Consistent truncation and de Sitter space from gravitational instantons

Dimitrios Tsimpis

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Based on:

- Robin Terrisse & DT, JHEP (2019)
- Robin Terrisse & DT, JHEP (2018)
- Bertrand Souères & DT, Phys. Rev. D (2018)

Motivation

- Within the context of effective actions arising from critical superstrings: realize a positive cosmological constant (dark energy) in a controlled setting has turned out to be difficult
- Quantum ingredients bring about control issues
- Various no-go theorems in two-derivative supergravities with classical ingredients
- Gaugino condensation in heterotic strings does not seem to lead to a positive c.c.
- Fermion condensation seems more promising in IIA

Some background: IIA supereravities

- Dimensional reduction of CJS supergravity
  Giani & Pernici, Phys.Rev.D (1984)
  Campbell & West, Nucl.Phys.B (1984)
  Huq & Namazie, Class.Quant.Grav. (1985)
- Romans Supergravity
  Romans, Phys.Lett.B (1986)
- HLW Supergravity
  Howe, Lambert & West, Phys.Lett.B (1998)
- 10d (massive) superspace
  DT, JHEP (2005)

Effective action

- The vacuum is given by:

$$\left.\frac{\delta S_{\text{eff}}}{\delta\phi}\right|_{\langle\phi\rangle} = 0$$

- The type IIA 10d string effective action:

$$\mathcal{L}_{\text{eff}} = \frac{1}{g_s^2}\left(\mathcal{L}_{\text{IIA}} + \alpha'^3\left(I_0(R) - \frac{1}{8}I_1(R) + \cdots\right) + \mathcal{O}(\alpha'^4)\right)$$
$$+\left(\alpha'^3\left(I_0(R) + \frac{1}{8}I_1(R) + \cdots\right) + \mathcal{O}(\alpha'^4)\right)$$
$$+\mathcal{O}\left(g_s^2\right)$$

- At low energies $(\alpha' \to 0)$, $\mathcal{L}_{\text{IIA}}$ is perturbatively exact in g_s

Fermionic condensates

- Nonperturbatively, fermion condensation may form in the vacuum

$$\langle\lambda\rangle = 0\ ; \quad \langle\bar{\lambda}\lambda\rangle := \int [\mathcal{D}\phi](\bar{\lambda}\lambda)e^{-S[\phi]} \neq 0$$

- Potential applications, to cosmology
- Single quartic dilatino term in $\mathcal{L}_{\rm IIA}$ (positive c.c. ? yes!)
- Quartic terms identical in massive and massless IIA (conjectured by Romans; easily shown in superspace)
- Could « simply » be read off from the original literature

