Vacuum stability in open string models with broken supersymmetry

Thibaut Coudarchet[1]

Work in progress in collaboration with S. Abel[2], E. Dudas[1] and H. Partouche[1]

Humboldt Kolleg Frontiers in Physics, Corfu, September 17, 2019

[1]Centre de physique théorique, École Polytechnique and CNRS

[2] Institute for Particle Physics Phenomenology, Durham University

Introduction

Introduction

- Phenomenologically, supersymmetry (SUSY) must be broken
- Generically, classical flat space is destabilized

$\rightarrow \mathcal{V}_{\text{1-loop}} \sim M_{\text{s}}^d$ if hard breaking

$\rightarrow$ If spontaneous breaking at scale M in classically flat space

$\downarrow$

No-scale model [Cremmer, Ferrara, Kounnas, Nanopoulos,'83]

$\mathcal{V}_{\text{tree}}$ is independent of M

$\mathcal{V}_{\text{1-loop}} \sim M^d$

Unless in specific cases, **still too high**

Introduction

- Phenomenologically, supersymmetry (SUSY) must be broken
- Generically, classical flat space is destabilized

$\to \mathcal{V}_{\text{1-loop}} \sim M_s^d$ if hard breaking

$\to$ If spontaneous breaking at scale M in classically flat space

$$\downarrow$$

No-scale model [Cremmer, Ferrara, Kounnas, Nanopoulos,'83]

$\mathcal{V}_{\text{tree}}$ is independent of M

$\mathcal{V}_{\text{1-loop}} \sim M^d$

Unless in specific cases, **still too high**

Introduction

- Phenomenologically, supersymmetry (SUSY) must be broken
- Generically, classical flat space is destabilized

$\rightarrow \mathcal{V}_{\text{1-loop}} \sim M_{\text{s}}^d$ if hard breaking

$\rightarrow$ If spontaneous breaking at scale M in classically flat space

$$\downarrow$$

No-scale model [Cremmer, Ferrara, Kounnas, Nanopoulos,'83]

$\mathcal{V}_{\text{tree}}$ is independent of M

$\mathcal{V}_{\text{1-loop}} \sim M^d$

Unless in specific cases, **still too high**

Introduction

- Phenomenologically, supersymmetry (SUSY) must be broken
- Generically, classical flat space is destabilized

$\to \mathcal{V}_{\text{1-loop}} \sim M_{\text{s}}^d$ if hard breaking

$\to$ If spontaneous breaking at scale M in classically flat space

$$\downarrow$$

No-scale model [Cremmer, Ferrara, Kounnas, Nanopoulos,'83]

$\mathcal{V}_{\text{tree}}$ is independent of M

$\mathcal{V}_{\text{1-loop}} \sim M^d$

Unless in specific cases, **still too high**

Purpose of the study

- We work in type I strings compactified on $T^2 \times T^4/\mathbb{Z}_2$ with spontaneously broken supersymmetry ($\mathcal{N} = 2 \to 0$)
- Breaking induced by a stringy Scherk-Schwarz mechanism
  $\to$ SUSY breaking scale $M = \frac{1}{2R}$
- At one-loop, we want to have a **positive potential** and try to lower its order of magnitude
- We address the question of **stability**

  $\to$ Tadpoles

  $\to$ Tachyonic moduli
- All this is a follow-up of a study done in d-dimensions with $\mathcal{N} = 4 \to 0$ [Abel, Dudas, Lewis, Partouche,'18]

Purpose of the study

- We work in type I strings compactified on $T^2 \times T^4/\mathbb{Z}_2$ with spontaneously broken supersymmetry ($\mathcal{N} = 2 \to 0$)
- Breaking induced by a stringy Scherk-Schwarz mechanism $\to$ SUSY breaking scale $M = \frac{1}{2R}$
- At one-loop, we want to have a **positive potential** and try to lower its order of magnitude
- We address the question of **stability**

  $\to$ Tadpoles

  $\to$ Tachyonic moduli
- All this is a follow-up of a study done in d-dimensions with $\mathcal{N} = 4 \to 0$ [Abel, Dudas, Lewis, Partouche,'18]

Purpose of the study

- We work in type I strings compactified on $T^2 \times T^4/\mathbb{Z}_2$ with spontaneously broken supersymmetry ($\mathcal{N} = 2 \to 0$)
- Breaking induced by a stringy Scherk-Schwarz mechanism
  $\to$ SUSY breaking scale $M = \frac{1}{2R}$
- At one-loop, we want to have a **positive potential** and try to lower its order of magnitude
- We address the question of **stability**

  $\to$ Tadpoles

  $\to$ Tachyonic moduli
- All this is a follow-up of a study done in d-dimensions with $\mathcal{N} = 4 \to 0$ [Abel, Dudas, Lewis, Partouche,'18]

Purpose of the study

- We work in type I strings compactified on $T^2 \times T^4/\mathbb{Z}_2$ with spontaneously broken supersymmetry ($\mathcal{N} = 2 \to 0$)
- Breaking induced by a stringy Scherk-Schwarz mechanism
  $\rightarrow$ SUSY breaking scale $M = \frac{1}{2R}$
- At one-loop, we want to have a **positive potential** and try to lower its order of magnitude
- We address the question of **stability**

  $\rightarrow$ Tadpoles

  $\rightarrow$ Tachyonic moduli
- All this is a follow-up of a study done in d-dimensions with $\mathcal{N} = 4 \to 0$ [Abel, Dudas, Lewis, Partouche,'18]

Review of the $\mathcal{N} = 4$ case

The potential is given by $\mathcal{V}_{\text{1-loop}} = -\frac{M_{\mathsf{s}}^d}{2(2\pi)^d}\left(\mathcal{T} + \mathcal{K} + \mathcal{A} + \mathcal{M}\right)$

$$\boxed{\mathcal{V}_{\text{1-loop}} \propto \int \mathrm{d}\tau_2 \, \mathrm{Str}\, e^{-\pi\tau_2 m^2}}$$

$\rightarrow$ The lightest states produces the dominant contribution

$\Rightarrow$ Up to exponentially suppressed terms, **if there is no mass scale lower than the SUSY breaking scale** $M = \frac{1}{2R}$

$$\boxed{\mathcal{V}_{\text{1-loop}} = (n_{\mathsf{F}} - n_{\mathsf{B}})\xi M^d + \mathcal{O}\left(\left(cM_{\mathsf{s}}M\right)^{\frac{d}{2}} e^{-cM_{\mathsf{s}}/M}\right)}$$

with $\xi > 0$ and cM_{s} a large scale. n_{F} and n_{B} count the fermionic and bosonic massless degrees of freedom

Review of the $\mathcal{N} = 4$ case

The potential is given by $\mathcal{V}_{\text{1-loop}} = -\frac{M_{\mathsf{s}}^d}{2(2\pi)^d}\left(\mathcal{T} + \mathcal{K} + \mathcal{A} + \mathcal{M}\right)$

$$\boxed{\mathcal{V}_{\text{1-loop}} \propto \int \mathrm{d}\tau_2 \,\mathrm{Str}\, e^{-\pi\tau_2 m^2}}$$

$\rightarrow$ The lightest states produces the dominant contribution

$\Rightarrow$ Up to exponentially suppressed terms, **if there is no mass scale lower than the SUSY breaking scale $M = \frac{1}{2R}$**

$$\boxed{\mathcal{V}_{\text{1-loop}} = (n_{\mathsf{F}} - n_{\mathsf{B}})\xi M^d + \mathcal{O}\left((cM_{\mathsf{s}}M)^{\frac{d}{2}} e^{-cM_{\mathsf{s}}/M}\right)}$$

with $\xi > 0$ and cM_{s} a large scale. n_{F} and n_{B} count the fermionic and bosonic massless degrees of freedom

Review of the $\mathcal{N} = 4$ case

The potential is given by $\mathcal{V}_{\text{1-loop}} = -\frac{M_{\mathsf{s}}^d}{2(2\pi)^d}\left(\mathcal{T} + \mathcal{K} + \mathcal{A} + \mathcal{M}\right)$

$$\boxed{\mathcal{V}_{\text{1-loop}} \propto \int \mathrm{d}\tau_2 \, \mathrm{Str}\, e^{-\pi\tau_2 m^2}}$$

$\rightarrow$ The lightest states produces the dominant contribution

$\Rightarrow$ Up to exponentially suppressed terms, **if there is no mass scale lower than the SUSY breaking scale** $M = \frac{1}{2R}$

$$\boxed{\mathcal{V}_{\text{1-loop}} = (n_{\mathsf{F}} - n_{\mathsf{B}})\xi M^d + \mathcal{O}\left((cM_{\mathsf{s}}M)^{\frac{d}{2}} e^{-cM_{\mathsf{s}}/M}\right)}$$

with $\xi > 0$ and cM_{s} a large scale. n_{F} and n_{B} count the fermionic and bosonic massless degrees of freedom

