspicuous. We know now that there are three generations. The number of generations was not important at the time.

2. Quarks. Also in three generations, each appearing in three colors r, g, b :

$$q_L = \left(\begin{array}{c} u^\alpha \\ d^\alpha \end{array} \right)_L, \left(\begin{array}{c} c^\alpha \\ s^\alpha \end{array} \right)_L, \left(\begin{array}{c} t^\alpha \\ b^\alpha \end{array} \right)_L \text{ with } I = \frac{1}{2}, Y = \frac{1}{3}, \alpha = r, g, b.$$

$$q_R = (u_R^\alpha, c_R^\alpha, t_R^\alpha), Y = 4/3, \kappa_R = (d_R^\alpha, s_R^\alpha, b_R^\alpha), Y = -2/3; I = 0, \alpha = r, g, b.$$

3. Scalars.

An isodoublet of complex scalars (Higgs) and its adjoint:

$$\phi = \left(\begin{array}{c} \phi^0 \\ \phi^- \end{array} \right), I = \frac{1}{2}, Y = -1 \text{ and } \bar{\phi} = i\tau_2\phi^* = \left(\begin{array}{c} \phi^+ \\ \phi^{*0} \end{array} \right), I = \frac{1}{2}, Y = 1.$$