The quartic fermion terms are

$$
\begin{aligned}
e^{-1}L_{(4)} = {} & \tfrac{1}{4}(\bar\psi_{\mu_1}\Gamma^{\mu_1}\psi^{\mu_3})(\bar\psi_{\mu_2}\Gamma^{\mu_2}\psi_{\mu_3}) \\
& -\tfrac{1}{32}(\bar\psi_{\mu_1}\Gamma_{\mu_2}\psi_{\mu_3})(2\bar\psi^{\mu_1}\Gamma^{\mu_2}\psi^{\mu_3}+4\bar\psi^{\mu_1}\Gamma^{\mu_3}\psi^{\mu_2}-\bar\psi_{\nu_1}\Gamma^{\mu_1\mu_2\mu_3\nu_1\nu_2}\psi_{\nu_2}) \\
& +\tfrac{1}{32}(\bar\psi_{\mu_1}\Gamma^{11}\psi_{\mu_2})(2\bar\psi^{\mu_1}\Gamma^{11}\psi^{\mu_2}+\bar\psi_{\nu_1}\Gamma^{11}\Gamma^{\mu_1\mu_2\nu_1\nu_2}\psi_{\nu_2}) \\
& -\tfrac{1}{64}(\bar\psi_{\mu_1}\Gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6}\psi_{\mu_2})(\bar\psi_{\mu_3}\Gamma_{\mu_4\mu_5}\psi_{\mu_6}) \\
& -\tfrac{3}{16}(\bar\psi^{\mu_1}\Gamma^{\mu_2\mu_3}\psi^{\mu_4})(\bar\psi_{[\mu_1}\Gamma_{\mu_2\mu_3}\psi_{\mu_4]}) \\
& -\tfrac{1}{16}(\bar\psi_{\mu_1}\Gamma^{11}\Gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5}\psi_{\mu_2})(\bar\psi_{\mu_3}\Gamma^{11}\Gamma_{\mu_4}\psi_{\mu_5}) \\
& +\tfrac{1}{16}(\bar\psi^{\mu_1}\Gamma^{11}\Gamma^{\mu_2}\psi^{\mu_3})(\bar\psi_{\mu_1}\Gamma^{11}\Gamma_{\mu_2}\psi_{\mu_3}+2\bar\psi_{\mu_2}\Gamma^{11}\Gamma_{\mu_3}\psi_{\mu_1}) \\
& +\tfrac{1}{16}(\bar\lambda\Gamma^{\mu_1\mu_2\mu_3}\lambda)[\bar\psi_{\mu_1}\Gamma_{\mu_2}\psi_{\mu_3}-\tfrac{1}{3}\sqrt{\tfrac{1}{2}}\,\bar\lambda\Gamma^{11}\Gamma_{\mu_1\mu_2}\psi_{\mu_3}] \\
& +\tfrac{1}{2}(\bar\lambda\Gamma^{11}\Gamma^{\mu_1}\Gamma^{\mu_2}\psi_{\mu_1})(\bar\lambda\Gamma^{11}\psi_{\mu_2}) \\
& -\tfrac{1}{32}\sqrt{\tfrac{1}{2}}(\bar\lambda\Gamma^{11}\Gamma^{\mu_1}\Gamma^{\mu_2\mu_3\mu_4\mu_5}\psi_{\mu_1})(\bar\psi_{\mu_2}\Gamma_{\mu_3\mu_4}\psi_{\mu_5} \\
& -\tfrac{1}{3}\sqrt{\tfrac{1}{2}}\,\bar\lambda\Gamma^{11}\Gamma_{\mu_2\mu_3\mu_4}\psi_{\mu_5}-\tfrac{1}{72}\bar\lambda\Gamma_{\mu_2\mu_3\mu_4\mu_5}\lambda) \\
& -\tfrac{3}{128}(\bar\lambda\Gamma^{\mu_1\mu_2\mu_3\mu_4}\lambda)(\bar\psi_{\mu_1}\Gamma_{\mu_2\mu_3}\psi_{\mu_4}-\tfrac{1}{3}\sqrt{\tfrac{1}{2}}\bar\lambda\Gamma_{\mu_1\mu_2\mu_3}\psi_{\mu_4}) \\
& +\tfrac{1}{8}\sqrt{\tfrac{1}{2}}(\bar\lambda\Gamma^{\mu_1}\Gamma^{\mu_2\mu_3\mu_4}\psi_{\mu_1})(\bar\psi_{\mu_2}\Gamma_{\mu_3}\Gamma^{11}\psi_{\mu_4} \\
& -\sqrt{\tfrac{1}{2}}\bar\lambda\Gamma_{\mu_2\mu_3}\psi_{\mu_4}+\tfrac{7}{72}\bar\lambda\Gamma^{11}\Gamma_{\mu_2\mu_3\mu_4}\lambda) \\
& +\tfrac{3}{16}\sqrt{\tfrac{1}{2}}(\bar\lambda\Gamma^{\mu_1}\Gamma^{\mu_2\mu_3}\psi_{\mu_1})(\bar\psi_{\mu_2}\Gamma^{11}\psi_{\mu_3}-3\sqrt{\tfrac{1}{2}}\bar\lambda\Gamma_{\mu_2}\psi_{\mu_3}+\tfrac{17}{24}\bar\lambda\Gamma^{11}\Gamma_{\mu_2\mu_3}\lambda) \\
& +\tfrac{1}{16}\sqrt{\tfrac{1}{2}}(\bar\psi_{\mu_1}\Gamma^{11}\psi_{\mu_2})(3\bar\lambda\Gamma^{\mu_3}\Gamma^{\mu_1\mu_2}\psi_{\mu_3}+5\sqrt{\tfrac{1}{2}}\bar\lambda\Gamma^{11}\Gamma^{\mu_1\mu_2}\lambda) \\
& -\tfrac{3}{32}(\bar\lambda\Gamma_{\mu_1}\psi_{\mu_2})(3\bar\lambda\Gamma^{\mu_3}\Gamma^{\mu_1\mu_2}\psi_{\mu_3}+\tfrac{13}{9}\sqrt{\tfrac{1}{2}}\bar\lambda\Gamma^{11}\Gamma^{\mu_1\mu_2}\lambda) \\
& +\tfrac{1}{1536}[\tfrac{1}{4}(\bar\lambda\Gamma^{\mu_1\mu_2\mu_3\mu_4}\lambda)(\bar\lambda\Gamma_{\mu_1\mu_2\mu_3\mu_4}\lambda)-\tfrac{821}{54}(\bar\lambda\Gamma^{\mu_1\mu_2\mu_3}\lambda)(\bar\lambda\Gamma_{\mu_1\mu_2\mu_3}\lambda) \\
& -\tfrac{82}{3}(\bar\lambda\Gamma^{11}\Gamma^{\mu_1\mu_2}\lambda)(\bar\lambda\Gamma^{11}\Gamma_{\mu_1\mu_2}\lambda)]\,. \qquad (24)
\end{aligned}
$$

