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Primordial Black Hole Mimickers in Quadratic Gravity (2-2 Holes) as Dark Matter

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Overview: What are 2-2 holes about? Differences from BHs? DM candidate as relics. Cues on quantum gravity?

- Horizonless ultracompact objects (HUCOs) from quadratic gravity, a candidate theory for quantum gravity. A family of solutions in the classical theory.

- Unlike Black Holes (BHs): they do not posses horizon; they exhibit different thermodynamical behaviour than BHs; back-reaction of thermal matter leads to radiation in the classical level; entropically favoured over BHs in certain circumstances.

- Possess a lower bound for its mass! Therefore, natural remnants of primordial objects as DM candidate. In BH case, no sign from the classical GR on the existence of remnants. Generally assumed that QG might stop the evaporation, leading to relics. In the case of 2-2 holes on the other hand, definite signs from the classical physics that evaporation indeed stops; although the full mechanism for termination of evaporation still requires quantum treatment.

- They resemble BHs from exterior down to a Planck length of would-be-horizon. Thus, they look similar to BHs astrophysically in the leading order of observations; consistent with the current GW and M87 implications. On the other hand, there exist predictions separating 2-2 holes (or HUCOs in general) from BHs.

- With improved observational techniques, a future sign on their existence would have direct Implications on the nature of quantum gravity; in this case in favour of Quantum Quadratic Gravity.

Beyond General Relativity

- **GR is an outstandingly successful theory.** Explains astrophysical phenomena well. Passed ubiquitous number of tests so far; Post-Newtonian tests, gravitational lensing, growth of galaxy clusters, and many more. GWs recently observed. Provides successful cosmology with FRW background.

- **But not free from problems.** Non-renormalizable. Can be considered as an effective field theory. Higher order corrections occur as the energy increases.

$$\mathcal{S}_{GR} = \int d^4x\sqrt{-g}\left[m_{Pl}^2\left(-\Lambda + \frac{1}{2}R\right) + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + ...\right]$$

- **As an effective theory, it is expected to be replaced at the UV by a well-behaved theory.** Hence the search for the theory of quantum gravity.

- **Expected to provide a resolution to another apparent problem in GR: information loss paradox in black holes.**

Beyond General Relativity

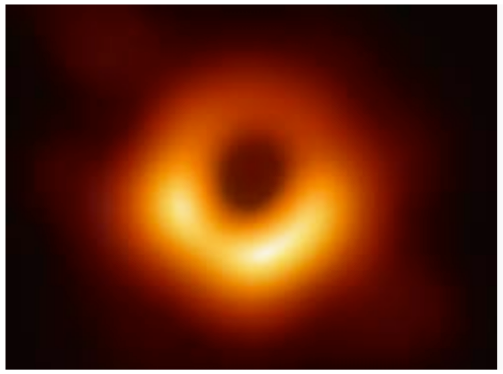
- **Event Horizon of a BH** → Point of no return - information loss - how to recover?

- **One possibility of resolution:** No horizons at all; horizonless ultracompact objects (HUCO), active area of research for decades.

- **Theory of quantum gravity**
  - ↗ BHs (hence horizons) exist and information loss is dealt with?
  - → Does not allow BHs horizons at all and so deals with the problem from the get-go?
  - ↘ Or allowing both BH and HUCO solutions but perhaps physically favors the latter?

HUCOs are still in the picture, phenomenologically!

- **Looks very similar to BHs astrophysically;** the radiation is trapped in high-redshift region due to strong gravitational field. Would look dark in the electromagnetic window within the current resolution of the observations.

