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FOR FUNDAMENTAL PHYSICS

CFTs at Large Charge

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University of Bern

based on arXiv:1505.01537, 1610.04495, 1707.00710, 1809.06371, 1902.09542, 1905.00026, 1909.02571, and work in progress with:

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F. Sannino (Odense), M. Watanabe (Bern)

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Large charge Q becomes controlling parameter in a perturbative expansion!

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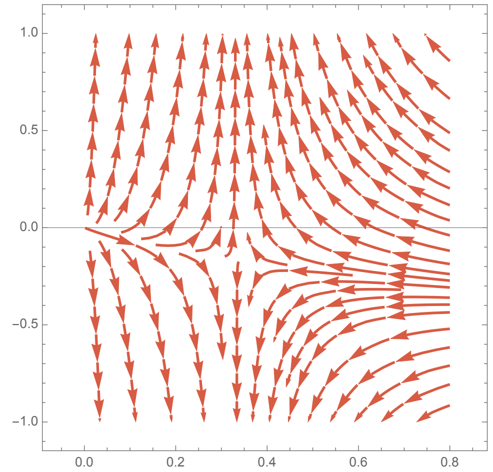

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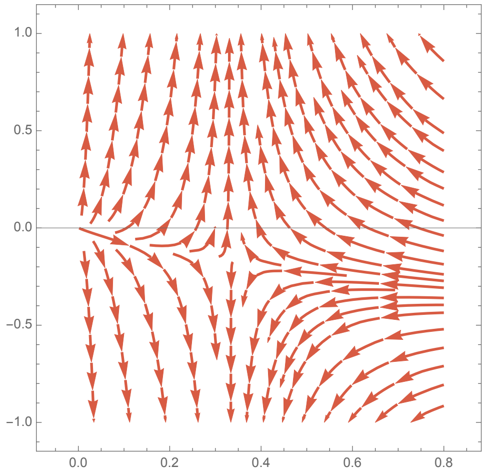

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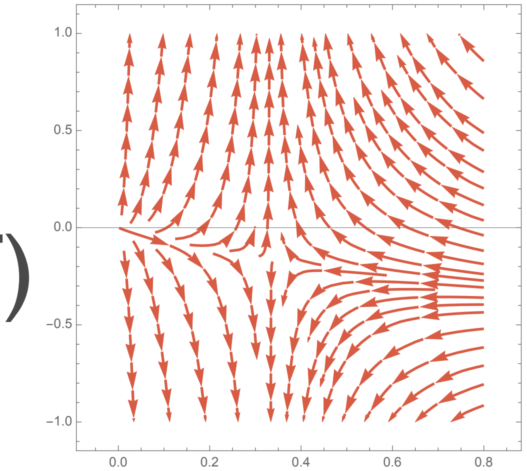

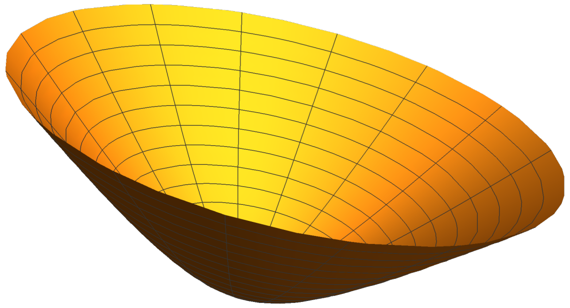

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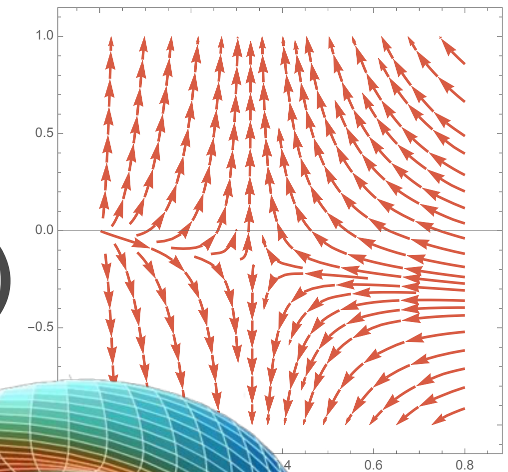

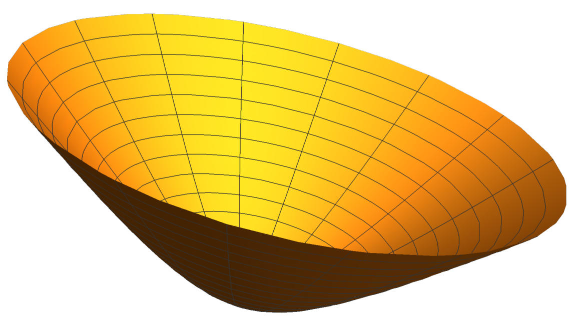

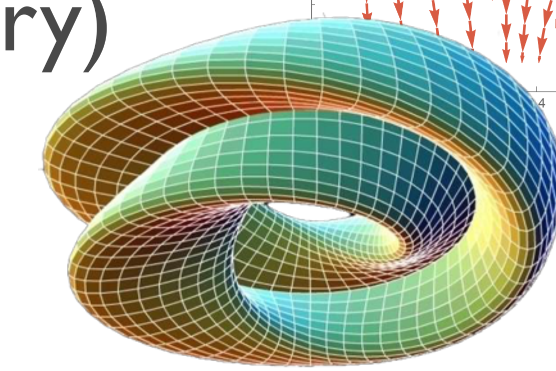

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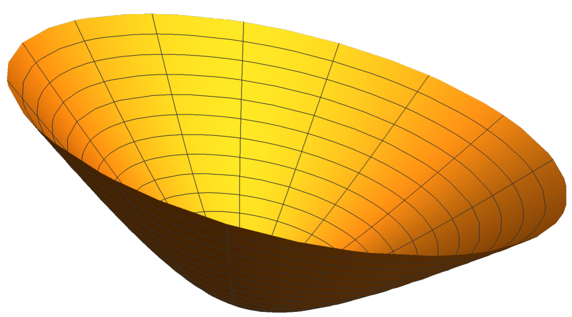

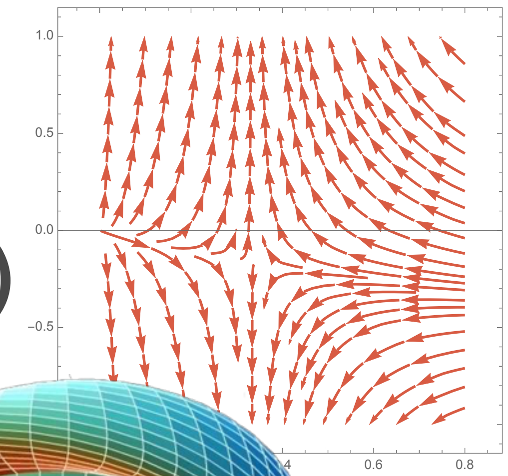

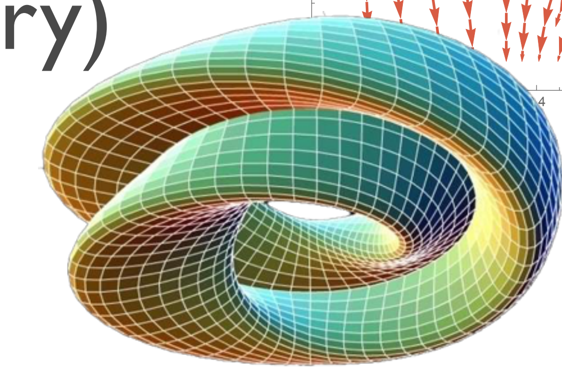

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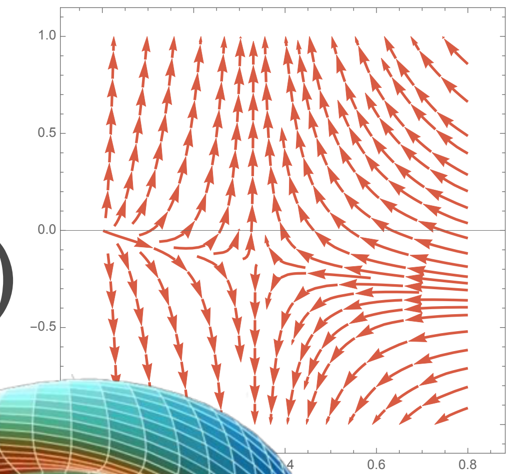

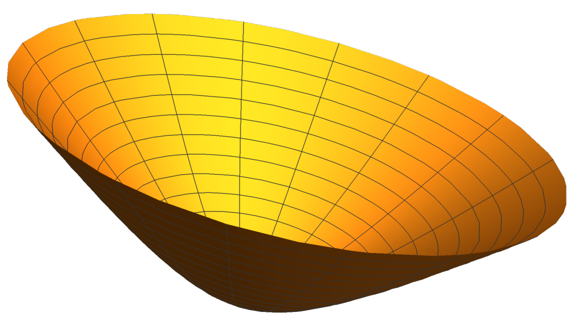

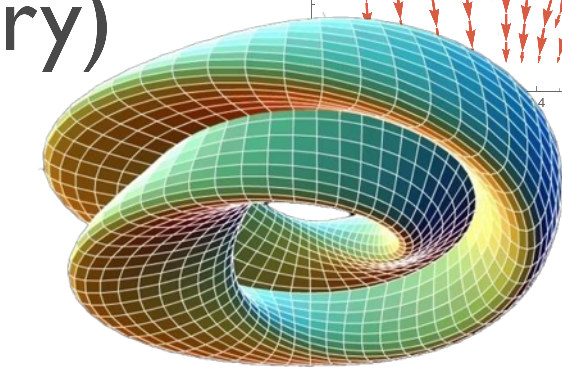

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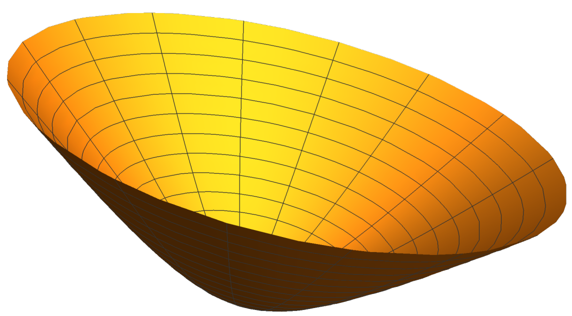

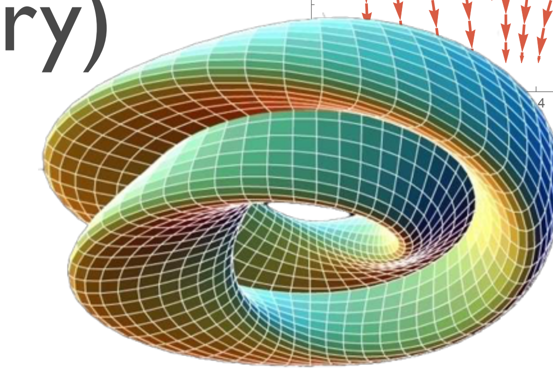

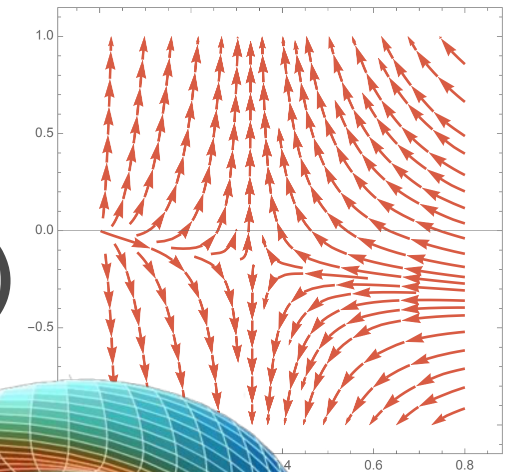

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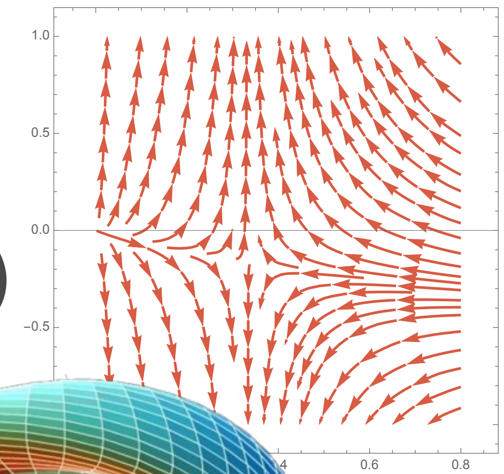

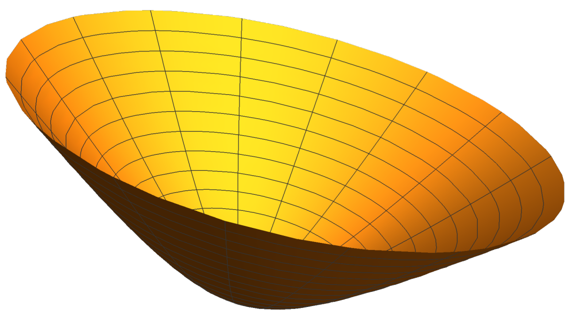

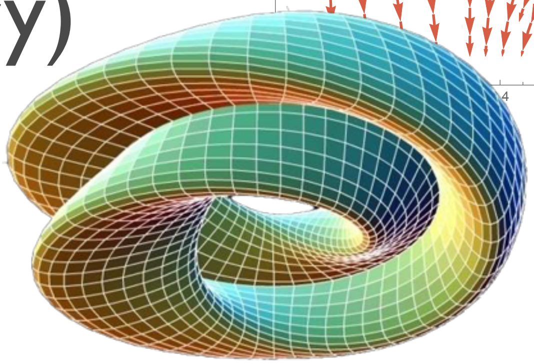

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Possibilities: analytic (2d), conformal bootstrap (d>2), lattice calculations, non-perturbative methods…
Prime candidate for the large-charge approach!
(Also: they come with a lot of space-time symmetry that will help us in practice to constrain the eff. action.)