Quartic fermion terms

- The literature:
  Giani & Pernici, Phys.Rev.D (1984)
  Campbell & West, Nucl.Phys.B (1984)
  Huq & Namazie, Class.Quant.Grav. (1985)
  Nicoletti & Orazi, Int.J.Mod.Phys.A (2011)

  does not seem to be in agreement!
  S.Theisen & DT, (2013)

- Calculate from scratch in superspace gives agreement with GP
  B.Souères & DT, (2018)

Gravitino condensates in IIA

- What is the origin of the condensate ?
- In the 4d path integral over metrics, fermion condensates arise from zero modes of the Dirac operator in gravitational instanton backgrounds

  Gibbons, Hawking & Perry, Nucl. Phys. B (1978)

- The dominant instanton contribution comes from the ALE spaces, classified according to their Hirzebruch signature τ
- In the ALE background there are 2τ spin-3/2 zero modes and no spin-1/2 modes

Gravitino condensates in IIA

- Construct a 4d universal consistent truncation of IIA on CY in the presence of fluxes & gravitino condensates
- Interesting/novel in its own right
- Captures the universal sector (common in all CY) of the effective action
- Facilitates the search for vacua

Review of 4d IIA CY compactification

- One gravity multiplet (metric, one vector)
- $h^{1,1}$ vector multiplets (one vector, two real scalars)
- $h^{2,1}+1$ hypermultiplets (four real scalars)

- B-field and Kähler deformations

$$B=\beta(x)+\sum_{A=1}^{h^{1,1}}\chi^A(x)e^A(y)\ ;\quad i\delta g_{a\bar{b}}=\sum_{A=1}^{h^{1,1}}v^A(x)e^A_{a\bar{b}}(y)$$

- RR-potentials and complex-structure deformations

$$\delta g_{\bar{a}\bar{b}}=\sum_{\alpha=1}^{h^{2,1}}\zeta^\alpha(x)\Omega^{*cd}{}_{\bar{a}}\Phi^\alpha_{cd\bar{b}}(y)\ ;\quad C_1=\alpha(x)\ ;$$

$$C_3=-\frac{1}{2}\Big(\xi(x)\mathrm{Im}\Omega+\xi'(x)\mathrm{Re}\Omega\Big)+\sum_{A=1}^{h^{1,1}}\gamma^A(x)\wedge e^A(y)+\Big(\sum_{\alpha=1}^{h^{2,1}}\xi^\alpha(x)\Phi^\alpha(y)+\mathrm{c.c.}\Big)$$

Review of 4d IIA CY compactification

- One gravity multiplet $(g_{\mu\nu}, \alpha)$
- $h^{1,1}$ vector multiplets (γ^A, v^A, χ^A)
- $h^{2,1}+1$ hypermultiplets (ξ, ξ', ϕ, b), $(\zeta^\alpha, \xi^\alpha)$

- B-field and Kähler deformations

$$B = \beta(x) + \sum_{A=1}^{h^{1,1}} \chi^A(x) e^A(y) \ ; \quad i\delta g_{a\bar{b}} = \sum_{A=1}^{h^{1,1}} v^A(x) e^A_{a\bar{b}}(y)$$

- RR-potentials and complex-structure deformations

$$\delta g_{\bar{a}\bar{b}} = \sum_{\alpha=1}^{h^{2,1}} \zeta^\alpha(x) \Omega^{*cd}{}_{\bar{a}} \Phi^\alpha_{cd\bar{b}}(y) \ ; \quad C_1 = \alpha(x) \ ;$$

$$C_3 = -\frac{1}{2}\Big(\xi(x)\mathrm{Im}\Omega + \xi'(x)\mathrm{Re}\Omega\Big) + \sum_{A=1}^{h^{1,1}} \gamma^A(x) \wedge e^A(y) + \Big(\sum_{\alpha=1}^{h^{2,1}} \xi^\alpha(x)\Phi^\alpha(y) + \text{c.c.}\Big)$$

Review of 4d IIA CY compactification

- One gravity multiplet $(g_{\mu\nu}, \alpha)$
- $h^{1,1}$ vector multiplets (γ^A, v^A, χ^A)
- $h^{2,1}+1$ hypermultiplets (ξ, ξ', ϕ, b), $(\zeta^\alpha, \xi^\alpha)$

- Universal sector

$(g_{\mu\nu}, \alpha)$, (γ, v, χ), (ξ, ξ', ϕ, b)