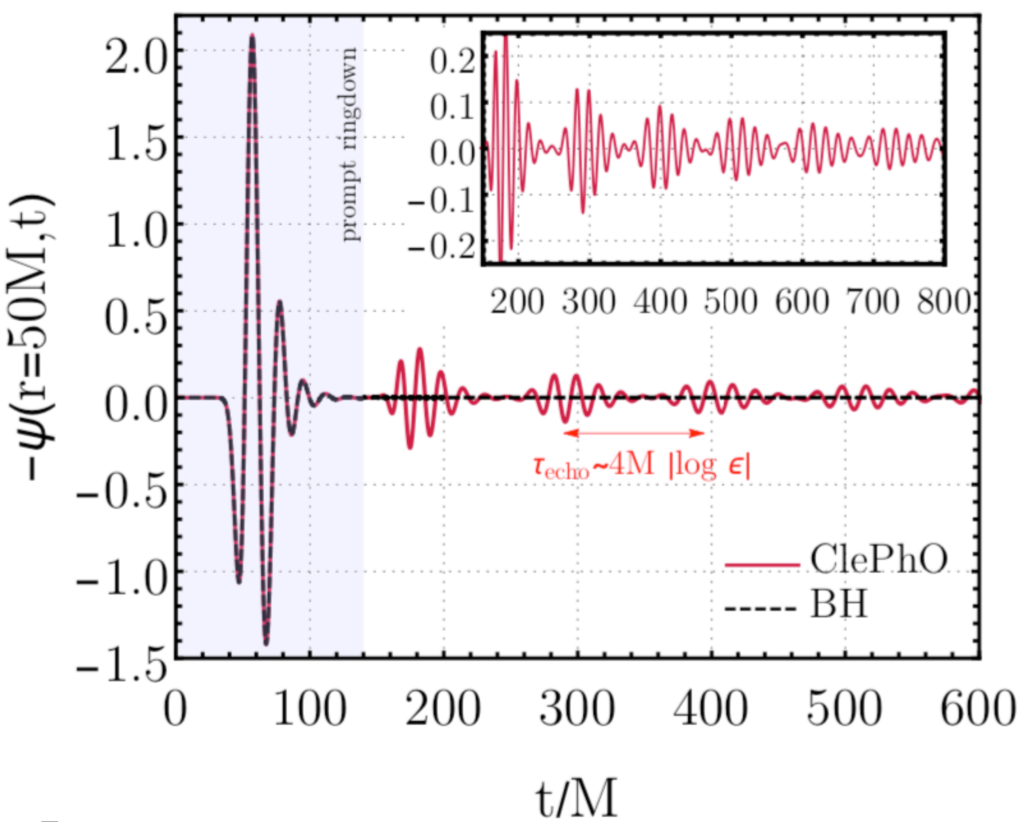


M87 shadow

For the latest status report, see:
Cardoso & Pani (2019)

See also **Abramowicz, Kluzniak, Lasota (2002)** for difficulties in confirming existence of a horizon from observations.

- **GW echoes:** Main GW signal of the post-merger ringdown phase virtually identical between BHs and 2-2 holes cases. But 2-2 holes have distinctive predictions for after the ringdown; echos of decreasing amplitudes because of the reflection of the pulse off inner surface (or center).



→ From **Cardoso & Pani (2019).** For a generic HUCO.

Evidence for echoes from the LIGO signals!
Conklin, Holdom, Ren (2017), Abedi et.al (2016)

Although the significance seems to be low:
Westerweck et.al, (2017)

See **Abedi et.al (2018),** for the response to Westerweck et.al, (2017).

Quantum Quadratic Gravity: A novel horizonless object

- **An old idea for quantum gravity.** Extending GR by including all the quadratic invariants in the action.

$$\mathcal{S} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(m_{Pl}^2 R - \alpha C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + \beta R^2\right)$$

$$m_2 = m_{\rm Pl}/\sqrt{2\alpha}$$
$$m_0 = m_{\rm Pl}/\sqrt{6\beta}$$

- **Renormalizable (Stelle-1977), but comes with baggage;** a ghost-like massive degree of freedom. On the other hand, It has been argued in literature that there might be solutions.

A recently proposed resolution: Non-perturbative effects due to possible strong coupling at the UV removes the ghost from the physical spectrum. GR emerges as the leading term in the low energy effective theory.
Holdom & Ren, Phys.Rev. D93 (2016) no.12, 124030, arXiv:1512.05305

It was discussed in the literature that ghosts might be benign in a quantum theory — not causing any unitarity problems.
(**Smilga, J.Phys. A47 (2014) no.5, 052001, arXiv: 1306.6066)**

Quadratic Gravity: 2-2 holes, information loss paradox, and dark matter

- **The classical solutions provide an object similar to black hole but with no horizon, a black hole mimicker: 2-2 hole**

  Sourced by dense, compact matter like black hole but possesses quite different features.

- **Different scaling relations, different thermodynamical behaviour.**

  Different law for Hawking radiation, entropy, energy. Anomalous features of BH thermodynamics emerge in the large-mass limit from the ordinary gas dynamics. In the small-mass limit, it looks just like a regular thermodynamical system.

