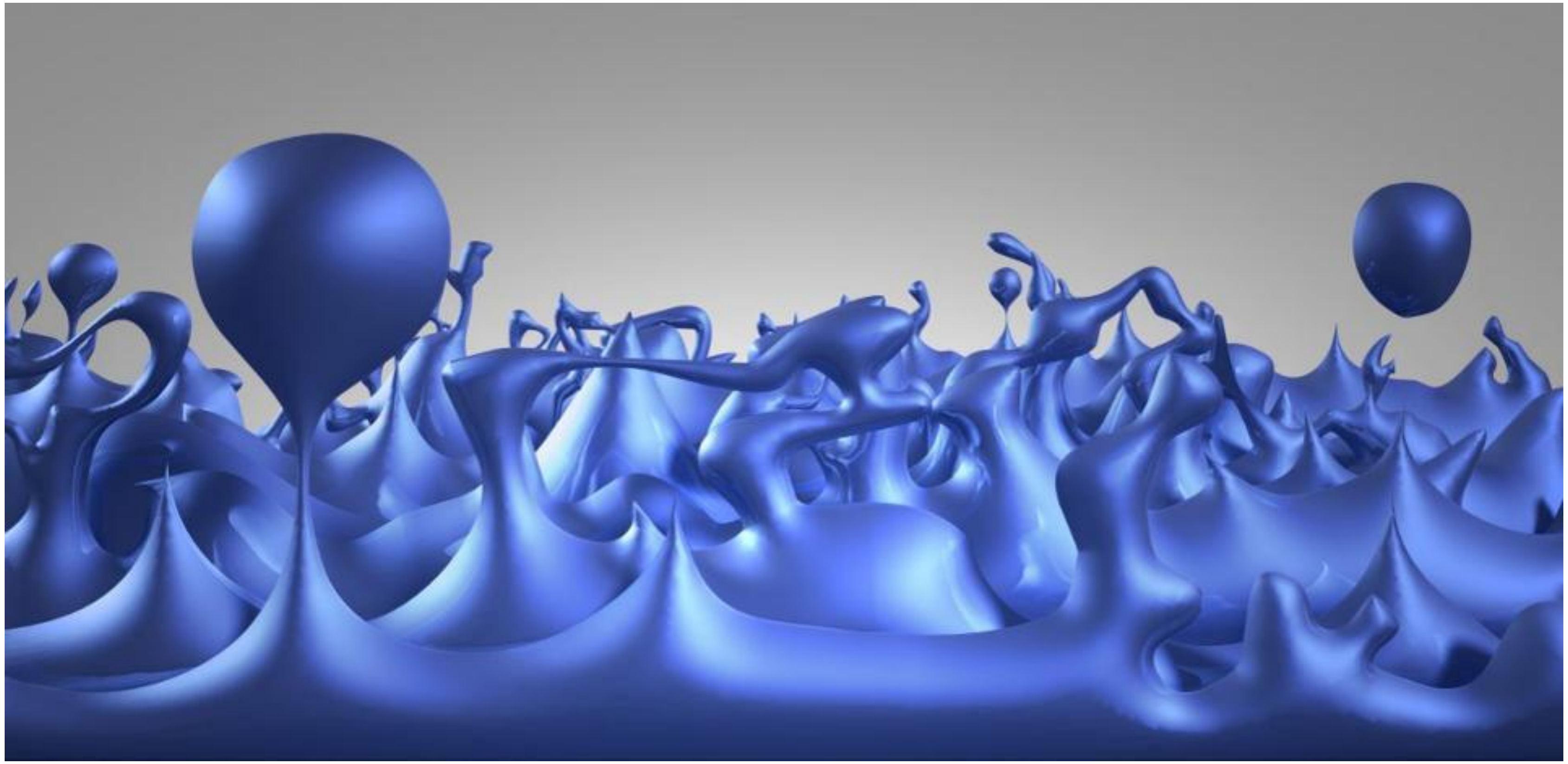
Instantons, Euclidean Wormholes and AdS/CFT

Thomas Van Riet – K.U.Leuven



Corfu, 2019

Partially based on

1812.05986

The holographic dual to supergravity instantons in $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_\mathrm{k}$

S. Katmadas[a], D. Ruggeri [b], M. Trigiante[b] and T. Van Riet[a] [1]

1811.12690

Euclidean axion wormholes have multiple negative modes

Thomas Hertog, Brecht Truijen, Thomas Van Riet

1. Supergravity instantons from axions
2. Holography
3. Euclidean Wormholes
4. Conclusions

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→ There exists a 'simple' path-integral argument [many papers].

→ RR axion is a theta angle on D3 brane $\chi\mathrm{Tr}[F \wedge F]$

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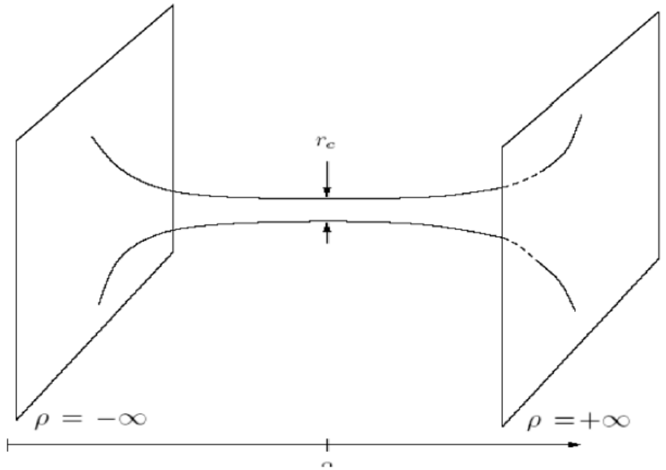
$$\boxed{\begin{aligned}
c<0\ &:\ e^{\frac{1}{2}\beta\phi} = \frac{Q}{\sqrt{-c}}|\sin(\tfrac{1}{2}\sqrt{-c}\,\beta\,h)|\,, \quad \chi = -\frac{2\sqrt{-c}}{\beta Q}\cot(\tfrac{1}{2}\sqrt{-c}\,\beta\,h) + c_0 \\
c=0\ &:\ e^{\frac{1}{2}\beta\phi} = \frac{1}{2}\beta\,Q|h|\,, \quad \chi = -\frac{4}{\beta^2 Q h} + c_0\,, \\
c>0\ &:\ e^{\frac{1}{2}\beta\phi} = \frac{Q}{\sqrt{c}}|\sinh(\tfrac{1}{2}\sqrt{c}\,\beta\,h)|\,, \quad \chi = -\frac{2\sqrt{c}}{\beta Q}\coth(\tfrac{1}{2}\sqrt{c}\,\beta\,h) + c_0\,.
\end{aligned}}$$

h is harmonic function.