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Assume that also in the IR, we have the same order parameter and that it transforms the same way under the global symmetry.

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- compute the quantum fluctuations to verify that they are parametrically small when Q >> 1.

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Wilsonian action has only a handful of terms that are not suppressed by the large charge. Useful!

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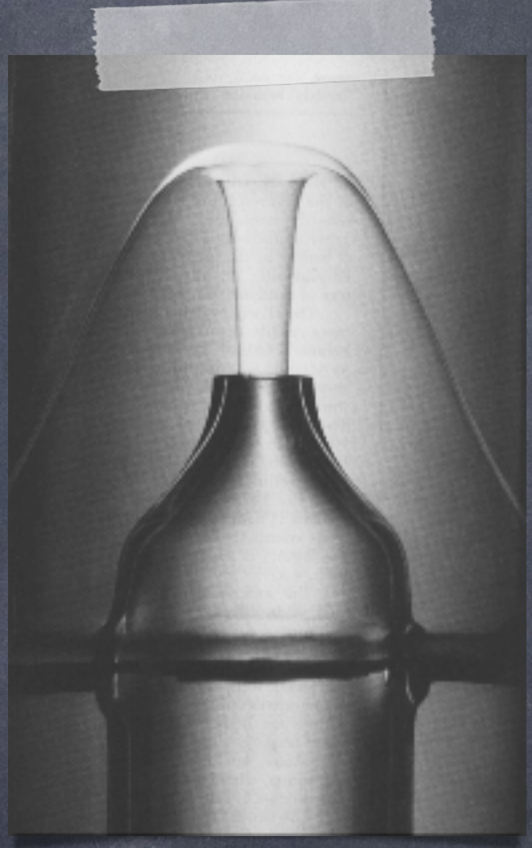

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Classical solution at lowest energy and fixed global charge becomes the vacuum of the quantum theory.

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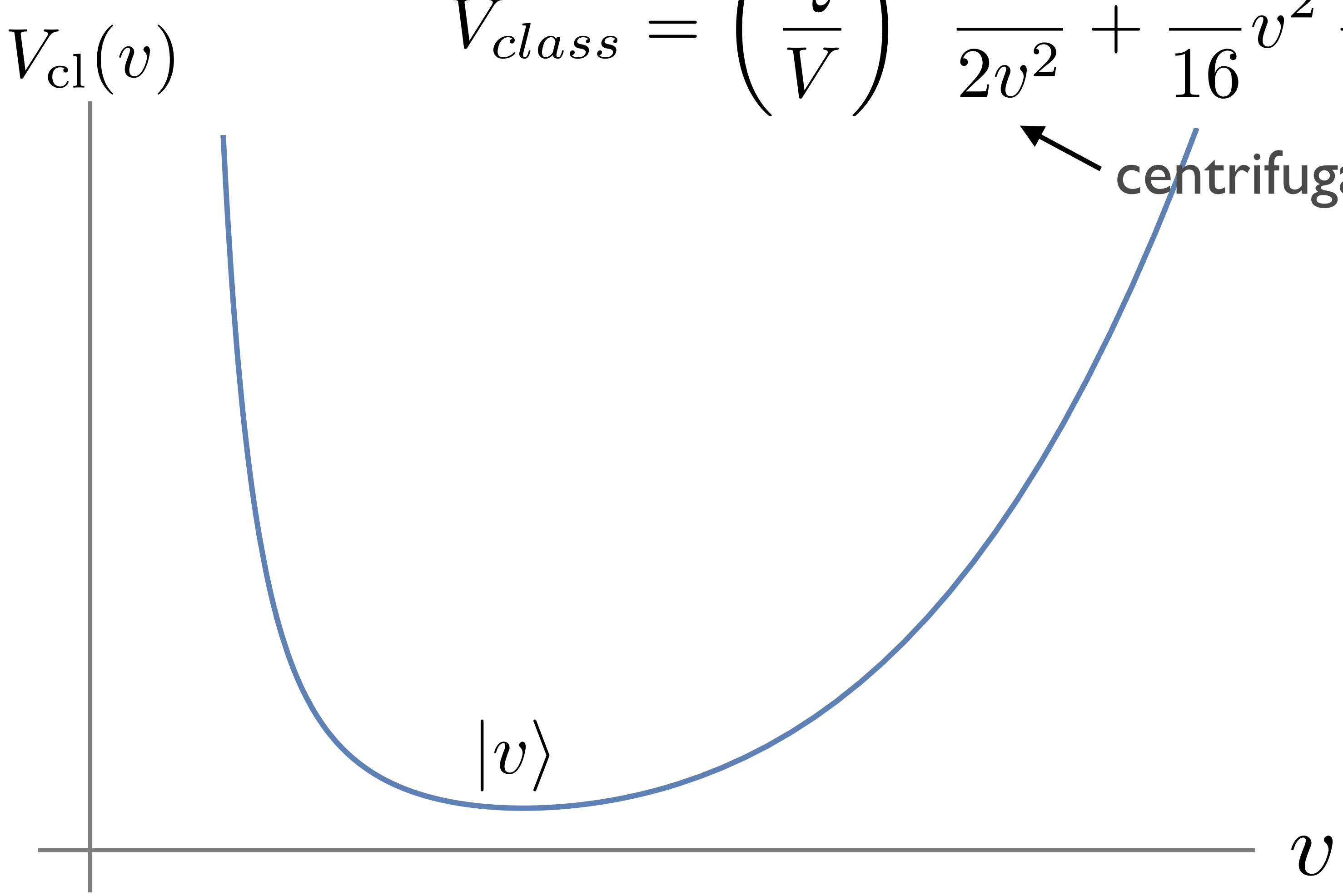

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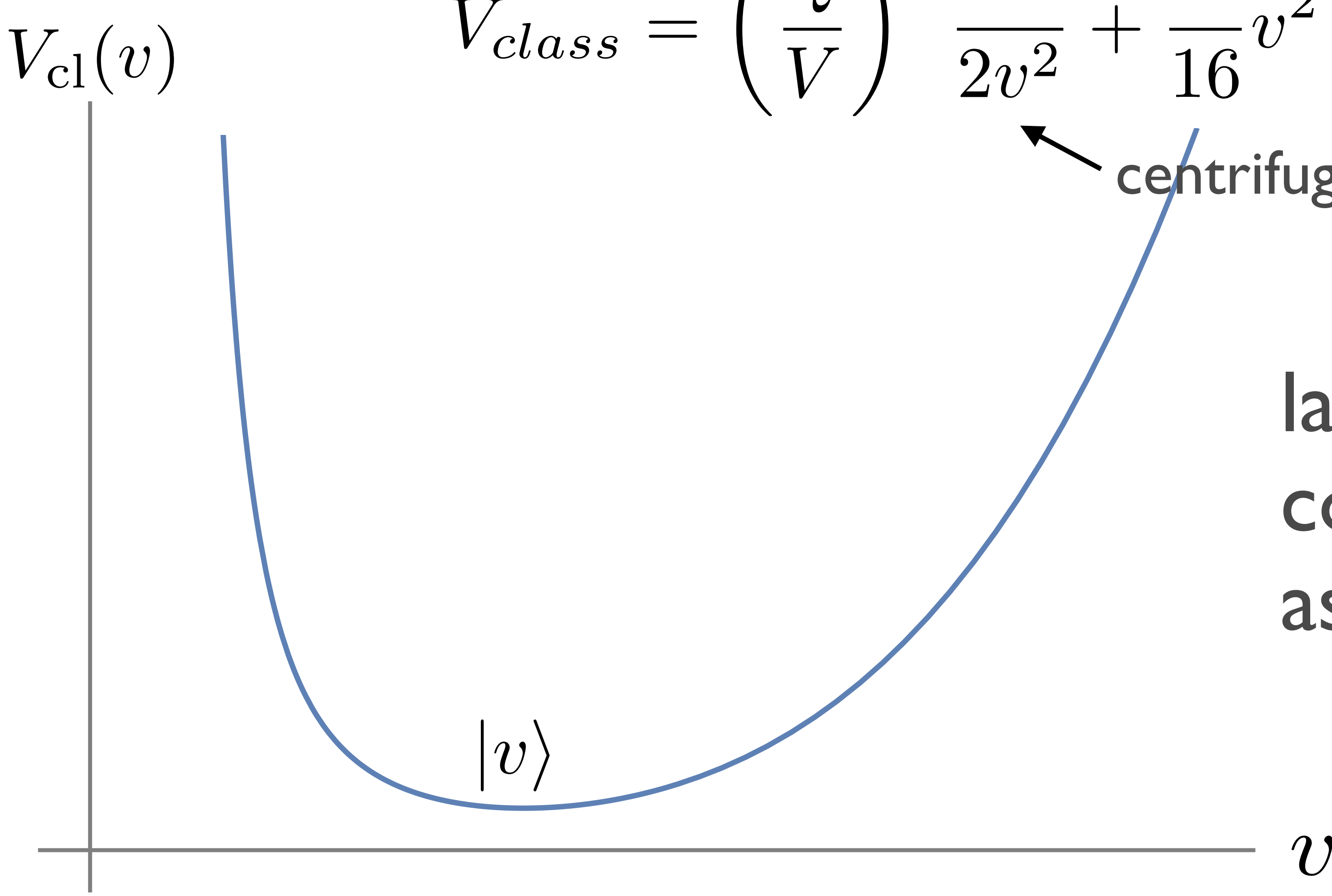

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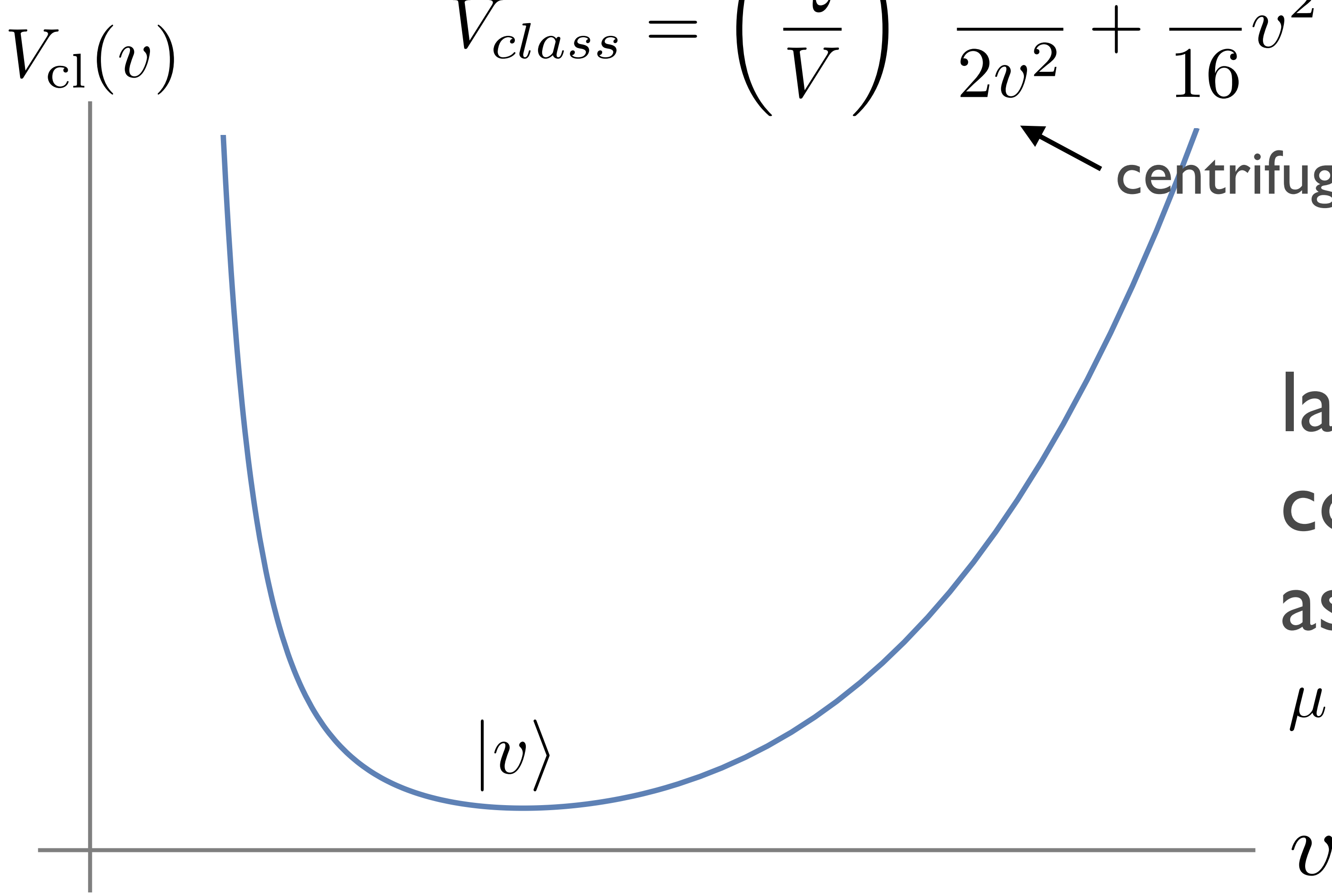

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For homogeneous solutions, there are no other terms contributing to the effective Lagrangian at non-negative ρ-scaling for d>1.