Reduction ansatz (bosonic)

- Metric

$$ds^2_{(10)} = e^{2A(x)}\left(e^{2B(x)} g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}\mathrm{d}y^m \mathrm{d}y^n\right)$$

- Fluxes

$$F = \mathrm{d}\alpha \ ; \quad H = \mathrm{d}\chi_\wedge J + \mathrm{d}\beta \ ;$$

$$G = \varphi \mathrm{vol}_4 + \tfrac{1}{2}c_0 J_\wedge J + J_\wedge(\mathrm{d}\gamma - \alpha \wedge \mathrm{d}\chi) - \tfrac{1}{2}\mathrm{d}\xi_\wedge \mathrm{Im}\Omega - \tfrac{1}{2}\mathrm{d}\xi'_\wedge \mathrm{Re}\Omega$$

- Automatically obey the Bianchi identities

$$\mathrm{d}F = 0 \ ; \quad \mathrm{d}H = 0 \ ; \quad \mathrm{d}G = H \wedge F$$

Reduction ansatz (bosonic)

- Internal Einstein eom

$$0 = e^{-8A-2B}\nabla^\mu\left(e^{8A+2B}\partial_\mu A\right) - \frac{1}{32}e^{3\phi/2-2A-2B}\mathrm{d}\alpha^2 + \frac{1}{8}e^{-\phi-4A}(\partial\chi)^2 - \frac{1}{48}e^{-\phi-4A-4B}h^2$$
$$- \frac{1}{32}e^{\phi/2-6A-2B}(\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi)^2 + \frac{1}{16}e^{\phi/2-6A}\Big[(\partial\xi)^2 + (\partial\xi')^2\Big]$$
$$+ \frac{3}{16}e^{\phi/2-6A-6B}\varphi^2 + \frac{7}{16}e^{\phi/2-6A+2B}c_0^2$$

- External Einstein eom

$$R^{(4)}_{\mu\nu} = g_{\mu\nu}\left(\nabla^2 A + \nabla^2 B + 8(\partial A)^2 + 2(\partial B)^2 + 10\partial A\cdot\partial B\right)$$
$$- 8\partial_\mu A\partial_\nu A - 2\partial_\mu B\partial_\nu B - 16\partial_{(\mu}A\partial_{\nu)}B + 8\nabla_\mu\partial_\nu A + 2\nabla_\mu\partial_\nu B$$
$$+ \frac{3}{2}e^{-\phi-4A}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2}e^{3\phi/2-2A-2B}\mathrm{d}\alpha^2_{\mu\nu} + \frac{1}{4}e^{\phi-4A-4B}h^2_{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi$$
$$+ \frac{1}{2}e^{\phi/2-6A}(\partial_\mu\xi\partial_\nu\xi + \partial_\mu\xi'\partial_\nu\xi') + \frac{3}{2}e^{\phi/2-6A-2B}(\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi)^2_{\mu\nu}$$
$$+ \frac{1}{16}g_{\mu\nu}\Big(-\frac{1}{2}e^{3\phi/2-2A-2B}\mathrm{d}\alpha^2 - \frac{1}{3}e^{\phi-4A-4B}h^2 - 3e^{\phi/2-6A}\Big[(\partial\xi)^2 + (\partial\xi')^2\Big]$$
$$- 6e^{-\phi-4A}(\partial\chi)^2 - 5e^{\phi/2-6A-6B}\varphi^2 - 9c_0^2e^{\phi/2-6A+2B} - \frac{9}{2}e^{\phi/2-6A-2B}(\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi)^2\Big)$$

Reduction ansatz (bosonic)

- Dilaton eom

$$0 = e^{-10A-4B}\nabla^\mu\left(e^{8A+2B}\partial_\mu\phi\right) - \frac{1}{4}e^{\phi/2-8A-2B}\left[(\partial\xi)^2 + (\partial\xi')^2\right] - \frac{3}{8}e^{3\phi/2-4A-4B}\mathrm{d}\alpha^2$$
$$+ \frac{3}{2}e^{-\phi-6A-2B}(\partial\chi)^2 + \frac{1}{12}e^{-\phi-6A-6B}h^2$$
$$+ \frac{1}{4}e^{\phi/2-8A-8B}\varphi^2 - \frac{3}{4}c_0^2 e^{\phi/2-8A} - \frac{3}{8}e^{\phi/2-8A-4B}(\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi)^2$$

- F-form eom

$$\mathrm{d}(e^{3\phi/2+6A}\star_4\mathrm{d}\alpha) = \varphi e^{\phi/2+2A-4B}\mathrm{d}\beta - 3e^{\phi/2+2A}\mathrm{d}\chi\wedge\star_4(\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi)$$