What kind of geometries?

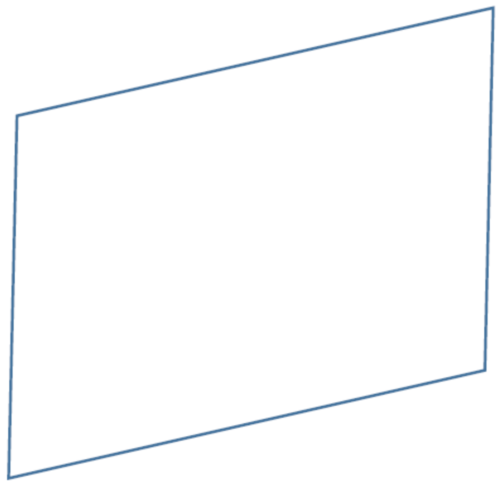
"Over-extremal" c < 0



- Regular geometry
- Regular scalars only if:

$$\boxed{\beta^2 < \frac{2(D-2)}{D-1}}$$

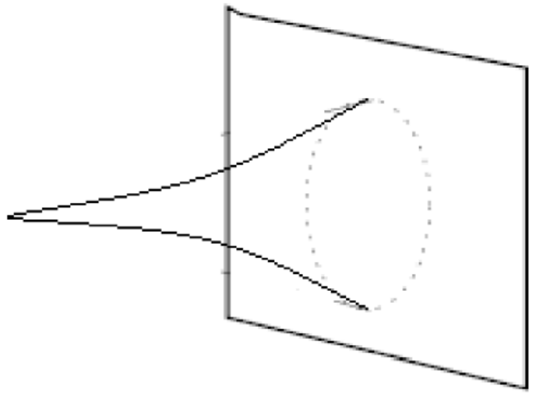
"Extremal" c = 0 :



- Regular geometry
- Singular scalars (standard ``Coulomb singularity")

= The D-instanton

"Under-extremal" c > 0 :



- Singular geometry
- Singular scalars (standard ``Coulomb singularity")

The on-shell actions are

$$S_{c=0} \sim |Q|\mathrm{e}^{-\frac{\beta}{2}\phi(\infty)} \Longrightarrow \text{BPS}$$

$$S_{c>0} \sim |Q|\mathrm{e}^{-\frac{\beta}{2}\phi(\infty}\sqrt{1+\frac{c}{Q^2}\mathrm{e}^{\beta\phi(\infty)}}$$

$$S_{c<0} \underbrace{\sim |Q|\left[\mathrm{e}^{-\frac{\beta}{2}\phi_L}\sqrt{1+\frac{c}{Q^2}\mathrm{e}^{\beta\phi_L}}\right.}_{\text{Half a wormhole}} \left. + \mathrm{e}^{-\frac{\beta}{2}\phi_R}\sqrt{1+\frac{c}{Q^2}\mathrm{e}^{\beta\phi_R}}\right]$$

Multi-field extension is surprisingly simple.

[For notational simplicity we fix D=5]

$$S = -\frac{1}{2\kappa_D^2} \int \sqrt{|g_D|}\left(R_D - \tfrac{1}{2}G_{ij}\partial\phi^i\partial\phi^j\right)$$

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- Scalars only dependent on radial variable.

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$$G_{ij}\dot\phi^i\dot\phi^j = c$$

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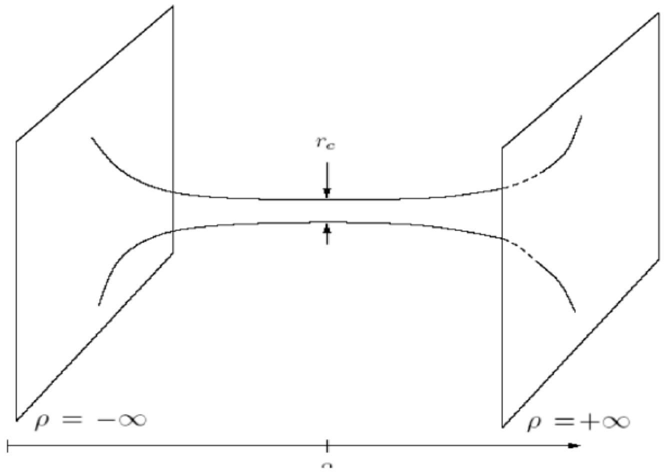
- The energy-momentum only sees the number c → same metrics as before.

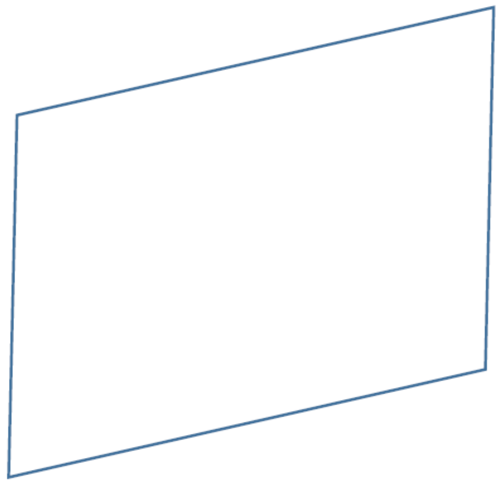
$$\frac{\dot{a}^2}{f^2} = \frac{c}{24}a^{-6} + 1$$

“Over-extremal” c < 0



``Time-like” geodesics

“Extremal” c = 0 :



``Light-like’’ geodesics

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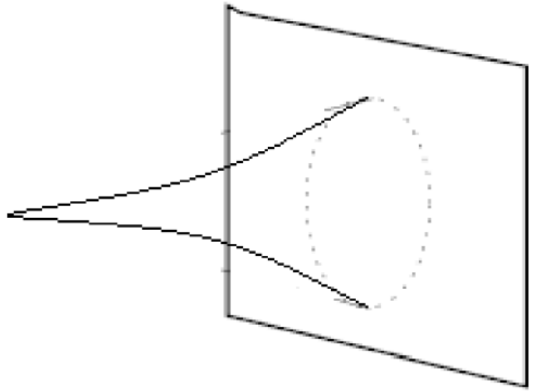