- **Candidate for dark matter**

  Its relic constitutes a candidate for dark matter if it is formed in the early universe, just like primordial black holes (PBHs). Primordial 2-2 holes part ways with PBHs in terms of implications and observational consequences due to its different thermodynamical behaviour

2-2 holes:

Holdom & Ren (2017)

- **Classical solutions for dense matter sources for the fully non-linear theory.**

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \qquad S_{\rm CQG} = \frac{1}{16\pi}\int d^4x \sqrt{-g}\left(m_{\rm Pl}^2 R - \alpha C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + \beta R^2\right)$$

- **Numerically solved for static, spherically symmetric, asymptotically flat spacetime.**

Non-Schwarzschild-like solutions.

Series expansion around origin. Solved for the spherical thin shell model.

$$A(r) = a_s r^s + a_{s+1} r^{s+1} + a_{s+2} r^{s+2} + \ldots,$$
$$B(r) = b_t (r^t + b_{t+1} r^{t+1} + b_{t+2} r^{t+2} + \ldots)$$

(s,t)	behavior at $r=0$	generic CQG	$\beta = 0$ CQG	GR
$(0,0)$	non-singular	a_2, b_2	b_2	none
$(1,-1)$	Schd-like	a_1, a_4, b_2	a_1, a_4	a_1
$(2,2)$	vanishing metric	a_2, a_5, b_3, b_4, b_5	a_2, b_3, b_4	NA
$(2,2)_E$		a_2, b_4	a_2	

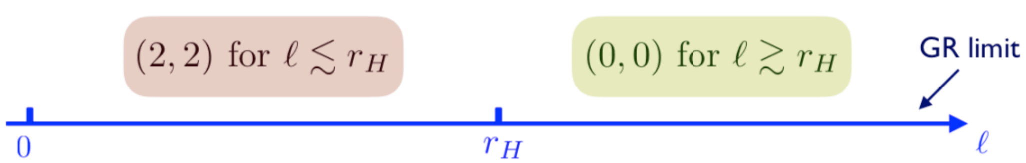
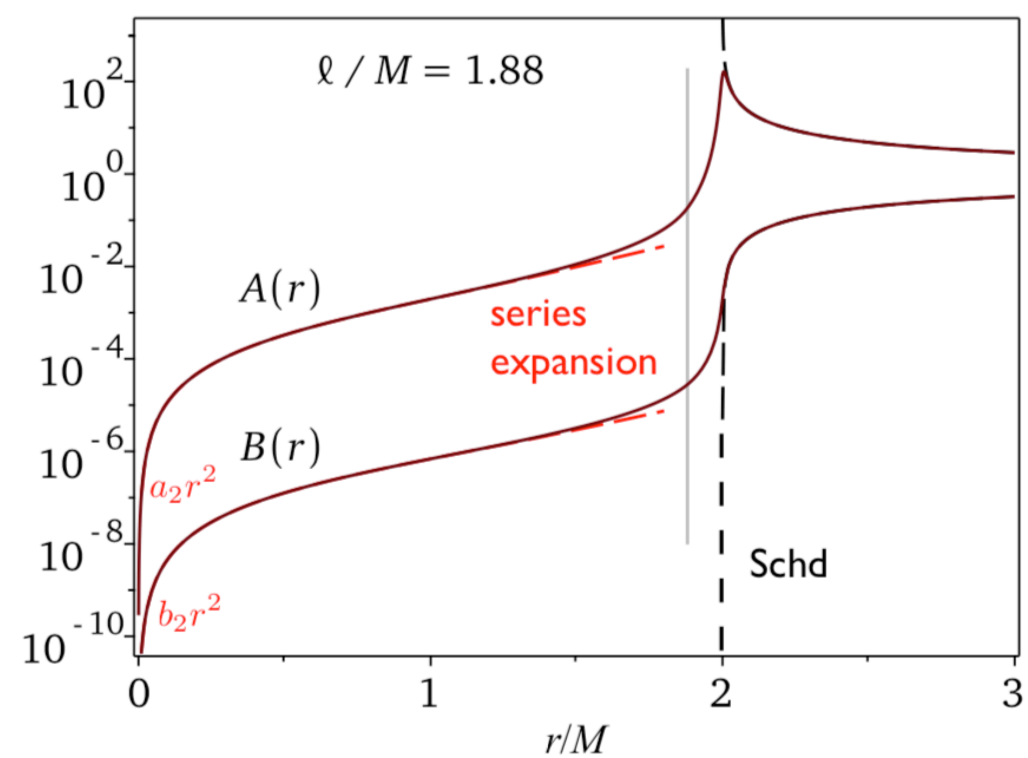


FIG. 1. A schematic illustration of asymptotically-flat horizonless solutions that couple to a thin-shell with physical mass M and shell-radius ℓ. $r_H = 2M/m_{\rm Pl}^2$ denotes the would-be horizon.