- H-form eom

$$\mathrm{d}\left(e^{-\phi+4A+2B}\star_4\mathrm{d}\chi\right) = c_0\varphi\,\mathrm{vol}_4 + (\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi)\wedge(\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi)$$
$$- e^{\phi/2+2A}\mathrm{d}\alpha\wedge\star_4(\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi)$$

$$\mathrm{d}\left(e^{-\phi+4A-2B}\star_4\mathrm{d}\beta\right) = 3c_0(\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi) - \mathrm{d}\xi\wedge\mathrm{d}\xi' + e^{\phi/2+2A-4B}\varphi\mathrm{d}\alpha$$

Reduction ansatz (bosonic)

- G-form eom

$$\mathrm{d}\left(e^{\phi/2+2A+2B}\star_4\mathrm{d}\xi\right)=h_\wedge\mathrm{d}\xi'$$

$$\mathrm{d}\left(e^{\phi/2+2A+2B}\star_4\mathrm{d}\xi'\right)=-h_\wedge\mathrm{d}\xi$$

$$\mathrm{d}\left(e^{\phi/2+2A}\star_4(\mathrm{d}\gamma-\alpha\wedge\mathrm{d}\chi)\right)=2\mathrm{d}\chi_\wedge\mathrm{d}\gamma+c_0\mathrm{d}\beta$$

together with the constraint:

$$\varphi=-3c_0\chi e^{-\phi/2-18A}$$

- Action

$$S_4=\int\mathrm{d}^4x\sqrt{g}\Big(R-24(\partial A)^2-\tfrac{1}{2}(\partial\phi)^2-\tfrac{3}{2}e^{-4A-\phi}(\partial\chi)^2-\tfrac{1}{2}e^{-6A+\phi/2}\left[(\partial\xi)^2+(\partial\xi')^2\right]$$

$$-\tfrac{1}{4}e^{3\phi/2+6A}\mathrm{d}\alpha^2-\tfrac{3}{4}e^{\phi/2+2A}(\mathrm{d}\gamma-\alpha_\wedge\mathrm{d}\chi)^2-\tfrac{1}{12}e^{-\phi+12A}\mathrm{d}\beta^2-\tfrac{9}{2}e^{-\phi/2-18A}c_0^2\chi^2-\tfrac{3}{2}e^{\phi/2-14A}c_0^2\Big)$$

$$+\int 3c_0\mathrm{d}\gamma_\wedge\beta+3c_0\chi\;\alpha_\wedge\mathrm{d}\beta+3\chi\;\mathrm{d}\gamma_\wedge\mathrm{d}\gamma-\beta_\wedge\mathrm{d}\xi_\wedge\mathrm{d}\xi'$$

Reduction ansatz (bosonic)

- In terms of the axion

$$\mathrm{d}\beta = e^{\phi-12A} \star_4 \left[\mathrm{d}b + \tfrac{1}{2}(\xi \mathrm{d}\xi' - \xi' \mathrm{d}\xi) + 3c_0(\gamma - \chi\alpha)\right]$$

- Action

$$\begin{aligned} S_4 = \int \mathrm{d}^4x\sqrt{g}\Big(& R - 24(\partial A)^2 - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{3}{2}e^{-4A-\phi}(\partial\chi)^2 - \tfrac{1}{2}e^{-6A+\phi/2}\left[(\partial\xi)^2 + (\partial\xi')^2\right] \\ & - \tfrac{1}{4}e^{3\phi/2+6A}\mathrm{d}\alpha^2 - \tfrac{3}{4}e^{\phi/2+2A}(\mathrm{d}\gamma - \alpha{\wedge}\mathrm{d}\chi)^2 - \tfrac{1}{2}e^{\phi-12A}(\mathrm{d}b + \omega)^2 \\ & - \tfrac{9}{2}e^{-\phi/2-18A}c_0^2\chi^2 - \tfrac{3}{2}e^{\phi/2-14A}c_0^2\Big) + \int 3\chi\, \mathrm{d}\gamma{\wedge}\mathrm{d}\gamma \end{aligned}$$

where

$$\omega := \tfrac{1}{2}(\xi \mathrm{d}\xi' - \xi' \mathrm{d}\xi) + 3c_0(\gamma - \chi\alpha)$$

Reduction ansatz (bosonic)

