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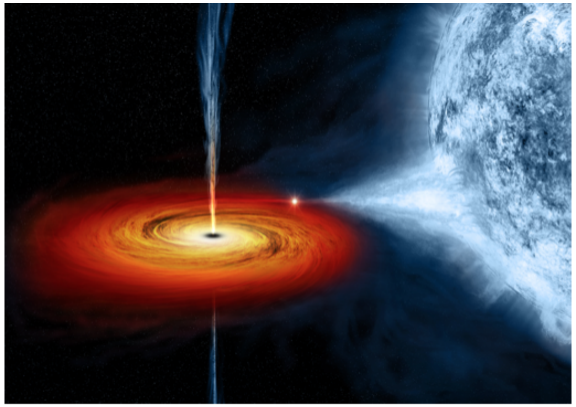
Supersymmetric spinning black holes in AdS_4 and their CFT duals



New developments in Strings and Gravity '19, Corfu, September 14, 2019

Based on work in collaboration with K. Hristov, S. Katmadas

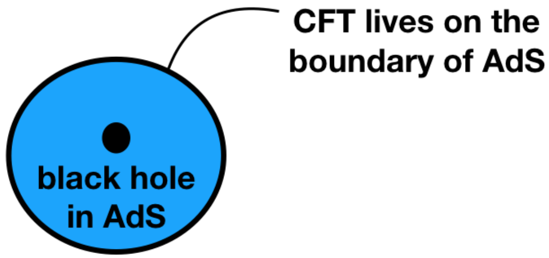
Supersymmetric (BPS) black holes

Black holes preserving susy provide a very valuable framework

- one can construct explicit solutions (most often analytical)
- String theory allows to identify the microscopic d.o.f. responsible for their entropy

Many studies in the context of asymptotically flat black holes have shown a remarkable agreement between macroscopic and microscopic picture

Quite recently, this was extended to asymptotically AdS black holes: entropy related to the **counting of states** in the dual CFT, living on the boundary.



Exact quantities (i.e. partition function, indices) computed via *supersymmetric localization* in the dual theory

Entropy matching for static AdS_4 black holes

Recent success: microstate counting for susy AdS_4 black holes [Cacciatori, Klemm '09]

- black holes are flows from AdS_4 to $AdS_2 \times \Sigma_g$ near horizon geometry
- magnetic gauge field cancels spin connection in the susy equations (topological twist)

Boundary is $S^1 \times \Sigma_g$: ABJM partition function on $S^1 \times \Sigma_g$ with magnetic fluxes s_i on Σ_g computed via susy localization, in the large N limit [Benini, Hristov, Zaffaroni '15], [Benini, Zaffaroni '16]

$$\log Z_{S^1 \times S^2} \approx -\frac{2\pi N^{3/2}}{3} \sqrt{2 m_1 m_2 m_3 m_4} \sum_{i=1}^{4} \frac{s_i}{m_i} \qquad \sum m_i = 2\pi$$

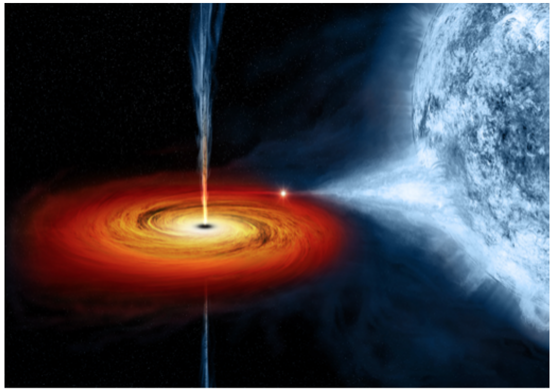
reproduces Bekenstein-Hawking entropy upon extremization on m_i.

Rotating BPS AdS black holes

In AdS_4 extremal black holes with angular momentum can preserve susy!

Not possible in 4D Minkowski

Extremal rotating AdS_4 black holes have Near Horizon geometry in the same class as the Near-Horizon Extremal Kerr, present in our universe.