Review of the $\mathcal{N} = 4$ case

Now introduce Wilson lines (WL) a_r^I along the I-th compactified circle for the r-th Cartan

If the WL do not introduce a mass scale lower than M, then the potential reads [Kounnas, Partouche,'16][Coudarchet, Partouche,'18]

$$\boxed{\mathcal{V}_{\text{1-loop}} = (n_{\mathsf{F}} - n_{\mathsf{B}})\xi M^d + \#(T_{\mathcal{R}_{\mathsf{B}}} - T_{\mathcal{R}_{\mathsf{F}}})(a_r^I)^2 + \cdots}$$

with $\# > 0$ and $T_{\mathcal{R}_{\mathsf{B}}}$ and $T_{\mathcal{R}_{\mathsf{F}}}$ are the Dynkin indices of the representations $\mathcal{R}_{\mathsf{B}}$ and $\mathcal{R}_{\mathsf{F}}$ in which the bosons and fermions live

- no mass scale below M ensures no linear term $\Rightarrow$ no tadpole
- **"life is not easy": the more positive the potential is, the more tachyonic it tends to be and conversely**

Review of the $\mathcal{N} = 4$ case

Now introduce Wilson lines (WL) a_r^I along the I-th compactified circle for the r-th Cartan

If the WL do not introduce a mass scale lower than M, then the potential reads [Kounnas, Partouche,'16][Coudarchet, Partouche,'18]

$$\boxed{\mathcal{V}_{\text{1-loop}} = (n_{\mathsf{F}} - n_{\mathsf{B}})\xi M^d + \#(T_{\mathcal{R}_{\mathsf{B}}} - T_{\mathcal{R}_{\mathsf{F}}})(a_r^I)^2 + \cdots}$$

with $\# > 0$ and $T_{\mathcal{R}_{\mathsf{B}}}$ and $T_{\mathcal{R}_{\mathsf{F}}}$ are the Dynkin indices of the representations $\mathcal{R}_{\mathsf{B}}$ and $\mathcal{R}_{\mathsf{F}}$ in which the bosons and fermions live

- no mass scale below M ensures no linear term $\Rightarrow$ no tadpole
- **"life is not easy": the more positive the potential is, the more tachyonic it tends to be and conversely**

Review of the $\mathcal{N} = 4$ case

Now introduce Wilson lines (WL) a_r^I along the I-th compactified circle for the r-th Cartan

If the WL do not introduce a mass scale lower than M, then the potential reads [Kounnas, Partouche,'16][Coudarchet, Partouche,'18]

$$\boxed{\mathcal{V}_{\text{1-loop}} = (n_{\mathsf{F}} - n_{\mathsf{B}})\xi M^d + \#(T_{\mathcal{R}_{\mathsf{B}}} - T_{\mathcal{R}_{\mathsf{F}}})(a_r^I)^2 + \cdots}$$

with $\# > 0$ and $T_{\mathcal{R}_{\mathsf{B}}}$ and $T_{\mathcal{R}_{\mathsf{F}}}$ are the Dynkin indices of the representations $\mathcal{R}_{\mathsf{B}}$ and $\mathcal{R}_{\mathsf{F}}$ in which the bosons and fermions live

- no mass scale below M ensures no linear term $\Rightarrow$ no tadpole
- **"life is not easy": the more positive the potential is, the more tachyonic it tends to be and conversely**

Review of the $\mathcal{N} = 4$ case

The goal is to find models without tachyons and with $n_{\mathsf{F}} = n_{\mathsf{B}}$

$\rightarrow$ Super no-scale models [Kounnas, Partouche,'15]

In the $\mathcal{N} = 4$ model:

- compactification on T^{10-d}
- Scherk-Schwarz mechanism along the ninth direction

It is useful to visualize things in the type I' theory, T-dualized along the internal torus

- 2^{10-d} $O(d-1)$-planes located at the corners of the **"internal box"**
- 32 $D(d-1)$-branes are located on the O-planes to ensure the absence of tadpoles

Review of the $\mathcal{N} = 4$ case

The goal is to find models without tachyons and with $n_{\mathsf{F}} = n_{\mathsf{B}}$

$\rightarrow$ Super no-scale models [Kounnas, Partouche,'15]

In the $\mathcal{N} = 4$ model:

- compactification on T^{10-d}
- Scherk-Schwarz mechanism along the ninth direction

It is useful to visualize things in the type I' theory, T-dualized along the internal torus

- 2^{10-d} $O(d-1)$-planes located at the corners of the **"internal box"**
- 32 $D(d-1)$-branes are located on the O-planes to ensure the absence of tadpoles

Review of the $\mathcal{N} = 4$ case

The goal is to find models without tachyons and with $n_{\mathsf{F}} = n_{\mathsf{B}}$

$\rightarrow$ Super no-scale models [Kounnas, Partouche,'15]

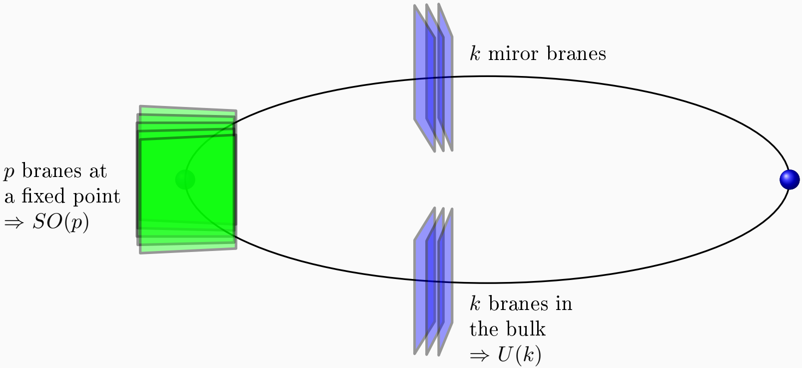
In the $\mathcal{N} = 4$ model:

- compactification on T^{10-d}
- Scherk-Schwarz mechanism along the ninth direction

It is useful to visualize things in the type I' theory, T-dualized along the internal torus

- 2^{10-d} $O(d-1)$-planes located at the corners of the **"internal box"**
- 32 $D(d-1)$-branes are located on the O-planes to ensure the absence of tadpoles

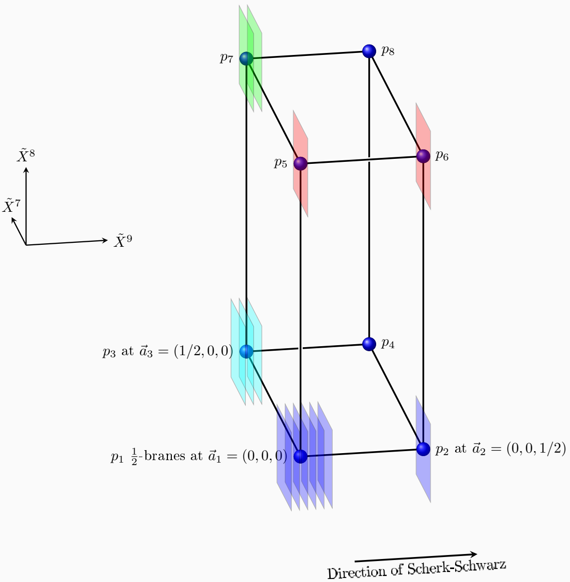
Review of the $\mathcal{N} = 4$ case



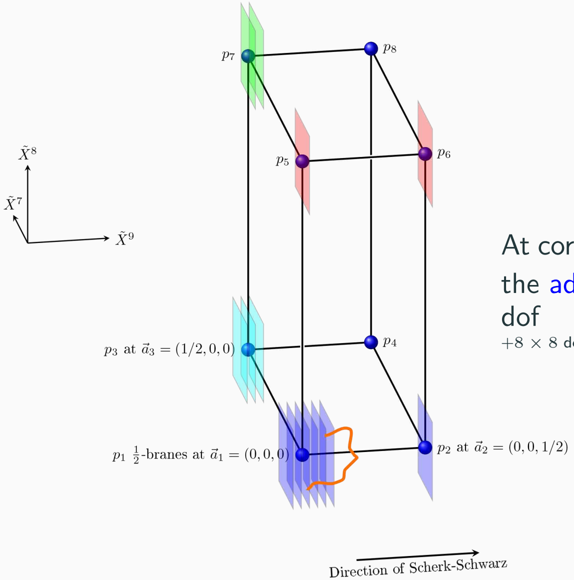
$\rightarrow$ a single brane at a fixed point is frozen