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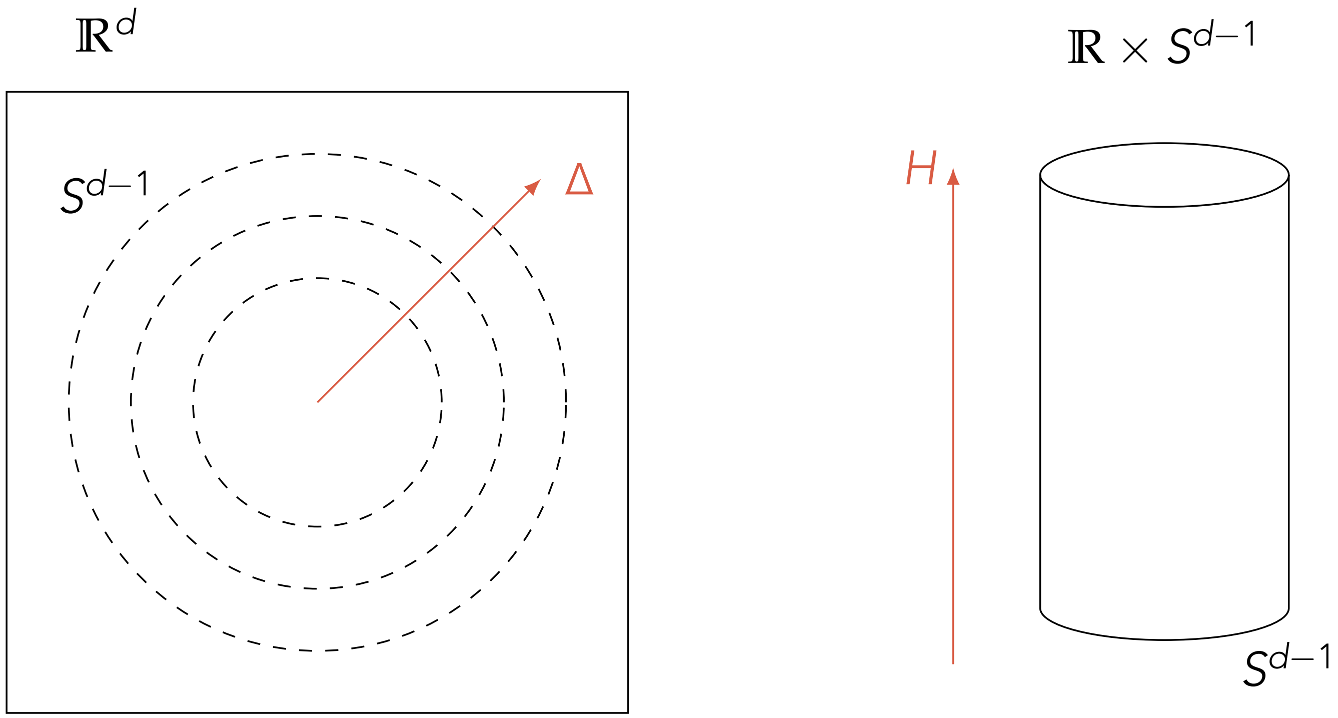

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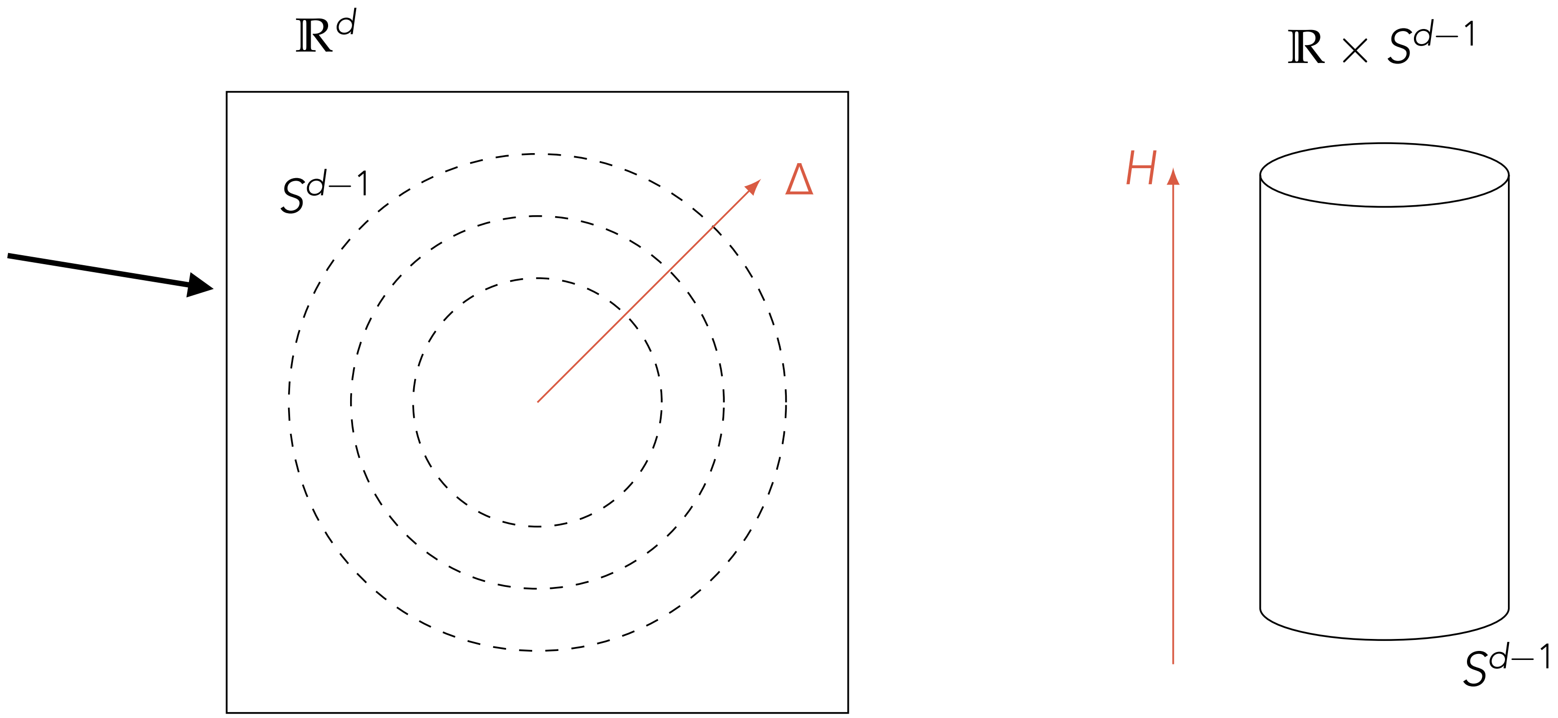

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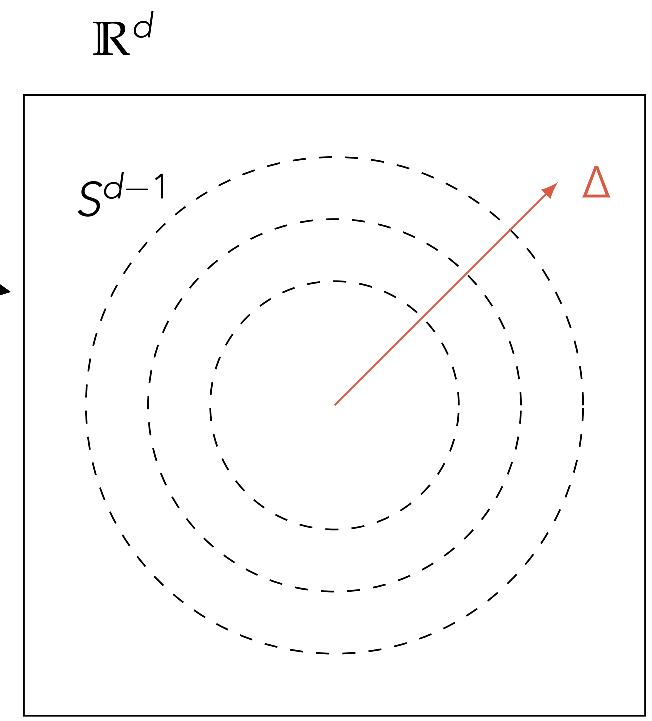

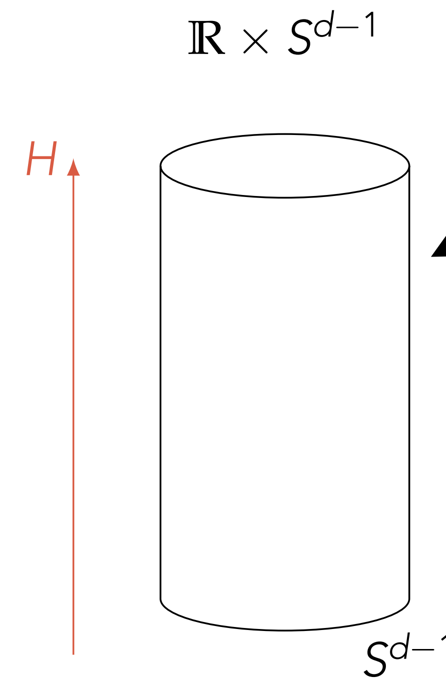

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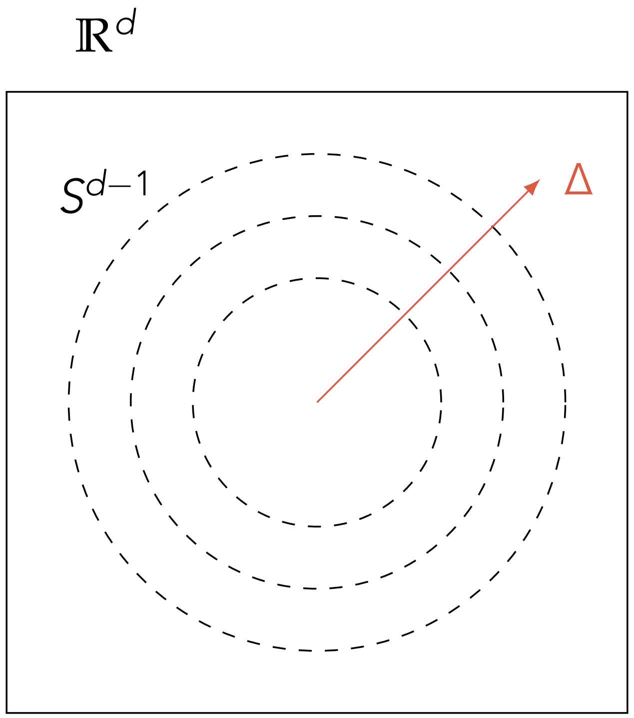