``Space-like” geodesics

$$\boxed{ds^2 = \left(1 + \frac{c}{2(D-1)(D-1)}\tau^{-2(D-2)}\right)^{-1} d\tau^2 + \tau^2 d\Omega^2}$$

Can we solve for scalars? → Often, in top-down models, the scalar manifold is a symmetric spaces. Geodesic problem is integrable and solvable ☺

Instanton solutions also exist in Euclidean AdS

$$\Lambda = -\frac{(D-1)(D-2)}{l^2}$$

$$S = -\frac{1}{2\kappa_D^2}\int\sqrt{|g_D|}\left(R_D - \tfrac{1}{2}G_{ij}\partial\phi^i\partial\phi^j - \Lambda\right)$$

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Scalars again trace out geodesic curves on pseudo-Riemannian manifold:

$$G_{ij}\dot{\phi}^i\dot{\phi}^j = c$$

The harmonic function h is affine parameter and feels the cosmological constant.

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- Moduli inside AdS are coupling constants for exactly marginal operators in the dual field theory: they label the family of CFT's = *conformal manifold.*

- Metric G_{ij} on moduli space corresponds to the `Zamolodchikov' metric g_{ij} defined by the two-point functions:

$$g_{ij}(\varphi) = x^{2\Delta} \langle O_i(x) O_j(0) \rangle_{S[\varphi]}$$

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Top down constructions of ?

$$S = -\frac{1}{2\kappa_D^2}\int\sqrt{|g_D|}\Big(R_D - \tfrac{1}{2}G_{ij}\partial\phi^i\partial\phi^j - \Lambda\Big)$$

In other words: ***what are AdS moduli?*** Question has been recently revived in the SUGRA literature ☺ [Triendl, Louis, Luest, Westphal, McAllister,….]

moduli space of $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_{\mathrm{k}}$: $$\frac{\mathrm{SU}(1,k)}{\mathrm{S}[\mathrm{U}(1)\times\mathrm{U}(\mathrm{k})]} \Longrightarrow$$ [Corrado, Gunaydin, Warner, Zagermann 2002, Louis, Triendl, Zagermann 2015,]

2k real scalars.

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- Dual theory is N=2 “necklace quiver CFT” [Kachru, Silverstein ‘98] and has k gauge nodes → hence k complex couplings (k theta-angles), which form the conformal manifold. For our story, this is the only thing we need:

$$\mathcal{L} \supset \sum_{\alpha=0}^{k-1}\left(-\frac{1}{4g_\alpha^2}\mathrm{Tr}[F_\alpha^2] - i\frac{\theta_\alpha}{32\pi^2}\mathrm{Tr}[F_\alpha \wedge F_\alpha]\right)$$

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- There are exactly k commuting shift symmetries in the AdS moduli space → Wickrotation is fixed [Hertog, Trigiante, VR 2017]

$$\frac{\mathrm{SU}(1,k)}{\mathrm{S}[\mathrm{U}(1)\times\mathrm{U}(\mathrm{k})]} \Longrightarrow \frac{\mathrm{SL}(k+1,\mathbb{R})}{\mathrm{GL}(k,\mathbb{R})}\,.$$

Metric can be rewritten using symmetric coset formalism into

$$G_{IJ}(\phi) = \frac{1}{2}\,\mathrm{Tr}(M^{-1}\partial_I M\,M^{-1}\partial_J M)\,,$$

and geodesic problem can be solved via exponential map:

$$\boxed{M(\phi(\tau)) \;=\; \exp(2\,Q\,\tau)} \qquad\qquad Q \in \mathfrak{sl}(k+1)$$

Explicit solutions in [Ruggeri, Trigiante, VR 2017]

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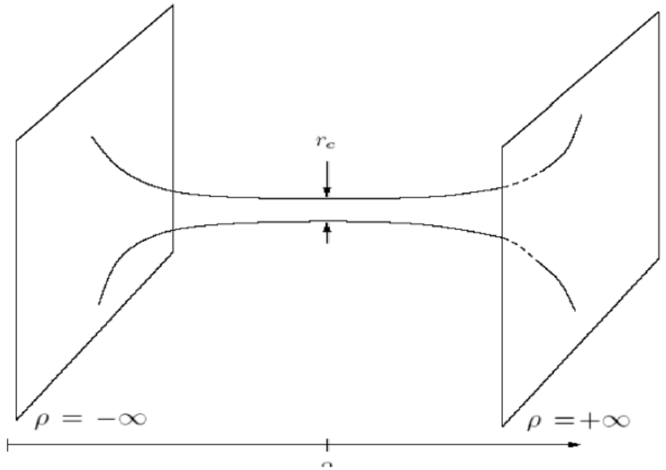

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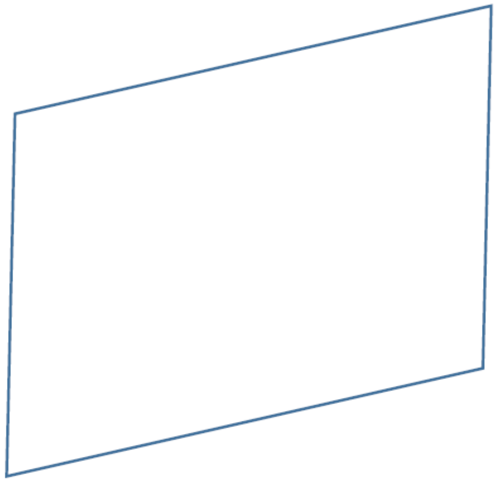

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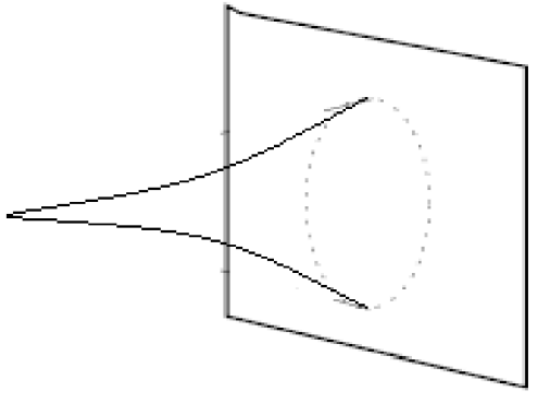

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All *extremal* solutions fall into two classes (nilpotent orbits): [Ruggeri, Trigiante, VR 2017]

- $Q^2 = 0$: These solutions are ½ BPS (8 supercharges).
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When k=1, i.e. for $AdS_5 \times S^5$ there is only the SUSY class. Holography well studied.