$\rightarrow$ produces a trivial group factor schematically written $SO(1)$

Review of the $\mathcal{N} = 4$ case



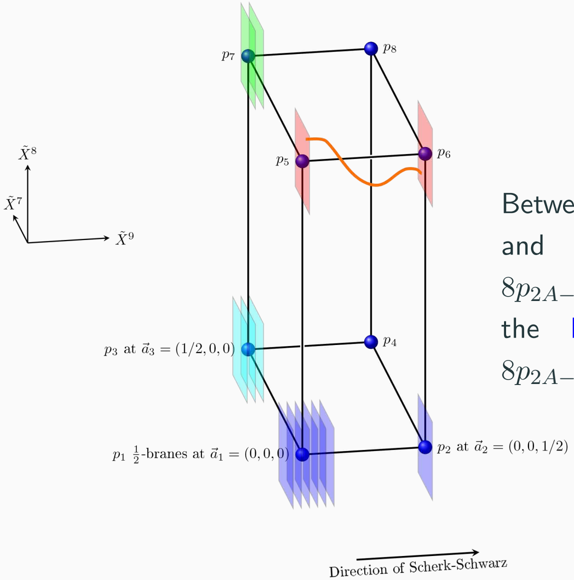
Review of the $\mathcal{N} = 4$ case



At corner p_A, 8 bosons in the adjoint $\Rightarrow 8\frac{p_A(p_A-1)}{2}$ dof

$+8 \times 8$ dof from the closed string sector

Review of the $\mathcal{N} = 4$ case



Between corners p_{2A-1} and p_{2A}, $8p_{2A}$ and $8p_{2A-1}$ fermions in the bifundamental $\Rightarrow$ $8p_{2A-1}p_{2A}$ dof

Review of the $\mathcal{N} = 4$ case

It is all the information needed to find stable and super no-scale configurations

- **A lot of models** have a negative potential with $(n_{\mathsf{F}} - n_{\mathsf{B}}) < 0$
- Only **a few** have $(n_{\mathsf{F}} - n_{\mathsf{B}}) = 0$
    - Enough O-planes must be present $\Rightarrow d \leq 5$
    - Configurations with gauge groups up to $SO(5)$
- Possible to reach $(n_{\mathsf{F}} - n_{\mathsf{B}}) = 64$ with no gauge group
- Closed string moduli are flat directions

All these algebraic computations can be recovered from an explicit computation of the potential

Review of the $\mathcal{N} = 4$ case

It is all the information needed to find stable and super no-scale configurations

- **A lot of models** have a negative potential with $(n_\mathsf{F} - n_\mathsf{B}) < 0$
- Only **a few** have $(n_\mathsf{F} - n_\mathsf{B}) = 0$
  - Enough O-planes must be present $\Rightarrow d \leq 5$
  - Configurations with gauge groups up to $SO(5)$
- Possible to reach $(n_\mathsf{F} - n_\mathsf{B}) = 64$ with no gauge group
- Closed string moduli are flat directions

All these algebraic computations can be recovered from an explicit computation of the potential

Review of the $\mathcal{N} = 4$ case

It is all the information needed to find stable and super no-scale configurations

- **A lot of models** have a negative potential with $(n_\mathsf{F} - n_\mathsf{B}) < 0$
- Only **a few** have $(n_\mathsf{F} - n_\mathsf{B}) = 0$
  - Enough O-planes must be present $\Rightarrow d \leq 5$
  - Configurations with gauge groups up to $SO(5)$
- Possible to reach $(n_\mathsf{F} - n_\mathsf{B}) = 64$ with no gauge group
- Closed string moduli are flat directions

All these algebraic computations can be recovered from an explicit computation of the potential

Review of the $\mathcal{N} = 4$ case

It is all the information needed to find stable and super no-scale configurations

- **A lot of models** have a negative potential with $(n_\mathsf{F} - n_\mathsf{B}) < 0$
- Only **a few** have $(n_\mathsf{F} - n_\mathsf{B}) = 0$
  - Enough O-planes must be present $\Rightarrow d \leq 5$
  - Configurations with gauge groups up to $SO(5)$
- Possible to reach $(n_\mathsf{F} - n_\mathsf{B}) = 64$ with no gauge group
- Closed string moduli are flat directions

All these algebraic computations can be recovered from an explicit computation of the potential

Review of the $\mathcal{N} = 4$ case

It is all the information needed to find stable and super no-scale configurations

- **A lot of models** have a negative potential with $(n_{\mathsf{F}} - n_{\mathsf{B}}) < 0$
- Only **a few** have $(n_{\mathsf{F}} - n_{\mathsf{B}}) = 0$
  - Enough O-planes must be present $\Rightarrow d \leq 5$
  - Configurations with gauge groups up to $SO(5)$
- Possible to reach $(n_{\mathsf{F}} - n_{\mathsf{B}}) = 64$ with no gauge group
- Closed string moduli are flat directions

All these algebraic computations can be recovered from an explicit computation of the potential

$\mathcal{N} = 2 \to 0$ model

Setup

We start from the Gimon-Polchinski-Pradisi-Sagnotti model

- Compactification on $T^2 \times T^4/\mathbb{Z}_2$, directions $(4, 5, 6, 7, 8, 9)$
- The circle in the T^2 in direction 5 is used to implement the Scherk-Schwarz breaking

The RR tadpole cancellation condition requires:

- 32 $D9$-branes $\to N$
- 32 $D5$-branes orthogonal to the T^4 $\to D$

No vectors running in the Möbius partition function $\Rightarrow$ unitary representations

Setup

We start from the Gimon-Polchinski-Pradisi-Sagnotti model

- Compactification on $T^2 \times T^4/\mathbb{Z}_2$, directions $(4, 5, 6, 7, 8, 9)$
- The circle in the T^2 in direction 5 is used to implement the Scherk-Schwarz breaking

The RR tadpole cancellation condition requires:

- 32 $D9$-branes $\to N$
- 32 $D5$-branes orthogonal to the T^4 $\to D$

No vectors running in the Möbius partition function $\Rightarrow$ unitary representations

Setup

We start from the Gimon-Polchinski-Pradisi-Sagnotti model

- Compactification on $T^2 \times T^4/\mathbb{Z}_2$, directions $(4,5,6,7,8,9)$
- The circle in the T^2 in direction 5 is used to implement the Scherk-Schwarz breaking

The RR tadpole cancellation condition requires:

- 32 $D9$-branes $\rightarrow N$
- 32 $D5$-branes orthogonal to the $T^4 \rightarrow D$

No vectors running in the Möbius partition function $\Rightarrow$ unitary representations

Setup

- We add discrete Wilson lines for the $D9$-branes and for the orthogonal part of the $D5$-branes
- We add discrete positions for the $D5$-branes inside the T^4 $\rightarrow$ they are at corners of the internal box

All discrete Wilson lines can be seen as discrete positions in the correct T-dual picture

Corner labelling: In total, $2^4 \times 2^2 = 64$ corners

Label by ii', where $\rightarrow i = 1, \ldots, 16$ corresponds to the T^4
$\rightarrow i' = 1, \ldots, 4$ corresponds to the T^2

$(2i' - 1)$ and $2i'$ are opposite corners along the Scherk-Schwarz direction

Setup

- We add discrete Wilson lines for the $D9$-branes and for the orthogonal part of the $D5$-branes
- We add discrete positions for the $D5$-branes inside the T^4
  $\rightarrow$ they are at corners of the internal box

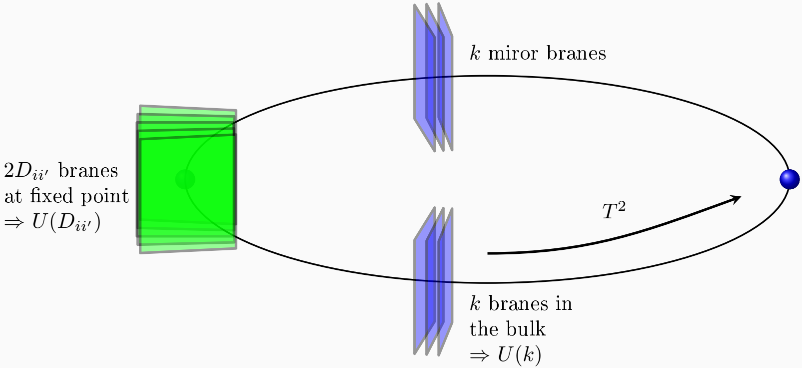
All discrete Wilson lines can be seen as discrete positions in the correct T-dual picture