- Include background three-flux

$$F = \mathrm{d}\alpha \;; \quad H = \mathrm{d}\chi{\scriptstyle\wedge} J + \mathrm{d}\beta + \frac{1}{2}\mathrm{Re}\big(b_0 \Omega^*\big)$$

$$G = \varphi \mathrm{vol}_4 + \frac{1}{2} c_0 J{\scriptstyle\wedge} J + J{\scriptstyle\wedge}(\mathrm{d}\gamma - \alpha \wedge \mathrm{d}\chi) - \frac{1}{2} D\xi{\scriptstyle\wedge}\mathrm{Im}\Omega - \frac{1}{2} D\xi'{\scriptstyle\wedge}\mathrm{Re}\Omega$$

where

$$D\xi := \mathrm{d}\xi + b_1 \alpha \;; \quad D\xi' := \mathrm{d}\xi' + b_2 \alpha \;; \quad b_0 = i b_1 + b_2$$

- Action

$$S_4 = \int \mathrm{d}^4 x \sqrt{g} \Big(R - 24 (\partial A)^2 - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{3}{2} e^{-4A-\phi} (\partial\chi)^2 - \tfrac{1}{2} e^{-6A+\phi/2} \left[(D\xi)^2 + (D\xi')^2 \right]$$
$$- \tfrac{1}{4} e^{3\phi/2+6A} \mathrm{d}\alpha^2 - \tfrac{3}{4} e^{\phi/2+2A} (\mathrm{d}\gamma - \alpha{\scriptstyle\wedge}\mathrm{d}\chi)^2 - \tfrac{1}{2} e^{\phi-12A} \left(\mathrm{d}b + \omega\right)^2$$
$$- \tfrac{1}{2} e^{-\phi/2-18A} (3c_0\chi - \Xi)^2 - \tfrac{1}{2} e^{-\phi-12A} |b_0|^2 - \tfrac{3}{2} e^{\phi/2-14A} c_0^2 \Big) + \int 3\chi \; \mathrm{d}\gamma{\scriptstyle\wedge}\mathrm{d}\gamma$$

where

$$\omega := \tfrac{1}{2}(\xi \mathrm{d}\xi' - \xi' \mathrm{d}\xi) + 3 c_0 (\gamma - \chi\alpha) + \Xi\alpha \;; \quad \Xi := b_2 \xi - b_1 \xi'$$

IIA consistent truncation with condensates

- The IIA (pseudo)action

$$S = S_b + \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{g}\Big\{2(\tilde{\Psi}_M\Gamma^{MNP}\nabla_N\Psi_P) + \frac{1}{2}e^{5\phi/4}m(\tilde{\Psi}_M\Gamma^{MN}\Psi_N)$$
$$- \frac{1}{2\cdot 2!}e^{3\phi/4}F_{M_1M_2}(\tilde{\Psi}^M\Gamma_{[M}\Gamma^{M_1M_2}\Gamma_{N]}\Gamma_{11}\Psi^N)$$
$$- \frac{1}{2\cdot 3!}e^{-\phi/2}H_{M_1\dots M_3}(\tilde{\Psi}^M\Gamma_{[M}\Gamma^{M_1\dots M_3}\Gamma_{N]}\Gamma_{11}\Psi^N)$$
$$+ \frac{1}{2\cdot 4!}e^{\phi/4}G_{M_1\dots M_4}(\tilde{\Psi}^M\Gamma_{[M}\Gamma^{M_1\dots M_4}\Gamma_{N]}\Psi^N) + L_{\Psi^4}\Big\}$$

where

$$S_b = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{g}\Big(- R + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2\cdot 2!}e^{3\phi/2}F^2$$
$$+ \frac{1}{2\cdot 3!}e^{-\phi}H^2 + \frac{1}{2\cdot 4!}e^{\phi/2}G^2 + \frac{1}{2}m^2e^{5\phi/2}\Big) + \mathrm{CS}$$

- Gives the correct bosonic eoms in the presence of gravitino condensates (parameters of the effective action)
- Fermionic eoms automatically satisfied

IIA consistent truncation with condensates

- Gravitino ansatz for 4d condensates

$$\Psi_m = 0 \; ; \quad \Psi_{\mu+} = \psi_{\mu+} \otimes \eta - \psi_{\mu-} \otimes \eta^c \; ; \quad \Psi_{\mu-} = \psi'_{\mu+} \otimes \eta^c - \psi'_{\mu-} \otimes \eta$$