Holographic dual of D(-1) in AdS_5 x S5 are self-dual SUSY instantons in N=4 SYM [Banks/ Green, Dorey/ Khoze/ Mattis/ Vandoren/ Hollowood, Chu/ Ho/ Wu, Balasubramanian/ Kraus/ Lawrence/ Trivedi, … , 1998]

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Some simple correspondences:

- Charges (quantisation): instanton charge= Potryagin index.
- Moduli space (zero modes).
- **On-shell action: real and imaginary part.**
- **(Holographic) one-point functions.**

Main result from [Katmadas, Ruggeri, Trigiante, VR, 2018] (aside subtleties) are based on on-shell actions and one-point functions for

$$\mathrm{Tr}[F^2] \text{ and } \mathrm{Tr}[F \wedge F]$$

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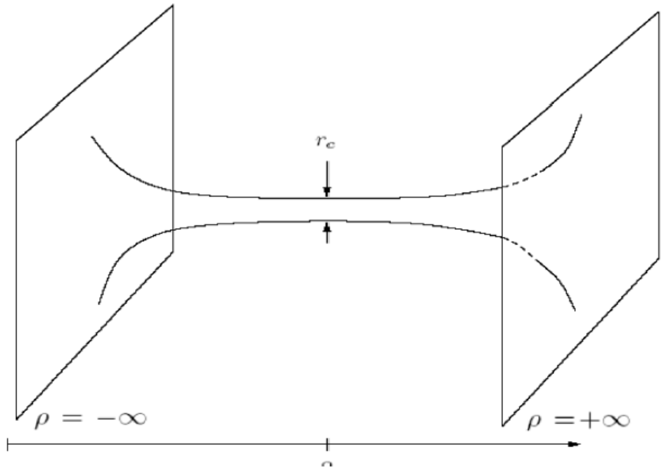
- **<u>non-SUSY solutions</u>** ($Q^3=0$): Some of them can be interpreted and match so called "quasi-instantons" [Imaanpur 2008]. *These are solutions which are self-dual in each separate gauge node, but orientations differ from node to node. Very simple way of SUSY-breaking!*

$$\mathrm{Tr}[F_\alpha^2] = \mathrm{sign}(N_\alpha)\mathrm{Tr}[F_\alpha \wedge F_\alpha]$$

Potryagin index = axion charge quantum

“Over-extremal” c < 0



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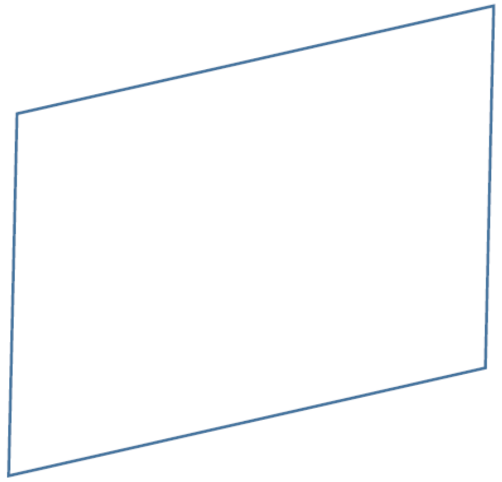

``Light-like” geodesics

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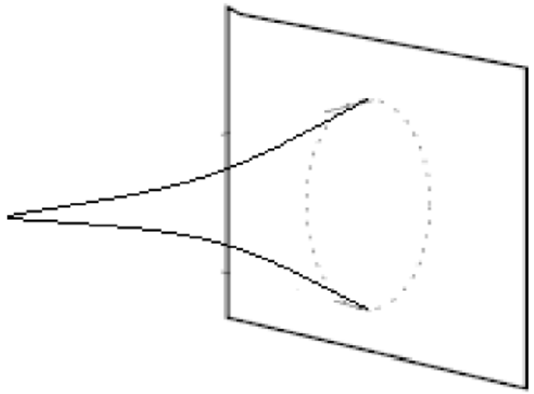

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- Suggestion for holographic dual from computing one point functions & action.

non-self dual YM instantons…

[Bergshoeff, Collinucci, Ploegh, Vandoren, VR 2005]

$$A_\mu^{\mathrm{SU(N)}} = \begin{pmatrix} A_\mu^{\mathrm{SU(2)}} & 0 & \cdots & 0 \\ 0 & A_\mu^{\mathrm{SU(2)}} & & 0 \\ \vdots & & \ddots & \\ 0 & & & \overline{A}_\mu^{\mathrm{SU(2)}} \end{pmatrix} .$$

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→ Essentially same story for k>1, just multiple gauge nodes. [Katmadas, Ruggeri, Trigiante, VR, 2018]

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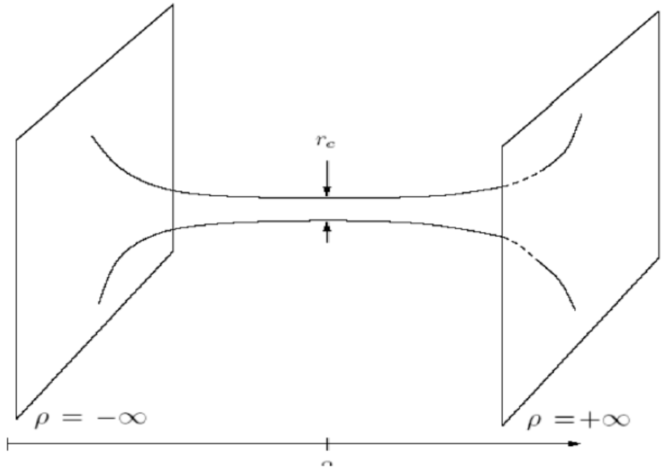

Wormhole originally found by [Giddings/ Strominger 1987]