Corner labelling: In total, $2^4 \times 2^2 = 64$ corners

Label by ii', where $\rightarrow i = 1, \ldots, 16$ corresponds to the T^4
$\rightarrow i' = 1, \ldots, 4$ corresponds to the T^2

$(2i' - 1)$ and $2i'$ are opposite corners along the Scherk-Schwarz direction

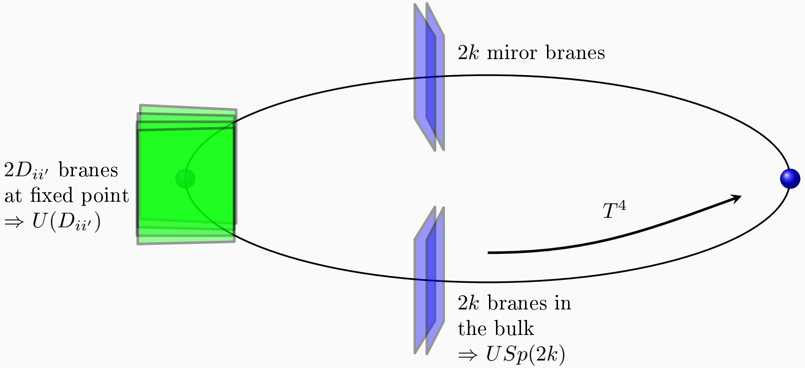
Along the T^2:



$\rightarrow$ branes cannot be frozen anymore because the group must be unitary

Dynamical branes

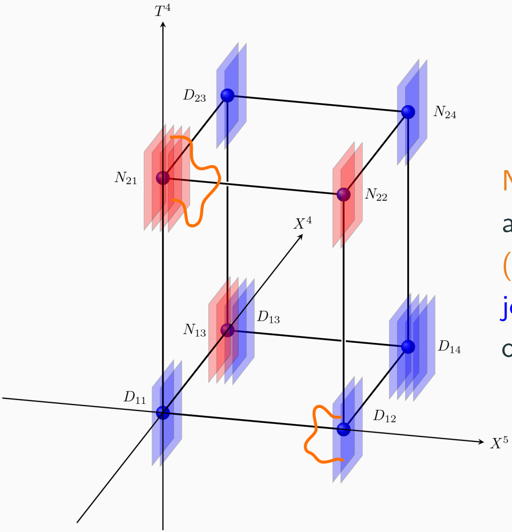
Along the T^4: The orbifold cuts again half of the dof



$\rightarrow$ Branes are always in pairs $\Rightarrow N_{ii'} = 2n_{ii'}$ and $D_{ii'} = 2d_{ii'}$

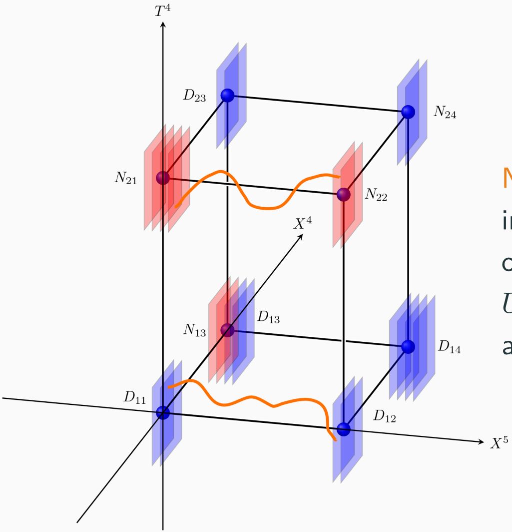
$\rightarrow$ They can only move by multiple of four

Massless spectrum



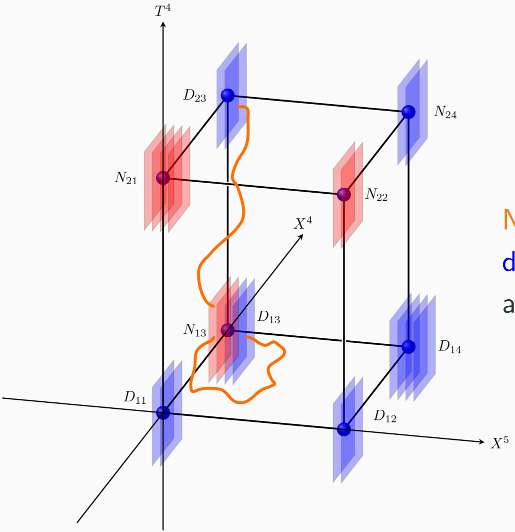
Neuman-Neuman (NN) and Dirichlet-Dirichlet (DD) bosons in the adjoint and antisymmetric of $U(n_{ii'})$ and $U(d_{ii'})$

Massless spectrum



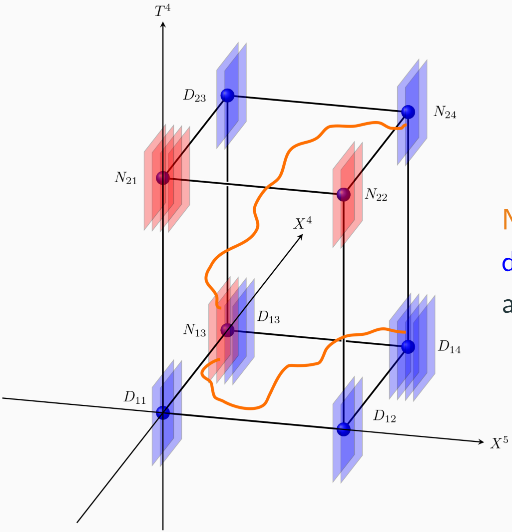
NN and DD fermions in the bifundamental of $U(n_{i(2i'-1)})$ and $U(n_{i(2i')})$ or $U(d_{i(2i'-1)})$ and $U(d_{i(2i')})$

Massless spectrum



ND bosons in the bifundamental of $U(n_{i(2i')})$ and $U(d_{j(2i')})$

Massless spectrum



ND fermions in the bifundamental of $U(n_{i(2i'-1)})$ and $U(d_{j(2i')})$

Massless spectrum

From the partition function, in total we count in the open string sector:

$$n_{\mathsf{B}}^{\mathsf{open}} = 8\left(n_{ii'}^2 + d_{ii'}^2 + \frac{1}{2} n_{ii'} d_{ji'} - 16\right)$$

$$\begin{aligned} n_{\mathsf{F}}^{\mathsf{open}} = 8\Big(& n_{i(2i'-1)} n_{i(2i')} + d_{i(2i'-1)} d_{i(2i')} \\ & + \frac{1}{2} n_{i(2i'-1)} d_{j(2i')} + n_{i(2i)} d_{j(2i'-1)}\Big) \end{aligned}$$

The closed string spectrum is the bosonic content (because **fermions are massive**) of the $T^4/\mathbb{Z}_2$ orientifold which is in six dimensions:

- 1 gravity multiplet g_{MN}, B_{MN}^+
- 1 tensor multiplet B_{MN}^-, ϕ $\longrightarrow$ 92 degrees of freedom
- 20 hypermultiplets $20 \times 4\phi$

Massless spectrum

From the partition function, in total we count in the open string sector:

$$n_{\mathsf{B}}^{\mathsf{open}} = 8\left(n_{ii'}^2 + d_{ii'}^2 + \frac{1}{2} n_{ii'} d_{ji'} - 16\right)$$

$$\begin{aligned} n_{\mathsf{F}}^{\mathsf{open}} = 8\Big(& n_{i(2i'-1)} n_{i(2i')} + d_{i(2i'-1)} d_{i(2i')} \\ & + \frac{1}{2} n_{i(2i'-1)} d_{j(2i')} + n_{i(2i)} d_{j(2i'-1)} \Big) \end{aligned}$$

The closed string spectrum is the bosonic content (because **fermions are massive**) of the $T^4/\mathbb{Z}_2$ orientifold which is in six dimensions:

- 1 gravity multiplet g_{MN}, B^+_{MN}
- 1 tensor multiplet B^-_{MN}, ϕ $\longrightarrow 92$ degrees of freedom
- 20 hypermultiplets $20 \times 4\phi$

Super no-scale and stability conditions

Super no-scale:

$$\left(n_{i(2i'-1)} - n_{i(2i')}\right)^2 + \left(d_{i(2i'-1)} - d_{i(2i')}\right)^2 + \frac{1}{2}\left(n_{i(2i'-1)} - n_{i(2i')}\right)\left(d_{j(2i'-1)} - d_{j(2i')}\right) = 4$$

Stability

$$\mathcal{N} = 4 \to 0 \xrightarrow{\text{orbifold}} \mathcal{N} = 2 \to 0$$

$$\begin{tikzcd} & \text{Vector multiplet} \\ \text{Vector multiplet} \arrow[ur] \arrow[r] \arrow[dr] & \text{Hypermultiplet} \\ & \text{Twisted sector} \end{tikzcd}$$