For $l \gtrsim r_H$ (low compactness):
When the shell is very thin, the deviation of (0,0) from GR-solution (star) is very small. When ℓ is large, GR has no horizonless solution but Quadratic Gravity has (0,0).

For $l \lesssim r_H$: (high compactness):
(2-2) solutions take over..

We are interested in $l \lesssim r_H$ case: (2-2) solutions.
Resembles Schd solution in the exterior down to a distance Planck length away from the would-be-horizon.
Transition to high curvature interior solution.



Ren's talk (2019), Tsinghua U.

2-2 holes with relativistic thermal gas

Ren (2019)

- **In-falling relativistic particles in thermal equilibrium, p=3ρ.**
- **αC^2 term is necessary, βR^2 does not play an essential role.**

β=0 gives us Einstein-Weyl gravity

In the large mass limit: $r_H/\lambda_2 \gg 1$ **BH thermodynamics reproduced up to some factor.**

$$T_\infty \approx 1.35\, N^{-1/4}\sqrt{\frac{\lambda_2}{\ell_{\rm Pl}}}\, T_H, \quad S \approx 0.74\, N^{1/4}\sqrt{\frac{\ell_{\rm Pl}}{\lambda_2}}\, S_{\rm BH}.$$

$$T_\infty = T(r)\sqrt{-g_{00}(r)}$$

Tolman law for local temperature

$$T_H = 1/4\pi r_H$$

Hawking temperature

$$S_{\rm BH} = \pi r_H^2/\ell_{\rm Pl}^2$$

Bekenstein-Hawking entropy

Thermal gas

$$T_{\mu\nu} = \text{diag}\left(B\rho, Ap, r^2 p, r^2 s_\theta^2 p\right),$$

$$\rho = \frac{N}{(2\pi)^3}\int_0^\infty \frac{E}{e^{E/T}-\epsilon} 4\pi p^2 dp,$$

$$p = \frac{N}{3(2\pi)^3}\int_0^\infty \frac{p^2/E}{e^{E/T}-\epsilon} 4\pi p^2 dp,$$

↓ m=0

Relativistic limit

$$\rho = 3p = N\frac{3}{\pi^2}T^4$$

In the small mass limit: $r_H/\lambda_2 \gtrsim 1$

$$T_\infty \propto \frac{1}{r_b}\sqrt{\frac{r_a}{\ell_{\rm Pl}}}, \quad S_{\rm in} \propto \left(\frac{r_a^2}{\ell_{\rm Pl}^2}\right)^{3/4},$$

$r_b \equiv 1/\sqrt{b_2}$

$r_a \equiv 1/\sqrt{a_2}$ Interior radius

would-be horizon size $r_H = 2M\ell_{\rm Pl}^2$ (M is the physical mass)

No solution exits for $\frac{r_H}{\lambda_2} \lesssim 1$

So; minimum allowed mass for 2-2 holes!

$$M_{min} \sim \frac{m_{Pl}^2}{m_2}$$!

Black Hole evaporation

- The most important information comes from radiation of primordial (black or 2-2) holes and the formation mass.

Surface gravity → κ

$$T_{BH} = \frac{\kappa}{2\pi} = \frac{1}{8\pi G M} \sim 10^{-7} \left(\frac{M_\odot}{M} \right) \mathrm{K}$$

- Using the Stefan–Boltzmann law of black-body radiation

$$\frac{dM}{dt} \approx -\frac{\pi^2}{120} g_* \, 4\pi r_h^2 \, T_{BH}^4 \longrightarrow \tau_{\rm evap} \sim \frac{G^2 M^3}{g_*} \sim 10^{17} \left(\frac{M}{10^{15}\mathrm{g}} \right)^3 \mathrm{s}$$

So, for $M \sim 10^{15}\mathrm{g}$, $\tau_{\rm evap} \sim 10^{17}\mathrm{s} \sim \tau_{univ}$

Or, $M > 10^{9}\mathrm{g}$, $\tau_{\rm evap} > 1\mathrm{s} \sim \tau_{\rm BBN}$

- **What about 2-2 holes?**

Radiation Temperature, evaporation time for 2-2 holes

In the large-mass limit

$$T_{22} \approx 1.35\ g_*^{-1/4} \sqrt{\frac{\lambda_2}{l_{Pl}}}\ T_{BH} = 1.35\ g_*^{-1/4} \sqrt{\frac{m_{Pl}}{m_2}}\ T_{BH}$$