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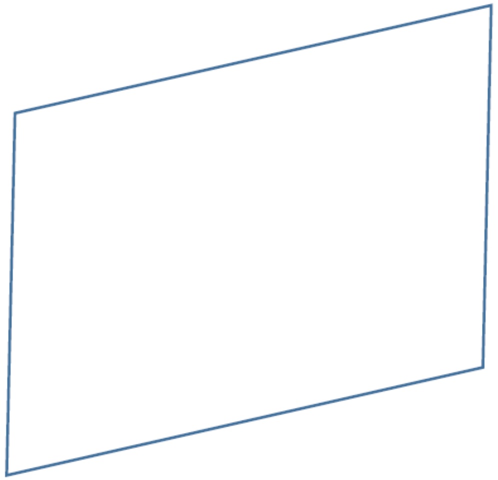

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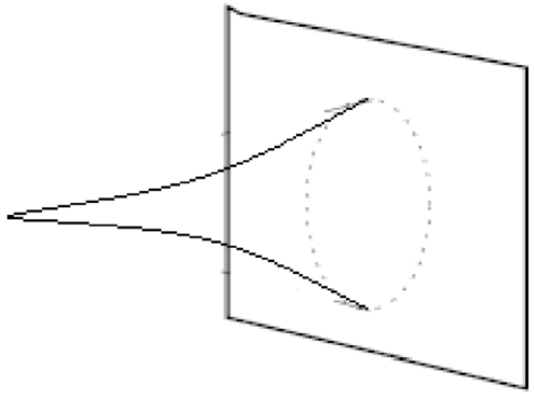
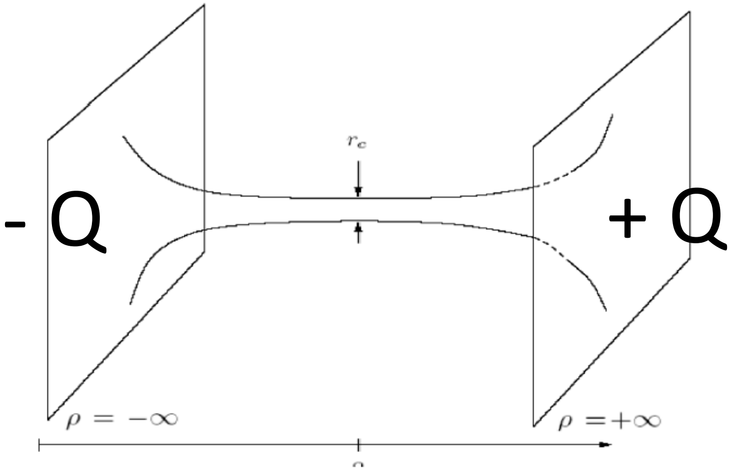


``Time-like” geodesics

Great review by Hebecker&Soler 2018.

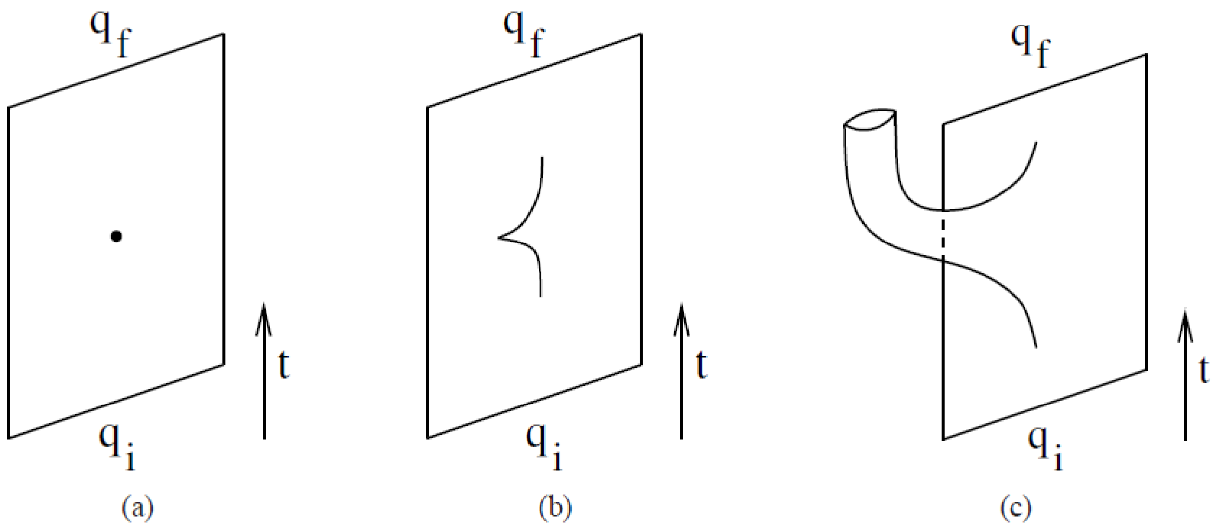
Full wormhole does not carry any axion charge (there is no field singularity). It is a ``charge conduit”



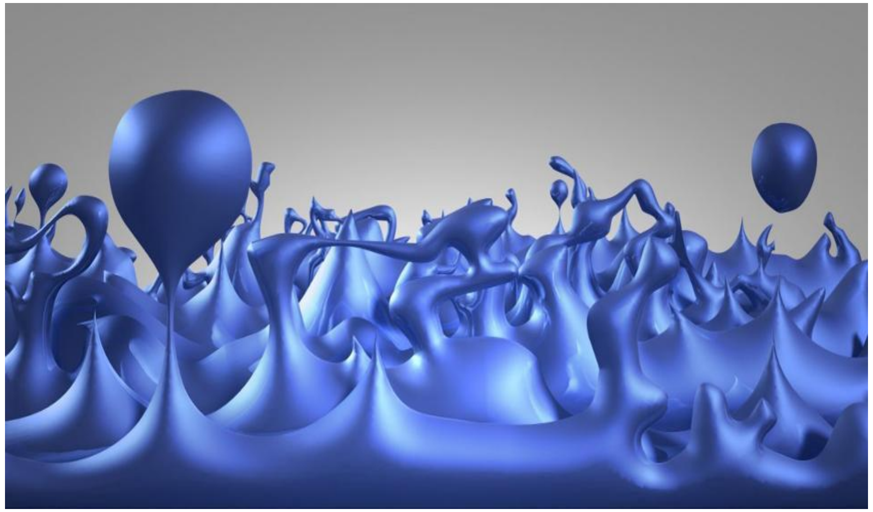
Hence ‘over-extremal’ might be a misnomer…Unless *cut in half* [consistently?]. Then

$$S_{c<0} \sim |Q|\left[\mathrm{e}^{-\frac{\beta}{2}\phi_L}\sqrt{1+\frac{c}{Q^2}\mathrm{e}^{\beta\phi_L}}\right]$$