- ALE zero modes (Euclidean signature)

$$\psi^{\mu}_{-} = \psi'^{\mu}_{-} = 0$$

- Set

$$\mathcal{A} := \left(\tilde{\psi}_{\mu+}\gamma^{\mu\nu}\psi'_{\nu+}\right) = -\left(\tilde{\psi}^{\mu}_{+}\psi'_{\mu+}\right)$$

$$\mathcal{B} := -\frac{3}{2}(\tilde{\psi}_{[\mu}\psi'_{\nu]})^2 + (\tilde{\psi}^{\mu}\gamma_{\rho\nu}\psi'_{\mu})(\tilde{\psi}^{\rho}\psi'^{\nu}) + 3(\tilde{\psi}_{[\mu_1}\gamma_{\mu_2\mu_3}\psi'_{\mu_4]})^2$$

where

$$L_{\Psi^4} = e^{-4A-4B}\mathcal{B}$$

- Rewrite the 10d (pseudo)action in terms of $\mathcal{A}$, $\mathcal{B}$

IIA consistent truncation with condensates

- Expected form of the 4d condensates

$$\mathcal{A} \sim \langle \tilde{\psi}_\mu \psi^\mu \rangle \propto M_\mathrm{P}\, e^{-S_0}$$

Hawking & Pope, Nucl.Phys.B (1978)
Konishi, Magnoli & Panagopoulos, Nucl.Phys.B (1988)
where

$$S_0 = \frac{\beta_1 \alpha'}{2\kappa^2} \int_{M_4} \mathrm{d}x^4 \sqrt{g} R_{\kappa\lambda\mu\nu} R^{\kappa\lambda\mu\nu}$$

is the 4d effective action arising from compactification of the 10d S_eff (including one-loop and eight-derivative corrections) evaluated at the instanton solution $M_4 \times Y$, and

$$\tau = \frac{1}{48\pi^2} \int \mathrm{d}x^4 \sqrt{g} R_{\kappa\lambda\mu\nu} R^{\kappa\lambda\mu\nu} \;\; ; \quad \beta_1 = \frac{1}{\pi^2 l_s^2} \int_Y \mathrm{d}^6 x \sqrt{g}\, R^2_{mnkl}$$

so that

$$\mathcal{A} \propto l_s^{-1} e^{-c\,(l_Y/l_s)^2} \;\; ; \quad \mathcal{B} \propto l_s^{-2} e^{-2c\,(l_Y/l_s)^2}$$

IIA consistent truncation with condensates

- Internal Einstein eom

$$0 = e^{-8A-2B}\nabla^{\mu}\left(e^{8A+2B}\partial_{\mu}A\right) + \ldots$$
$$+\frac{1}{4}\left(\varphi e^{\phi/4-4A-4B} - c_0 e^{\phi/4-4A}\right)\mathcal{A} - \frac{1}{8}e^{2A+2B}L_{\Psi^4}$$

- External Einstein eom

$$R^{(4)}_{\mu\nu} = \cdots - \frac{1}{2}g_{\mu\nu}e^{\phi/4-4A-4B}\varphi\mathcal{A}$$

- Dilaton eom

$$0 = e^{-10A-4B}\nabla^{\mu}\left(e^{8A+2B}\partial_{\mu}\phi\right) + \cdots + \tfrac{1}{4}(3c_0 e^{\phi/4+2A} + \varphi e^{\phi/4+2A-4B})\mathcal{A}$$

- F-form, H-form eoms

$$\mathrm{d}(e^{3\phi/2+6A}\star_4 \mathrm{d}\alpha) = \ldots + e^{\phi/4+4A-2B}\mathcal{A}\,\mathrm{d}\beta$$
$$\mathrm{d}\left(e^{-\phi+4A-2B}\star_4 \mathrm{d}\beta\right) = \ldots + e^{\phi/4+4A-2B}\mathcal{A}\,\mathrm{d}\alpha$$

- G-form eom unchanged except for the constraint

$$\varphi = e^{-\phi/2-18A}\left(\Xi - 3c_0\chi - e^{\phi/4+12A}\mathcal{A}\right)$$

IIA consistent truncation with condensates

- Action

$$S_4 = \int \mathrm{d}^4x\sqrt{g}\Big(R - 24(\partial A)^2 - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{3}{2}e^{-4A-\phi}(\partial\chi)^2 - \tfrac{1}{2}e^{-6A+\phi/2}\left[(D\xi)^2 + (D\xi')^2\right]$$
$$- \tfrac{1}{4}e^{3\phi/2+6A}\mathrm{d}\alpha^2 - \tfrac{3}{4}e^{\phi/2+2A}(\mathrm{d}\gamma - \alpha{\wedge}\mathrm{d}\chi)^2 - \tfrac{1}{2}e^{\phi-12A}(\mathrm{d}b + \tilde{\omega})^2 - V\Big) + \int 3\chi\ \mathrm{d}\gamma{\wedge}\mathrm{d}\gamma$$

and

$$V(\chi, \xi, \xi'\phi, A) = \tfrac{3}{2}c_0^2 e^{\phi/2-14A} + \tfrac{1}{2}|b_0|^2 e^{-\phi-12A} - 3c_0\mathcal{A}e^{\phi/4-4A} + e^{6A}\mathcal{B}$$
$$+ \tfrac{1}{2}\Big(\mathcal{A}e^{3A} + (3c_0\chi - \Xi)e^{-\phi/4-9A}\Big)^2$$