Super no-scale and stability conditions

Super no-scale:

$$\left(n_{i(2i'-1)} - n_{i(2i')}\right)^2 + \left(d_{i(2i'-1)} - d_{i(2i')}\right)^2 + \frac{1}{2}\left(n_{i(2i'-1)} - n_{i(2i')}\right)\left(d_{j(2i'-1)} - d_{j(2i')}\right) = 4$$

Stability

$$\mathcal{N} = 4 \to 0 \qquad \xrightarrow{\text{orbifold}} \qquad \mathcal{N} = 2 \to 0$$

Vector multiplet $\longrightarrow$ Vector multiplet

Vector multiplet $\longrightarrow$ Hypermultiplet

Vector multiplet $\longrightarrow$ Twisted sector

Super no-scale and stability conditions

$$\text{Moduli space} = \underbrace{\text{Coulomb branch} \times \text{Higgs branch}}_{\text{Wilson lines}} \times \text{Twisted scalars}$$

$$\times \underbrace{\text{Closed string moduli}}_{\text{WL, flat directions}}$$

Dynkin indices:

	Representation $\mathcal{R}$	$\mathcal{T}_{\mathcal{R}}$
$SU(q)$, $q \geq 2$	fundamental	1
	$\overline{\text{fundamental}}$	1
	adjoint	$2q$
	antisymmetric	$q-2$
	$\overline{\text{antisymmetric}}$	$q-2$

Super no-scale and stability conditions

$$\text{Moduli space} = \underbrace{\text{Coulomb branch} \times \text{Higgs branch}}_{\text{Wilson lines}} \times \text{Twisted scalars}$$

$$\times \underbrace{\text{Closed string moduli}}_{\text{WL, flat directions}}$$

Dynkin indices:

	Representation $\mathcal{R}$	$\mathcal{T}_{\mathcal{R}}$
$SU(q)$, $q \geq 2$	fundamental	1
	$\overline{\text{fundamental}}$	1
	adjoint	$2q$
	antisymmetric	$q-2$
	$\overline{\text{antisymmetric}}$	$q-2$

Super no-scale and stability conditions

Example for $U(n_{i(2i')})$

Bosons:

- 4 in the adjoint
- 4 in the antisymmetric
- 4 in the $\overline{\text{antisymmectric}}$
- $2\sum_j d_{j(2i')}$ in the fund
- $2\sum_j d_{j(2i')}$ in the $\overline{\text{fund}}$

Fermions:

- $2 \times 4n_{i(2i'-1)}$ in the fund
- $2 \times 4n_{i(2i'-1)}$ in the $\overline{\text{fund}}$
- $2\sum_j d_{j(2i'-1)}$ in the fund
- $2\sum_j d_{j(2i'-1)}$ in the $\overline{\text{fund}}$

- We can write the same thing for $U(n_{i(2i'-1)})$
- We have something similar with $n \leftrightarrow d$ for $U(d_{i(2i')})$ and $U(d_{i(2i'-1)})$

Super no-scale and stability conditions

Example for $U(n_{i(2i')})$

Bosons:

- 4 in the adjoint
- 4 in the antisymmetric
- 4 in the $\overline{\text{antisymmectric}}$
- $2\sum_j d_{j(2i')}$ in the fund
- $2\sum_j d_{j(2i')}$ in the $\overline{\text{fund}}$

Fermions:

- $2 \times 4n_{i(2i'-1)}$ in the fund
- $2 \times 4n_{i(2i'-1)}$ in the $\overline{\text{fund}}$
- $2\sum_j d_{j(2i'-1)}$ in the fund
- $2\sum_j d_{j(2i'-1)}$ in the $\overline{\text{fund}}$

- We can write the same thing for $U(n_{i(2i'-1)})$
- We have something similar with $n \leftrightarrow d$ for $U(d_{i(2i')})$ and $U(d_{i(2i'-1)})$

Super no-scale and stability conditions

Higgs branch:

$$n_{i(2i'-1)} - n_{i(2i')} - 1 \geq 0 \quad \text{for} \quad n_{i(2i'-1)} \geq 2$$

$$n_{i(2i')} - n_{i(2i'-1)} - 1 \geq 0 \quad \text{for} \quad n_{i(2i')} \geq 2$$

Coulomb branch:

$$4(n_{i(2i')} - n_{i(2i'-1)}) + \sum_{j=1}^{16}(d_{j(2i')} - d_{j(2i'-1)}) - 4 \geq 0 \quad \text{for} \quad n_{i(2i')} \geq 1$$

$$4(n_{i(2i'-1)} - n_{i(2i')}) + \sum_{j=1}^{16}(d_{j(2i'-1)} - d_{j(2i')}) - 4 \geq 0 \quad \text{for} \quad n_{i(2i'-1)} \geq 1$$

Super no-scale and stability conditions

Higgs branch:

$$n_{i(2i'-1)} - n_{i(2i')} - 1 \geq 0 \quad \text{for} \quad n_{i(2i'-1)} \geq 2$$
$$n_{i(2i')} - n_{i(2i'-1)} - 1 \geq 0 \quad \text{for} \quad n_{i(2i')} \geq 2$$

Coulomb branch:

$$4(n_{i(2i')} - n_{i(2i'-1)}) + \sum_{j=1}^{16}(d_{j(2i')} - d_{j(2i'-1)}) - 4 \geq 0 \quad \text{for} \quad n_{i(2i')} \geq 1$$
$$4(n_{i(2i'-1)} - n_{i(2i')}) + \sum_{j=1}^{16}(d_{j(2i'-1)} - d_{j(2i')}) - 4 \geq 0 \quad \text{for} \quad n_{i(2i'-1)} \geq 1$$

Explicit computation

Wilson lines for $D9$-branes and $D5$-branes:

$$a_\alpha^{\mathcal{I}} = \langle a_\alpha^{\mathcal{I}} \rangle + \epsilon_\alpha^{\mathcal{I}}, \qquad \langle a_\alpha^{\mathcal{I}} \rangle \in \left\{0, \frac{1}{2}\right\}, \quad \alpha = 1, \dots, 32, \ \mathcal{I} = 4, \dots, 9$$

$$b_\alpha^{\mathcal{I}} = \langle b_\alpha^{\mathcal{I}} \rangle + \xi_\alpha^{\mathcal{I}}, \qquad \langle b_\alpha^{\mathcal{I}} \rangle \in \left\{0, \frac{1}{2}\right\}, \quad \alpha = 1, \dots, 32, \ \mathcal{I} = 4, \dots, 9$$

$\mathcal{I}$ is split into $I = 6, \dots, 9 \longrightarrow$ Higgs branch

and $I' = 4, 5 \longrightarrow$ Coulomb branch

No intermediate mass scale:

$G_{\mathcal{I}\mathcal{J}}$ is the metric of the internal space, $\sqrt{G_{55}} = R$

$$G^{55} \ll |G_{ij}| \ll G_{55}, \quad |G_{5j}| \ll \sqrt{G_{55}}, \quad G_{55} \gg 1 \quad i, j \neq 5$$

$$\boxed{\mathcal{V}_{\text{1-loop}} = \frac{\Gamma\left(\frac{5}{2}\right)}{\pi^{\frac{13}{2}}} M^4 \sum_{l_5} \frac{\mathcal{N}_{2l_5+1}(\epsilon, \xi, G)}{|2l_5 + 1|^5} + \cdots}$$

Explicit computation

Wilson lines for $D9$-branes and $D5$-branes:

$$a_\alpha^{\mathcal{I}} = \langle a_\alpha^{\mathcal{I}} \rangle + \epsilon_\alpha^{\mathcal{I}}, \qquad \langle a_\alpha^{\mathcal{I}} \rangle \in \left\{0, \frac{1}{2}\right\}, \quad \alpha = 1, \dots, 32, \ \mathcal{I} = 4, \dots, 9$$

$$b_\alpha^{\mathcal{I}} = \langle b_\alpha^{\mathcal{I}} \rangle + \xi_\alpha^{\mathcal{I}}, \qquad \langle b_\alpha^{\mathcal{I}} \rangle \in \left\{0, \frac{1}{2}\right\}, \quad \alpha = 1, \dots, 32, \ \mathcal{I} = 4, \dots, 9$$

$\mathcal{I}$ is split into $I = 6, \dots, 9 \ \longrightarrow$ Higgs branch
and $I' = 4, 5 \longrightarrow$ Coulomb branch

No intermediate mass scale:

$G_{\mathcal{I}\mathcal{J}}$ is the metric of the internal space, $\sqrt{G_{55}} = R$

$$G^{55} \ll |G_{ij}| \ll G_{55}, \quad |G_{5j}| \ll \sqrt{G_{55}}, \quad G_{55} \gg 1 \quad i, j \neq 5$$

$$\boxed{\mathcal{V}_{\text{1-loop}} = \frac{\Gamma\left(\frac{5}{2}\right)}{\pi^{\frac{13}{2}}} M^4 \sum_{l_5} \frac{\mathcal{N}_{2l_5+1}(\epsilon, \xi, G)}{|2l_5+1|^5} + \cdots}$$

Explicit computation

Wilson lines for $D9$-branes and $D5$-branes:

$$a_\alpha^{\mathcal{I}} = \langle a_\alpha^{\mathcal{I}} \rangle + \epsilon_\alpha^{\mathcal{I}}, \qquad \langle a_\alpha^{\mathcal{I}} \rangle \in \left\{0, \frac{1}{2}\right\}, \quad \alpha = 1, \ldots, 32, \; \mathcal{I} = 4, \ldots, 9$$

$$b_\alpha^{\mathcal{I}} = \langle b_\alpha^{\mathcal{I}} \rangle + \xi_\alpha^{\mathcal{I}}, \qquad \langle b_\alpha^{\mathcal{I}} \rangle \in \left\{0, \frac{1}{2}\right\}, \quad \alpha = 1, \ldots, 32, \; \mathcal{I} = 4, \ldots, 9$$

$\mathcal{I}$ is split into $I = 6, \ldots, 9 \longrightarrow$ Higgs branch

and $I' = 4, 5 \longrightarrow$ Coulomb branch

No intermediate mass scale:

$G_{\mathcal{I}\mathcal{J}}$ is the metric of the internal space, $\sqrt{G_{55}} = R$

$$G^{55} \ll |G_{ij}| \ll G_{55}, \quad |G_{5j}| \ll \sqrt{G_{55}}, \quad G_{55} \gg 1 \quad i, j \neq 5$$

$$\boxed{\mathcal{V}_{\text{1-loop}} = \frac{\Gamma\left(\frac{5}{2}\right)}{\pi^{\frac{13}{2}}} M^4 \sum_{l_5} \frac{\mathcal{N}_{2l_5+1}(\epsilon, \xi, G)}{|2l_5 + 1|^5} + \cdots}$$

Explicit computation

$$\begin{aligned}
\mathcal{N}_{2l_5+1}(\epsilon,\xi,G) = 2\Bigg\{ &-\sum_{(\alpha,\beta)\in L_N} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon^{I'}_\alpha-\epsilon^{I'}_\beta\right)\right] \\
&\times \mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon^I_\alpha-\epsilon^I_\beta)G^{IJ}(\epsilon^J_\alpha-\epsilon^J_\beta)+(\epsilon^4_\alpha-\epsilon^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) \\
&-\sum_{(\alpha,\beta)\in L_D} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\xi^{I'}_\alpha-\xi^{I'}_\beta\right)\right] \\
&\times \mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[\frac{1}{2}(\xi^I_\alpha-\xi^I_\beta)G_{IJ}(\xi^J_\alpha-\xi^J_\beta)+(\xi^4_\alpha-\xi^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) \\
&-\frac{1}{2}\sum_{(\alpha,\beta)\in L_{N\text{-}D}} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon^{I'}_\alpha-\epsilon^{I'}_\beta\right)\right]\mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon^4_\alpha-\epsilon^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) \\
&+\sum_{\alpha}\cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\epsilon^{I'}_\alpha\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\epsilon^I_\alpha G^{IJ}\epsilon^J_\alpha+\left(\epsilon^4_\alpha\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) \\
&+\sum_{\alpha}\cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\xi^{I'}_\alpha\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\frac{1}{2}\xi^I_\alpha G_{IJ}\xi^J_\alpha+\left(\xi^4_\alpha\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) - 23\Bigg\}
\end{aligned}$$

$L_N, L_D, L_{N-D} \Rightarrow$ massless

F, fermion number

$\hat{G}^{44} = G^{44} - \frac{G^{45}}{G^{55}}G^{55}\frac{G^{54}}{G^{55}}$

$\mathcal{H}_\nu(z) = \frac{2}{\Gamma(\nu)}z^\nu K_\nu(2z)$

Explicit computation

$$\mathcal{N}_{2l_5+1}(\epsilon,\xi,G) = 2\Bigg\{ -\sum_{(\alpha,\beta)\in L_N} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon^{I'}_\alpha - \epsilon^{I'}_\beta\right)\right]$$

$$\times\,\mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon^I_\alpha-\epsilon^I_\beta)G^{IJ}(\epsilon^J_\alpha-\epsilon^J_\beta)+(\epsilon^4_\alpha-\epsilon^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$-\sum_{(\alpha,\beta)\in L_D} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\xi^{I'}_\alpha - \xi^{I'}_\beta\right)\right]$$

$$\times\,\mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[\frac{1}{2}(\xi^I_\alpha-\xi^I_\beta)G_{IJ}(\xi^J_\alpha-\xi^J_\beta)+(\xi^4_\alpha-\xi^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$-\frac{1}{2}\sum_{(\alpha,\beta)\in L_{N\text{-}D}} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon^{I'}_\alpha - \epsilon^{I'}_\beta\right)\right]\mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon^4_\alpha-\epsilon^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$+\sum_{\alpha}\cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\epsilon^{I'}_\alpha\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\epsilon^I_\alpha G^{IJ}\epsilon^J_\alpha+\left(\epsilon^4_\alpha\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$+\sum_{\alpha}\cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\xi^{I'}_\alpha\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\frac{1}{2}\xi^I_\alpha G_{IJ}\xi^J_\alpha+\left(\xi^4_\alpha\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)-23\Bigg\}$$

$L_N, L_D, L_{N-D} \Rightarrow$ massless

F, fermion number

$\hat{G}^{44} = G^{44} - \frac{G^{45}}{G^{55}}G^{55}\frac{G^{54}}{G^{55}}$

$\mathcal{H}_\nu(z) = \frac{2}{\Gamma(\nu)}z^\nu K_\nu(2z)$

Explicit computation

$$
\begin{aligned}
\mathcal{N}_{2l_5+1}(\epsilon,\xi,G) = 2\Bigg\{ & -\sum_{(\alpha,\beta)\in L_N} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon^{I'}_\alpha-\epsilon^{I'}_\beta\right)\right] \\
& \times \mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon^I_\alpha-\epsilon^I_\beta)G^{IJ}(\epsilon^J_\alpha-\epsilon^J_\beta)+(\epsilon^4_\alpha-\epsilon^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) \\
& -\sum_{(\alpha,\beta)\in L_D} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\xi^{I'}_\alpha-\xi^{I'}_\beta\right)\right] \\
& \times \mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[\frac{1}{2}(\xi^I_\alpha-\xi^I_\beta)G_{IJ}(\xi^J_\alpha-\xi^J_\beta)+(\xi^4_\alpha-\xi^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) \\
& -\frac{1}{2}\sum_{(\alpha,\beta)\in L_{N\text{-}D}} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon^{I'}_\alpha-\epsilon^{I'}_\beta\right)\right]\mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon^4_\alpha-\epsilon^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) \\
& +\sum_{\alpha}\cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\epsilon^{I'}_\alpha\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\epsilon^I_\alpha G^{IJ}\epsilon^J_\alpha+\left(\epsilon^4_\alpha\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) \\
& +\sum_{\alpha}\cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\xi^{I'}_\alpha\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\frac{1}{2}\xi^I_\alpha G_{IJ}\xi^J_\alpha+\left(\xi^4_\alpha\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) - 23\Bigg\}
\end{aligned}
$$

$L_N, L_D, L_{N-D} \Rightarrow$ massless

F, fermion number

$\hat{G}^{44} = G^{44} - \frac{G^{45}}{G^{55}}G^{55}\frac{G^{54}}{G^{55}}$

$\mathcal{H}_\nu(z) = \frac{2}{\Gamma(\nu)}z^\nu K_\nu(2z)$

Explicit computation

$$\mathcal{N}_{2l_5+1}(\epsilon,\xi,G) = 2\Bigg\{ -\sum_{(\alpha,\beta)\in L_N} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon^{I'}_\alpha - \epsilon^{I'}_\beta\right)\right]$$

$$\times\, \mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon^I_\alpha-\epsilon^I_\beta)G^{IJ}(\epsilon^J_\alpha-\epsilon^J_\beta)+(\epsilon^4_\alpha-\epsilon^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$-\sum_{(\alpha,\beta)\in L_D} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\xi^{I'}_\alpha - \xi^{I'}_\beta\right)\right]$$

$$\times\, \mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[\frac{1}{2}(\xi^I_\alpha-\xi^I_\beta)G_{IJ}(\xi^J_\alpha-\xi^J_\beta)+(\xi^4_\alpha-\xi^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$-\frac{1}{2}\sum_{(\alpha,\beta)\in L_{N\text{-}D}} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon^{I'}_\alpha - \epsilon^{I'}_\beta\right)\right]\mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon^4_\alpha-\epsilon^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$+\sum_\alpha \cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\epsilon^{I'}_\alpha\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\epsilon^I_\alpha G^{IJ}\epsilon^J_\alpha+\left(\epsilon^4_\alpha\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$+\sum_\alpha \cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\xi^{I'}_\alpha\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\frac{1}{2}\xi^I_\alpha G_{IJ}\xi^J_\alpha+\left(\xi^4_\alpha\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)-23\Bigg\}$$