Hawking temperature

Dependence on g_* and m_2. m_2 dependence is important.

$$\tau^{22}_{\text{evap}} = g_* \frac{m_2^2}{m_{Pl}^2} \tau^{BH}_{\text{evap}} = m_2^2\ M^3\ G^3$$

Extra dependence of g_* Canceling the initial g_*

For $m_2 \sim m_{Pl}$, $\tau^{22}_{\text{evap}} \sim \tau^{BH}_{\text{evap}}$ Strong coupling at the UV

For $m_2 \ll m_{Pl}$, $\tau^{22}_{\text{evap}} \ll \tau^{BH}_{\text{evap}}$ Weak coupling at the UV

Primordial (Black or 2-2) holes

- Black hole formation in early universe the corresponding implications are heavily studied in the literature. Initiated by **Carr (1975).**

- It is formed when highly over-dense regions of inhomogeneities gravitationally collapse.

- Main reason that it draws this much attention is its possible role as dark matter.

- Could be formed with a variety of masses or the production could peak at some scale depending on the scenario.

- Possible mass spans a wide mass range from the Planck mass to 10^{15} solar masses.

- Constraints and implications are directly related to the mass and the evaporation time.

2-2 hole relics as dark matter

- Even though black holes with masses smaller than $\sim 10^{15}\ \mathrm{gr}$ are expected to have evaporated by now and hence thought not to be able account for the missing mass, there is another possibility which has also heavily studied in the literature.

- Even though in GR there seems not to exists corresponding suitable mechanism, it is possible that black holes do not evaporate into nothing, but leave some remnants behind. Then, these relics may contribute significantly to DM abundance under some conditions.

- In the case of a **2-2 hole**, this is actually a natural scenario since they possess a minimum possible mass, signalling that there might indeed be a mechanism at small scales stopping the evaporation at some stage.

$$M_{rem} \sim M_{min} \sim \frac{m_{Pl}^2}{m_2}$$

Dark matter?

Formation mass:

$$M(t_f) \sim M_H(t_f) = \frac{m_{Pl}^2}{2H} = m_{Pl}^2 t_f$$

- In the inflationary scenario $H \lesssim H_{end} \lesssim 10^{-5} m_{Pl}$
- So formation mass is bounded from below: $M \gtrsim 1\mathrm{g}$
- Same bound applies for the 2-2 hole.
- Note that this does not give us information on M_{min} since the mass can go lower than 1 g by evaporation, down to the Planck mass.

Fractional abundance

See **Dalianis & Tringas (2019)**, for the recent progress in PBH relics physics.

- Fractional abundance of the remnants are given as

$$f_{rem} \sim \frac{M_{rem}}{M}\beta\left(\frac{M_{eq}}{M}\right)^{1/2}$$

$M_{eq} \sim 10^{50}\mathrm{g}$ is the horizon mass at the radiation-matter equality.

M is the formation (initial) mass of the hole.

β is the formation rate.

For $f_{rem} \sim 1$, a lower limit can be obtained for M_{rem} by assuming minimum $M \sim 1\mathrm{g}$ and maximum $\beta \sim 1$.

$$M_{rem} > 10^{-24}\mathrm{g}$$

- An upper limit can also be obtained since it is difficult to satisfy BBN constraint $\beta < 10^{-22}$ for $M_{rem} \ll M$.
  For PBHs
- So we want evaporation stops before BBN time scale, ~1 s. $\tau_{BH} \lesssim 1\mathrm{s} \rightarrow M \lesssim 10^9\mathrm{g}$, so $M_{rem} \ll 10^9\mathrm{g}$
- But for 2-2 holes the situation is slightly different. We have factor of $g_*^{-1/3}\left(\frac{m_{Pl}}{m_2}\right)^{2/3}$ on the RHS.

  Weakens the upper bound but the dependence is not that strong.