Interpretation as tunneling instantons describing nucleation of baby universes → only if cut in half:



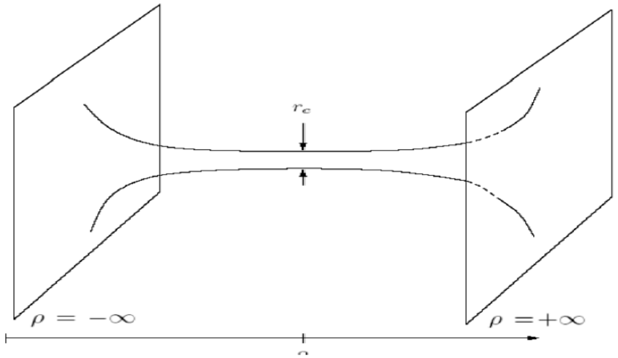
[Giddings/Strominger 1987,
Lavrelashvili/Tinyakov/Rubakov 1998,
Hawking 1987, …]

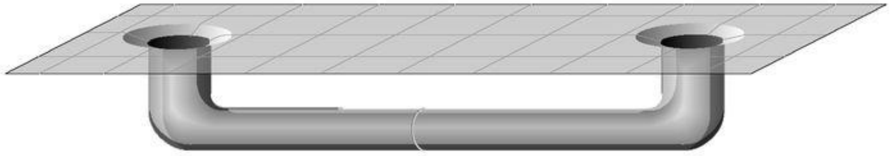


→Full wormhole describes emission *and* subsequent absorption o baby universe. Tunneling probability Planckian suppressed. (Planckian sized universes only)

An observer detects a violation of axion charge conservation. (Not surprising since it is global symmetry.) [Cf. Swampland ideas.]

If one glues the two boundaries into one space-time:





then wormholes represent a breakdown of (macroscopic) locality?: the effective action would include operators of the form

$$S_{WH} = -\frac{1}{2}\sum_{IJ}\int d^Dx\, d^Dy\, \mathcal{O}_I(x) C_{IJ} \mathcal{O}_J(y) \;;$$

[Coleman 1998]: Not really since

$$e^{-S_{WH}} = \int d\alpha_I\, e^{-\frac{1}{2}\alpha_I (C^{-1})_{IJ}\alpha_J} e^{-\int d^Dx \sum_I \alpha_I \mathcal{O}_I(x)} \;.$$

- Still contrived, no support from AdS/CFT [Arkani-Hamed/ Orgera/ Polchinski 2007, Maldacena/ Maoz 2004]

  → Dual field theory has no sign of Coleman's α parameters.

  → Consider axion wormhole connecting the same boundary at far separated Euclidean time distances T. In the CFT one can show (if vacuum is unique and gapped):

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \langle 0|\mathcal{O}_1|0\rangle \langle 0|\mathcal{O}_2|0\rangle + O(e^{-ET/2})$$

  Not reproduced by bulk computation if bulk is described by α parameters .

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \int d\alpha\, e^{-\frac{1}{2}\alpha_I (C^{-1})_{IJ}\alpha_J} \langle 0|\mathcal{O}_1|0\rangle_\alpha \langle 0|\mathcal{O}_2|0\rangle_\alpha + O(e^{-E_\alpha T/2})$$

  → See also recent work of [Betzios, Kiritsis,Papadoulaki 2019] !

- Another paradox: holographic one-point function give violation of positivity [Katmadas, Ruggeri, Trigiante, VR, 2018]:

$$|\mathrm{Tr}[F_\alpha^2]| < |\mathrm{Tr}[F_\alpha \wedge F_\alpha]|\,.$$

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- Computations did not use the right gauge-invariant variables. Interpretation as path integral for axion-charge transitions is 100% crucial.

Axion momentum eigenstate

$$K \equiv \langle \Pi_F | \exp(-HT) | \Pi_I \rangle$$

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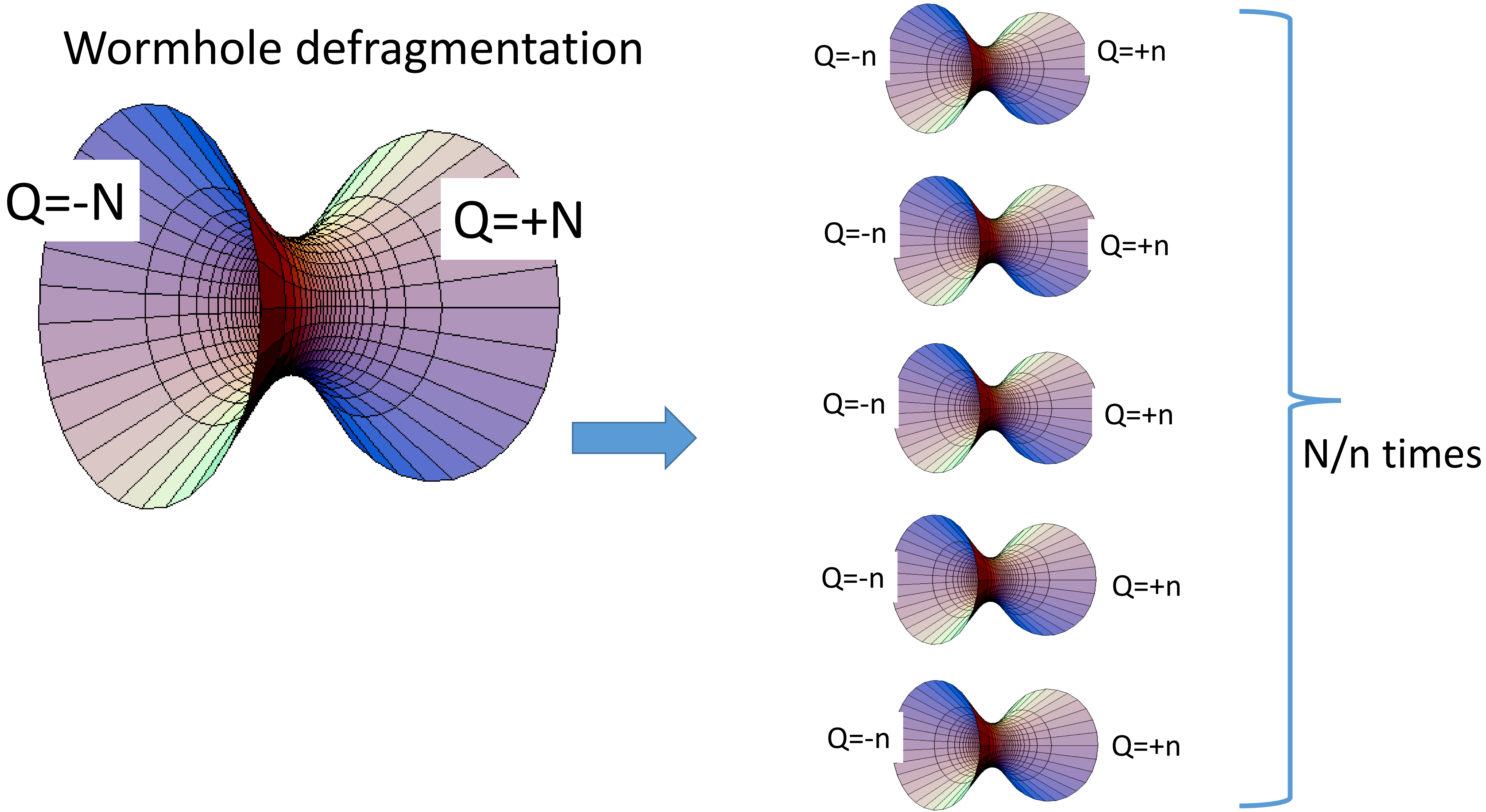
- Taking this into account gives well-behaved quadratic action. *No conformal factor problem (no Hawking-Perry rotation). S-wave mode is not dynamical*. All modes with angular momentum >2 have quadratic actions that are not strickly positive. Renormalisable fluctuations near wormhole throat lower the action! Interpretation:

Wormhole defragmentation



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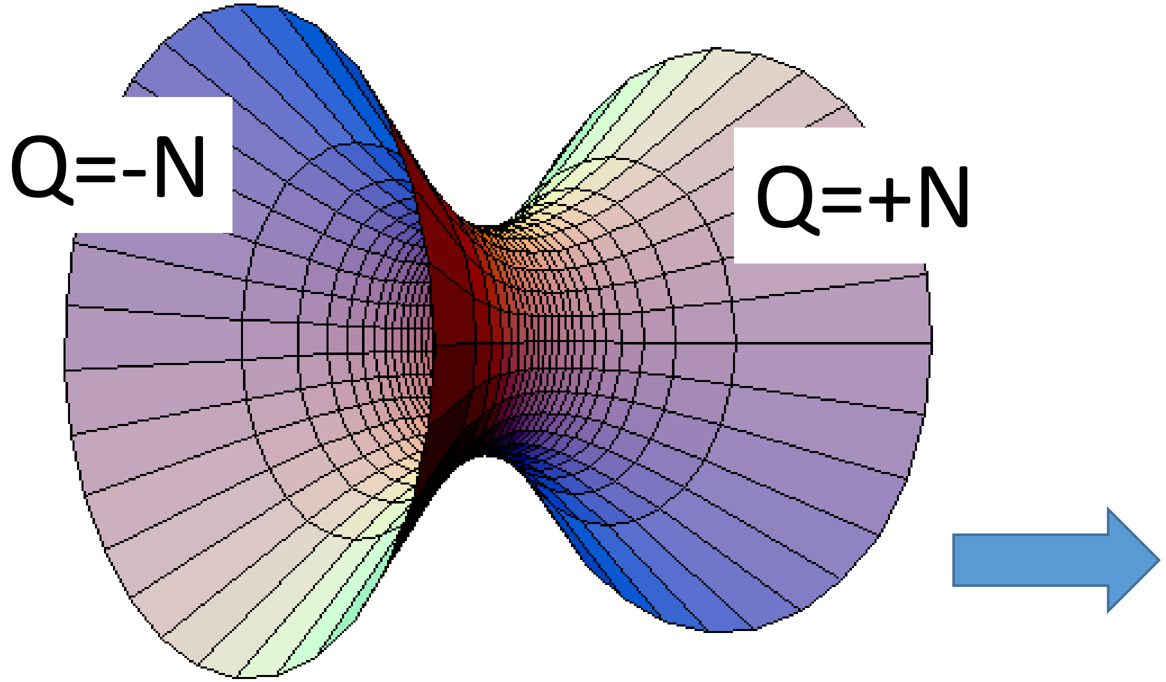