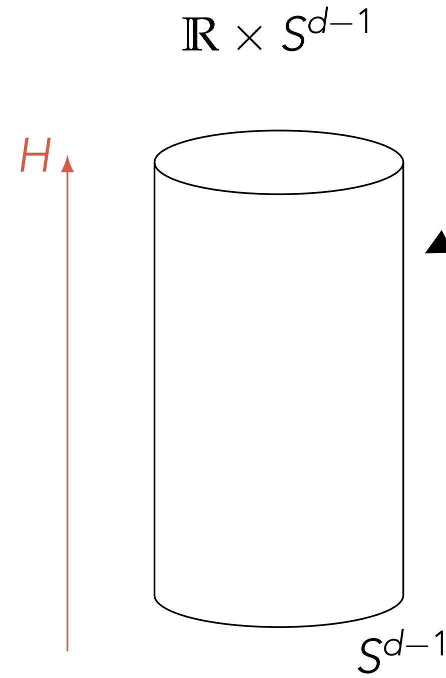

energy

Conformal dimension of lowest operator of charge Q:

The O(2) model

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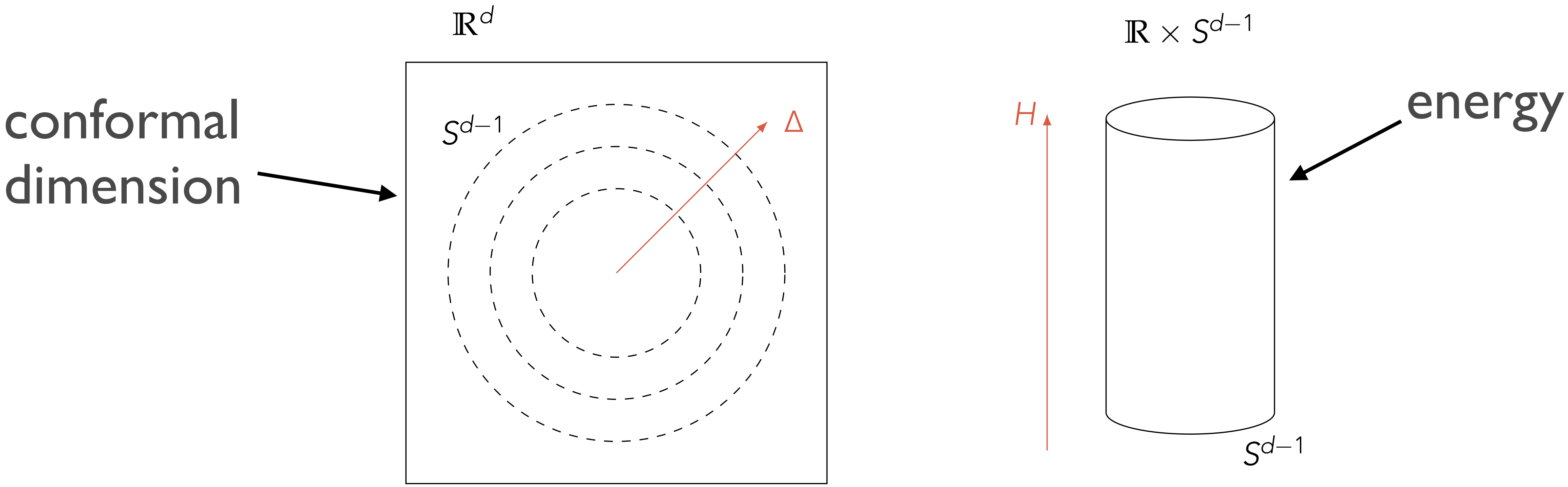

Conformal dimension of lowest operator of charge Q:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

S. Hellerman, D. Orlando, S. R., M. Watanabe, arXiv:1505.01537 [hep-th]

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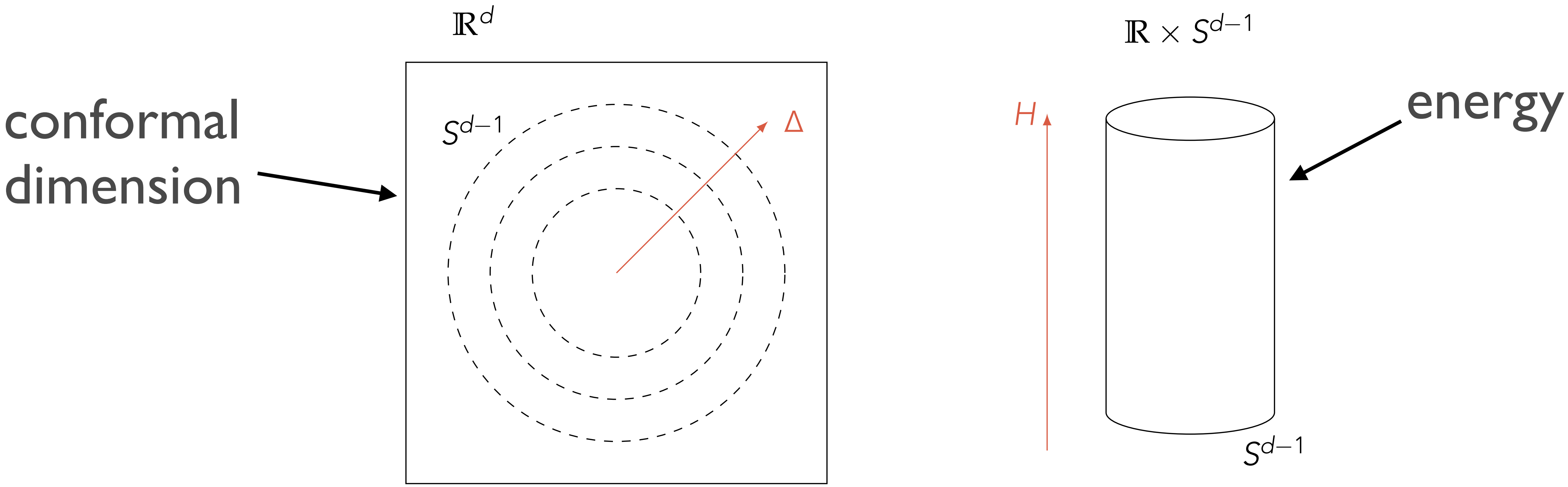

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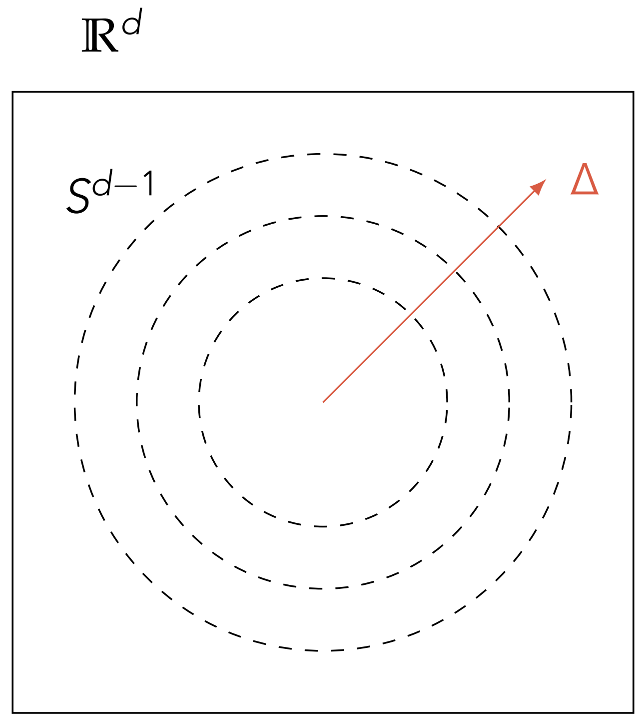

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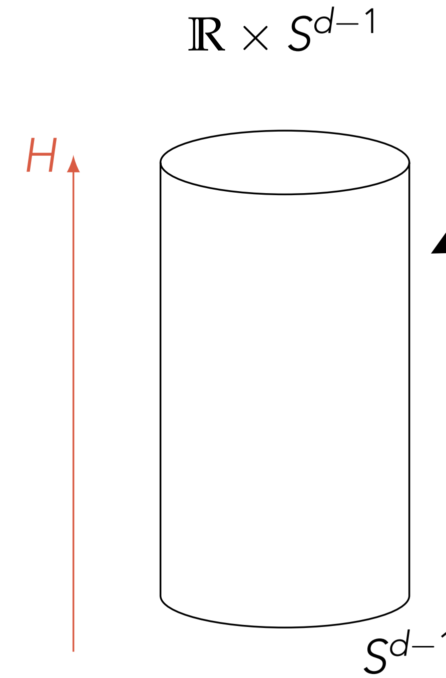

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$$E_{\text{VAC}} = \frac{1}{2\sqrt{2}r} \int \frac{\mathrm{d}\omega}{2\pi} \sum_{l=0}^{\infty} (2l+1) \log(\omega^2 + l(l+1)) = \frac{1}{2\sqrt{2}r} \zeta(-1/2 | S^2) = -\frac{0.0937\ldots}{r}$$

The O(2) model

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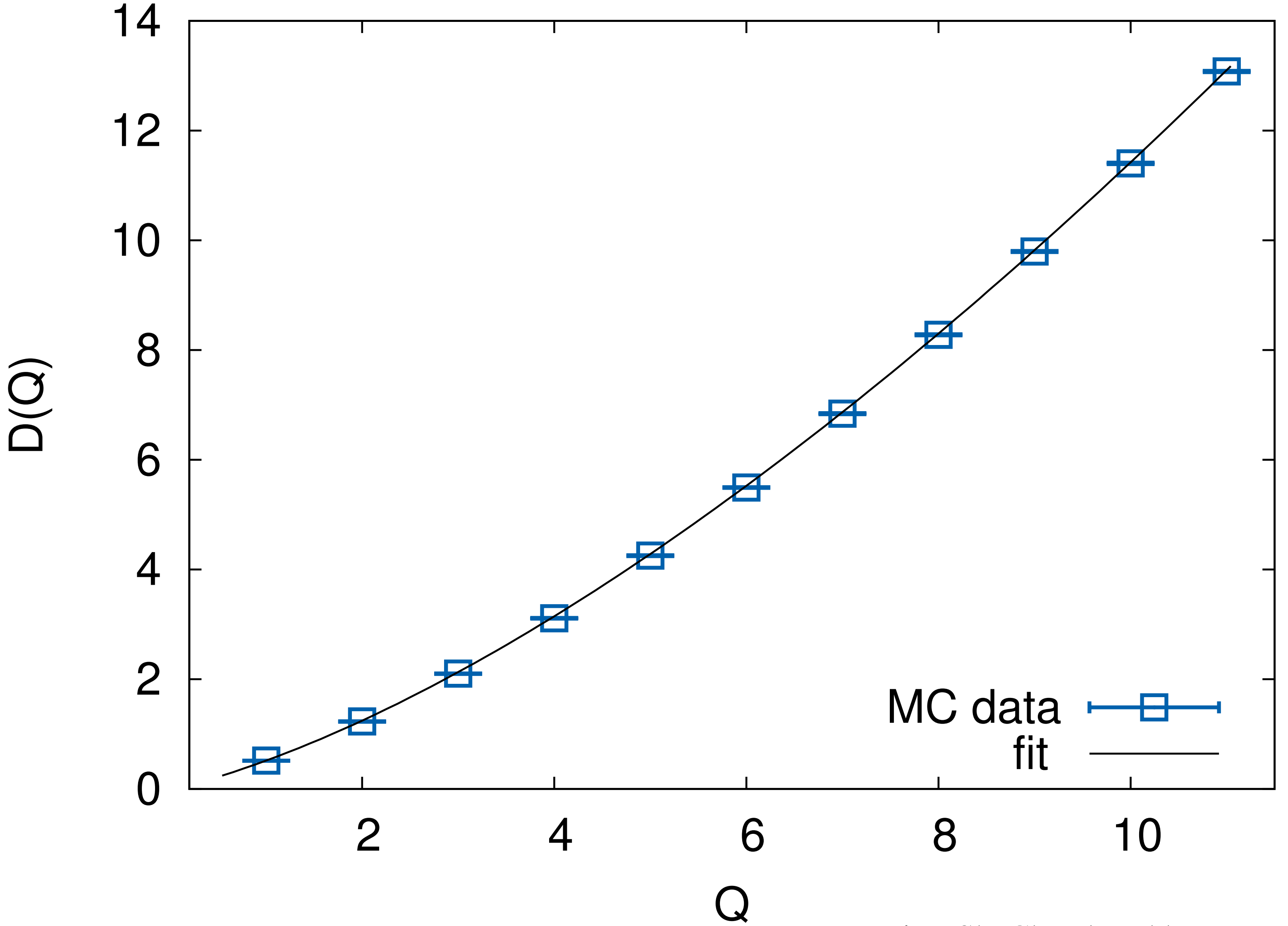