Focus here on AdS_4 black holes. Other dimensions studied as well, esp AdS_5/CFT_4.

AdS_5 rotating BPS black holes [Kunduri, Lucietti, Reall, Gutowski '04] entropy was found to be exceeding the number of states counted by [Minwalla et al, '07]. Recently lots of progress (Supersymmetric Casimir Energy, Superconformal index) but some puzzles still remain.

Refinement by angular momentum

On the field theory perspective:

- Field theory on **product** manifolds $S^1 \times \Sigma_g$
  $\rightarrow$ Static (magnetic) black holes

- Refinement by **angular momentum**

$$ds^2 = d\theta^2 + f(\theta)(d\phi - \zeta dt)^2 + dt^2$$

  $\rightarrow$ Rotating magnetic BPS black holes

SUGRA: find these rotating solutions [Hristov, Katmadas, CT '18]

Rotating supersymmetric black holes

First studies in minimal 4d gauged supergravity. **Two classes** of solutions:

- electric: supersymmetric Kerr–Newman AdS black hole with no static limit [Kostelecky,Perry '92]. Charges satisfy

$$M = \frac{J}{l_{AdS}} + Q$$

- magnetically charged black holes can be static, however need to be supported by non-constant scalars. Minimal case produce naked singularities [Caldarelli,Klemm'98].

U(1) FI gauged supergravity + vector multiplets: only isolated examples [Cvetic et al,'05],[Klemm '11]. Lack of systematic! Solutions with both compact horizon and static limit possible.

Adding multiplets help in identifying the *entropy function*, to be matched with the CFT index

$\mathcal{N} = 2$ U(1) gauged sugra coupled to vector multiplets

Gravity multiplet coupled to n_V vector multiplets: bosonic fields are the graviton, $(n_V + 1)$ vector fields, n_V complex scalars z^i expressed in terms of holomorphic sections $X^I(z)$.

Gauging specified by Fayet-Iliopoulos parameters $G = (g^I, g_I)$
Scalar potential $V(z) \rightarrow$ can have supersymmetric AdS black holes

$$S = \int d^4x \sqrt{g} \Big[R + g_{i\bar{j}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} - I_{\Lambda\Sigma} F^\Lambda_{\mu\nu} F^{\mu\nu,\Sigma} + R_{\Lambda\Sigma} \epsilon^{\mu\nu\rho\sigma} F^\Lambda_{\mu\nu} F^\Sigma_{\rho\sigma} + V(z) \Big]$$

BPS equations: susy variations of fermions are zero

$$\delta_\epsilon \psi^I_\mu = 0 \qquad \delta_\epsilon \lambda^i_I = 0$$

Matter-coupled rotating black holes

Start from metric with timelike Killing vector

$$ds^2 = -e^{2U}(dt+\omega)^2 + e^{-2U} ds_3^2 \qquad F^I = d(\xi(dt+\omega)) + d\mathcal{A}$$

It is convenient to repackage the equations in terms of the symplectic variables

$$\mathcal{I} = e^{-U} \mathrm{Im} \begin{pmatrix} X^I \\ F_I \end{pmatrix} \qquad \mathcal{F}_{\mu\nu} = \begin{pmatrix} F^I_{\mu\nu} \\ G_{I,\mu\nu} \end{pmatrix}$$

Focus on models with

$$F = \frac{X^1 X^2 X^3}{X^0}$$

Rewrite BPS equations of [Meessen,Ortin '12] in terms of the symplectic sections $X^I, F_I = \partial F/\partial X^I$ and other invariants built from derivatives of a symplectically invariant quartic form I_4

$$I_4(\Gamma) = -(q_0 p^0 - p^i q_i)^2 + 4 q_0 q_1 q_2 q_3 + 4 p^0 p^1 p^2 p^3 + 4(p^1 p^2 q_1 q_2 + p^1 p^3 q_1 q_3 + p^2 p^3 q_2 q_3)$$