Constraints from diffuse photon flux and minimum mass of 2-2 hole

See **Bai & Orlofsky (2019)** for PeBH case

$$\Phi_\gamma \approx \frac{M_{DM}}{M_{22}} \mathcal{J} \frac{dN_\gamma}{dE_\gamma dt}$$

Generic for each galaxy

Emission rate for photon. Depends on temperature

$$\mathcal{J} = \int \rho^2_{DM}(l,\Omega) dl d\Omega$$ *J* - factor

$$\frac{d^2 N_\gamma}{dE_\gamma dt} \approx \frac{\pi^2}{60} 4\pi r_h^2 \frac{T_{22}^4}{\langle E_\gamma \rangle^2} \qquad \langle E_\gamma \rangle \sim T_{22}$$

$$\Phi_\gamma \lesssim 10^{-4}\ \mathrm{cm}^{-2}\mathrm{sr}^{-1}\mathrm{s}^{-1}\mathrm{MeV}^{-1}$$

For Milky way

Temperature of 2-2 hole in the small mass limit:

$$T_{22} = \frac{8 \times 10^{-11}}{g_*^{1/4}} \left(\frac{M_{min}}{\mathrm{g}}\right)^{-1/6} \left(1 - 0.007 \ln\left[7 \times 10^4 \frac{M_{min}}{\mathrm{g}}\right]\right)^{-7/12} \frac{\mathrm{MeV}}{k_B}$$

$g_* = 11$

$$M_{min} \lesssim 7 \times 10^{14} \mathrm{g}$$

Similar calculation for a regular BH with Hawking temperature gives us

$$M_{rem} \lesssim 7 \times 10^{12} \mathrm{g}$$

Recall : $M_{min} \sim \frac{m_{Pl}^2}{m_2} \longrightarrow m_2 \gtrsim 10^{-20} m_{Pl}$

What is next?

- Differences that new scaling relations of 2-2 holes bring in compared to PBHs.

- Detailed study of primordial 2-2 hole relics as dark matter. What kind of information can we infer more?

- Formation of 2-2 holes and its initial mass. Inflation? Phase transitions?

- Effects on the evolution of the universe. Formation in the radiation era or dust era?

- More radical questions? Can observations of ultracompact objects give clues on quantum gravity? Astrophysics(large scales) — Quantum gravity (small scales)

Summary:

- Quadratic gravity, a candidate quantum gravity theory, provides novel classical horizonless configurations for ultracompact sources, namely 2-2 holes.

- Horizonless nature of these objects in the framework of quantum gravity suggests a resolution to information loss paradox.

- It has been argued that in LIGO GW data there are traces of post-merger echoes, which would be a smoking gun signal for horizonless ultra compact objects, if confirmed. Could shed light on the nature of quantum gravity.

- These objects exhibits different thermodynamical behaviour than black holes.

- Remnants of the primordial 2-2 holes constitute candidate for dark matter in similarity to primordial black holes.

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Thank you…

Extra slides

Thin-shell models

Holdom&Ren (2017)

$$T_{\mu\nu} = \mathrm{diag}(T_{tt},\ T_{rr},\ T_{\theta\theta},\ T_{\theta\theta}\sin^2\theta)$$

$T_{tt}(r) = B(r)\rho(\ell)\delta(r-\ell), \quad T_{rr}(r) = 0$. and from $\nabla^\mu T_{\mu r} = 0 \longrightarrow T_{\theta\theta}(r) = \frac{r^3 B'(r)}{4B^2(r)} T_{tt}(r)$.

We can also set $T_{\theta\theta}(r) = r^2\tilde{p}(\ell)\delta(r-\ell)$. The vacuum solutions inside and outside the shell are matched at $r=\ell$ with five conditions, where A, A', B'/B, B''/B are continuous while A'' has a jump as determined by the energy density on the shell,

$$A''_{\mathrm{out}}(\ell) - A''_{\mathrm{in}}(\ell) = 2\pi A^3(\ell)\ell\rho(\ell)\frac{(\alpha-3\beta)B'(\ell)\ell - 2(\alpha+6\beta)B(\ell)}{9\alpha\beta B(\ell)}\,.$$

The derivatives are respect to r, evaluated at ℓ. The value of B itself is trivially continuous at the shell.