D. Banerjee, Sh. Chandrasekharan, D. Orlando [hep-th/1707.00711]

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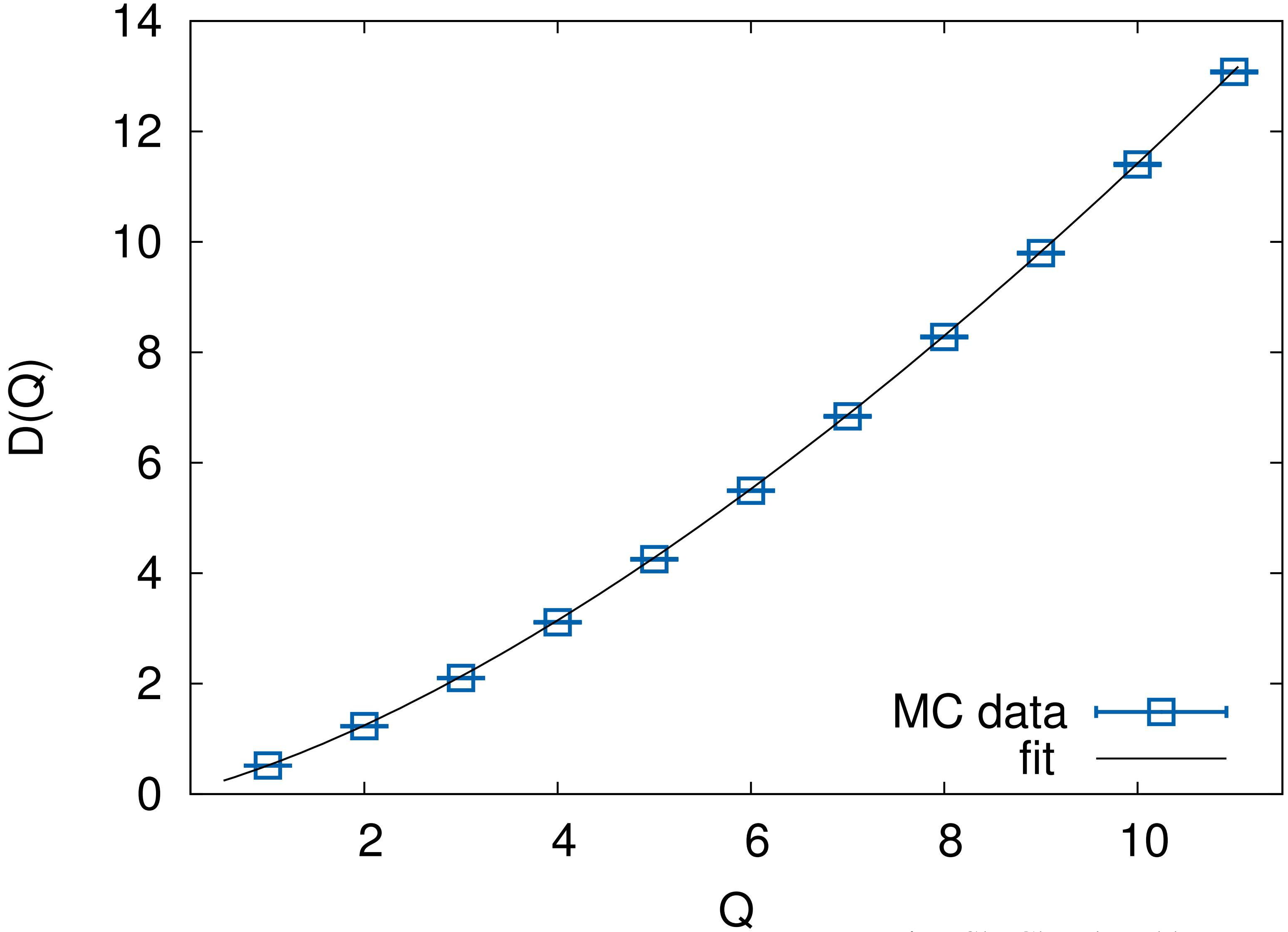

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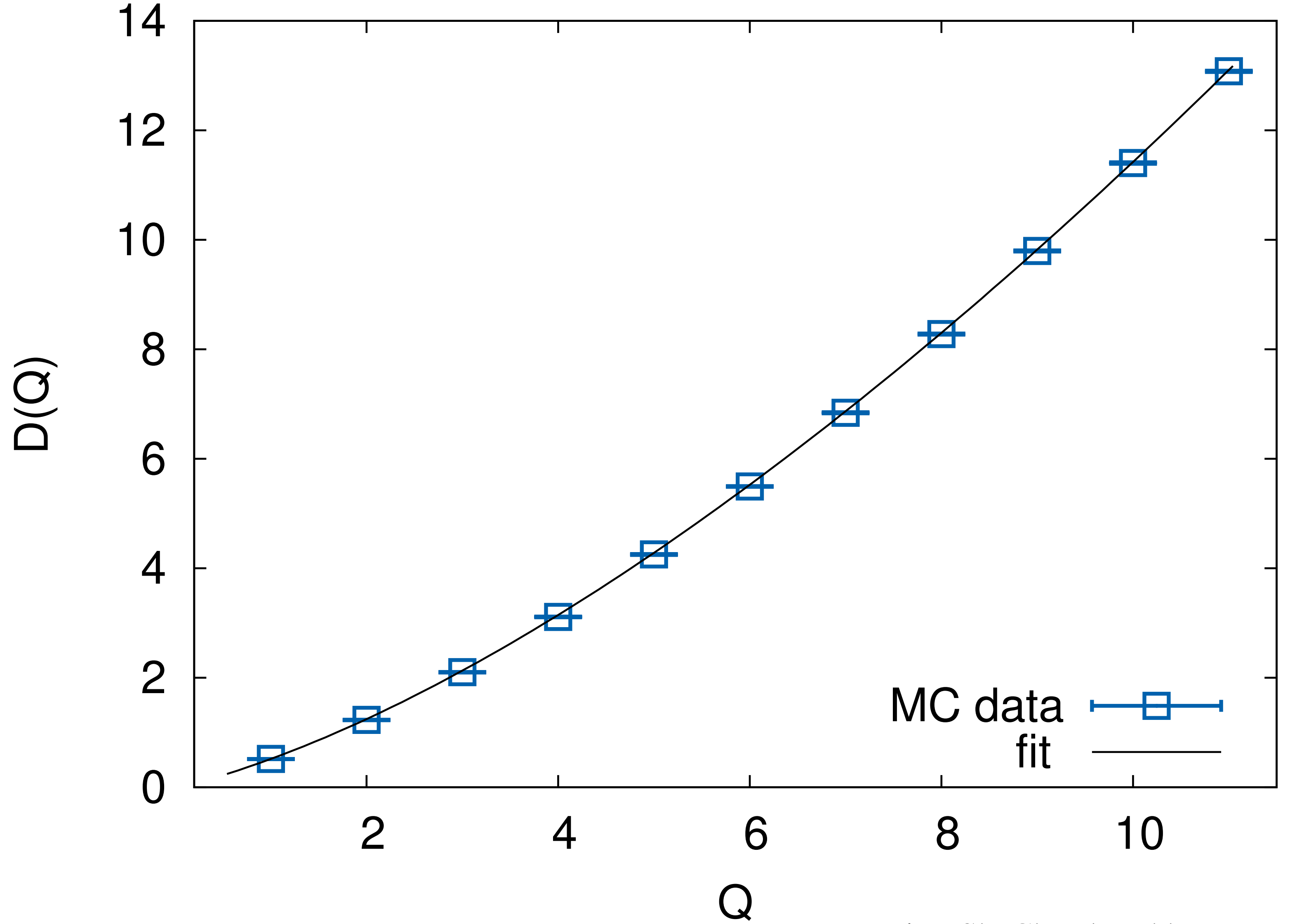

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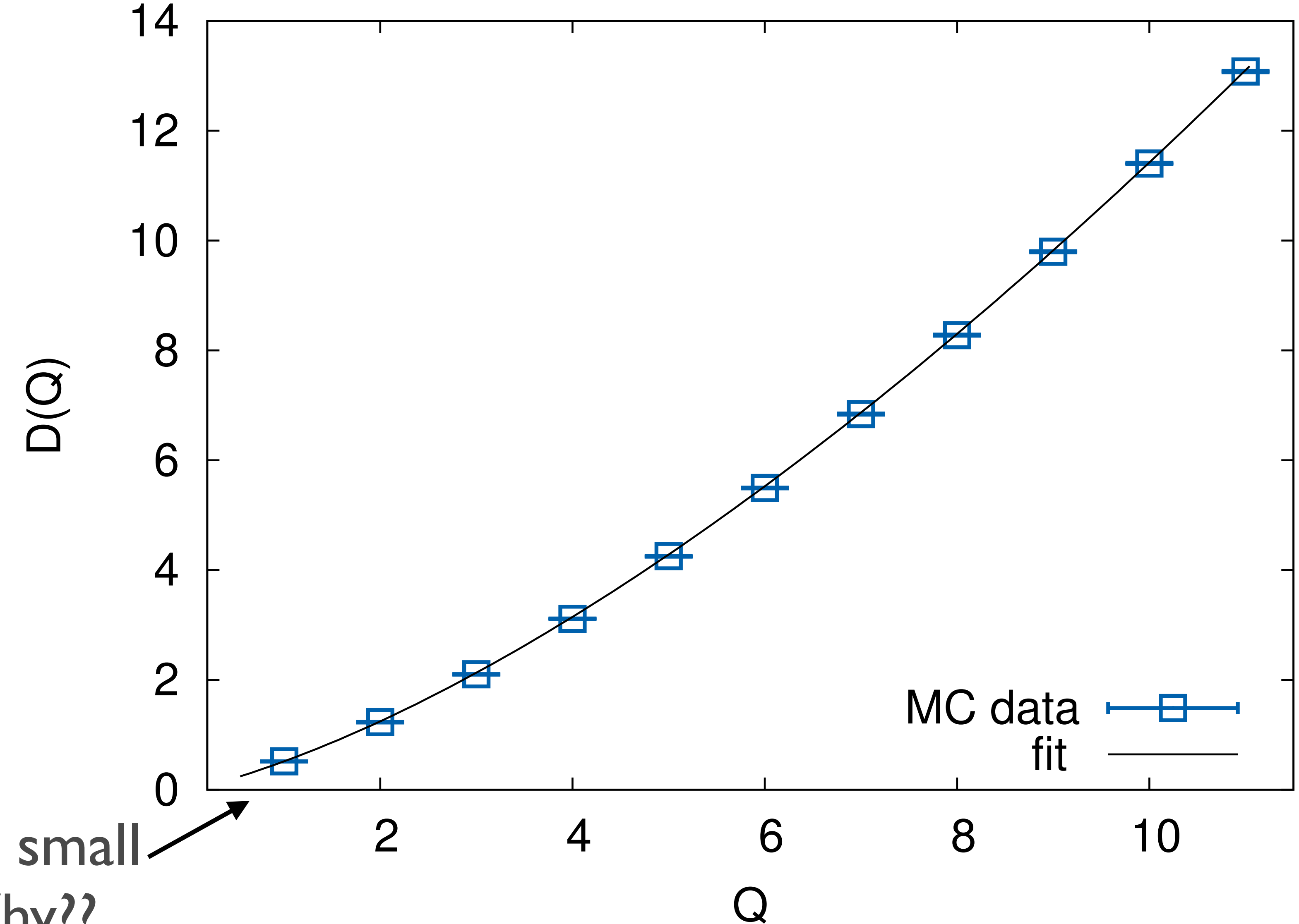