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$$F = \frac{X^1X^2X^3}{X^0}$$

Such that

$$\mathrm{Re}\begin{pmatrix} X^I \\ F_I \end{pmatrix} \sim I_4'\left(\mathrm{Im}\begin{pmatrix} X^I \\ F_I \end{pmatrix}\right) \qquad e^{2U} \sim \sqrt{I_4(\mathcal{R})}$$

BPS equations [Meessen,Ortin '12] for solutions with a timelike Killing vector

$$de^x - \langle \tilde{G}, \mathcal{I} \rangle \wedge e^x + \epsilon^{xyz} \langle \mathcal{A}, \tilde{G}^y \rangle \wedge e^z = 0$$

$$\star d\mathcal{I} + \langle \star\tilde{G}, \mathcal{I} \rangle \mathcal{I} - \frac{1}{4} I_4'(\mathcal{I}, \mathcal{I}, \star\tilde{G}) - \rho d\omega G + \mathcal{F} = 0$$

$$\star d\omega = \langle d\mathcal{I}, \mathcal{I} \rangle - \frac{1}{2} \langle \tilde{G}, I_4'(\mathcal{I}) \rangle$$

Interested in solutions that implement the topological twist. Take as 3d base

$$ds_3^2 = dr^2 + e^{2\psi(r)} ds_\Sigma^2$$

we get $\psi' = \langle G, \mathcal{I} \rangle$ and

$$\tilde{\omega}^{ab} = \epsilon^{ab} \langle G, \mathcal{A} \rangle \qquad \rightarrow \qquad \langle G, \Gamma \rangle = \int_\Sigma R = \kappa$$

Near horizon Solution and sections

Near horizon geometry:

$$ds^2 = -e^{2u}(r dt + \omega_0)^2 + e^{-2u}\frac{dr^2}{r^2} + e^{-2u}\left(\frac{d\theta^2}{\Delta(\theta)} + \nu^2\Delta(\theta) f_\kappa^2(\theta) d\phi^2\right)$$

with Δ, ω_0, u depending only on θ; $f_k(\theta)$ specifies the topology of the horizon

BPS equations give

$$d \star d\omega_0 = R^{(2)} \star \omega_0 \qquad \to \qquad \omega_0 = -\frac{j}{\nu}\Delta(\theta) f_\kappa^2(\theta) d\phi$$

Solution for $\kappa = 1$

$$d(e^{\psi}\mathcal{I}) = -j \sin\theta G \qquad \to \qquad e^{\psi}\mathcal{I} = H_0 + j\cos\theta G$$

$$\nu = \langle G, H_0 \rangle \qquad \Delta(\theta) = 1 + I_4(G) j^2 \sin^2\theta$$

Full Solution and sections

Full geometry:

$$ds^2 = -e^{2U}(dt+\omega)^2 + e^{-2U}dr^2 + e^{-2U+2\psi}\left(\frac{d\theta^2}{\Delta(\theta)} + \Delta(\theta) f_\kappa^2(\theta) d\phi^2\right)$$

with ω, U, ψ depending on r, θ; $f_k(\theta)$ specifies the topology of the horizon

BPS equations give

$$\omega = \left(\omega_\infty(\theta) - j e^{-\psi}\Delta(\theta)\right) f_\kappa^2(\theta) d\phi$$

Solution for $\kappa = 1$

$$e^{\psi}\mathcal{I} = H_0 + H_\infty r + j\cos\theta G$$

$$\omega_\infty = j I_4(G)^{1/4} \qquad H_\infty = \frac{1}{2} I_4(G)^{-3/4} I_4(G)'$$