- The axionic scalar b and two out of χ, ξ, ξ' remain flat directions

Vacua

- Maximally-symmetric solutions are obtained for

$$\alpha = \gamma = 0$$

and

$$\vec{\nabla} V(\chi_0, \xi_0, \xi_0', \phi_0, A_0) = 0$$

- The Einstein equations give

$$R = 3c_0^2 e^{\phi_0/2 - 14A_0} + |b_0|^2 e^{-\phi_0 - 12A_0} - 3c_0 \mathcal{A} e^{\phi_0/4 - 4A_0}$$
$$+ (3c_0\chi_0 - \Xi_0)^2 e^{-\phi_0/2 - 18A_0} + (3c_0\chi_0 - \Xi_0)\mathcal{A} e^{-\phi_0/4 - 6A_0}$$

Vacua

- For $c_0 \neq 0,\ \mathcal{B} > 0$ this system admits de Sitter solutions parameterized by the condensates and $\phi_0,\ A_0$

$$\chi_0 = -\frac{1}{3c_0}\left(\mathcal{A} g_s^{1/4} e^{12A_0} - \Xi_0\right)$$
$$c_0 = \frac{1}{20}\, g_s^{-1/4} e^{10A_0}\left(7\mathcal{A} \pm \sqrt{49\mathcal{A}^2 + 80\mathcal{B}}\right)$$
$$|b_0|^2 = \frac{3}{400}\, g_s e^{18A_0}\left(40\mathcal{B} - 21\mathcal{A}^2 \mp 3\mathcal{A}\sqrt{49\mathcal{A}^2 + 80\mathcal{B}}\right)$$

where $g_s := e^{\phi_0}$ and we may absorb e^{A_0} in l_Y ; $\mp\mathcal{A} > 0$

- Taking into account the expected form of the condensates, the Einstein equation determines the scalar curvature to be

$$R_{\mathrm{dS}} = 3 g_s^{-1} |b_0|^2 \propto l_s^{-2} e^{-2c\,(l_Y/l_s)^2}$$

- We have verified numerically that the nonzero eigenvalues of the Hessian are positive at the solution

Vacua

- Four-flux quantization $\frac{1}{l_s^3}\int_{\mathcal{C}_A} G \in \mathbb{Z}$ implies

$$n_A \propto g_s^{-1/4}\left(\frac{l_Y}{l_s}\right)^4 e^{-c\,(l_Y/l_s)^2}\mathrm{vol}(\mathcal{C}_A)$$

  where $\mathrm{vol}(\mathcal{C}_A)$ is the volume of $\mathcal{C}_A$ in units of l_Y
  Can be solved for $\mathrm{vol}(\mathcal{C}_A)$ of order one by tuning $g_s \ll 1$

- Note that $g_s,\, l_Y$ can be tuned independently
- Given n_A this fixes the Kähler moduli in units of l_Y
  (so that the value modulus remains unconstrained)
- Higher-order flux corrections are controlled by $|g_s G| \sim g_s^{3/4}$
- Three-flux quantization $\frac{1}{l_s^2}\int_{\mathcal{C}_\alpha} H \in \mathbb{Z}$ fixes the complex structure

Conclusions

- We constructed a 4d consistent universal truncation of IIA on CY in the presence of background fluxes and gravitino condensates induced by (ALE) gravitational instantons
- The truncation admits de Sitter solutions (which are local minima of the scalar potential) in the perturbative regime, supported by the gravitino condensates
- The solutions rely on the quartic condensate being positive. It is crucial to check whether or not this condition holds! Need the form of the Dirac zero modes in the background of the second instanton in the ALE series

Conclusions

- The condensates are controlled by the l_Y/l_s ratio
- String loop corrections can be tuned to be negligible
- The l_Y/l_s ratio can be tuned in accordance with current cosmological data. This requires

$$l_{4d}^{-2} \sim R_{\rm dS} \propto l_s^{-2} e^{-2c\,(l_Y/l_s)^2}$$

  and can be achieved for $l_Y/l_s \sim 10,\ c \sim 1$, assuming

$$\frac{R_{\rm dS}}{M_{\rm P}^2} \sim \left(\frac{l_s}{l_{4d}}\right)^2 \sim 10^{-122}$$

- 4d higher-order corrections are controlled by l_s/l_{4d}, and can be tuned to be small
- $(l_Y/l_s)^2$ corrections at the two-derivative order are 1% or less

Conclusions

- Embed in 4d (gauged) supergravity
- Include other light modes — stability
- Fluxed instantons
- Calculate the condensates in string theory