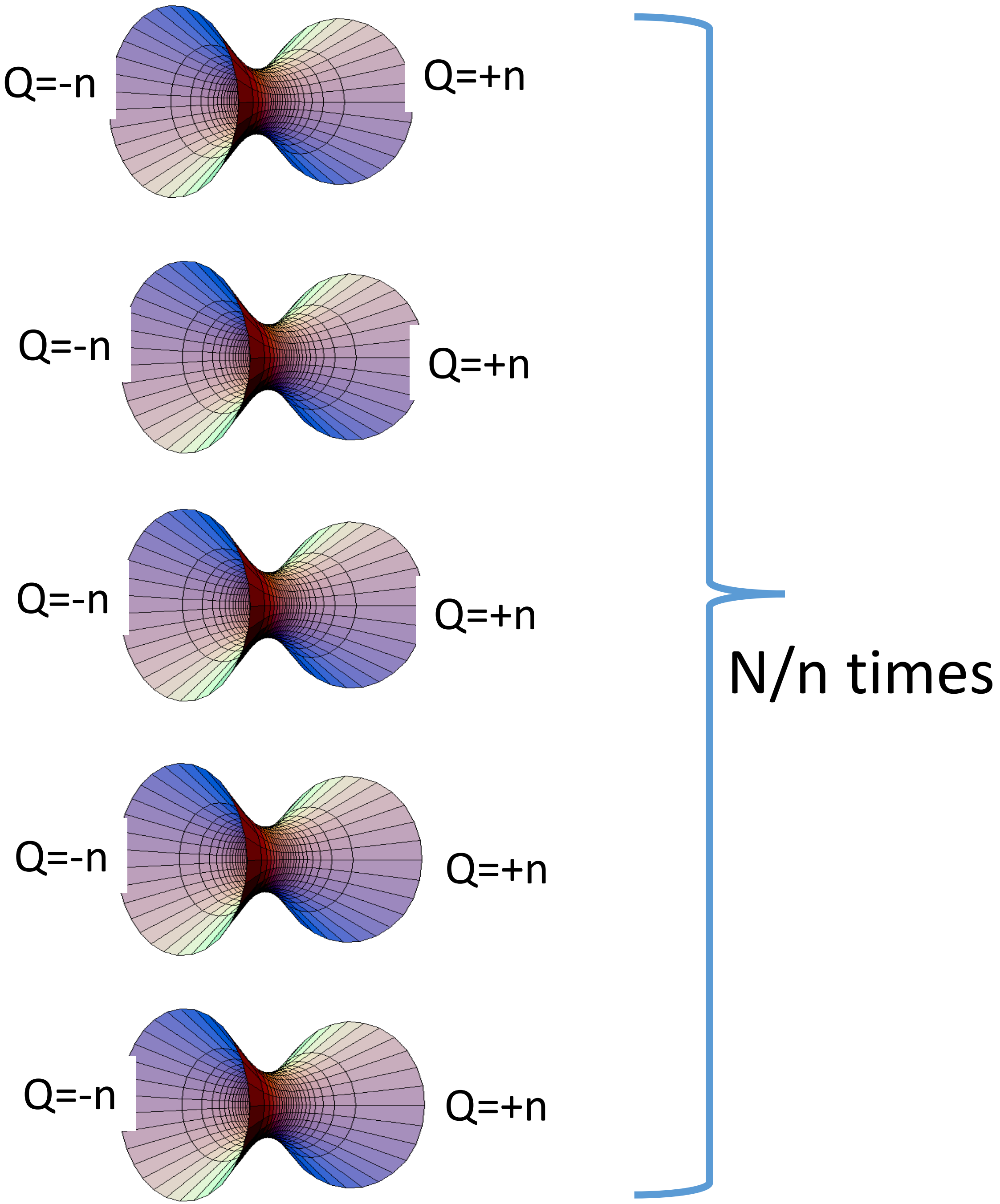

- Completely analogues to over-extremal electric objects. They defragment into microscopic constituents….
- Microscopic “wormholes” cannot be argued to not contribute.

1. Supergravity instantons from axions
2. Holography
3. Euclidean Wormholes
4. Conclusions

Holography suggest that some *geodesic curves on the conformal manifold correspond to instantons of the CFT*.

Orbifolds of AdS5 x S5 are a great testing ground.

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- Match between SUSY grav. instantons and N=2 quiver instantons survives simple checks.

- Even some non-SUSY “extremal”instantons seem to have dual description (“quasi inst).

- The wormhole case is less clear. But we have found clear evidence that they are “spurious” solutions and do not contribute to the path integral. This solves long standing puzzles.

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Future?

- Geodesics on moduli space of AdS3 x S3 x T^4 (K3) ?
- Link with recent work of Shenker et al on JT gravity and "sum over wormholes"?

EXTRA

[Corrado, Gunaydin, Warner, Zagermann 2002, Louis, Triendl, Zagermann 2015,]

moduli space of $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_\mathrm{k}$ $\quad \dfrac{\mathrm{SU}(1,k)}{\mathrm{S}[\mathrm{U}(1)\times\mathrm{U(k)}]} \Longrightarrow \dfrac{\mathrm{SL}(k+1,\mathbb{R})}{\mathrm{GL}(k,\mathbb{R})}$.

Moduli space metric: $\quad \mathrm{d}s^2 = 4\mathrm{d}U^2 + e^{-4U}\mathcal{N}^2 + 2e^{-2U}\sum_{i=1}^{k-1}[(\mathrm{d}\zeta^i)^2 + (\mathrm{d}\tilde{\zeta}_i)^2]$

$$\mathcal{N} \equiv da + \mathcal{Z}^M \mathbb{C}_{MN} d\mathcal{Z}^N \qquad \mathcal{Z}^M \equiv (\zeta^i,\, \tilde{\zeta}_i)$$

2k real scalars: $U, a, \zeta^i, \tilde{\zeta}_i$ where $i = 1 \ldots k-1.$

- Dual gauge theory has k gauge nodes → k complex couplings (k theta-angles).
- There are exactly k commuting shift symmetries → Wickrotation: [Hertog, Trigiante, VR 2017]

$$\mathrm{d}s^2 = 4\mathrm{d}U^2 - e^{-4U}\mathcal{N}^2 + 2e^{-2U}\sum_{i=1}^{k-1}[(\mathrm{d}\zeta^i)^2 - (\mathrm{d}\tilde{\zeta}_i)^2]$$

AdS/CFT paradoxes

[Maldacena/ Maoz 2004]:

AdS/CFT=sum over all geometries with fixed boundary conditions is the same as partition function of CFT living on the boundary. What if disconnected boundaries are connected through bulk? *Correlation functions in the CFT's factorize between the different boundaries, but not via AdS/CFT computation*?

→ Is there some coupling between the CFT's?

→ Maybe after summing over all geometries correctly the factorization happens?

→ <u>*Or are all wormholes ``unstable" and do not contribute?*</u> Some counter-examples found, not but of axion wormhole type…(no regular axion solution found)

*<u>Interpretation & meaning of instantons depends on **stability** [Coleman]:</u>*

- Perform "Gaussian approximation" of path integral around saddle point:

$$Z = e^{-S[\Phi_0]} \int \mathrm{D}\phi \, e^{-\delta^2 S[\Phi_0,\phi] + \mathcal{O}(\phi^3)} \qquad \delta^2 S = \tfrac{1}{2} \int \phi \hat{\mathrm{M}} \phi$$

- Solve eigenvalue problem: $\dfrac{1}{X}\hat{\mathrm{M}}\phi_n = \lambda_n \phi_n \, , \quad \displaystyle\int X \, \phi_n \phi_m = \delta_{nm}$

- To find: $Z \sim e^{-S[\Phi_0]} \displaystyle\int \Pi_n \mathrm{d}z_n \, e^{-\frac{1}{2}\sum_n \lambda_n z_n^2} \sim \dfrac{e^{-S[\Phi_0]}}{\sqrt{\Pi_n \lambda_n}} \, .$