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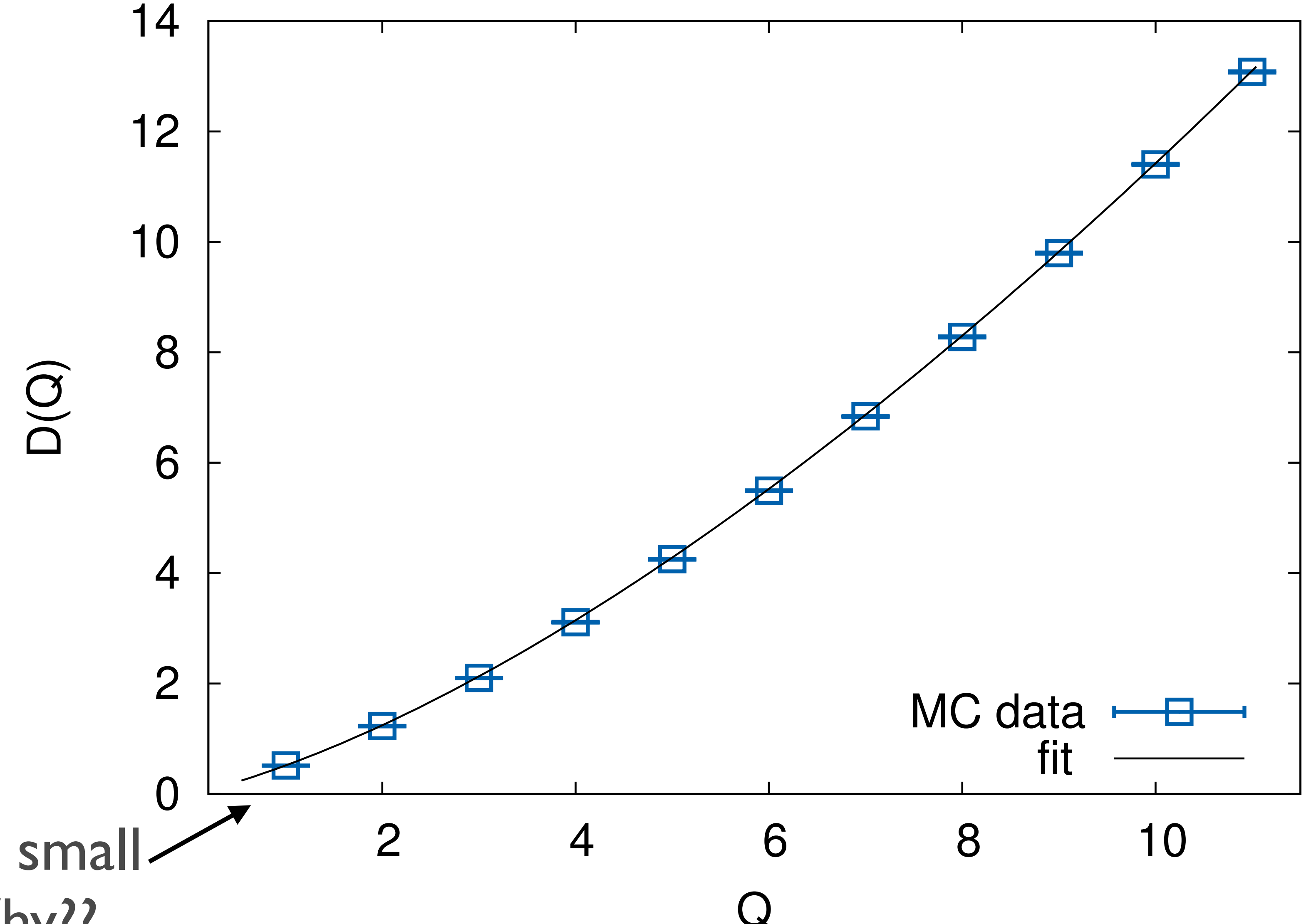

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Beyond O(2)

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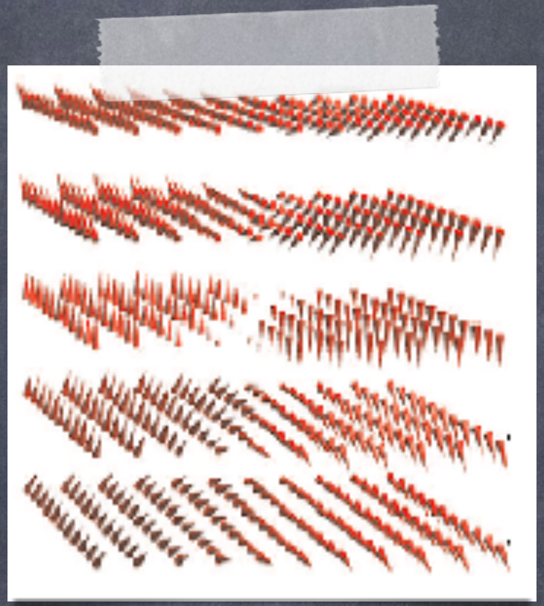

Beyond O(2): 3d O(2n) vector model

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For quantum description, write effective theory for fluctuations around the ground state.

Expand Lagrangian around the ground state

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U(1) sector: $\varphi_k = \frac{1}{\sqrt{2}}\, e^{i\mu t + i\hat{\phi}_{2k}/v}\left(v + \hat{\phi}_{2k-1}\right)$ $\quad$ $\begin{cases}\hat{\phi}_{2k-1} \to \hat{\phi}_{2k-1}\\ \hat{\phi}_{2k} \to \hat{\phi}_{2k} + \theta\,,\end{cases}$

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Find inverse propagators and dispersion relations.

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Non-relativistic Goldstones have no zero-point energy and do not contribute to the conformal dimensions. Ground-state energy again determined by a single relativistic Goldstone.

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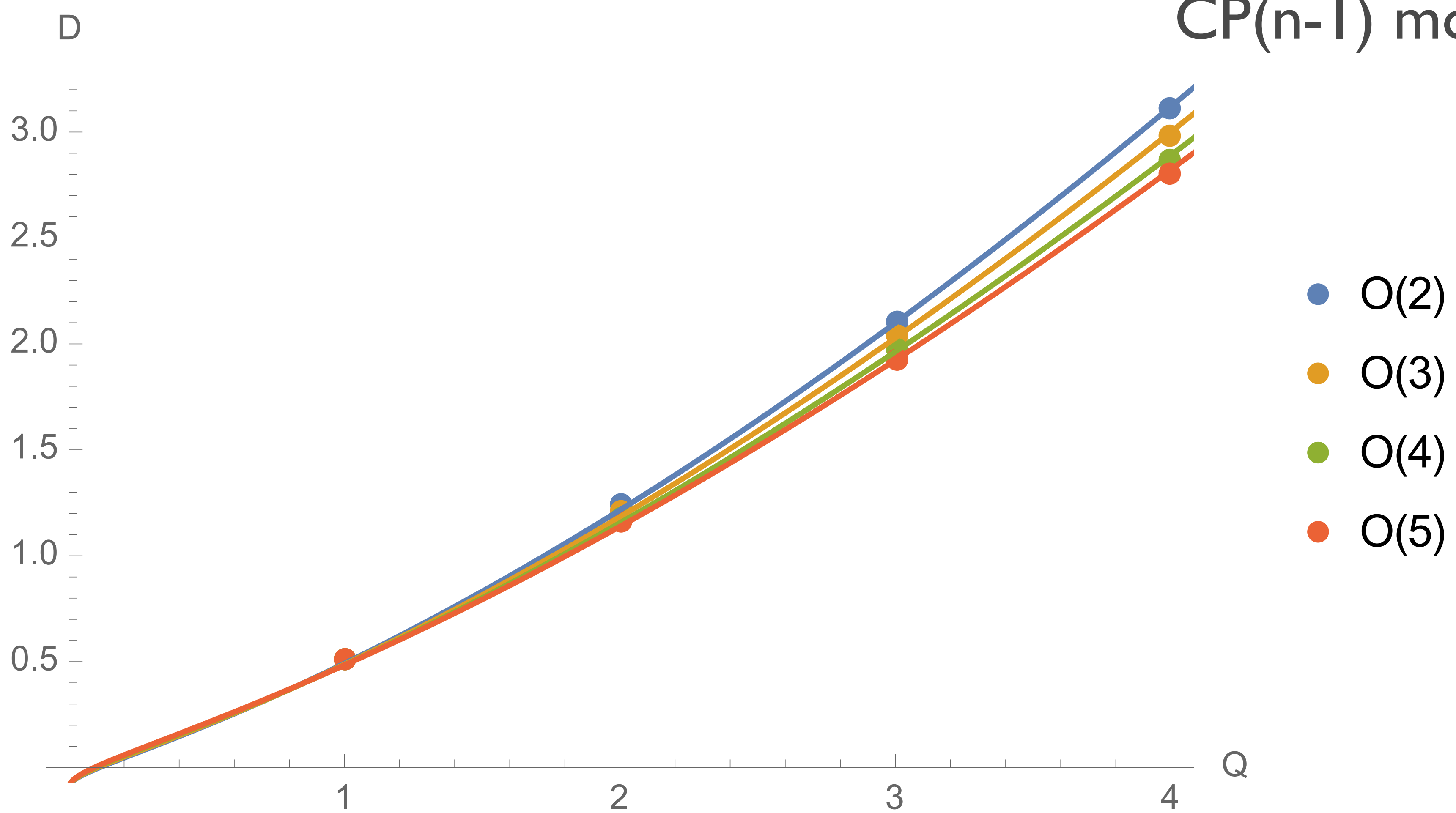

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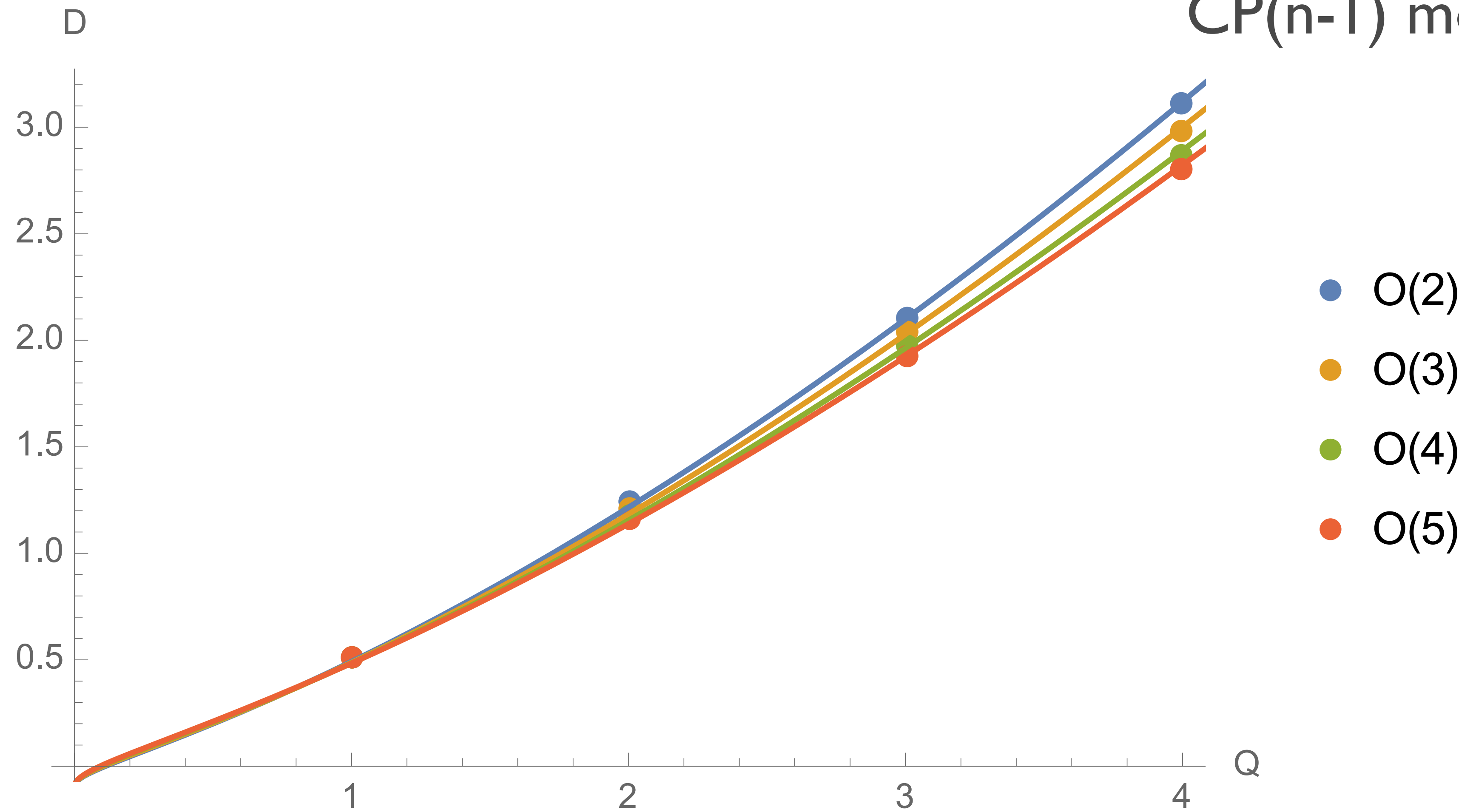

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$c_{3/2}$ decreases, $c_{1/2}$ increases with increasing n

The O(2n) vector model

New lattice data for O(4) model:

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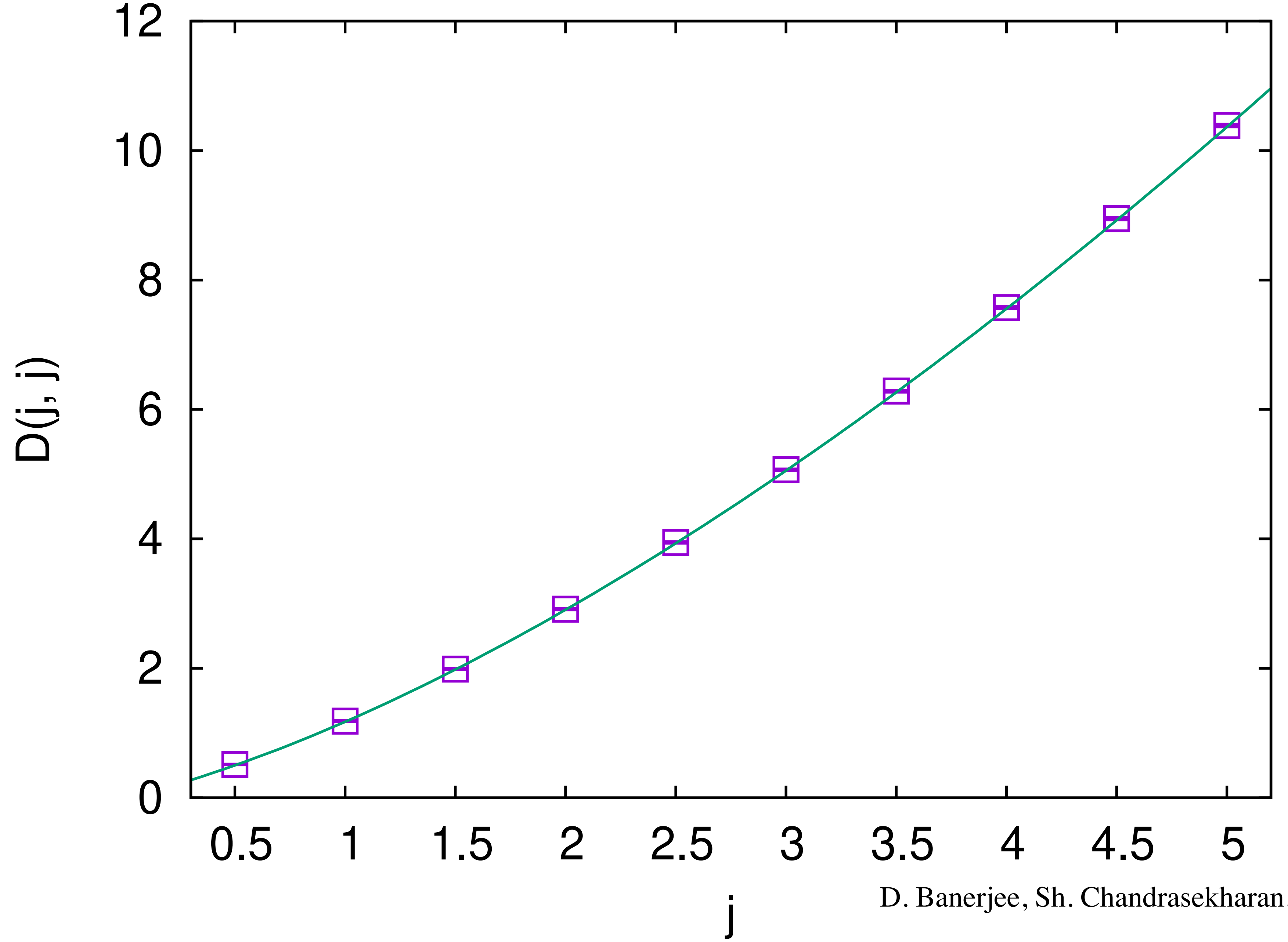

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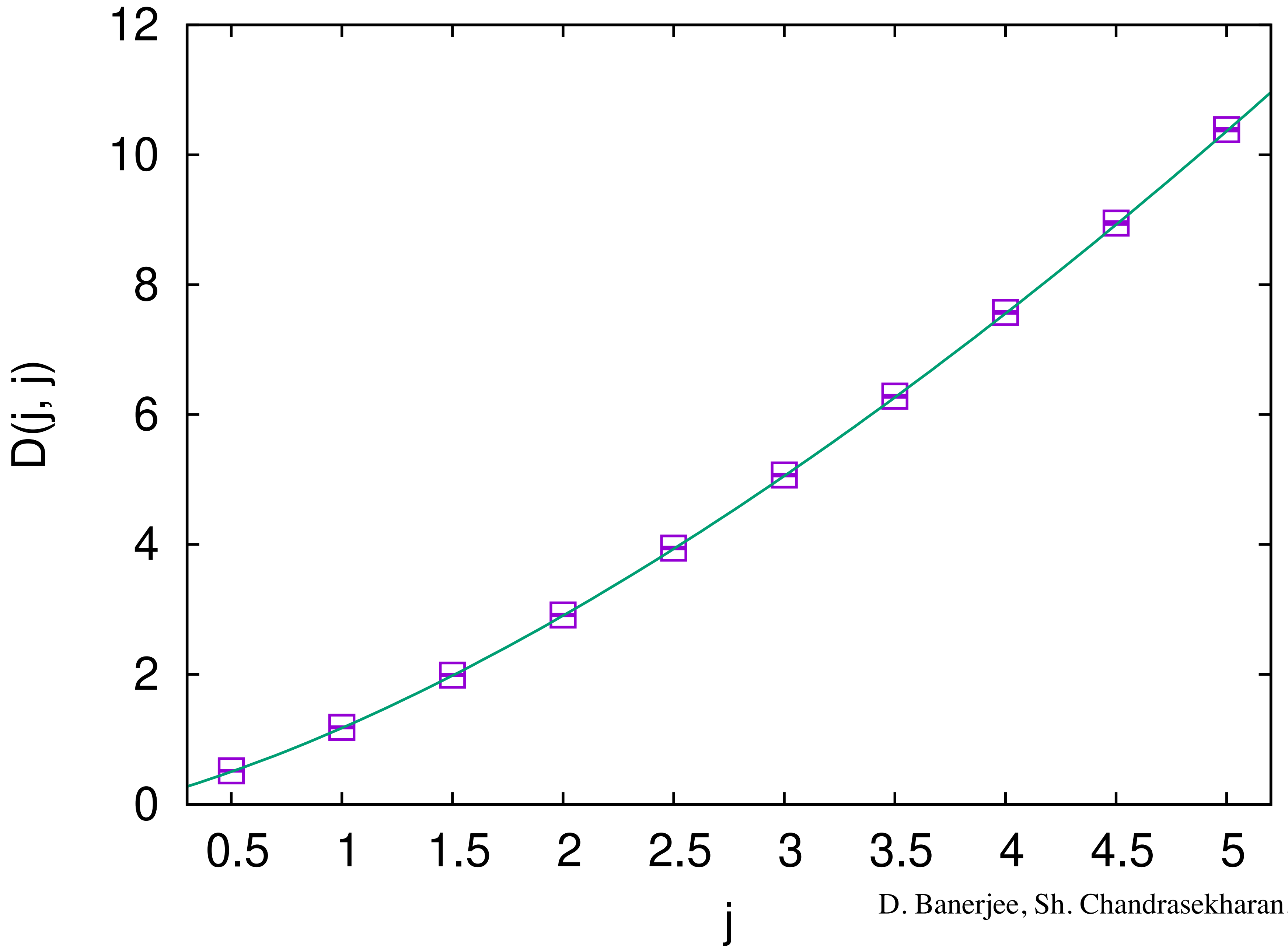

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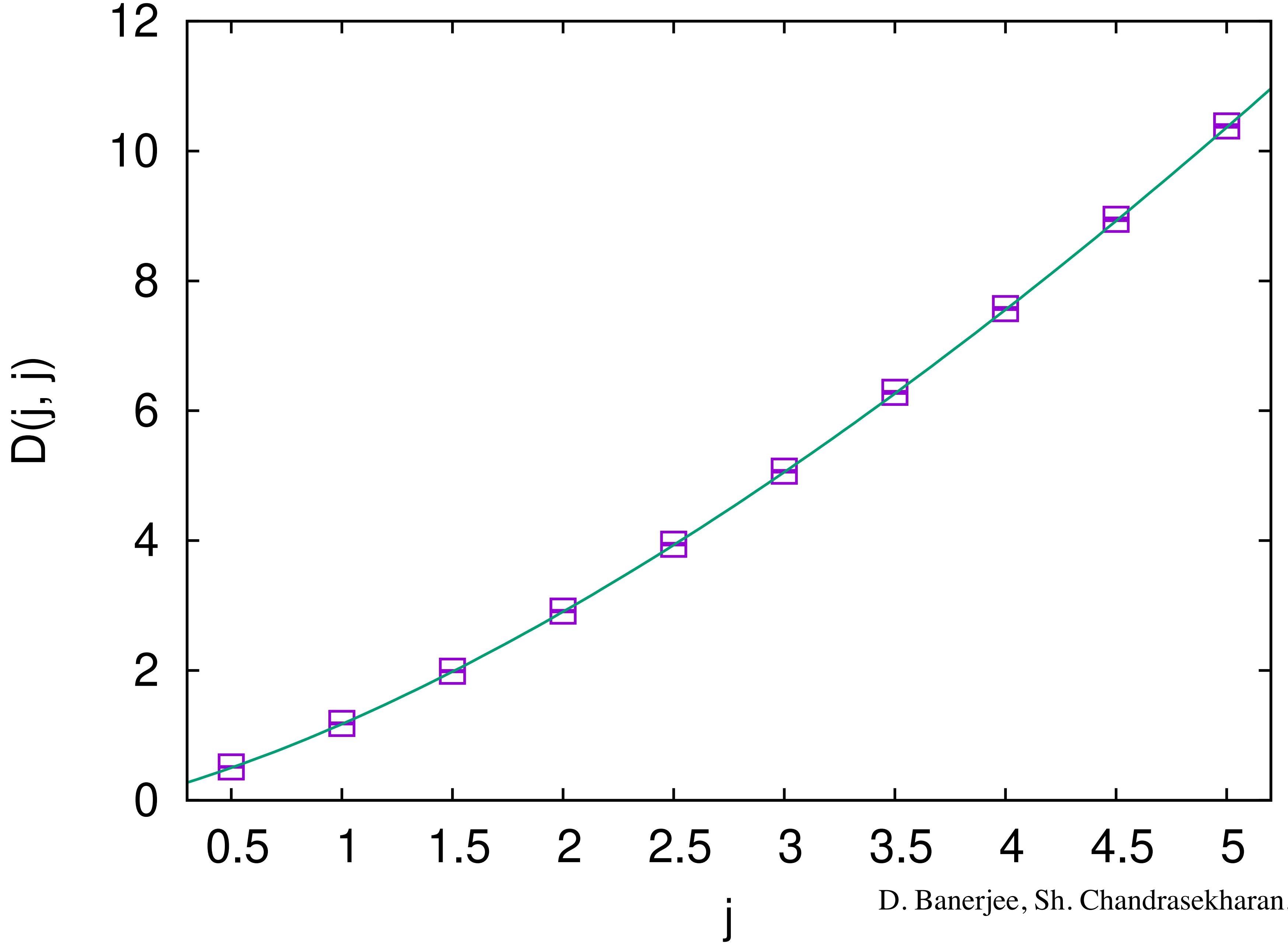

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Again excellent agreement with large-Q prediction!