$L_N, L_D, L_{N-D} \Rightarrow$ massless

F, fermion number

$$\hat{G}^{44} = G^{44} - \frac{G^{45}}{G^{55}}G^{55}\frac{G^{54}}{G^{55}}$$

$$\mathcal{H}_\nu(z) = \frac{2}{\Gamma(\nu)}z^\nu K_\nu(2z)$$

Explicit computation

$$\mathcal{N}_{2l_5+1}(\epsilon,\xi,G) = 2\Bigg\{ -\sum_{(\alpha,\beta)\in L_N} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon_\alpha^{I'}-\epsilon_\beta^{I'}\right)\right]$$

$$\times\,\mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon_\alpha^I-\epsilon_\beta^I)G^{IJ}(\epsilon_\alpha^J-\epsilon_\beta^J)+(\epsilon_\alpha^4-\epsilon_\beta^4)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$-\sum_{(\alpha,\beta)\in L_D} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\xi_\alpha^{I'}-\xi_\beta^{I'}\right)\right]$$

$$\times\,\mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[\frac{1}{2}(\xi_\alpha^I-\xi_\beta^I)G_{IJ}(\xi_\alpha^J-\xi_\beta^J)+(\xi_\alpha^4-\xi_\beta^4)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$-\frac{1}{2}\sum_{(\alpha,\beta)\in L_{N\text{-}D}} (-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon_\alpha^{I'}-\epsilon_\beta^{I'}\right)\right]\mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon_\alpha^4-\epsilon_\beta^4)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$+\sum_{\alpha}\cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\epsilon_\alpha^{I'}\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\epsilon_\alpha^I G^{IJ}\epsilon_\alpha^J+\left(\epsilon_\alpha^4\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)$$

$$+\sum_{\alpha}\cos\left[4\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\xi_\alpha^{I'}\right]\mathcal{H}_{\frac{5}{2}}\left(2\pi|2l_5+1|\frac{\left[\frac{1}{2}\xi_\alpha^I G_{IJ}\xi_\alpha^J+\left(\xi_\alpha^4\right)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right)-23\Bigg\}$$

$L_N, L_D, L_{N-D} \Rightarrow$ massless

F, fermion number

$$\hat{G}^{44} = G^{44} - \frac{G^{45}}{G^{55}}G^{55}\frac{G^{54}}{G^{55}}$$

$$\mathcal{H}_\nu(z) = \frac{2}{\Gamma(\nu)}z^\nu K_\nu(2z)$$

Explicit computation

Dynamical degrees of freedom:

$$\epsilon_r^I, \xi_r^I \qquad I = 6, \ldots, 9, \quad \text{and} \quad r = 1, \ldots, \sum_{i=1}^{16} \sum_{i'=1}^{4} \left\lfloor \frac{n_{ii'}}{2} \right\rfloor$$

$$\epsilon_{r'}^{I'}, \xi_{r'}^{I'}, \quad I' = 4, 5, \qquad \text{and} \quad r' = 1, \ldots, 16$$

$$\mathcal{N}_{2l_9+1} = 32\pi^2 (2l_9+1)^2 \Big\{ \mathcal{O}\left(\epsilon^0, \xi^0\right) + \sum_r \left(n_{i(r)i'(r)} - n_{i(r)\tilde{i}'(r)} - 1\right) \epsilon_r^I \Delta^{IJ} \epsilon_r^J$$

$$+ \sum_{r'} \left(n_{i(r')i'(r')} - n_{i(r')\tilde{i}'(r')} - 1 + \frac{1}{4} \sum_i \left(d_{ii'(r')} - d_{i\tilde{i}'(r')} \right) \right) \epsilon_{r'}^{I'} \Delta^{I'J'} \epsilon_{r'}^{J'}$$

$$+ \sum_r \left(d_{i(r)i'(r)} - d_{i(r)\tilde{i}'(r)} - 1 \right) \xi_r^I \Delta_{IJ} \xi_r^J$$

$$+ \sum_{r'} \left(d_{i(r')i'(r')} - d_{i(r')\tilde{i}'(r')} - 1 + \frac{1}{4} \sum_i \left(n_{ii'(r')} - n_{i\tilde{i}'(r')} \right) \right) \xi_r^{I'} \Delta^{I'J'} \xi_{r'}^{J'} + \mathcal{O}\left(\epsilon^4, \xi^4\right) \Big\}$$

Explicit computation

Dynamical degrees of freedom:

$$\epsilon_r^I, \xi_r^I \qquad I = 6, \dots, 9, \quad \text{and} \quad r = 1, \dots, \sum_{i=1}^{16} \sum_{i'=1}^{4} \left\lfloor \frac{n_{ii'}}{2} \right\rfloor$$

$$\epsilon_{r'}^{I'}, \xi_{r'}^{I'}, \quad I' = 4, 5, \qquad \text{and} \quad r' = 1, \dots, 16$$

$$\mathcal{N}_{2l_9+1} = 32\pi^2(2l_9+1)^2 \Big\{ \mathcal{O}\left(\epsilon^0, \xi^0\right) + \sum_r \left(n_{i(r)i'(r)} - n_{i(r)\tilde{i}'(r)} - 1\right) \epsilon_r^I \Delta^{IJ} \epsilon_r^J$$

$$+ \sum_{r'} \left(n_{i(r')i'(r')} - n_{i(r')\tilde{i}'(r')} - 1 + \frac{1}{4} \sum_i \left(d_{ii'(r')} - d_{i\tilde{i}'(r')} \right) \right) \epsilon_{r'}^{I'} \Delta^{I'J'} \epsilon_{r'}^{J'}$$

$$+ \sum_r \left(d_{i(r)i'(r)} - d_{i(r)\tilde{i}'(r)} - 1 \right) \xi_r^I \Delta_{IJ} \xi_r^J$$

$$+ \sum_{r'} \left(d_{i(r')i'(r')} - d_{i(r')\tilde{i}'(r')} - 1 + \frac{1}{4} \sum_i \left(n_{ii'(r')} - n_{i\tilde{i}'(r')} \right) \right) \xi_r^{I'} \Delta^{I'J'} \xi_{r'}^{J'} + \mathcal{O}\left(\epsilon^4, \xi^4\right) \Big\}$$

Twisted scalars

How to find the mass of the twisted scalars?

- The twisted scalars are not Wilson lines
- They are Neuman-Dirichlet states

Idea:

- A **string computation** of their one-loop two-point function
  - Tedious, requires the technology of twist fields and their correlators [Atick, Dixon, Griffin, Nemeschansky,'87]
  - Extracting only the sign of the mass is maybe doable

$\rightarrow$ maybe a field theory approach would be simpler

Twisted scalars

How to find the mass of the twisted scalars?

- The twisted scalars are not Wilson lines
- They are Neuman-Dirichlet states

Idea:

- A **string computation** of their one-loop two-point function
  - Tedious, requires the technology of twist fields and their correlators [Atick, Dixon, Griffin, Nemeschansky,'87]
  - Extracting only the sign of the mass is maybe doable

$\rightarrow$ maybe a field theory approach would be simpler

Twisted scalars

How to find the mass of the twisted scalars?