Attractor and entropy

Attractor relating H_0 to charges Γ [Hristov, Katmadas, CT '18]

$$\Gamma = \frac{1}{4} I_4'(H_0, H_0, G) + \frac{1}{2} j^2 I_4'(G)$$

Conserved charges computed via Komar intergrals. i.e. angular momentum

$$J = \frac{1}{16\pi} \int_{S^2} dS^{\mu\nu} \nabla_\mu \xi_\nu$$

Allow to express entropy S_{BH} in function of charges, i.e. for T^3 model

$$S_{BH} = \pi \frac{l_{AdS}^2}{\sqrt{2}} \sqrt{\sqrt{(1+12p^1)(1+4p^1)^3 - 4J^2 l_{AdS}^{-4}} - (24(p^1)^2 + 12 g_1 p^1 + 1)}$$

J bounded from above. Reduces to static case for $J \to 0$

Holography

Found rotating attractors which extend to 1/4 BPS rotating black holes with boundary

$$ds^2 = r^2\Delta(\theta)\left[-\frac{dt^2}{l_{AdS}^2} + \frac{d\theta^2}{\Delta(\theta)^2} + \frac{\sin\theta^2}{\Delta(\theta)}\left(d\phi + \frac{j}{l_{AdS}^3}dt\right)^2\right]$$

where $l_{AdS}^2 = (I_4(G))^{-1/2}$. Squashing of Σ in parameterized by $\Delta(\theta)$.

Entropy to be reproduced by Large N twisted index with angular momentum refinement for ABJM [Benini, Zaffaroni '16]

- Difficult to compute, though some progress in [Closset, Kim, Willett '18]
- On gravity, provide an entropy function coming from on-shell action (need to find $T > 0$ solution and perform BPS limit)

Electric matter-coupled Kerr-Newman black hole

Start from general base

$$ds_3^2 = d\rho + e^{2\phi}(dx^2 + dy^2)$$

Before: $e^{2\phi} = \Phi(x)e^{2\psi(\rho)}$. Radial coordinate $\rho = r$.

Electric matter coupled Kerr-Newman solutions: choose

$$e^{2\phi} = Q(q)P(p) \qquad \rho = qp \qquad x = \alpha(q) + \beta(p)$$

such that

$$ds_3^2 = (q^2P(p) + p^2Q(q))\left(\frac{dp^2}{P(p)} + \frac{dq^2}{Q(q)}\right) + Q(q)P(p)dy^2$$

q radial variable, p, y are coordinates on the sphere. No twist, $g_I P^I = 0$

Electric matter-coupled Kerr-Newman black hole

Entropy obtained by extremizing an *entropy function* with respect to variables m^I conjugate to the electric charges, ω conjugate to J

$$\mathcal{S}(\omega, m^I) = -2\frac{F(m_I)}{\omega} + \sum_I m^I q_I + \omega J \qquad \sum_I 2 g_I m^I - \omega - 2\pi i = 0$$

$F(m^I)$ is the prepotential of the model.

This is the form conjectured by [Choi et al. '18] and tested by them on the only known solutions [Cvetic et al, '05].

Confirmed for full family of new solutions [Hristov, Katmadas, CT '19].

Comparison with the superconformal index

Quite recently [Cabo-Bizet et al '18] [Cassani, Papini '19] showed that the entropy function for rotating AdS black holes is the supergravity on-shell action in a particular extremal limit.

Extremal, BPS limit following a supersymmetric trajectory in the space of complexified solutions $\rightarrow$ complex chemical potentials

Electric case: entropy to be reproduced by the superconformal index.

- Superconformal index found to be $\sim O(1)$ [S. Kim, '09] when fugacities are real
- Should be instead $\sim N^{3/2}$ to match black hole entropy. There is $N^{3/2}$ scaling for complex fugacities [Choi, Hwang, Kim, '19] in Cardy limit. Large black hole entropy matches (**Exciting!**) but further checks needed.

Conclusions and perspectives

- Understand the thermodynamic origin of the entropy functions
  - Going beyond extremality and compute on-shell action
- Elucidate the extremization principle in terms of the attractor mechanism for rotating black holes
- Compute the twisted Witten index in presence of angular momentum refinement
  - useful because direct connection with static case

the end. Thank you!