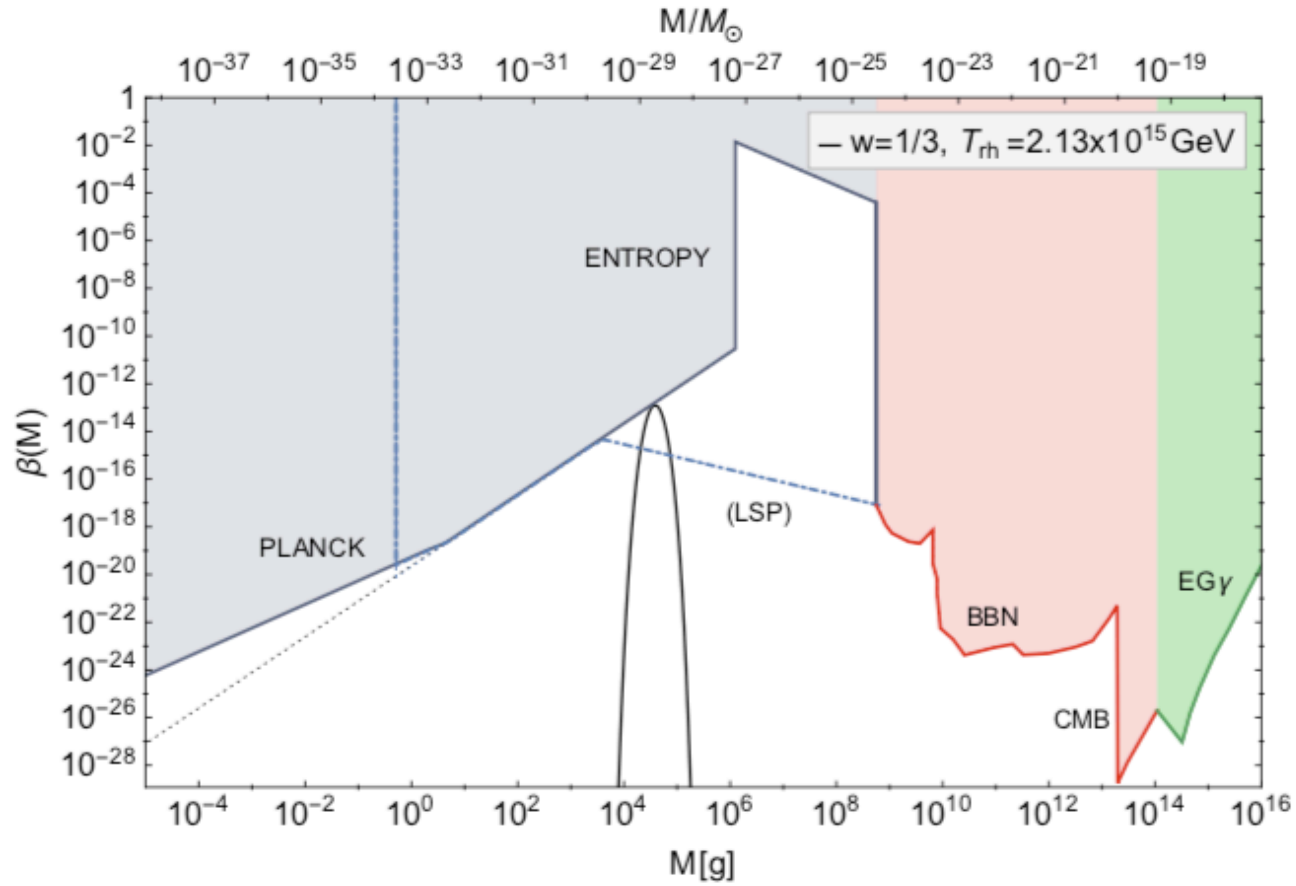

FIG. 9: The PBH formation rate $\beta(M)$ estimated by the Eq. (3.11) for the model (5.9) with the parameters listed in the table III and $\mathcal{P}_{\mathcal{R}}(k)$ depicted in Fig. 5 for the RD case. The central mass of the PBHs is $M \simeq 4 \times 10^4$g that evaporate leaving behind Planck mass remnants with $f_{\rm rem} = 1$. The dotted black lines depicts the constraints if the reheating temperature was $T_{\rm rh} > 10^{15}$ GeV. The LSP upper bound is not applicable since the PBH remnants comprise the total dark matter in our scenarios.