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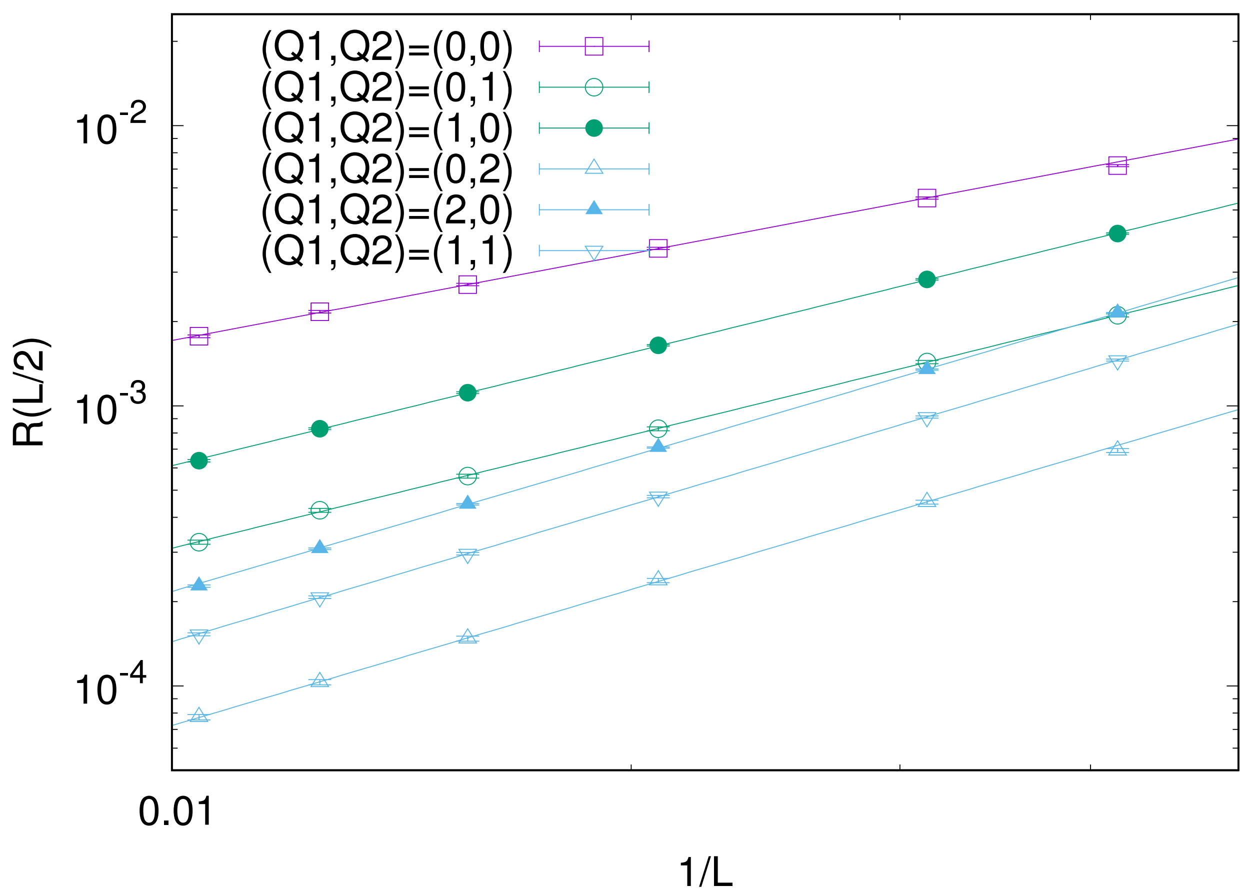

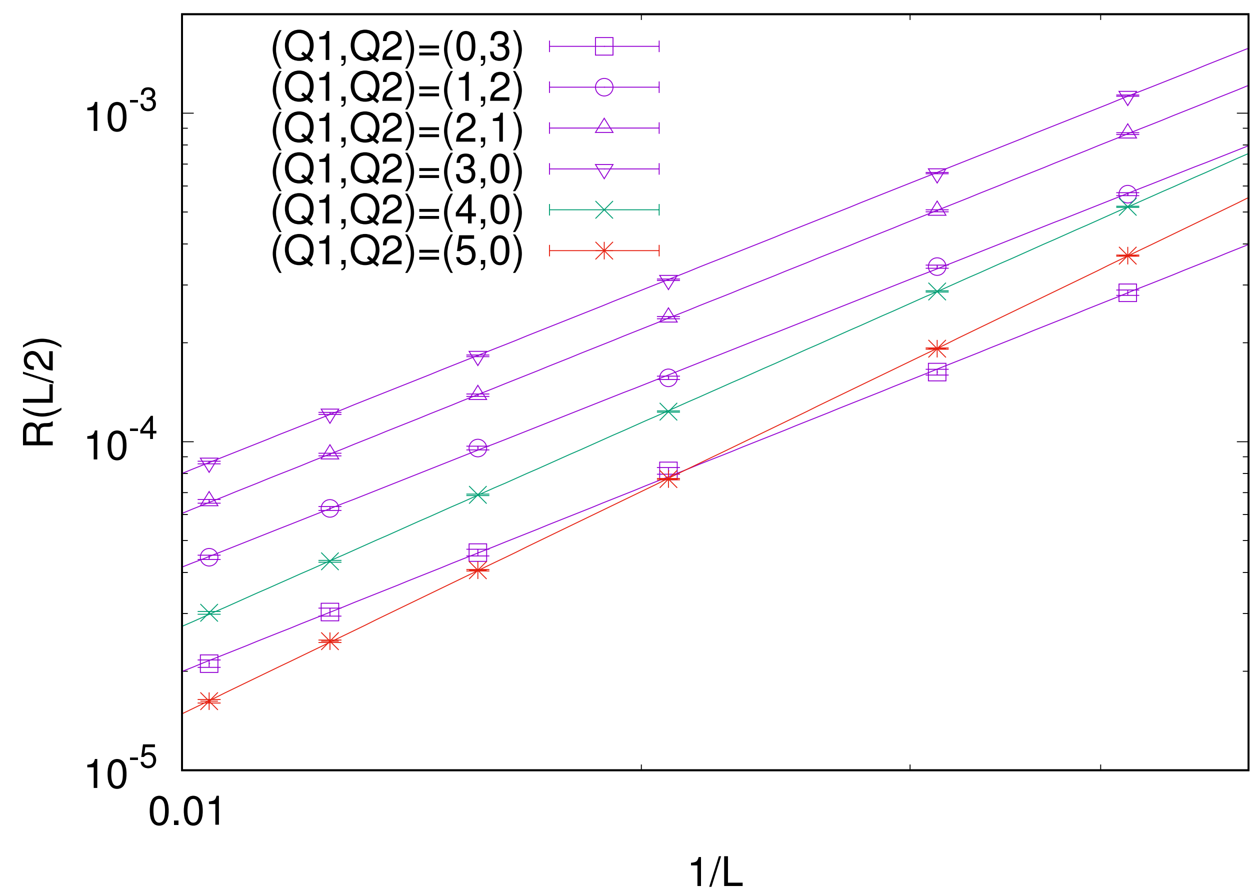

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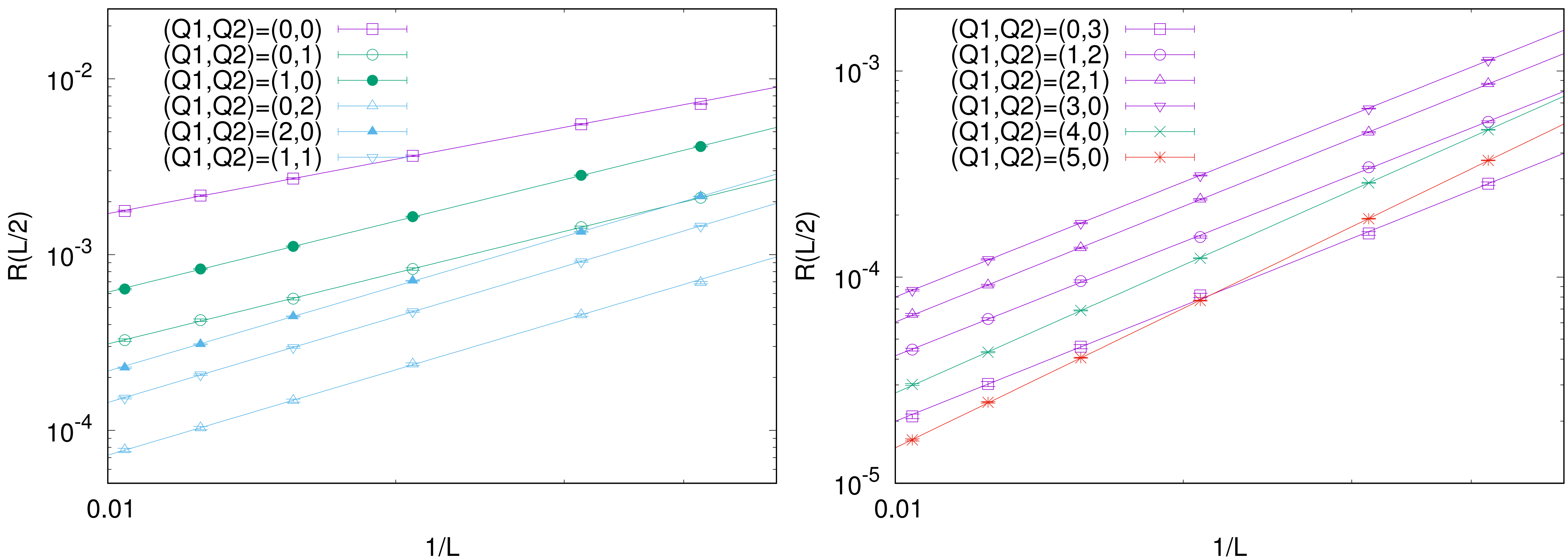

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Parallel lines in log/log plot: conformal dimensions are the same!

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  - we checked U(1), O(2n) vector models, SU(N) matrix models

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- Further lattice simulations: inhomogeneous sector, general O(N) Chandrasekharan et al.

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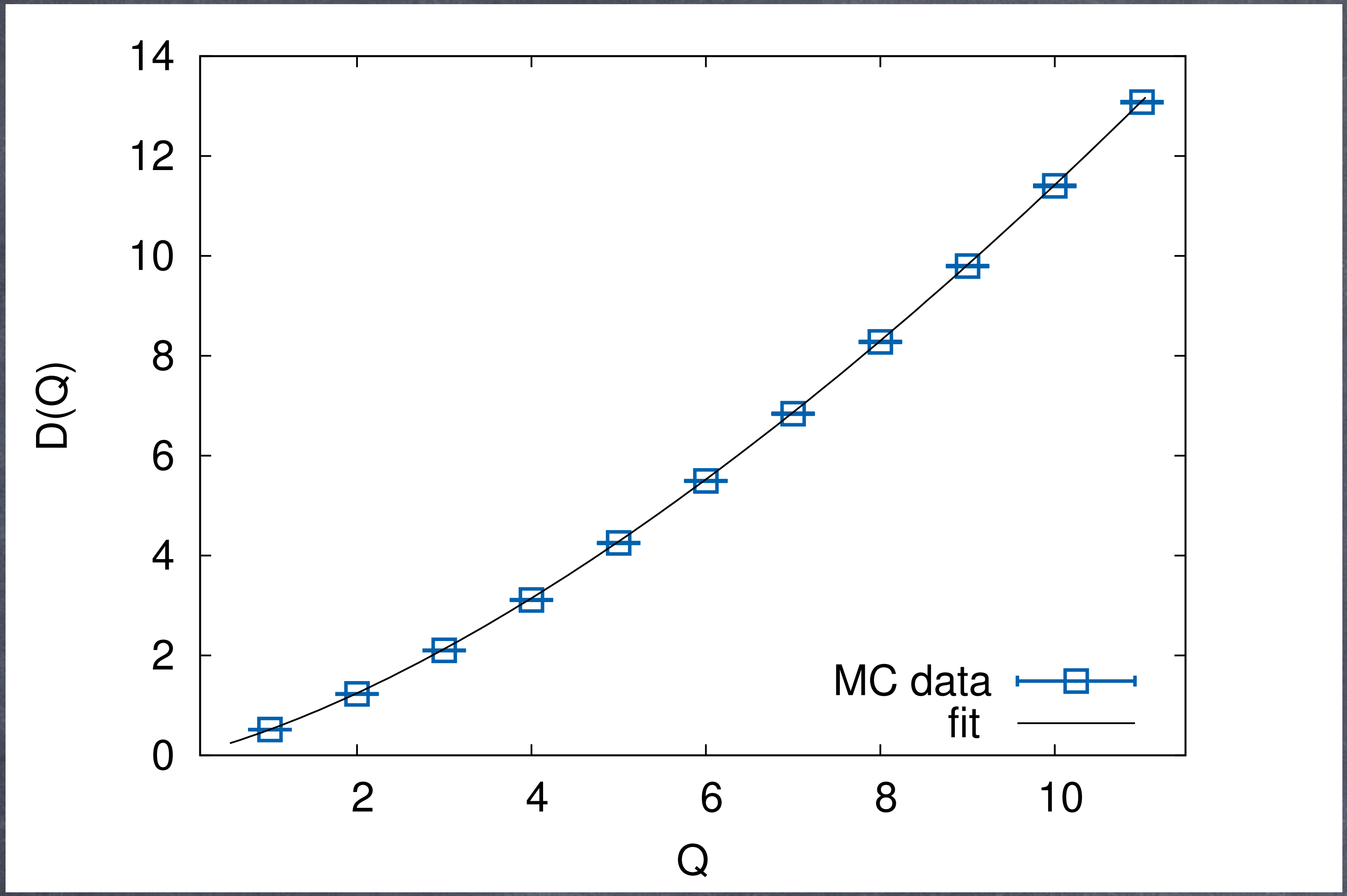
  Arias-Tamargo, Rodriguez-Gomez, Russo; Badel, Cuomo, Monin, Rattazzi; Watanabe
- strongly coupled CFTs in 4d at IR fixed point
- going away from the conformal point
- Fishnet CFTs (non-unitary)

Further directions

- Chern-Simons matter theories @large charge

  Watanabe
- 4-ε expansion @large charge

  Arias-Tamargo, Rodriguez-Gomez, Russo; Badel, Cuomo, Monin, Rattazzi; Watanabe
- strongly coupled CFTs in 4d at IR fixed point
- going away from the conformal point
- Fishnet CFTs (non-unitary)
- Study fermionic theories. Can large-charge approach be used for QCD (e.g. large baryon number)?



Thank you for your attention!