- The twisted scalars are not Wilson lines
- They are Neuman-Dirichlet states

Idea:

- A **string computation** of their one-loop two-point function
  - Tedious, requires the technology of twist fields and their correlators [Atick, Dixon, Griffin, Nemeschansky,'87]
  - Extracting only the sign of the mass is maybe doable

$\rightarrow$ maybe a field theory approach would be simpler

Conclusions and outlooks

Conclusions and outlooks

- Following [Abel, Dudas, Lewis, Partouche,'18], we are looking for super no-scale model (**exponentially suppressed potential**) without moduli instabilities
- This in an open string $T^4/\mathbb{Z}_2$ model with broken supersymmetry
- We expressed the super no-scale condition *via* the counting of massless degrees of freedom
- We expressed stability conditions for the open string Wilson lines masses
- The mass of the twisted scalars must still be found to obtain the full constraints on the allowed brane configurations

Conclusions and outlooks

- Following [Abel, Dudas, Lewis, Partouche,'18], we are looking for super no-scale model (**exponentially suppressed potential**) without moduli instabilities
- This in an open string $T^4/\mathbb{Z}_2$ model with broken supersymmetry
- We expressed the super no-scale condition *via* the counting of massless degrees of freedom
- We expressed stability conditions for the open string Wilson lines masses
- The mass of the twisted scalars must still be found to obtain the full constraints on the allowed brane configurations

Conclusions and outlooks

- Following [Abel, Dudas, Lewis, Partouche,'18], we are looking for super no-scale model (**exponentially suppressed potential**) without moduli instabilities
- This in an open string $T^4/\mathbb{Z}_2$ model with broken supersymmetry
- We expressed the super no-scale condition *via* the counting of massless degrees of freedom
- We expressed stability conditions for the open string Wilson lines masses
- The mass of the twisted scalars must still be found to obtain the full constraints on the allowed brane configurations

Conclusions and outlooks

- Following [Abel, Dudas, Lewis, Partouche,'18], we are looking for super no-scale model (**exponentially suppressed potential**) without moduli instabilities
- This in an open string $T^4/\mathbb{Z}_2$ model with broken supersymmetry
- We expressed the super no-scale condition *via* the counting of massless degrees of freedom
- We expressed stability conditions for the open string Wilson lines masses
- The mass of the twisted scalars must still be found to obtain the full constraints on the allowed brane configurations

Conclusions and outlooks

- Following [Abel, Dudas, Lewis, Partouche,'18], we are looking for super no-scale model (**exponentially suppressed potential**) without moduli instabilities
- This in an open string $T^4/\mathbb{Z}_2$ model with broken supersymmetry
- We expressed the super no-scale condition *via* the counting of massless degrees of freedom
- We expressed stability conditions for the open string Wilson lines masses
- The mass of the twisted scalars must still be found to obtain the full constraints on the allowed brane configurations

Thank you for your attention!

Annulus partition function with discrete WL

$$
\begin{aligned}
\mathcal{A} = \frac{1}{4}\int_0^\infty \frac{\mathsf{d}\tau_2}{\tau_2^3}\Bigg\{ \Bigg[& (V_4O_4 + O_4V_4)\left(N_{ii'}N_{jj'}\,\frac{P^{(4)}_{\vec{m}+\vec{a}_i-\vec{a}_j}}{\eta^4} + D_{ii'}D_{jj'}\,\frac{W^{(4)}_{\vec{n}+\vec{a}_i-\vec{a}_j}}{\eta^4}\right) \\
& + (V_4O_4 - O_4V_4)\left(R^N_{ii'}R^N_{ij'} + R^D_{ii'}R^D_{ij'}\right)\left(\frac{2\eta}{\theta_2}\right)^2 + 2N_{ii'}D_{jj'}(O_4C_4 + V_4S_4)\left(\frac{\eta}{\theta_4}\right)^2 \\
& + 2e^{4i\pi\vec{a}_i\cdot\vec{a}_j}R^N_{ii'}R^D_{jj'}(O_4C_4 - V_4S_4)\left(\frac{\eta}{\theta_3}\right)^2\Bigg]\frac{P^{(2)}_{\vec{m}'+\vec{a}_{i'}-\vec{a}_{j'}}}{\eta^4} \\
& - \Bigg[(S_4S_4 + C_4C_4)\left(N_{ii'}N_{jj'}\,\frac{P^{(4)}_{\vec{m}+\vec{a}_i-\vec{a}_j}}{\eta^4} + D_{ii'}D_{jj'}\,\frac{W^{(4)}_{\vec{n}+\vec{a}_i-\vec{a}_j}}{\eta^4}\right) \\
& + (C_4C_4 - S_4S_4)\left(R^N_{ii'}R^N_{ij'} + R^D_{ii'}R^D_{ij'}\right)\left(\frac{2\eta}{\theta_2}\right)^2 + 2N_{ii'}D_{jj'}(S_4O_4 + C_4V_4)\left(\frac{\eta}{\theta_4}\right)^2 \\
& + 2e^{4i\pi\vec{a}_i\cdot\vec{a}_j}R^N_{ii'}R^D_{jj'}(S_4O_4 - C_4V_4)\left(\frac{\eta}{\theta_3}\right)^2\Bigg]\frac{P^{(2)}_{\vec{m}'+\vec{a}'_{\mathsf{s}}+\vec{a}_{i'}-\vec{a}_{j'}}}{\eta^4}\Bigg\}
\end{aligned}
$$

Möbius partition function with discrete WL

$$
\begin{aligned}
\mathcal{M} = -\frac{1}{4}\int_0^\infty \frac{\mathsf{d}\tau_2}{\tau_2^3}\Bigg\{ \Bigg[& (\hat{V}_4\hat{O}_4 + \hat{O}_4\hat{V}_4)\left(N_{ii'}\frac{P^{(4)}_{\vec{m}}}{\hat{\eta}^4} + D_{ii'}\frac{W^{(4)}_{\vec{n}}}{\hat{\eta}^4} \right) \\
& - (N_{ii'} + D_{ii'})(\hat{V}_4\hat{O}_4 - \hat{O}_4\hat{V}_4)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 \Bigg] \frac{P^{(2)}_{\vec{m}'}}{\hat{\eta}^4} \\
& - \Bigg[(\hat{C}_4\hat{C}_4 + \hat{S}_4\hat{S}_4)\left(N_{ii'}\frac{P^{(4)}_{\vec{m}}}{\hat{\eta}^4} + D_{ii'}\frac{W^{(4)}_{\vec{n}}}{\hat{\eta}^4} \right) \\
& - (N_{ii'} + D_{ii'})(\hat{C}_4\hat{C}_4 - \hat{S}_4\hat{S}_4)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 \Bigg] \frac{P^{(2)}_{\vec{m}'+\vec{a}'_{\mathsf{s}}}}{\hat{\eta}^4} \Bigg\}
\end{aligned}
$$

Massless spectrum

Bosons:

$$V_4O_4\left[n_{ii'}\bar{n}_{ii'}+d_{ii'}\bar{d}_{ii'}\right]$$

$$+O_4V_4\left[\frac{n_{ii'}(n_{ii'}-1)}{2}+\frac{\bar{n}_{ii'}(\bar{n}_{ii'}-1)}{2}+\frac{d_{ii'}(d_{ii'}-1)}{2}+\frac{\bar{d}_{ii'}(\bar{d}_{ii'}-1)}{2}\right]$$

$$+\frac{O_4C_4}{2}\left[\left(1-e^{4i\pi\vec{a}_i\cdot\vec{a}_j}\right)\left(n_{ii'}d_{ji'}+\bar{n}_{ii'}\bar{d}_{ji'}\right)+\left(1+e^{4i\pi\vec{a}_i\cdot\vec{a}_j}\right)\left(n_{ii'}\bar{d}_{ji'}+\bar{n}_{ii'}d_{ji'}\right)\right]$$

Fermions:

$$C_4C_4\left[n_{i(2i'-1)}\bar{n}_{i(2i')}+\bar{n}_{i(2i'-1)}n_{i(2i')}+d_{i(2i'-1)}\bar{d}_{i(2i')}+\bar{d}_{i(2i'-1)}d_{i(2i')}\right]$$

$$+S_4S_4\left[n_{i(2i'-1)}n_{i(2i')}+\bar{n}_{i(2i'-1)}\bar{n}_{i(2i')}+d_{i(2i'-1)}d_{i(2i')}+\bar{d}_{i(2i'-1)}\bar{d}_{i(2i')}\right]$$

$$+\frac{S_4O_4}{2}\Big[\left(1-e^{4i\pi\vec{a}_i\cdot\vec{a}_j}\right)\left(n_{i(2i'-1)}d_{j(2i')}+n_{i(2i')}d_{j(2i'-1)}+\bar{n}_{i(2i'-1)}\bar{d}_{j(2i')}+\bar{n}_{i(2i')}\bar{d}_{j(2i'-1)}\right)$$

$$+\left(1+e^{4i\pi\vec{a}_i\cdot\vec{a}_j}\right)\left(n_{i(2i'-1)}\bar{d}_{j(2i')}+n_{i(2i')}\bar{d}_{j(2i'-1)}+\bar{n}_{i(2i'-1)}d_{j(2i')}+\bar{n}_{i(2i')}d_{j(2i'-1)}\right)\Big]$$