Taken from arXiv: :1905.01741

PBH remnants as dark matter produced in thermal, matter and runaway-quintessence post-inflationary scenarios

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(Dated: May 7, 2019)

Schwarzschild Black Holes

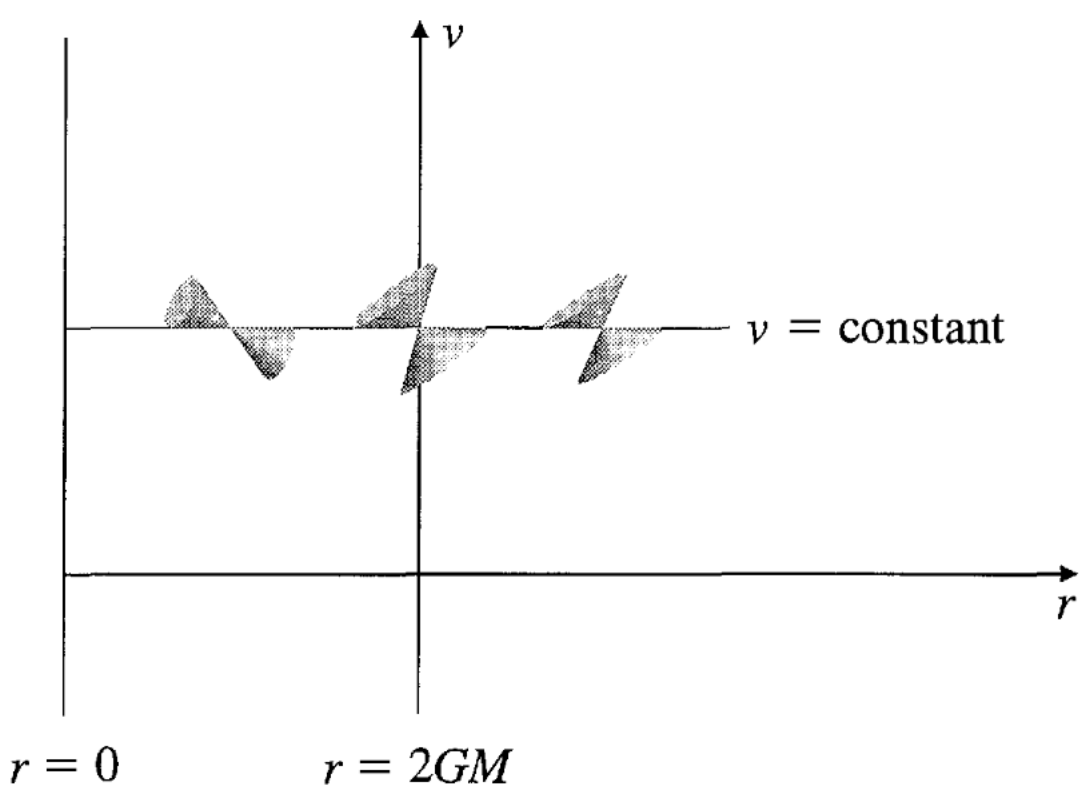
Eddington–Finkelstein coordinates

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

$$t \to v = t + r^*$$

$$r^* = r + 2GM \ln\left(\frac{r}{2GM} - 1\right)$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dv^2 + (dv\, dr + dr\, dv) + r^2 d\Omega^2$$



Entropy, total energy, (negligible) back-reaction on the metric in the case of 2-2 hole

Holdom & Ren (2017)
Holdom (2019)

$$S = \frac{(2\pi)^3}{45} \int_0^L T(r)^3 A(r)^{1/2} r^2 dr,$$

$$U = \frac{3}{4} T_\infty S = \frac{3}{8} M$$

$$T(r) \equiv T_\infty / \sqrt{B(r)}$$

Entropy exhibit Bekenstein-Hawking type area law for the large mass limit for 2-2 holes

Local (Tolman) temperature

Can backreaction on the metric be ignored and is the fixed background geometry justified? This scenario by all means is similar to ’t Hooft’s brick-wall model of the black hole entropy, **’t Hooft (1985)**, proposed to explain Bekenstein-Hawking entropy as contribution from a reflective wall around the black hole to the thermal energy of the corresponding quantum fields.

Mukohyama & Israel (1998) argues that the structure in the brick wall model represents exterior of a horizonless configuration instead of a black hole. Therefore, the ground state of corresponding quantum fields should be chosen as Boulware state (T_∞ =0), not Hawking-Hartle state (T_∞ =T_H). This leads to negative vacuum energy. Cancellations occur once the Boulware state and the corresponding thermal excitations. This is generally referred to as “topped-up” Boulware state and causes minor backreaction by construction. Similar arrangements can be made for the 2-2 hole case.

So far quantum corrections are not taken into account for the 2-2 hole case. The above discussion suggests that the total thermal energy, entropy, and temperature will be larger once the quantum corrections are taken into account but they are expected to be a factor of O(1). Recall, for instance, that classical T_∞ in the large mass limit for 2-2 hole is proportional to T_H.